MONETARY POLICY AND FOREIGN PRICE DISTURBANCES UNDER FLEXIBLE EXCHANGE RATES: A STOCHASTIC APPROACH

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1. INTRODUCTION

The analysis of monetary policy under flexible exchange rates has received extensive treatment during recent years. Particular attention has been devoted to the role of exchange rate expectations, with a general conclusion being that many of the propositions of the traditional static Fleming-Mundell model (see Fleming (1962), Mundell (1968)) require substantial modification once such expectations are taken into account.[1/] With few exceptions, the analysis has been carried out in a deterministic framework. The usual assumption made is that any monetary expansion which is introduced is initially (ex ante) unanticipated, but having taken place, it is then (ex post) expected to be permanent. This assumption underlies, for example, the recent contribution by Dornbusch (1976) and much of the subsequent work that stems from it.[2/]

This assumption, while very natural, is also a polar one. In practice, agents in the economy may attempt to anticipate monetary policy and the policies themselves may, and may be expected to, vary in their permanency. It is intuitively clear that the effects of monetary policy -- and for that matter any other policy -- will vary substantially, depending upon the extent to which it is predicted and the permanence with which it is expected to continue. This view would seem to be one of the central messages to emerge from the current rational expectations literature.

The prime objective of the present paper is to analyze the effects of monetary policy under flexible exchange rates, allowing for more

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general assumptions regarding these kinds of expectational issues. To do so, it is desirable to develop the analysis within a stochastic framework. But having constructed the framework, it is then straightforward to use it to investigate similar kinds of issues with respect to other exogenous disturbances. Of these, one of the more interesting concerns foreign price disturbances. This has also been receiving increased attention recently, with economists investigating the extent to which flexible exchange rates insulate the domestic economy against such disturbances.[3/] Yet precisely the same limitations characterizes the existing literature in this area; it deals with ex ante unanticipated, but ex post expected permanent, changes in the foreign price level. Thus a secondary objective of the paper is to generalize these kinds of assumptions about foreign price changes.

Like much of the current work on exchange rate determination expectations play a central role in our analysis. The expectations of the endogenous variables appearing in the model (the one period expectations of the exchange rate and the cost of living) are assumed to be consistent with the structure of the model, and conditional on the predictions of the exogenous variables (the money supply and foreign price level). Through this process, the predictions of all future exogenous variables impinge on the current state of the economy. Such a rational expectations framework is attractive and natural for analyzing the kinds of expectational issues we have noted, enabling us to distinguish explicitly between actual and anticipated changes in the one hand, and permanent and temporary changes on the other. Indeed the application to exchange rate determination is not new. This framework has been employed recently by

Frenkel (1976), Mussa (1976), Barro (1978) and Cox (1980) among others, and the present paper can be viewed as an extension of this work. In particular, more attention is devoted to analyzing the response of the complete system to alternative forms of monetary disturbances.[4/]

The rest of the paper proceeds as follows. Section 2 details the model and reduces it to a form convenient for further analysis. In order to determine the short-run equilibrium, it is first necessary to solve the system for the endogenously determined rational expectations and this is undertaken in Section 3. After summarizing the short-run equilibrium in Section 4, the following section analyzes the effects of a variety of anticipated and actual monetary disturbances. Section 6 briefly deals with the effects of foreign price disturbances, while our main conclusions are summarized in the final section.

2. STOCHASTIC MODEL OF AN OPEN ECONOMY UNDER FLEXIBLE EXCHANGE RATES

The model we shall develop shares much in common with the recent models emphasizing the monetary approach to exchange rate determination.[5] Two general features we wish to stress at the outset are the following. First, the country is assumed to produce a single (composite) commodity, part of which is consumed domestically, the remainder of which is exported. The price of this commodity is determined in the market for domestic output, so that the price of exports is endogenously determined. On the other hand, we assume that the country is sufficiently competitive in the market for its imports to take the (foreign) price of imports as given. Secondly, the domestic bond market is assumed to be perfectly integrated with that in the rest of the world. Thus the model is described by the following five equations

$$Y_t = d_1 Y_t - d_2[r_t - (C^*_{t+1,t} - C_t)] + d_3[Q_t + E_t - P_t] + u_{1t}$$

$$0 < d_1 < 1, \quad d_2 > 0, \quad d_3 > 0 \tag{1a}$$

$$C_t = \delta P_t + (1-\delta)(Q_t + E_t) \tag{1b}$$

$$M_t - C_t = \alpha_1 Y_t - \alpha_2 r_t + u_{2t} \qquad \alpha_1 > 0, \quad \alpha_2 > 0 \tag{1c}$$

$$r_t = \rho + E^*_{t+1,t} - E_t + u_{3t} \tag{1d}$$

$$Y_t = \alpha + \gamma_1[P_t - C^*_{t,t-1}] + \gamma_2[P_t - P^*_{t,t-1}] + u_{4t} \quad \gamma > 0 \tag{1e}$$

where Y_t = real domestic output at time t, expressed in logarithms,

r_t = domestic nominal interest rate at time t,

ρ = the mean of the foreign nominal interest rate, taken to be exogenous and fixed,

P_t = price of domestically produced good (in terms of domestic currency), expressed in logarithms,

Q_t = price of imported good (in terms of foreign currency), expressed in logarithms,

E_t = logarithm of the current exchange rate (measured in units of the domestic currency per unit of foreign currency),

C_t = domestic cost of living (measured in domestic currency), expressed in logarithms,

M_t = domestic nominal supply expressed in logarithms,

$E^*_{t+s,t}$ = expectations of E for time t+s, held at time t; s = 1,2,..., all t,

$C^*_{t+s,t}$ = expectation of C for time t+s, held at time t, s = 1,2,...; all t,

The specification of some variables in terms of logarithms is purely for convenience, enabling us to avoid problems of linearization, which would otherwise be necessary.

Equation (1a) describes the domestic economy's IS curve. The demand for domestic output varies positively with domestic income and the relative price of foreign to domestic goods.[6/] It varies inversely with the real rate of interest, which is defined to be the domestic nominal rate, less the expected rate of inflation of the domestic CPI. Equation (1b) defines the domestic CPI to be a multiplicatively weighted average of the price of domestic goods and the domestic price of foreign goods.

While we choose this form primarily for convenience, it does have some theoretical merit. It is the "true" cost of living index if the domesitc residents' utility function, defined with domestic goods and foreign goods as arguments, is Cobb-Douglas; see e.g. Samuelson and Swamy (1974). The relative weight δ can be used to parameterize the degree of openness of the economy; the larger δ, the less open the economy and vice versa.

The monetary sector is summarized in equations (1c) and (1d). The first of these equations describes the domestic LM curve, making the usual assumption that all domestic money is held by domestic residents who also hold no foreign currency.[7/] Note that the domestic stock of money is deflated by the overall CPI, reflecting the fact that part of the transactions demand for money is for imports. The specification of the real transactions variable by Y follows convention, although it is at best only a proxy. A more accurate measure of real transactions would be $(P'H + Q'E'M)/C'$, where M denotes real imports and the primes denote corresponding prices in natural units ($P = \ell nP'$ etc.). But this would be intractible, since H and M are themselves endogenous functions. In fact, some justification for (1c) can be obtained by appealing further to an underlying Cobb-Douglas utility function. In this case, the minimum (nominal) expenditure function corresponding to an exogenously given level of utility $\bar{U}$ is proportional to $C'\bar{U}$. Assuming that the transactions component for the (nominal) demand for money varies with minimum expenditures, and assuming that utility (welfare) is monotonically related to real income, taking logarithms yields a transactions demand of the form (1c).[8/] The assumption of perfect capital market integration is embodied in the interest rate parity condition, (1d). The domestic nominal interest rate r_t equals the exogenous world rate $\rho + u_{3t}$, where u_{3t} represents stochastic fluctuations in the foreign interest rate, plus the expected rate of exchange depreciation.

