SMALL ECONOMETRIC MODELS OF THE U.S. AND WEST GERMANY WITHOUT PRIOR RESTRICTIONS

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Introduction

In an earlier paper [1977b] I criticized large-scale macroeconometric models constructed by the usual methods, suggested alternative methods, and applied them to analysis of the German and U.S. economies. In interpreting any econometric model, one is implicitly or explicitly "identifying" the model, in the usual econometric sense of that word. That is, one is assuming some connection between the estimated parameters of the model and the behavior patterns of the people in the economy being modeled. While the earlier paper was quite explicit about what is wrong with the usual rules of identification as used in building standard models, it left the rules of identification underlying my interpretation of loosely structured models largely implicit. Because this makes it hard to discuss the limitations of the methods, and because certain aspects of these methods are being applied by other economists, this paper begins with a discussion of the assumptions underlying the most direct interpretation of what I call "innovation accounting".

As will become clear in the methodological discussion, it is critical to the reliability of some of the conclusions obtained in the earlier paper that the list of variables included in the model be rich enough. The latter part of the paper discusses empirical results obtained with a model similar in spirit to that of the earlier paper,

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but with a longer list of variables. For the U.S. at least (the German results are unfortunately not yet available at the time of writing this draft), many of the most important conclusions persist despite the increase in number of variables: Unpredictable movements in the money supply still are the dominant source of variation in the price level; a "monetarist Phillips curve", in which relatively short-lived (2-year) rises in real GNP and real wage follow a money shock again emerges; and the real quantity variables as a block cannot be taken exogenous, despite the fact that two of the three additional variables are components of GNP. On the other hand, the larger model is in some respects more hospitable to what might be labeled "Keynesian" views. In the larger model a slightly larger proportion of variance of real quantity variables is accounted for by innovations in real quantities, which focuses attention on an aspect of the earlier paper's results which received little comment in the text of that paper: Most variance in real GNP and unemployment is <u>not</u> accounted for by variables associated with monetary policy. The view that the main source of instability in the U.S. economy has been unpredictable policy shifts of the Federal Reserve looks implausible in the light of these results. Also, the "monetarist Phillips curve" is mediated by interest rate movements, with money supply movements which are not associated with interest movements having very different effects on real quantity variables.

Innovation Accounting

We begin by assuming we are dealing with an mxl vector time series $y(t)$. Denote by $\hat{y}(t)$ the best linear forecast of $y(t)$

based on $y(s)$, $s < t$.[1/] The innovation in $y(t)$ is then defined as $y(t) - \hat{y}(t)$. Because the innovation in $y(t)$, $e(t)$, is itself a linear combination of current and past values of $y(t)$ for each t, $e(t)$ is serially uncorrelated. Under rather general conditions, $y(t)$ can be expressed as a linear combination of current and past innovations. Under slightly more restrictive conditions, the coefficients on $e(t-s)$ in the formula expressing $y(t)$ as a linear combination of $e(t-s)$ for $s \geq 0$ depends only on s, not on t.[2/] The representation of $y(t)$ as a linear combination of current and past innovations, i.e. as a distributed lag on $e(t)$, is called the moving average representation (MAR) of y. We will assume further that $y(t)$ can be approximated well by a finite linear combination of past y's, with the weight on $y(t-s)$ dependent on s but not t.[3/] This leads us to a linear regression equation of the form

$$1) \qquad y(t) = \sum_{s=1}^{q} A(s)y(t-s) + e(t) .$$

If the equation is exact, i.e. if y is an exact function of q past y's, then the usual distribution theory applies to least-squares estimation of the A's in this equation asymptotically, if we assume

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[1] To be more precise, $\hat{y}(t)$ is the projection of $y(t)$ on the Hilbert space spanned by past $y(s)$ under the covariance inner product.

[2] Sufficient conditions are that y be a linearly regular covariance stationary process, in the terminology of Rozanov [1967]. However many types of non-stationary processes will also meet the conditions specified here.

[3] This condition, again, will hold for all covariance-stationary linearly regular processes, but in addition for some non-stationary processes.

$e(t)$ has constant finite variance and that y is the conditional expectation of y given past y.[4]

Once we have estimated the A's (the coefficients of what is called the autoregressive representation or AR) it is a straightforward exercise to compute the coefficients of the moving average representation. Suppose the MAR is written

$$2) \qquad y(t) = \sum_{s=1}^{\infty} B(s)e(t-s) .$$

Then it is not too hard to see that the i, j'th component of $B(s)$, $b_{ij}(s)$, when treated as a function of s, traces out the response of y_i over time to an initial condition in which all past values of all variables are zero except for $y_j(0)$, which is unity. Thus while the coefficients $A(s)$ in the AR are the coefficients of the reduced form of a standard econometric model, the coefficients in $B(s)$ can be thought of as obtained from simulations of the model.

