A NOTE ON "LEASE-ONLY"

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Of the many provocative homework problems that Milton Friedman appended to his price theory text, that which he titled, "Alcoa: The Secondhand Market and Monopoly Position"[1] has probably received the most attention in the professional economics literature.[2] Most of the studies that this problem has evoked have dealt with the basic issue that Friedman addressed, the quality of the economic analysis in a portion of Learned Hand's celebrated decision in the Alcoa case.[3]

In the literature evoked by Friedman's Alcoa problem, however, rather little attention has been paid to the direct implications for public policy of the "mock problems" that Friedman set up to aid in evaluating Hand's reasoning.[4] These problems deal with a durable good which its producer contemplates either (i) selling outright in which case a competitive secondhand market will arise or (ii) retaining ownership and renting them out. Suppose, Friedman effectively wrote, that the life of a machine can be prolonged by applying to it competitively obtainable repair services. "Prove," he asked, "that the company's optimum output will ... be different in cases (i) and (ii) and indicate

1. Milton Friedman, Price Theory 337 - 8 (1976).

2. See, for example, Darius W. Gaskin, Jr., Alcoa Revisited: The Welfare Implications of a Secondhand Market, 7 J. Econ. Theory 254 (1974).

3. US v Aluminum Co. of America, 148 F 2d 416 at 424 - 5 (1945).

4. Several studies have, however, tangentially attacked Friedman's mock problems. These studies have dealt mainly with the difference in the durability of a durable good that will result if it is produced by a monopolist on the one hand and a competitively organized industry on the other. The basic theoretical results developed below can be gleaned from two of these studies, Richard Schmalensee, Market Structure, Durability, and Maintenance Effort, 41 Rev. Econ. Studies 277 (1974), and Teddy T. Su, Durability of Consumption Goods Reconsidered, 65 Amer. Econ. Rev. 148 (1975).

in which case the total number of machines in existence will be larger, and hence the net rental value lower."

This mock problem is closely related to a once quite common business practice. To cite but three particularly prominent examples, American Can, International Business Machines, and United Shoe Machinery long refused to sell their patented equipment outright but rather allowed their customers only to lease this equipment. All three companies terminated this "lease-only" practice only as a result of antitrust suits shortly after World War II.[5]

The practice of lease-only and its termination raise several interesting questions. First, why did American Can, IBM, United Shoe, and many other firms follow the practice? More precisely, assuming that the practice was adopted in the expectation of higher profits than those obtainable through outright sale of equipment, were American Can, IBM, United Shoe, etc. justified in this expectation? Second, what were the social consequences of ending the practice? Did doing so result in lower effective prices for machine services? Did it reduce the deadweight losses resulting from monopolistic pricing of the services of patented machines? And how did it affect the profits of patent holders and hence the incentives to develop and patent new durable equipment?

Perhaps surprisingly, only the question about profits can be answered unambiguously: Lease-only almost certainly[6] resulted in higher

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5. US v American Can Co., 87 F Supp 18 (1949), US v International Business Machines Corporation, 1956 Trade Cases, paragraph 68,245, a consent decree filed in the Southern District of New York, and US v United Shoe Machinery Corp., 110 F Supp 295 (1953), affirmed per curiam, 347 US 521 (1954).

6. The equivocation here reflects a remote possibility: As is suggested by one of Friedman's mock problems that is not summarized above, if it were possible for American Can, IBM, United Shoe, etc. to build one-hoss shay-type machines that would collapse into a pile of dust at a carefully chosen point in time, these firms would be indifferent between selling and leasing their machines. However, as will be established below, lease-only definitely yields higher profits if the useful lives of machines can be extended by maintaining them appropriately.

profits than those obtainable through outright sale; ending the practice almost certainly diminished profits. Contrary to an implication of Friedman's statement of the problem, however, it is not possible to "indicate in which case the total number of machines in existence will be larger, and hence the net rental value lower"; whether prohibiting lease-only leads to lower net rental values for machines depends on the specific demand and maintenance cost functions to which machines are subject. If, as can be shown, circumstances exist under which sell-only would lead to higher net rental values than lease-only, sell-only would yield higher deadweight losses under these circumstances; sell-only could also lead to higher deadweight losses than lease-only even if sell-only results in lower net rental values.