Finally, the supply of domestic output is governed by equation (1e). This resembles a Lucas (1973) supply function, although as Flood (1979) has argued, with both international and intranational trading, this rationale is inappropriate. Rather, it may be justified in terms of the wage determination model of Gray (1976) and Fischer (1977). We assume that the nominal wage contract (the logarithm of the money wage) is set with the goal of equating the expected supply of labor to the expected demand. Assuming that workers are concerned with their real wages in terms of the CPI, their expected supply of labor, $N^{s*}_{t,t-1}$, is described by

$$N^{s*}_{t,t-1} = a + b[W_t - C^*_{t,\,t-1}] \qquad a > 0,\ b > 0 \tag{2a}$$

Assuming further a production function linking the logarithm of output and employment of the form

$$Y_t = \epsilon N_t \qquad 0 < \epsilon < 1 \tag{2b}$$

it follows that the expected demand for labor is determined by the marginal product condition

$$\ell n\epsilon + (\epsilon-1)N^{d*}_{t,t-1} = W_t - P^*_{t,t-1} \tag{2c}$$

In order to ensure that the labor market is expected to clear, W_t must be set to equate the expected demands and supply appearing in (2a) and (2c), namely

$$W_t = \frac{\ell n\epsilon + (\epsilon-1)a + (1-\epsilon)bC^*_{t,t-1} + P^*_{t,t-1}}{1 + b(1-\epsilon)} \tag{2d}$$

Actual employment, however, is assumed to depend upon the realized real wage in accordance with the firm's marginal product schedule

$$\ell n\epsilon + (\epsilon-1)N_t = W_t - P_t \tag{2c'}$$

Combining equations (2b), (2c'), (2d) yields

$$Y_t = \frac{\epsilon\,[a+b\ell n\epsilon]}{1+b(1-\epsilon)} + \frac{\epsilon}{1-\epsilon}\left[\frac{(1-\epsilon)b(P_t-C^*_{t,t-1}) + (P_t-P^*_{t,t-1})}{1 + b(1-\epsilon)}\right] \tag{2e}$$

which is of the form (1e).[9]

The model contains four stochastic disturbances $u_{1t},\ldots,u_{4t}$, all of which are assumed to have zero means, finite second moments, and to be independently distributed over time. The disturbances u_{1t}, u_{2t} refer to stochastic shifts in domestic demand; u_{3t} reflects stochastic foreign monetary disturbances, insofar as they impinge on the domestic economy through the interest rate parity relationship; u_{4t} represents stochastic shifts in domestic supply conditions.

The two key exogenous variables in the model upon which we wish to focus are M_t and Q_t. The endogenous variables to be determined by the system include Y_t, r_t, E_t, P_t, and C_t. These are determined on the assumption that expectations are rational in the sense that expectations of the endogenous variables $C^*_{t,\ t-1}$, etc. are equal to the predictions yielded by the model, conditional upon the expectations of the exogenous variables held at the same time. These latter expectations, since they pertain to variable exogenous to the model, can themselves be taken to be exogenous.

In order to simplify notation, it is convenient to define initial base levels of the exogenous variables, $\bar{M}$, $\bar{Q}$, and to consider subsequent changes in the system relative to the base level. Setting the random variables u_{it} all to zero, the stationary deterministic equilibrium of the system corresponding to $\bar{M}$, $\bar{Q}$ is attained where

$$\bar{Y} = d_1\bar{Y} - d_2\bar{r} + d_3[\bar{Q} + \bar{E} - \bar{P}] \tag{3a}$$

$$\bar{C} = \delta\bar{P} + (1-\delta)(\bar{Q} + \bar{E}) \tag{3b}$$

$$\bar{M} - \bar{C} = \alpha_1\bar{Y} - \alpha_2\bar{r} \tag{3c}$$

$$\bar{r} = \rho \tag{3d}$$

$$\bar{Y} = \alpha + \gamma_1[\bar{P} - \bar{C}] \tag{3e}$$

These equations determine the stationary deterministic equilibrium levels of the endogenous variables $\bar{r}$, $\bar{Y}$, $\bar{E}$, $\bar{P}$, $\bar{C}$, in terms of the exogenous variables and parameters. Taking differentials of (3a) - (3e) we obtain

$$\frac{d\bar{E}}{d\bar{M}} = \frac{d\bar{P}}{d\bar{M}} = \frac{d\bar{C}}{d\bar{M}} = 1; \qquad \frac{d\bar{Y}}{d\bar{M}} = 0 \tag{4a}$$

$$\frac{d\bar{E}}{d\bar{Q}} = -1; \quad \frac{d\bar{Y}}{d\bar{Q}} = \frac{d\bar{P}}{d\bar{Q}} = \frac{d\bar{C}}{d\bar{Q}} = 0 \tag{4b}$$

That is, a 1% increase in the domestic money supply will lead to a 1% increase in the equilibrium rate of exchange and in the price of domestic output and hence in the domestic CPI, leaving domestic real output unaffected. Likewise a 1% increase in the foreign price level will lead to a 1% decrease in the equilibrium rate of exchange, leaving the domestic variables $\bar{P}$, $\bar{Q}$, $\bar{Y}$ unaffected.

Subtracting (3) from (2), the behavior of the system about the initial equilibrium (3) can be expressed in deviation form

$$(1-d_1)y_t = -d_2[(e^*_{t+1,t}-e_t)-(c^*_{t+1,t}-c_t)] + d_3[e_t + q_t-p_t] + u_{1t} + d_2u_{3t} \tag{5a}$$

$$c_t = \delta p_t + (1-\delta)(q_t + e_t) \tag{5b}$$

$$m_t - c_t = \alpha_1 y_t - \alpha_2[e^*_{t+1,t} - e_t] + u_{2t} + \alpha_2 u_{3t} \tag{5c}$$

$$r_t - \bar{r} = e^*_{t+1,t} - e_t + u_{3t} \tag{5d}$$

$$y_t = \gamma[p_t - c^*_{t,t-1}] + \gamma_2[P_t - P^*_{t,t-1}] + u_{4t} \tag{5e}$$

where lower case letters denote deviations about the initial equilibrium; i.e. $y_t = Y_t - \bar{Y}$, $m_t = M_t - \bar{M}$ etc. The advantage of defining the system about its initial equilibrium is that it enables us to incorporate the constants and exogenous parameters of the system (such as ρ) in $\bar{E}$, $\bar{P}$, etc.

The basic scenario we shall analyze is a situation in which prior to some initial time period, $t = o$ say, the economy is in the equilibrium defined in (3). Then at time o, for a variety of reasons, individuals may begin to expect the exogenous variables to change over subsequent periods. These changes in expectations may come about through announcements, "leaks in information", observation of events taking place or expected to take place at home and abroad, etc. The details of what generate the changes in these expectations of exogenous variables are themselves exogenous to the model and need not concern us. Our task is to trace through the effects of changes in these expectations, as well as the effects of change in the actual variable (when and if they actually occur), on the endogenous variables of the economy.