There is a common-sense rule which states that it is dangerous to simulate a model's response to conditions widely different from what occurred in the sample period. Since $e(t)$ is serially uncorrelated, a unit shock of the type "simulated" in the $B(s)$ coefficients is not unlike the sample behavior of e as regards serial correlation. However, the components of e may be contemporaneously correlated. If

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[4] I.e., the condition that has characterized as "best linear predictors are best predictors". If (1) without the error term would be an unstable difference equation, then the usual distribution theory has an asymptotic justification, but in a special sense. See Sims [1978] and Fuller [1976].

these correlations are high, simulation of a shock to $y_j(0)$ while all other components of y are held at zero, may be misleading. E.g., if GNP and unemployment have negatively correlated innovations, simulation of the economy's response to a unit shock to GNP while unemployment remains fixed may imply a pattern of movement in y which is unlike anything which has occurred historically and is likely therefore to be unreliable.

Furthermore, because of the contemporaneous correlation among components of e, it is not possible to partition the variance of y into pieces accounted for by each innovation. For these two reasons it is appealing to apply an orthogonalizing transformation to e, to obtain $v(t) = Ge(t)$, where G is a matrix chosen to make the covariance matrix of $v(t)$ the identity. There are many ways one could choose G, but for reasons we will discuss below it is natural to consider first G's of triangular form. The triangular form preserves the connection of the elements of v with the corresponding variables in y, in the sense that, if G is lower triangular, $v_j(t)$ is the normalized error in forecasting $y_j(t)$ from past values of the y vector and current values of $y_i(t)$ for $i < j$. We can represent y as

$$3) \qquad y(t) = \sum_{s=1}^{\infty} B(s)G^{-1}v(t-s) .$$

The components of the matrix function $B(s)G^{-1}$ thus are still interprable as responses to "shocks" in particular variables, but now the shocks are of a more typical sort and, because the components of v

are uncorrelated, we can allocate the variance of each element in y to "sources" in elements of v. In particular, the sum of squares from $s = 0$ to $s = T$ of the i, j'th component of $B(s)G^{-1}$ represents the component of error variance in the $T + 1$ step-ahead forecast of y_i which is accounted for by innovations in y_j.

The moving average representation and the allocation of forecast error variance to innovations summarize the average behavior of the system over the sample period. One can also analyze particular historical episodes by locating the dates of unusually large innovations and using the MAR to determine what aspects of the subsequent evolution of the system the large innovations account for. My own work [1977b] and joint work with Sargent [1977] has concentrated on displaying the "average" characteristics of systems, while Robert J. Gordon [1977], and Gordon and J. A. Wilcox [1978], using closely related methods, has emphasized historical analysis of particular episodes.

As an example of how these methods get fleshed out with economic interpretation, suppose we were interested in whether an inflationary period beginning at date t can in fact be largely attributed to "wage push". A natural procedure would be to fit a vector AR system to a set of variables including wages, prices, and other relevant variables.[5] One then asks if there were large wage-innovations near date t. If not, it would not seem reasonable to attribute the inflation to "wage-push", since even if wage changes were large near t, they were explainable by reactions according to the usual pattern

[5] The question of how one decides what the "other relevant variables" are and what happens if some are omitted will be taken up below.

to past values of other variables in the system. If there was a large wage-innovation near t, one can then use the MAR to determine whether it had substantial influence on subsequent developments in prices. If not, it seems reasonable to claim that the wage "shock" at t does not explain the subsequent general inflation. We can even see a natural policy implication of such analysis: if it turned out that large wage-innovations account for most of inflation, historically, we might conclude that government intervention in the wage-bargaining process would be the most effective method of controlling general inflation. R. J. Gordon's [1977] international analysis inflation is in part in the spirit of this example.

As another example, we might interpret the proportion of inflation accounted for by innovations in the money stock (either in a historical simulation or in a partition of variance) as a measure of how much of inflation is accounted for by surprise shifts in monetary policy.[6] If a great deal of inflation were accounted for this way, we might then conclude that the case for rigidly specified money-supply rule (e.g. a fixed percentage growth rate) was strengthened.