Examining the behavior of a machine owner who strives to maintain it so as to minimize the cost of the stream of services it renders is a necessary first step in demonstrating the validity of these propositions. Suppose that a particular type of machine yields a constant flow of services through time provided only that it is suitably maintained. Let P be the initial price of the machine when purchased from its manufacturer and $M(t)$ be the total cost of maintaining the machine if it is to yield exactly t years of service. It seems plausible to suppose that $M(t)$ increases at an increasingly rapid rate with increases in t, i.e., that it would take a greater increase in maintenance outlays to get an additional year's services from a $t + 1$ than from a t year old machine. With this notation,

$$r = (P + M(t))/t \tag{1}$$

is the annual cost of a unit of machine services.[7]

Minimizing r can easily be shown to require selecting that value of t for which

$$m(t) = (P + M(t))/t \tag{2}$$

where $m(t) \cdot dt$ is the additional maintenance required to add an infinitesimal increment, dt, to the life of a t-year old machine.[8] Equation (2) says, simply enough, that cost minimization requires selecting that machine life for which the marginal (maintenance) cost of machine services equals the average cost of these services.

Letting t_S refer to the value of t which satisfies equation (2),

$$r_S = m(t_S) = (P + M(t_S))/t_S \tag{3a}$$

is the cost to a machine buyer of a year's machine services if P is the price of a new machine. Similar logic leads to

$$c = m(t_L) = (C + M(t_L))/t_L \tag{3b}$$

as the minimum cost, c, of a year's machine services to a producer who chooses to lease his machines if C is the unit cost of manufacturing them.

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7. Ignored here are both opportunity costs and the possibility that the maintenance required to obtain t years of service from a machine depends not just on t but also on how well the machine was built. It would be more realistic to discount flows of machine services and maintenance outlays and to allow both the price of the machine and the maintenance required to obtain t years of service from it to depend on an index of the durability built into it. Making these refinements would lead to considerably more complex algebra, however, and would not fundamentally alter the conclusions reached below. See Richard Schmalensee and Teddy T. Su, supra, note 5.

8. Differentiating equation (1) with respect to t and setting the result equal to zero yields:

$$dr/dt = m(t)/t - (P + M(t))/t^2 = 0$$

which reduces to equation (2) on multiplying through by t.

If a machine's price, P, is greater than its cost of production, C, then t_S will be greater than t_L; a machine purchaser would select a longer life for his machines than would a manufacturer who chooses to lease the machines. This is because the average fixed cost of a machine is greater at any given value of t for the buyer (P/t) than for the manufacturer-owner (C/t). This being the case, the value of $m(t_S)$ and hence t_S required to satisfy equation (3a) must necessarily be greater than the value of $m(t_L)$ and hence t_L required to satisfy equation (3b).[9]

The annual demand for machine services, $D(r)$, can reasonably be regarded as a function of their (implicit, when machines are sold) annual rental rate, r. If their monopolist-manufacturer is allowed to practice lease-only and chooses to do so, his annual profits can be written

$$\pi^L = (r - (C + M(t))/t)\,D(r) \quad . \tag{4}$$

In the interests of minimizing costs, the manufacturer can be expected to select that machine life which satisfies equation (3b). If he does,

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9. Proceeding formally, t in equation (2) can be regarded as a function of P. Differentiating equation (2) totally with respect to P yields

$$\frac{dm(t)}{dt}\cdot\frac{dt}{dP} = \frac{1 + m(t)\; dt/dP}{t} - \frac{(P + M(t))}{t^2}\,\frac{dt}{dP}$$

$$= \frac{1}{t} + \left(m(t) - \frac{P + M(t)}{t}\right)\frac{dt/dP}{t} = \frac{1}{t}$$