To solve the model it is further convenient to eliminate $(r_t - \bar{r})$, c_t, $c^*_{t,t-1}$, $c^*_{t+1,t-1}$ from equations (5a), (5c), (5d). This may be done by first taking conditional expectations of (5b) at time $t-1$

$$c^*_{t,t-1} = \delta p^*_{t,t-1} + (1-\delta)(q^*_{t,t-1} + e^*_{t,t-1}) \tag{5b'}$$

Then substituting from (5b), (5b'), (5d) we obtain

$$\begin{aligned}(1-d_1)y_t = -d_2[\delta(e^*_{t+1,t}-e_t)-\delta(p^*_{t+1,t}-p_t) - (1-\delta)(q^*_{t+1,t}-q_t)]\\ + d_3[e_t + q_t-p_t] + u_{1t} - d_2u_{3t}\end{aligned} \tag{6a}$$

$$\begin{aligned}y_t = \gamma_1[p_t-\delta p^*_{t,t-1} - (1-\delta)(q^*_{t,t-1} + e^*_{t,t-1})]\\ + \gamma_2[p_t - p^*_{t,t-1}] + u_{4t}\end{aligned} \tag{6b}$$

$$\begin{aligned}m_t = \delta p_t + (1-\delta)(q_t+e_t) + \alpha_1 y_t - \alpha_2[e^*_{t+1,t}-e_t]\\ + u_{2t} - \alpha_2 u_{3t}\end{aligned} \tag{6c}$$

Equations (6a), (6b), (6c) yield three stochastic difference equations in domestic output y_t, the price of the domestic good p_t and its expectations, the exchange rate e_t and its expectations, together with the exogenous variables m_t, q_t and their expectations.

3. SOLUTION FOR EXPECTATIONS

In order to determine the solutions for y_t, p_t and e_t, it is first necessary to solve for the expectations of the endogenous variables, appearing in (6a) - (6e). To do this, we take conditional expectations of (6a) - (6c) for an arbitrary date i, held at the initial time o

$$(1-d_1)y^*_{i,o} = -d_2[\delta(e^*_{i+1,o} - e^*_{i,o}) - \delta(p^*_{i+1,o} - p^*_{i,o}) - (1-\delta)(q^*_{i+1,o} - q^*_{i,o})] + d_3[e^*_{i,o}+q^*_{i,o}-p^*_{i,o}] \tag{7a}$$

$$y^*_{i,0} = \gamma_1(1-\delta)[p^*_{i,o} - e^*_{i,o} - q^*_{i,o}] \tag{7b}$$

$$m^*_{i,o} = \delta p^*_{i,o} + (1-\delta)(e^*_{i,o} + q^*_{i,o}) + \alpha_1 y^*_{i,o} - \alpha_2[e^*_{i+1,o}-e^*_{i,o}] \tag{7c}$$

where for any variable X say, $X^*_{t+1,t}$ denotes the prediction formed at time t (t = 0, 1,...) for time t+i (i = 0,1,...). In forming these expectations, we are using the familiar property of conditional expectations operators

$$\mathcal{E}_{t-1}[\mathcal{E}_t(X_{t+i})] = \mathcal{E}_{t-1}[X_{t+i}] = X^*_{t+i,t-1}$$

and we define $X^*_{t,t} = X_t$. This last assumption merely asserts that the prediction of the current value of a variable is the actual current value and is another way of saying that the variable is instantly observable.

The most convenient procedure for solving for the expectations of the endogenous variables is as follows. First, let us define the relative price of domestic to foreign goods at time o say

$$z_o \equiv p_o - q_o - e_o \tag{8}$$

so that the expectation of the relative price formed at time o for time i say is

$$z^*_{i,o} = p^*_{i,o} - q^*_{i,o} - e^*_{i,o} \tag{8'}$$

Substituting for $y^*_{i,o}$ from (7b) into (7a) and using the definition (8') yields the following first order difference equation in $z^*_{i,o}$

$$d_2 \delta z^*_{i+1,o} - [d_2\delta + \gamma_1(1-d_1)(1-\delta) + d_3]z^*_{i,o} = -d_2[q^*_{i+1,o} - q^*_{i,o}] \tag{9}$$

The general solution to this equation is

$$z^*_{i,o} = A(1/\mu)^i + \frac{\mu}{\delta}\sum_{j=1}^{\infty}[q^*_{i+j,o} - q^*_{i+j-1,o}]\mu^{j-1} \tag{10}$$

where A is an arbitrary constant and

$$\mu = \frac{d_2\delta}{d_2\delta + \gamma_1(1-d_1)(1-\delta) + d_3} < 1$$

Since $1/\mu > 1$, as long as $A \neq 0$, the expectations of the relative price held at time o over lengthening forecast horizons (i.e. as $i \to \infty$) will diverge. In order to rule out such behavior and thus ensure that $z^*_{i,o}$ remains bounded we set $A = 0$. This procedure, typical of rational expectations models, is often justified on the grounds that the instability which would otherwise occur would be inconsistent with observed behavior. Alternatively it may be justified more formally by appealing to transversality conditions from appropriate optimizing models, which in effect impose boundedness conditions on expected price movements.[10/] Thus setting $A = 0$, the solution reduces to

$$z^*_{i,o} = \frac{1}{\delta}\sum_{j=1}^{\infty}(\Delta q^*_{i+j,o})\ \mu^j \tag{11}$$

where

$$\Delta q^*_{i+j,o} \equiv q^*_{i+j,o} - q^*_{i+j-1,o}\ .$$

Substituting the solution for $z^*_{i,o}$ into (7b), the corresponding solution for the expected value of real output $y^*_{i,o}$ is given by

$$y^*_{i,o} = \frac{\gamma_1(1-\delta)}{\delta} \sum_{j=1}^{\infty} (\Delta q^*_{i+j,o})\mu^j \tag{12}$$

From (11) and (12) it is observed that the expectations $z^*_{i,o}$, $y^*_{i,o}$ of the real variables are independent of expectations about the nominal money supply $m^*_{j,o}$. They are also independent of any sustained uniform increase in the foreign price level which is expected to occur at or before time i. On the other hand, any increase in the foreign price level which is expected to occur after period i will influence these expectations. For example, a once and for all increase in the foreign price level which is expected to occur in period $i + J$ will have the following effects

$$\frac{\partial z^*_{i,o}}{\partial q^*_{i+J,o}} = \frac{1}{\delta}\mu^J; \quad \frac{\partial y^*_{i,o}}{\partial q^*_{i+J,o}} = \frac{\gamma_1(1-\delta)}{\delta}\mu^J$$

The reason is that the increase in $q^*_{i+J,o}$ generates a reduction in the expected real rate of interest for period $(i+J)$, thereby stimulating the expected demand and expected output for that period, and forcing up the expected relative price of domestic output. These effects taper off as $J \to \infty$.

Letting $i \to \infty$ in the solutions (11) and (12), we obtain

$$\lim_{i\to\infty} y^*_{i,o} = 0; \qquad \lim_{i\to\infty} z^*_{i,o} = 0$$

That is, the expected level of domestic output $Y^*_{i,o}$ and the expected level of the relative price $Z^*_{i,o}$ converge to the stationary equilibrium levels $\bar{Y}$, $\bar{Z} \equiv \bar{P} - \bar{Q} - \bar{E}$, respectively, defined by equations (3a) - (3e).

To determine the solutions for the expectations of the nominal variables $e^*_{i,o}$, $p^*_{i,o}$, we note (8') and substitute for $z^*_{i,o}$, $y^*_{i,o}$ into (7c). This yields the following equation in $e^*_{i,o}$

$$\alpha_2 e^*_{i+1,o} - (1+\alpha_2)e^*_{i,o} = [1 + \frac{\alpha_1\gamma_1(1-\delta)}{\delta} \sum_{j=1}^{\infty} (\Delta q^*_{i+j,o})\mu^j \quad (13)$$

$$+ q^*_{i,o} - m^*_{i,o}$$

With $\alpha_2 > 0$, the complementary function for this equation implies an unbounded solution for $e^*_{i,o}$. Thus in order to ensure stability, its coefficient in the general solution must be set to zero, yielding the following solution for $e^*_{i,o}$

$$e^*_{i,o} = \frac{1}{1+\alpha_2} [\sum_{k=o}^{\infty} (m^*_{i+k,o} - q^*_{i+k,o})\lambda^k]$$

$$- \frac{1}{1+\alpha_2} [1 + \frac{\alpha_1\gamma_1(1-\delta)}{\delta}][\sum_{k=0}^{\infty} \lambda^k \sum_{j=1}^{\infty} (\Delta q^*_{i+j+k,o})\mu^j] \quad (14)$$

where

$$\lambda = \frac{\alpha_2}{1+\alpha_2} < 1$$

The expectation of the exchange rate held at time 0 for time i say depends upon the time profile of the expectations of the money supply and foreign price level for all time periods beginning at time i and extending indefinitely into the future. For example, if the money supply is expected to undergo an increase dm^* beginning at time $i + J$ say, and if this increase is expected to continue permanently, the exchange rate at time i will be expected to devalue by an amount

$$\frac{\partial e^*_{i,o}}{\partial m^*} = \lambda^J \quad (15)$$

If $J = 0$ and the increase is expected to occur immediately, the percentage expected devaluation will be equal to the expected percentage increase in the money supply. Otherwise, the expected devaluation will be less than proportional and indeed as $J \to \infty$, the effect will dampen out.

Similarly, a foreign price increase dq^* which is expected to take place at or before time i will result in an exactly proportional expected revaluation of the domestic currency,

$$\frac{\partial e^*_{i,o}}{\partial q^*} = -1$$

If the increase is expected to take place some time in the future at $i + J$, $(J > 1)$, the response of the expected exchange rate is

$$\frac{\partial e^*_{i,o}}{\partial q^*} = -\lambda^J - \frac{1}{1+\alpha_2}\left[1 + \frac{\alpha_1\gamma_1(1-\delta)}{\delta}\right]\mu^J \frac{[1-(\lambda/\mu)^J]}{1-\lambda/\mu} < 0 \tag{16}$$

which again implies an unambiguous expected revaluation. The first term is essentially analogous to (15); the second is the expected real interest rate effect and operates through the expectations of the real variables $y^*_{i,o}$, $z^*_{i,o}$.

Finally, substituting the solution for $e^*_{i,o}$ into (11), and using (8'), we obtain the following solution for the expected price of domestic output

$$p^*_{i,o} = q^*_{i,o} + \frac{1}{1+\alpha_2}\sum_{k=0}^{\infty}[m^*_{i+k,o} - q^*_{i+k,o}]\lambda^k + \frac{1}{\delta}\sum_{j=1}^{\infty}(\Delta q^*_{i+j,o})\mu^j$$

$$- \frac{1}{1+\alpha_2}\left[1 + \frac{\alpha_1\gamma_1(1-\delta)}{\delta}\right]\left[\sum_{k=0}^{\infty}\lambda^k\sum_{j=1}^{\infty}(\Delta q^*_{i+j+k,o})^{\mu j}\right] \tag{17}$$

The effect of an expected permanent increase in the money supply starting at time $i + J$ is as in (15), raising the expected price by an amount

$$\frac{\partial p^*_{i,o}}{\partial m^*} = \lambda^J \tag{15'}$$

A foreign price increase which is expected to occur in period i and to continue permanently, will leave the expected price level for period i unaffected. If the increase is expected to take place some time beyond period i, the expected price level for period i will be affected, although the adjustment will dampen out as $J \rightarrow \infty$.

4. SHORT-RUN EQUILIBRIUM OF SYSTEM

The short-run solutions for the endogenous variables y_t, p_t, e_t obtained from equations (6a), (6b), (6c) can be expressed in terms of the conditional expectations $e^*_{t+1,t}$, $e^*_{t,t-1}$, $z^*_{t+1,t}$, $z^*_{t,t-1}$. It is clear that the choice of conditioning date 0 in Section 3 was made for notational convenience and analogous expressions to (11), (12), (14), (17) hold with respect to expectations formed at any arbitrary time t for any arbitrary time $t + i$. Specifically,

$$z^*_{t+i,t} = \frac{1}{\delta} \sum_{j=1}^{\infty} (\Delta q^*_{t+i+j,t}) \mu^j \tag{12'}$$

$$\begin{aligned} e^*_{t+i,t} &= \frac{1}{1+\alpha_2} \sum_{k=o}^{\infty} [m^*_{t+i+k,t} - q^*_{t+i+k,t}] \lambda^k \\ &\quad - \frac{1}{1+\alpha_2}[1 + \frac{\alpha_1\gamma_1(1-\delta)}{\delta}][\sum_{k=o}^{\infty} \lambda^k \sum_{j=1}^{\infty} \Delta q^*_{t+i+j+k,t} \mu^j] \end{aligned} \tag{14'}$$

Thus the solutions for y_t, p_t and e_t can be expressed most conveniently by writing (6a) - (6c) as the matrix equation

$$\begin{pmatrix} (1-d_1) & (d_3+d_2\delta) & -(d_3+d_2\delta) \\ -1 & \gamma & 0 \\ \alpha_1 & \delta & 1-\delta+\alpha_2 \end{pmatrix} \begin{pmatrix} y_t \\ p_t \\ e_t \end{pmatrix} \tag{18}$$

$$= \begin{pmatrix} d_2\delta z^*_{t+1,t} + d_2(q^*_{t+1,t}-q_t) + (d_3+d_2\delta)q_t + u_{1t} - d_2u_{3t} \\ (\gamma_1\delta + \delta_2)z^*_{t,t-1} + \gamma(q^*_{t,t-1} + e^*_{t,t-1}) - u_{4t} \\ m_t + \alpha_2 e^*_{t+1,t} - (1-\delta)q_t - u_{2t} + \alpha_2 u_{3t} \end{pmatrix}$$

where $\gamma \equiv \gamma_1 + \gamma_2$,

$q^*_{t+1,t}$, $q^*_{t,t-1}$ are exogenous and $z^*_{t+1,t}$, $z^*_{t,t-1}$, $e^*_{t+1,t}$ are obtained by setting $i = 1$ in (12'), (14'). Through these latter variables, the solutions for y_t, p_t, and e_t depend upon the expectations of all future values of the exogenous variables (as formed at times t-1, t), an aspect which is a central feature of rational expectations models.

The solutions obtained from (18) are also stochastic, depending upon the outcomes of the random variables $u_{1t},\ldots,u_{4t}$. Thus one can analyze the response of the system to these various stochastic shocks. Although this would be of interest, we do not pursue it here. Instead, we shall focus on the effects of actual and anticipated changes in the money supply and level of foreign prices, for given drawings of $u_{1t},\ldots,u_{4t}$. We shall interpret such changes in m as reflecting actual and anticipated autonomous policy changes. However, one could also postulate the money supply to be generated by some feedback control law, in which case it would follow some stochastic process. In this case, the anticipated change would be derived by calculating conditional expectations of the stochastic process embodied in the control law.[11/] This approach constrains the possible range of policy changes and for our purpuses it is more convenient to leave the specification of the domestic monetary process rather general. Finally, we could allow q to be generated by some specific stochastic process, but here too we find it useful to keep the specification of actual and foreign price changes quite open.