The last equality in this expression follows from equation (2) itself. Hence $dt/dP = 1/[t\; dm(t)/dt]$ which must be positive if, by assumption, $dm(t)/dt$ is positive.

equation (4) can be rewritten as

$$\pi^L = (r - c)\, D(r) \quad . \tag{4'}$$

Maximizing this expression can readily be shown[10] to require selecting that rental rate for which

$$(r_L - c)/r_L = 1/E \tag{5}$$

where E is the elasticity of demand for machine services. Equation (5) says, simply enough, that a profit-maximizing lessor of machine services should set that rental rate for them, r_L, which equates Lerner's index of monopoly power with the reciprocal of the elasticity of demand for them.

If the machine monopolist chooses (or is forced) to sell them outright, at a price of P, he can expect average annual sales of, say, $\Delta(P) = D(r_S)/t_S$ where t_S and hence r_S are values that cost-minimizing buyers of machines will select to satisfy equation (3a). With these annual sales, the monopolist's profits can be written

$$\pi^S = (P - C)\, D(r_S)/t_S \tag{6}$$

Multiplying equation (3a) through by t_S and rearranging terms yields an

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10. Differentiating equation (4) with respect to r and setting the result equal to zero yields

$$d\pi^L/dr = D(r) + (r - c)\, dD(r)/dr = 0 \quad .$$

Shifting $D(r)$ to the right-hand side of this expression and dividing through by $r\, dD(r)/dr$ yields equation (5).

expression that can be used to eliminate P from equation (6):

$$\pi^S = (r_S t_S - M(t_S) - C)\,D(r_S)/t_S$$

$$= (r_S - (M(t_S) + C)/t_S)\,D(r_S) \quad . \qquad (7)$$

Equation (7) is quite similar to equation (4). There is a crucial difference, however. In maximizing equation (4), the machine monopolist can separately select machine life, t_L, and the annual rental charge, r_L. The monopolist does not have this flexibility in maximizing (7), however. Once he sets P, his customers jointly determine t_S and r_S to satisfy equation (3a). That is, equation (3a) constrains the monopolist in striving to maximize equation (7) but not in striving to maximize equation (4).

In maximizing a function that is subject to a constraint, a maximizer could never do better than if the constraint did not exist and can only do as well with as without the constraint only if it turns out not to be binding. The constraint implied by equation (3a) would not be binding if, e.g., the monopolist could design his machines to break down irreparably after exactly t_L years of service. In the absence of such felicitous engineering, however, practicing lease-only would definitely yield greater profits than would selling as well as leasing machines.

Figures 1a and 1b respectively depict the alternative equilibria reached in the machine market under lease-only and sell-only. Figure 1a illustrates a conventional profit-maximizing-monopoly equilibrium with marginal cost (i.e., c) equal to marginal revenue at an annual rental rate of r_L and annual sales of D_L machine-years of services. In this equilibrium, the monopolist manufactures $D(r_L)/t_L$ machines annually