5. DOMESTIC MONETARY EXPANSION

It will be recalled that all variables in (18) have been measured as deviations about the initial deterministic equilibrium defined by equations (5). To analyze the effects of actual and anticipated monetary expansions it is convenient to assume that the foreign price level remains fixed, and is expected to remain fixed, at its initial level $\bar{Q}$. Hence we may set $q^*_{t+i,t} = q_t = 0$. The solutions for the (deviations of) domestic output, price level, and the exchange rate can be expressed in a variety of equivalent ways, probably the simplest and most easily interpreted being the following

$$y_t = \frac{\gamma(d_3+d_2\delta)}{J}[(m_t-m^*_{t,t-1}) + \sum_{k=1}^{\infty}(m^*_{t+k,t} - m^*_{t+k,t-1})\lambda^k] + e_{1t} \tag{19a}$$

$$p_t = \frac{(d_3+d_2\delta)}{J}[(m_t-m^*_{t,t-1}) + \sum_{k=1}^{\infty}(m^*_{t+k,t} - m^*_{t+k,t-1})\lambda^k] \tag{19b}$$

$$+ \frac{1}{1+\alpha_2}\sum_{k=0}^{\infty} m^*_{t+k,t-1}\lambda^k + e_{2t}$$

$$e_t = \frac{[(1-d_1)\gamma + (d_3+d_2\delta)]}{J}[(m_t-m^*_{t,t-1}) + \sum_{k=1}^{\infty}(m^*_{t+k,t}-m^*_{t+k,t-1})\lambda^k] \tag{19c}$$

$$+ \frac{1}{1+\alpha_2}\sum_{k=0}^{\infty} m^*_{t+k,t-1}\lambda^k + e_{3t}$$

where $J = \gamma(1-d_1)(1-\delta+d_2) + (d_3+d_2\delta)(1+\alpha_2+\alpha_1\delta) > 0$, and e_{1t}, e_{2t}, e_{3t} are disturbance terms which are linear functions of the original disturbance terms $u_{1t},\ldots,u_{4t}$, but the precise nature of which need not concern us.

Two critical factors appearing throughout these expressions are $(m_t - m^*_{t,t-1})$, and $(m^*_{t+k,t} - m^*_{t+k,t-1})$, $k=1,2,\ldots$ The first of these is simply the unanticipated component of the actual money supply at time

t, relative to what was anticipated at time $t-1$. The second term measures the amount by which the forecast of the money supply for time $t+k$ is revised between times $t-1$ and t, presumably on the basis of new information acquired during period t.

From equations (19) we can draw the following general conclusions. The level of domestic output, its price, and the rate of exchange all vary positively with the unanticipated component of the money supply at time t. They also vary positively with the discounted sum of the amounts by which the forecasts for all future periods are revised between time $t-1$ and time t. In addition, the monetary variables p_t and e_t depend upon the discounted sum (weighted average) of the expected money supplies for all future periods (as perceived at time $t-1$), with the weights summing to unity.

An important special case of (19) worth noting arises when the expectations for all future money supplies formed at a particular time are held uniformly over all future periods. Formally, this assumption is expressed by

$$m^*_{t+k,t} = m^*_t \qquad k = 1,\ldots;\ \text{all } t \tag{20}$$

where m^*_t denotes the uniformly held expectation at time t. With this assumption, the three solutions (19a) - (19c) simplify to 12/

$$y_t = \frac{\gamma(d_3+d_2\delta)}{J}\left[(m_t - m^*_{t-1}) + \alpha_2(m^*_t - m^*_{t-1})\right] + e_{1t} \tag{19a'}$$

$$p_t = \frac{(d_3+d_2\delta)}{J}\left[(m_t - m^*_{t-1}) + \alpha_2(m^*_t - m^*_{t-1})\right] + m^*_{t-1} + e_{2t} \tag{19b'}$$

$$e_t = -\frac{[(1-d_1)\gamma+(d_3+d_2\delta)]}{J}\left[(m_t-m^*_{t-1})+\alpha_2(m^*_t-m^*_{t-1})\right]+m^*_{t-1}+e_{3t} \tag{19c'}$$

Using equations (19) enables us to determine the effects of a multitude of types of actual and anticipated monetary disturbances on the economy. For obvious reasons, we restrict outselves to just a few examples which it is convenient to categorize as follows: (a) Partial expectations effects; (b) actual monetary expansions; (c) announcement effects.

A. Partial Expectations Effects

(i) The anticipation at time $t-1$ of an increase in the money supply for the single period $t+K$, which ex post at time t is no longer expected to occur (i.e. $dm^*_{t+K,t-1} > 0$, $dm^*_{t+k,t-1} = 0 \quad k \neq K$; $dm^*_{t+k,t} = 0$ all k) will cause the price of domestic output to rise and the level of domestic output to fall. The reason is that the expectation of the increase in the money supply will increase the expected price of domestic goods; see (17). This leads to an upward shift in the domestic supply function for period t, forcing the actual price of domestic goods to increase during the period, thereby reducing the demand for these goods and causing domestic output to fall. Both responses are proportional to λ^K, and so tend to zero as $K \to \infty$.

The effect on the exchange rate is indeterminate, being given by the expression

$$\frac{\partial e_t}{\partial m^*_{t+K,t-1}} = \frac{\gamma[\alpha_1(d_3+d_2\delta) - (1-d_1)\delta]\lambda^K}{J(1+\alpha_2)} \tag{20}$$

On the one hand, the endogenous increase in p_t will raise the demand for money; on the other hand, the decrease in y_t will reduce the demand for money. The net effect of these two offsetting influences can be shown to decrease the demand for money by an amount proportional to $[\alpha_1(d_3+d_2\delta) - (1-d_1)\delta]$.[13/] Thus, in the event that this term is positive, the demand for money will decrease. At the same time, the net effect of an increase in e_t, taking into account both its effect on the cost of living and its effect on the domestic interest rate, is to raise the demand for money.14/ Hence if $[\alpha_1(d_3+d_2\delta) - (1-d_1)\delta] > 0$, the domestic exchange rate must increase (depreciate) in order to increase the demand for money and maintain equilibrium with the exogenously given fixed nominal supply. The opposite argument applies if the sign of this term is reversed.

The changes in the variables y_t, p_t, and e_t are only transitory. In period $t+1$, when expectations are revised back down, these variables will return to their original levels. We may also observe that essentially the same adjustment applies if the initial increase in expectations occurring in period $t-1$ is expected to apply uniformly to all periods beyond $t+k$. The responses are simply scaled by a factor $1/(1-\lambda) = 1 + \alpha_2$.

(ii) The anticipation of an expansion in the money supply, initially expected in period t to take place in period $t+K$ $(dm^*_{t+k,t-1} = 0$, all k; $dm^*_{t+K,t} > 0$, $dm^*_{t+k,t} = 0$, $k \neq K)$ will raise all three variables, real domestic output, the price of domestic goods, and the exchange rate, during period t. The reason is that the immediate effect of the increase in $m^*_{t+K,t}$ is to raise both $p^*_{t+1,t}$ $e^*_{t+1,t}$ proportionately. The domestic nominal interest rate will rise, reducing the demand for money, although the domestic real interest rate will remain unchanged. With the nominal money supply fixed, the demand for money will have to be increased and this can occur through increases in y_t, e_t, or p_t. Assuming that the most rapid adjustment is in e_t, this will put an upward pressure on domestic prices, stimulating output. If in period $t+1$, expectations are revised back down to their original levels, y_{t+1}, p_{t+1}, e_{t+1} will respond as in case (i). In period $t+2$, they will be back to their original levels (apart from random disturbances, that is).