Lease-Only

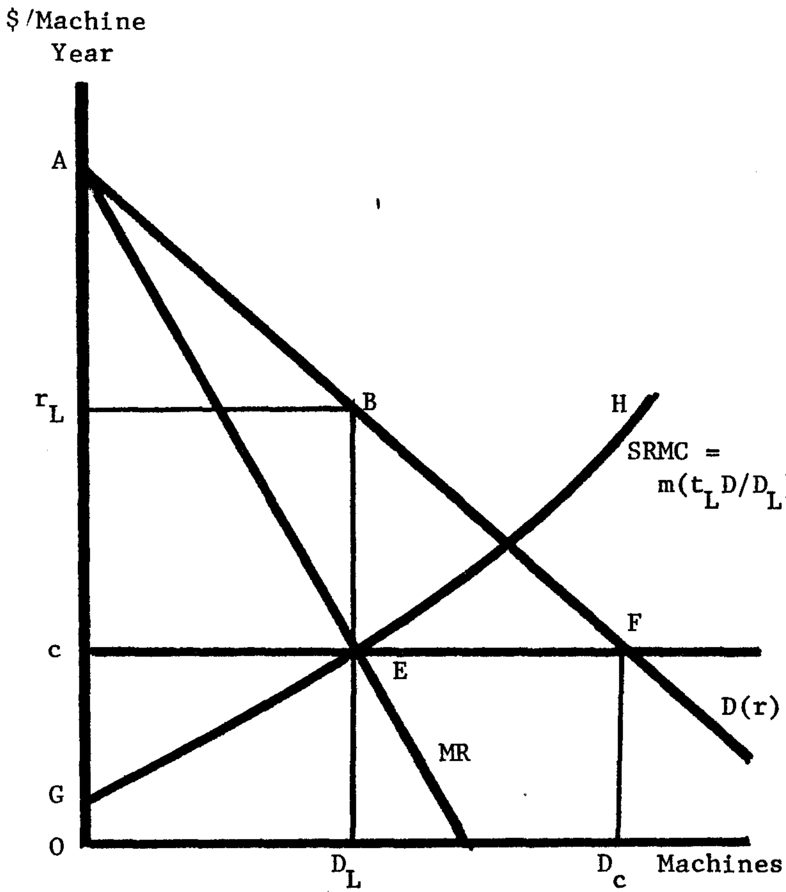

Figure 1a

Sell-Only

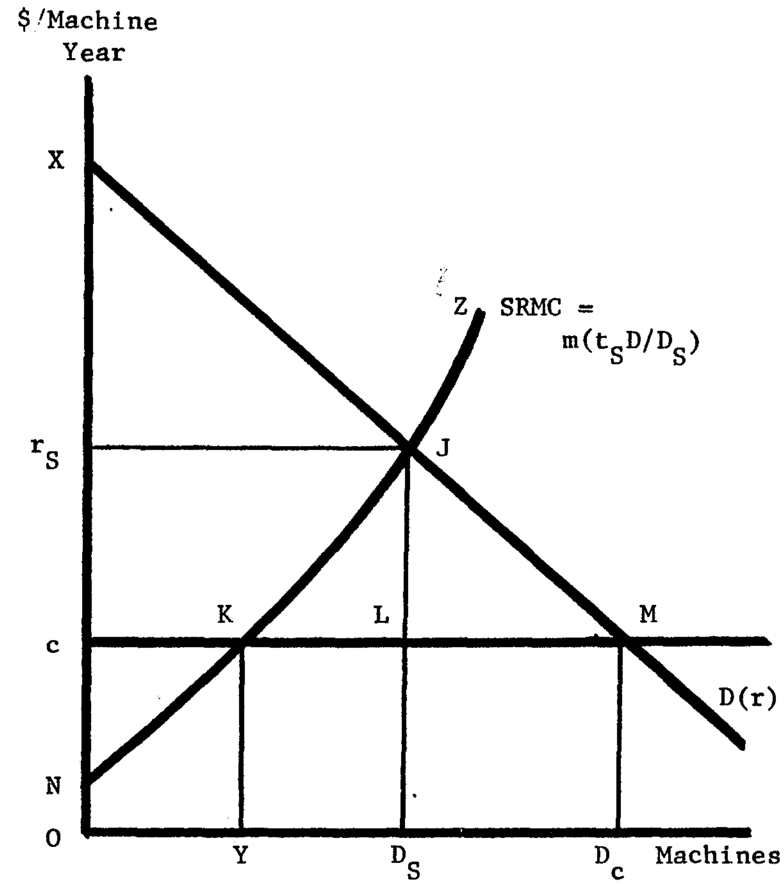

Figure 1b

with t_L selected so that $m(t_L) = c$, the minimum cost of providing a year's machine services. Annual maintenance outlays equal GED_L0 dollars, the area under the marginal maintenance cost schedule, while the capital outlays required to produce D_L/t_L machines is CEG , the area between the maintenance cost schedule and the minimum cost of producing machine services.[11] The annual benefits society derives from the existence of this market can therefore be regarded as the sum of consumers' surplus, ABr_L , and monopoly profits, r_LBEc . The annual deadweight loss from the existence of monopoly rather than competition in this market equals BFE .

Under sell-only, the monopolist produces $D(r_S)/t_S$ machines a year with t_S being selected by machine buyers to minimize $(P + M(t))/t$, thereby yielding an implicit rental rate for machine services of r_S . The area CKN represents the cost of producing D_S/t_S machines. The

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11. Since the monopolist manufactures D_L/t_L machines per year, the first unit of machine services provided each year can be viewed as obtained from a group of machines that average t_L/D_L years of age at a cost of $m(t_L/D_L)$, the second from a group of $2t_L/D_L$ year-old machines at a cost of $m(2t_L/D_L)$ and so forth. Total maintenance costs, area GED_L0 , can therefore be written

$$\int_0^{D_L} m(t_L D/D_L)\, dD = (D_L/t_L) \int_0^{D_L} m(t_L D/D_L)\, d(t_L D/D_L) = D_L M(t_L)/t_L$$

$$\text{Area } cEG = \text{Area } cED_L0 - \text{Area } GED_L0$$

$$= cD_L - D_L M(t_L)/t_L = D_L(C + M(t_L) - M(t_L))/t_L = D_L C/t_L$$

because

$$c = m(t_L) = (C + M(t_L))/t_L \quad .$$

annual cost of maintaining the $D_S = D(r_S)$ machines is NJD_S0[12] while the annual benefit society derives from this market is the sum of consumers' surplus, XJr_S, and monopoly profits, r_SJKc. The total deadweight loss in this market equals KJM. Of this total, JML reflects the consumers' surplus foregone by having D_S units of machine services used annually at an implicit rental rate of r_S rather than the competitive solution, D_c units at a rental rate of c. The remaining deadweight loss, KJL, is attributable to the excess maintenance of machines, i.e., to spending $M(t_S)$ rather than $M(t_L)$ dollars on maintenance over the life of each machine. Put differently, cLD_S0 is the minimum annual outlay that would be required to provide D_S machine years of service while $cKJD_S0$ is the actual annual outlay for these services under the sell-only equilibrium. The implications of the existence of this excess maintenance component of deadweight loss in the sell-only case are perhaps worth emphasizing. Given its existence, the aggregate benefits to society with lease-only could exceed those resulting from its prohibition even if prohibition leads to lower rental rates for machine services.

As Figure 1 is drawn, the profit maximizing monopoly rental rate is

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12. Point Y in figure 1(b) has a value of t_LD_S/t_S. Reasoning similar to that in footnote 11, supra, therefore leads to

$$\text{Area } NKYO = (D_S/t_S)\int_0^{t_LD_S/t_S} m(t_SD/D_S)\,d(t_SD/D_S) = D_SM(t_L)/t_S \quad .$$

Hence

$$\text{Area } cKN = \text{Area } cKYO - \text{Area } NKYO = m(t_L)D_St_L/t_S - D_SM(t_L)/t_S = D_SC/t_S$$

$$\text{Area } NJD_S0 = (D_S/t_S)\int_0^{D_S} m(t_SD/D_S)\,d(t_SD/D_S) = D_SM(t_S)/t_S \quad .$$

higher than the implicit rental rate when the monopolist is forced to sell machines rather than rent machine services. Intuition, Milton Friedman's hints in his Alcoa-case problem, and Judge Wyzanski's opinion in the United Shoe Machinery case[13] suggest that this would be the normal result of forcing a lease-only monopolist to sell his machines. Intuition would suggest, that is to say, that the competition to the monopolist from used machines that would result from forcing him to sell them would face him with a more elastic effective demand for machine services than with lease-only and hence would lead to a lower profit-maximizing rental rate.