(iii) Combining (i) and (ii) we see that an anticipation of an expansion in the money supply initially formed at time $t-1$ and maintained at time t $(dm^*_{t+K,t-1} = dm^*_{t+K,t})$ will have no effect on real income during period t, which we noted above responds to the rate of forecast revision. The current price of domestic output and the exchange rate will both rise by the same amount, leaving the relative price of domestic output unaffected.

B. Actual Monetary Expansions

(i) A purely unanticipated random increase in the level of the domestic money supply, which is not expected to continue beyond the current period $(dm^*_{t+k,t-1} = dm^*_{t+k,t} = 0$ all k, $dm_t > 0)$ has an expansionary effect on domestic output and leads to an increase in the domestic price level, coupled with a depreciation of the domestic currency. With expectations unchanged, the adjustment is quite conventional and accords with traditional analyses of monetary policy for flexible rate systems under perfect capital mobility and static expectations; see e.g. Fleming (1962), Mundell (1968).

(ii) One of the most interesting disturbances to consider is that of an increase in the domestic money supply which is initially unanticipated, but having occurred, is then expected to continue permanently $(dm^*_{t+k,t-1} = 0$ for all k; $dm_t = dm^*_{t+k,t} > 0$ for $k = 1,2,\ldots)$. This is precisely the monetary disturbance analyzed by Dornbusch (1976) and subsequent authors and has been shown in some cases to be associated with "overshooting" of the short-run exchange rate in relation to its long-run equilibrium proportionate response. In the present context such a disturbance leads to an unambiguous increase in the three variables, domestic output, the price of domestic output, and the exchange rate. The short-run percentage increase in the price of domestic output is less than that of the money supply, so that p_t undershoots its long-run proportionate response, just as it does in these other models.[15/] Rewriting (19c) we can show

$$\frac{de_t}{dm_t} = 1 + \frac{\gamma[\delta(1-d_1) - \alpha_1(d_3 + d_2\delta)]}{J} \tag{21}$$

Hence whether or not the exchange rate overshoots its ulitmate proportionate response depends upon $\delta(1-d_1) - \alpha_1(d_3 + d_2\delta)$. If $\delta(1-d_1) - \alpha_1(d_3 + d_2\delta) > 0$, it is clear from (21) that $de_t/dm_t > 1$ and short-run overshooting occurs; if $\delta(1-d_1)-\alpha_1(d_3+d_2\delta)$, $de_t/dm_t < 1$, and the short-run exchange rate adjusts only partially towards its equilibrium.

To understand the response of the exchange rate further, it is convenient to write the third equation appearing in (18) in the form

$$\alpha_1 y_t + \delta z_t = (1 + \alpha_2)(m_t - e_t) - u_{2t} + \alpha_2 u_{3t} \tag{22}$$

To derive (22) we have set $q_t = 0$; in addition (14') in conjunction the expectational assumption, enables us to set $e^*_{t+1,t} = m_t$. The expression on the left hand side is the net effect on the demand for money resulting from changes in the real variables income y_t, and the relative price z_t. Given the increase in y_t associated with this monetary disturbance, the relative price z_t must fall in order to stimulate demand and maintain product market equilibrium. Thus the net effect of these two changes on the demand for money is ambiguous and turns out to be proportional to $\alpha_1(d_3 + d_2\delta) - \delta(1-d_1)$.[16/] If this term is negative say, then these two components generate a net fall in the demand for money. In this case it follows from (22) that $de_t/dm_t > 1$, in order to generate sufficient additional demand to absorb the increased supply.

Hence the likelihood of overshooting decreases with d_1, d_2, d_3 and α_1 and will increase or decrease with δ according as $\alpha_1 d_2 - (1-d_1) \gtrless 0$. This is perfectly consistent with Dornbusch who takes $\delta = 1$ and throughout most of his analysis assumes output is fixed, which is equivalent to setting $\alpha_1 = 0$. Overshooting is then inevitable. With endogenous income and $\alpha_1 > 0$ the possibility of undershooting arises. In effect, the larger α_1, the more the burden of adjustment to an increase in the money supply is undertaken by income and the less need be borne by the exchange rate, reducing the likelihood of overshoot. In the limiting case when $\delta = o$, undershooting will always occur.

To conclude this case, two further points should be made. First, our propositions regarding overshooting or undershooting, while developed for an initially totally unexpected increase in the money supply, continue to hold as long as the initial disturbance is under-predicted $(dm_t > dm^*_{t,t-1})$. Secondly, assuming that the increase in the money supply undertaken at time t is still expected at time $t+1$ to continue permanently (i.e. $dm^*_{t+k,t+1} = dm_t$), we immediately see that $dy_{t+1} = 0$, $dp_{t+1} = de_{t+1} = dm_t$. The economy adjusts to its new equilibrium in just <u>one</u> period. The contrast with Dornbusch in this respect arises simply because of our different choice of supply function, which is the source of the gradual adjustment in his analysis.

We turn now briefly to three other disturbances.

(iii) An anticipated increase in the money supply, which ex ante is expected to be permanent, but which ex post is expected to be only transitory $(dm^*_{t+k,t-1} = dm_t,\ k = 0,1,..;\ dm^*_{t+k,t} = 0,\ k = 1,..)$ has a contractionary effect on income. It also leads to short-run increases in both p_t and e_t, although in both cases the percentage increase will be less than that of the money supply. This is because the contractionary effect of a given expected increase in the money supply more than outweighs the expansionary effect resulting from the actual increase. The fall in output must be met by a rise in the relative price z_t in order to maintain product market equilibrium. Thus p_t must rise by more than e_t and since with $dm^*_{t+k,t}$ the response of p_t is necessarily less than that of m_t (this can be shown from (19b)), the same must be true of e_t.

(iv) A fully anticipated increase in the money supply, which both ex ante and ex post is expected to be only transitory ($dm^*_{t,t-1} = dm_t$; $dm^*_{t+k,t-1} = dm^*_{t+k,t} = 0$, $k = 1,\ldots$) will have no effect on real output, and as in (iii) will lead to partial increases in both p_t and e_t.

(v) Finally, a fully anticipated increase in the money supply, which both ex ante and ex post is expected to be permanent ($dm^*_{t+k,t-1} = dm^*_{t+k,t} = dm_t$, all k) will have no effect on real output. Both the price of domestic output and the exchange rate will rise immediately by the full amount of the monetary increase.

C. Announcement Effects

The effect of an announcement of a future monetary expansion on the current exchange rate has been investigated in the context of the Dornbusch model in two recent papers by Gray and Turnovsky (1979) and Wilson (1979). These authors demonstrate that the announcement at time 0 say of a monetary expansion to take effect at a future time T will lead to upward jump in the exchange rate at time 0. Thereafter, the exchange rate will continue to rise in a continuous exponential manner until the stable arm of the saddle is reached, which is then followed towards the new equilibrium.[17/] The announcement moderates the initial jump and indeed with sufficient lead time, the initial overshooting may no longer occur. In this case a lesser degree of overshooting will take place at some point before the monetary expansion actually takes effect.