Plausible though this reasoning might seem to be, it is not a necessary consequence of ending lease only. Forcing American Can, IBM, United Shoe Machinery, and, by implication, all other producers of patented durables to sell as well as to lease their products <u>may</u> have resulted in lower implicit rental rates to their customers. Doing so did not necessarily have this result, however. Rather, whether ending lease only has increased or decreased the implicit or explicit rental rate buyers pay for a particular durable depends on the specific maintenance cost and demand schedules associated with that durable.

Providing a general proof of this proposition is a difficult task.[14] Therefore, rather than attempt such a proof, we rely on two specific algebraic examples. In both, we assume a constant-elasticity demand schedule for machine services -- $D(r) = Ar^{-a}$. In the first example, the cost of manufacturing a machine is assumed to be C dollars while the total cost of maintaining the machine for a t-year life is $M(t) = Bt^b$; in the second example, $C + M(t) = Be^{bt}$.

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13. <u>Supra</u>, note 5.

14. Schmalensee has provided a heuristic proof. See <u>supra</u>, note 5, at 282-5.

Expressions for the ratio of the rental rate implicit in a profit-maximizing price for machines under sell-only to the profit-maximizing rental rate under lease-only are: [15]

$$\text{Example 1:}\quad r_S/r_L = (a - 1)\, x^{1-1/b}/a \tag{8a}$$

$$\text{Example 2:}\quad r_S/r_L = (a - 1)\, e^{y-1}/a \tag{8b}$$

where

$$x = (ab - a + 1)/(ab - a - b + 1)$$

$$y = (1 + [1 + 4/(a - 1)]^{\frac{1}{2}})/2$$

For example 1, as a and b increase, r_S/r_L approaches (but never reaches) one from below. Values of equation (8a) for a sample of a and b values are:

	a		
b	1.1	2	4
1.1	0.14	0.63	0.86
2	0.42	0.87	0.97
4	0.67	0.94	0.99

For example 2, values of equation (8b) for a sample of values of a are:

a:	1.1	1.19963	2	4
r_S/r_L:	1.35	1.00001	0.93	0.98

Thus, for a less than about 1.2, r_S exceeds r_L; circumstances do exist under which forcing monopolists to sell as well as to lease

15. The algebra involved in developing these expressions and those that follow is tedious but straightforward. We therefore only briefly sketch the steps involved. Differentiating $(P + M)/t$ and $(C + M)/t$ with respect to t and setting the results equal to zero yield expressions for t_S and t_L as functions of P and C respectively. Inserting these expressions in $r_S = m(t_S)$ and $c = m(t_L)$ yields expressions for r_S and c as functions of P and C respectively. From the fact that, for a simple profit-maximizing monopolist, (price - marginal cost)/price = $1/E$ where E is the elasticity of demand for his product, it follows that $r_L = ac/(a - 1)$. Inserting the expressions for r_S and t_S as functions of P into equation (7), differentiating with respect to P and setting the result equal to zero yields an expression for P as a function of C and other system parameters which can be substituted for P in $r_S = m(t_S)$.

their patented durables would yield no reduction in the effective rental rates for the services of these durables.

Examples 1 and 2 differ in their implications not just for the possibility of having r_S exceed r_L but also for the effect of ending lease-only on the deadweight losses attributable to the existence of monopoly in durable equipment markets and on the profitability of this sort of monopoly. Expressions for the ratio of DWL(S) to DWL(L), the deadweight loss attributable respectively to sell-only and lease-only are:[16]

$$\text{Example 1:} \quad \frac{DWL(S)}{DWL(L)} = \frac{1 - x^{(1-b)(a-1)/b}(1 + 1/x)}{1 - [a/(a-1)]^{1-a} - [(a-1)/a]^{a}} \qquad (9a)$$

$$\text{Example 2:} \quad \frac{DWL(S)}{DWL(L)} = \frac{1 + e^{(1-a)(y-1)}[(a-1)/y-a]}{1 - [a/(a-1)]^{1-a} - [(a-1)/a]^{a}} \qquad (9b)$$

Values of these expressions for a sample of a and b values are:

	a		
	1.1	2	4
b	Example 1		
1.1	0.23	0.54	0.77
2	0.71	0.92	0.98
4	0.87	0.97	0.99
	Example 2		
all	1.28	1.02	1.01

For example 1, ending lease-only reduces deadweight losses. The ratios of sell-only to lease-only deadweight losses are particularly small when

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16. In both parts of Figure 1, the deadweight loss can be measured as the difference between (a) the area bounded by the vertical axis, the demand schedule, and machine rental rates of c and r_L or r_S, and (b) monopoly profits. If $D(r) = A r^{-a}$, the former area is $A(c^{1-a} - r_i^{1-a})/(a-1)$ where $i = L$ or S. Profits are determined by inserting expressions for r_L, r_S, t_L, and t_S obtained as indicated in note 15, <u>supra</u> in equations (4) and (7).

a, the price-elasticity of demand for machine services, and b which can be interpreted as the elasticity of maintenance costs with respect to machine life, both have small values. For example 2, however, ending lease-only increases deadweight losses. Furthermore, the smaller is the price-elasticity of demand, the greater is the ratio of DWL(S) to DWL(L).

The monopoly positions of IBM, United Shoe, and many other firms which refused to sell their durable equipment rested in substantial measure on patent protection. The research and development underlying patents are generally time-consuming and costly activities -- activities in which business firms would not engage unless rewarded for doing so by, e.g., temporary monopoly profits. This being the case, while a reduction in deadweight losses from ending lease-only may be a Good Thing from the standpoint of static efficiency, it may well reflect a Bad Thing from the standpoint of future incentives to improve durable equipment.

Expressions for the ratios of π^S to π^L, profits under sell-only and lease-only respectively are:[17]

$$\text{Example 1:} \quad \pi^S/\pi^L = x^{(a-ab-1)/b}[a/(a-1)]^a \tag{10a}$$

$$\text{Example 2:} \quad \pi^S/\pi^L = (a-1)^{1-a}\, a^a\, e^{(1-a)(y-1)}(1 - 1/y) \tag{10b}$$

Values of these expressions for a sample of a and b values are:

	a		
	1.1	2	4
b	Example 1		
1.1	0.12	0.27	0.45
2	0.57	0.77	0.88
4	0.80	0.98	0.99
	Example 2		
all	0.78	0.82	0.90

17. Obtained as indicated in note 16, *supra*.

Thus, for example 1, the substantial proportionate reductions in deadweight losses from ending lease-only that are associated with low values of a and b are also associated with substantial reductions in monopoly profits and hence in the incentives to develop new durable equipment. For example 2, however, while ending lease-only increases deadweight losses, its effects on potential monopoly profits and hence incentives are modest regardless of the price-elasticity of demand for machine services.

Normally, precious few substantive conclusions can be drawn from the numerical analysis of algebraic examples that have no factual basis. This is not entirely true, however, of the numerical illustrations described in the preceding few pages. A conclusion can be drawn from these illustrations, albeit one of a totally negative sort. It is that two types of doubts exist as to whether the Supreme Court's elimination in 1954[18] of lease-only as a legal way of exploiting patent monopolies should be deemed a Good Thing by society. Even if we knew the extent of society's willingness to trade static deadweight losses for the incentives to progress that the unfettered exploitation of patents may provide, the optimum treatment of a specific case of lease-only would still not be known. Rather, this treatment would depend on the specific factual circumstances of the case. The numerical examples developed above suggest that, depending on these circumstances, the results of ending lease-only in a specific case could range from substantial reductions in both deadweight losses and profits through modest reductions in both to actual increases in losses accompanied by modest reductions in profits.

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18. By approving *per curiam* Judge Wyzanski's 1953 decision in the United Shoe Machinery case. See note 5, *supra*.