The solution we have given in (19) can easily be used to analyze-ammouncement effects in the present context. Suppose that the monetary authority announces at time 0 a monetery expansion of dm to be introduced at time T. Assuming this is believed by the public, we may write

$$\begin{aligned} dm^*_{t',t} &= 0 \qquad && t' = 1,\ldots,T-1, \quad \text{all} \quad t < t' \\ &= dm && t' = T,\ T+1,\ldots \\ dm_t &= 0 && t = 1,\ldots T-1 \\ &= dm && t = T,\ T+1,\ldots \end{aligned} \tag{23}$$

Now substituting these values into (19a) - (19c) we obtain the following solutions for the changes in the three endogenous variables

$$\begin{aligned} dy_t &= 0 \qquad && t = 1,\ldots,T-1 \\ &= 0 && t = T,\ldots \end{aligned} \tag{24a}$$

$$\begin{aligned} dp_t &= \lambda^{T-t}dm \qquad && t = 1,\ldots,T-1 \\ &= dm && t = T,\ldots \end{aligned} \tag{24b}$$

$$\begin{aligned} de_t &= \lambda^{T-t}dm \qquad && t = 1,\ldots,T-1 \\ &= dm && t = T,\ldots \end{aligned} \tag{24c}$$

The behavior of the economy is very simple. In the period following the announcement both the price of domestic output and the exchange rate will rise by an amount λ^{T-1}. Thereafter, they will both increase monotonically at the rate (per period) $1/\alpha_2$ until time T when the monetary expansion takes place, when they will have risen by the full amount of the monetary expansion dm.[18] In contrast to the Gray-Turnovsky and Wilson analyses, there is no overshoot of the exchange rate. The reason for the difference is that we do not take the price level to be fixed in the short run, thereby allowing it to bear some of the necessary adjustment. Note that if $T = 1$, the adjustment is completed within one period; the present analysis essentially reduces to case B(v) discussed above.

The other feature to observe is that real output remains fixed throughout the entire adjustment process. The reason for this is evident from (19a), where we have commented that real output responds to forecast errors and to the revision of forecasts for future periods. But with the future monetary expansion announced, neither of these phenomena occur. The monetary expansion at time T is perfectly foreseen. Also, during the transition (i.e. before time T) there is no reason to update forecasts. Thus while the announcement of the monetary expansion leads to immediate responses in the nominal variables p_t and e_t, these adjustments turn out to be identical, leaving the <u>real</u> variables such as the relative price and the level of output, unaffected. In this respect, Wilson's inference (drawn from his full employment model) that the announcement of a monetary expansion may generate real effects is misleading. In a rational world, such an announcement will affect the monetary, but <u>not</u> the real variables.

To conclude this discussion we may note that if the announcement occurs at some time τ after 0, then output <u>at</u> the announcement date τ itself, will respond to the announcement, as previously held forecasts are revised. Thereafter, however, the analysis will be as above, with no real effects taking place.

6. CHANGES IN FOREIGN PRICE LEVEL

With $e^*_{t+1,t}$ in particular being a rather complex function of the expected foreign price level for all future periods (see (14')) a detailed analysis of the effects of future expected price disturbances on the current behavior of the system becomes rather tedious. We shall therefore restrict our attention to the case where these expectations formed at time t are held uniformly for all future periods. That is,

$$q^*_{t+k,t} = q^*_t \qquad k = 1,\ldots,;\ \text{all } t \tag{25}$$

where q^*_t denotes the uniformly held expectation at time t. It is also convenient to assume that the money supply remains, and is expected to remain, at its original equilibrium level $\bar{M}$, enabling us to set $m_t = m^*_{t+k,t} = 0$. With these assumptions, the two critical expectations (12'), (14') simplify drastically to

$$z^*_{t+1,t} = 0 \tag{12''}$$

$$e^*_{t+1,t} = -q^*_t \tag{14''}$$

Substituting (12"), (14") into the matrix equation (18), the solution for y_t, p_t and e_t are

$$y_t = \frac{\gamma}{J}\,[d_3\alpha_2 - d_2(1-\delta)(1 + d_2)](q_t - q^*_t) + e_{1t} \tag{26a}$$

$$p_t = \frac{1}{J}\,[d_3\alpha_2 - d_2(1-\delta)(1+d_2)](q_t - q^*_t) + e_{2t} \tag{26b}$$

$$e_t = -q_t + \eta(q_t - q^*_t) \tag{26c}$$

where $\qquad \eta = \frac{1}{J}\,[[(1-d_1)\gamma + d_3 + d_2\delta]\alpha_2 + (\delta + \alpha_1\gamma)d_2] > 0$

With the assumption of uniformity of expectations, the initially held expectations of q (i.e. those formed at time $t-1$) play no role. The reason is that any increase in q^*_{t-1} which would influence the system through the domestic supply function is exactly offset by a decrease in $e^*_{t,t-1}$ (see (14")), leaving the supply function and the system unaffected. Indeed the critical factor determining the behavior of the system is the term $(q_t - q^*_t)$. This measures the extent to which q_t is expected to change uniformly during subsequent periods. It therefore describes the <u>expected transitory</u> component of the current foreign price level (relative to the future). If the current foreign price level is expected to continue permanently, then $q_t = q^*_t$; if the current level is expected to be a purely transitory positive deviation from $\bar{Q}$ then $q_t > 0$, $q^*_t = 0$.

From (26a) - (26c) we find that any increase in the foreign price-level q_t which at time t is expected to be permanent $(dq^*_t = dq_t)$, will have no effect on both real domestic output and its price.[19/] The exchange rate will appreciate equally, leaving the relative price z_t also unchanged. By contrast, an increase in q_t which is expected to be at least partly only temporary $(dq_t - dq^*_t)$ will have an impact on both y_t and p_t. However, these effects are ambiguous and depend upon the sign of the expression $[d_3\alpha_2 - d_2(1-\delta)(1+\alpha_2)]$. The sources of the indeterminacy are primarily the real $(r_{e,t})$ and nominal (r_t) interest rates, which using (14") can be written respectively as

$$r_{e,t} = \delta z_t + (q_t - q^*_t) + u_{3t}$$

$$r_t = z_t - p_t + (q_t - q^*_t) + u_{3t}$$

For given values of z_t, p_t, an increase in $(q_t - q_t^*)$ will lead to corresponding increases in both the real and nominal rate. The first of these will tend to reduce the demand for domestic goods, causing its price and hence supply to fall. This is reflected in the term, $-d_2(1-\delta + \alpha_2)$. At the same time the increase in the nominal rate will reduce the demand for money. With the supply of money fixed, output and the price level will have to increase in order to restore demand and maintain money market equilibrium. This is reflected in the term $(d_3 + d_2\delta)\alpha_2$. Finally, the response of the exchange rate given in (26c) implies that $de_t/dq_t > -1$; that is the exchange rate will appreciate by less than the amount of the foreign price increase.

7. CONCLUSIONS

In this paper we have developed a stochastic model of a small open economy operating under a flexible exchange rate and have considered its response to two types of exogenous disturbances, namely a domestic monetary expansion and a foreign price increase. With respect to the former, we have shown that the level of domestic output, its price, and the exchange rate, at a given time all vary positively with: (i) the unanticipated component of the money supply during that period, and (ii) the discounted sum of the amounts by which the forecasts for all future money supplies are revised during the period, from what they were in the previous period. In addition, the latter two variables depend upon the discounted sum (with weights adding to unity) of the expected money supplies for all future periods, as perceived at the end of the previous period.[20] When these factors are all taken into account, any given monetary disturbance can generate a variety of short-run effects, depending upon the accuracy with which it is predicted and how it causes expectations to be revised. It may be associated with overshooting or undershooting of the exchange rate, depending in part upon how accurately it is predicted, and may even have perverse short-run effects on output. Such will be the case if the monetary expansion is intially perceived as being permanent, but having taken place, is then expected to be only temporary.

In equilibrium, the model possesses the usual monetary neutrality properties, in that a one percent increase in the money supply will have no effect on real output and will raise the price level and exchange rate proportionately. The transitional dynamics to a given monetary disturbance can take various forms, depending particularly upon how rapidly expectations of future money supplies adjust, and the time period between the announcement (if any) of the policy and its implementation. In contrast to previous analyses, the announcement of a future monetary expansion eliminates the overshooting of the exchange rate. And while it will give rise to immediate adjustments in the nominal variables, it will leave the real behavior of the system unaffected.

The responses to a foreign price change may be most usefully summarized by relating them to the discussion concerning the insulation properties of flexible exchange rates against such disturbances, and mentioned in Section 1. As noted there, this literature is conducted in a deterministic framework and has focused on initially unanticipated permanent increases in the foreign price level. Much of the emphasis has been on the role of domestic resident's wealth held in domestic and foreign denominated currencies. A principal conclusion is that flexible rates will provide only partial insulation against a foreign price increase as long as domestic residents hold some foreign securities in their portfolios. Unfortunately to include wealth effects in a log-linear framework such as this is not easily accomplished, so that our analysis does not incorporate this aspect. Under the assumptions of the present model, the existing literature draws the conclusion that flexible exchange rates will provide complete insulation against foreign price disturbances.

The analysis of Section 6 has shown that in the absence of wealth effects in the asset demand functions, flexible exchange rates will provide perfect insulation against increases in the foreign price level which are expected to be permanent (they need not actually be, however). Moreover, this proposition is true irrespective of the accuracy with which the foreign price level is initially anticipation. This latter proposition depends upon the uniformity assumption upon which Section 6 is based and need not hold if expectations vary between the various future periods. On the other hand, even abstracting from wealth effects, a flexible rate does not provide complete insulation against a foreign price disturbance which ex post is expected to be only transitory. Indeed the more transitory it is expected to be, the less the insulation. This raises the question of what the optimal exchange rate regime -- or what the optimal intervention policy in the face of such disturbances -- ought to be. But this is a subject for future research.

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Footnotes

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1. See e.g. Niehans (1975), Dornbusch (1976), and Turnovsky (1977), among others.

2. This statement refers to the bulk of the literature which deals with unannounced monetary expansions. Recently, some authors have analyzed the effects of a pre-announced monetary expansion; see e.g. Gray and Turnovsky (1979), Wilson (1979). Obviously this is anticipated by the time it actually takes place.

3. See e.g. Kouri (1976), Laidler (1977), Floyd (1978), Turnovsky (1979).

4. In some respects our analysis parallels Fischer's (1979) recent paper analyzing the neutrality of monetary disturbances in a closed economy.

5. See e.g. Dornbusch (1976), Barro (1978), Parkin (1978), Gray and Turnovsky (1979).

6. We abstract from wealth effects, which are difficult to incorporate adequately in a log-linear framework, due to the fact that wealth is an arithmetic sum of separate components.

7. This means that we are abstracting from the possibility of "currency substitution", an issue, which is receiving increasing attention in the international monetary literature; see e.g. Girton and Roper (1976), Miles (1978), Bilson (1979).

8. In any event, our general conclusions are reasonably robust with respect to alternative specifications of the money demand function.

9. Note that (1e) reduces to an exact analogue of the Lucas Function when $\delta = 1$. The general conclusions of our analysis continue to hold if (1e) is replaced by an expectations augmented Philips curve,

although in this case, the lags embodied in the price adjustment process, will introduce additional lags into the system.

10. See e.g. Brock (1974).

11. For example, if the money supply is generated by the feedback rule

$$m_t = \rho_1 m_{t-1} + \rho_2 y_{t-1} + \rho_3 p_{t-1} + \rho_4 e_{t-1}$$

Then the expectation of the money supply must satisfy

$$m^*_{t+i+1,t-1} = \rho_1 m^*_{t+i,t-1} + \rho_2 y^*_{t+i,t-1} + \rho_3 p^*_{t+i,t-1} + \rho_4 e^*_{t+i,t-1}$$

In order to determine the conditional expectations $y^*_{t+i,t-1}$ the procedure outlined in Section 3, must now take into account the conditional expectations of the control law itself.

12. With this simplification it is also useful to write (19a') in the form

$$y_t = \frac{\gamma d_3}{J}\,[(1+\alpha_2)(m_t - m^*_{t-1}) - \alpha_2(m_t - m^*_t)]$$

Written in this way we can see that the money supply at time t varies positively with $(m_t - m^*_{t-1})$ and negatively with $(m_t - m^*_t)$. The first of these measures the unanticipated component of the current money supply; the second measures the extent to which the money supply undertaken during period t is ex post expected to change uniformly in subsequent periods. It therefore describes the expected transitory component of the current money supply.

13. The net effect on the demand for money (m^d) resulting from the changes in p_t and y_t is

$$\frac{\partial m^d_t}{\partial m^*_{t+K,t-1}} = \frac{\delta \partial p_t}{\partial m^*_{t+K,t-1}} + \frac{\alpha_1 \partial y_t}{\partial m^*_{t+K,t-1}}$$

Calculating the appropriate partial derivatives from (19a), (19b), and substituting, yields

$$\frac{\partial m_t^d}{\partial m^*_{t+K,t-1}} = \frac{-\gamma(1-\delta+\alpha_2)[\alpha_1(d_3+d_2\delta)-(1-d_1)\delta]}{J(1+\alpha_2)}$$

14. The net effect of an increase in e_t on the demand for money is

$$\frac{\partial m_t^d}{\partial e_t} = 1-\delta + \alpha_2 > 0$$

15. In other words an unanticipated permanent change in the money supply is not neutral. The same conclusion, arising for the same reason is obtained in the Fischer model, when supply is assumed to be generated by a Lucas supply function; see Fischer (1979, p. 246).

16. In this case we have

$$\delta \frac{\partial z_t}{\partial m_t} + \alpha_1 \frac{\partial y_t}{\partial m_t} = \frac{-\delta\gamma(1-\alpha_1)(1+\alpha_2) + \gamma\alpha_1(d_3+d_2\delta)(1+\alpha_2)}{J}$$

$$= \frac{\gamma(1+\alpha_2)[\alpha_1(d_3+d_2\delta) - (1-d_1)\delta]}{J}$$

17. Because of the lagged price adjustment embodied in the Phillips curve, the dynamics of the systems considered by these authors is second order, having one stable and one unstable root, thus exhibiting saddle-point type instability.

18. The rate of change per unit period = $(1-\lambda)/\lambda = 1/\alpha_2$.

19. Strictly speaking, our analysis of foreign price disturbances is incomplete. In practice q_t is an endogenous variable in the rest of the world, as is the foreign interest rate $\rho_t \equiv \rho + u_{3t}$. Thus any stochastic shift in q_t is likely to be accompanied by

a stochastic change in ρ_t, as both variables respond to the common random influence. The degree of correlation between these two foreign variables will vary, depending upon the source of the foreign disturbance. By ignoring accompanying changes in the interest rate, we are making a special assumption about the nature of the underlying disturbance.

20. We have restricted our attention to the price of domestic output. The response of the overall domestic CPI can be easily derived using (5b) from that of the price of domestic output and the exchange rate. Indeed it is just an average of that of p_t and $(q_t + e_t)$ and for that reason is not discussed separately.