Using a model to generate policy conclusions is by definition the process of identifying a structure for the model, in the sense of, e.g., Hurwicz [1960]. In the next section the rules of identification underlying the foregoing examples are made explicit and compared with the usual rules of identification in econometrics.

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[6] Specifications in which this connection of money innovations to monetary policy is assumed play a major part in work by T. J. Sargent [1976].

Wold-Style Identification

Interpretation of MAR's along the lines of the foregoing example rests on an assumed connection between innovations in a variable (or more generally in a block of variables) and events originating in a behaviorally distinct sector. Thus in the example we assume that wage-innovations arise in the wage-bargaining mechanism and that policy measures exist which might affect such events without changing the behavior of other "sectors" of the economy.[7] As has already been mentioned, the AR is a conventional "reduced form" if a model has no exogenous variables. When there are exogenous variables, we can obtain an AR by appending to the conventional reduced form an AR for the exogenous variables. Thus the style of interpretation we are discussing involves making a connection between reduced form residuals (the innovations) and sectors. In the conventional style of econometric identification there is no such connection. In the conventional style sectors are connected to equations of the "structural form" of the model. Residuals from structural form equations might then be interpreted as shocks originating in the corresponding sector, but in general structural-form residuals are related to reduced-form residuals by an arbitrary linear transformation. Each reduced-form residual is an unrestricted mixture of shocks to all sectors. From this point of view there is no sense in the innovation-accounting scheme, since the innovation in a particular variable has no structural interpretation.

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[7] This conclusion about possible policies might appear in a negative form: if most inflation originates in wage-innovations and we suppose that policy cannot effectively influence wage-bargaining, then we might conclude that policy cannot affect inflation.

In practice, however, it is quite reasonable to make connections between variables and sectors, as reflected in the usual names of equations in macro-economic models: "investment", "consumption", "price", "wage", etc. One names equations for the most part according to the variable appearing on the left-hand side, though from the point of view of statistical theory the left-hand-side variable could be any endogenous variable in the equation. The equation corresponding to a sector must have on the left a variable which we are sure enters the equation with a non-zero coefficient on the current value. Generally there are only a small number of variables, or even only one, for which we can be sure of this. In other words, generally there are only a few variables which we are sure reflect immediately disturbances originating in a given sector. If it should in fact be true that only a few variables reflect immediately disturbances in the sector, then only those few variables appear contemporaneously in the structural form equations for that sector. In the conventional notation, the structural system

$$\Gamma y(t) + \beta X(t) = u(t) \; , \tag{4}$$

where $X(t)$ is the vector of predetermined variables, has a block diagonal Γ matrix. Under these conditions, the reduced form residuals for a block are linear combinations of the structural equation residuals for the block; the innovations in the block's variables are determined by shocks originating in that sector. This situation justifies the interpretation of innovation accounting along the lines of the examples.

While this argument does not require any assumptions on correlations among innovations, if in fact innovations show strong correlations across sectors, identification along these lines is strained. We might suppose, for example that wage innovations pertain to a "wage-bargaining" sector and money innovations to a "monetary policy" sector; but if these two innovations are strongly correlated our assumption that these two sectors are in fact distinct becomes questionable. Thus it seems reasonable to append to the assumption that only a few variables react immediately to shocks in a given sector the assumption that shocks from distinct sectors should be only weakly correlated.

With this additional specification, the connection of innovations to sectors becomes testable. Most directly, we can simply check to see whether correlations of innovations in different sectors are "large". In a model with many variables, however, it may not be clear how to decide what is a "large" set of correlations. An operational criterion can be provided by making the triangular decomposition of the covariance matrix which we discussed as part of the innovation accounting scheme. If our connection of variables to sectors is correct and if correlations of shocks across sectors are small, conclusions based on the innovation-accounting scheme should not change much when the ordering of the sectors is varied before the triangular orthogonalization.

In practice, it turns out that some conclusions are robust to the ordering of variables and some are not. When conclusions shift according to the ordering of variables, we need not necessarily abandon efforts at interpretation. For example, one can imagine correlations between money innovations and investment innovations arising from attempts by the monetary authority to stabilize interest rates. It could even happen that the main source of "surprises" in the money stock was shifts in investment demand. It is natural to check for this possibility by observing how the system responds to a money innovation -- if a money innovation results in a drop in interest rates, it is not likely that it is mainly a passive response to an increase in investment demand. In making this kind of interpretation, we begin to use detailed a priori hypotheses about the form of the equations in the various sectors. While in this paper's empirical work we do not do so, one can extend this method of interpreting innovations according to a priori notions of what the structure looks like, until in the limit one is testing the fit of a model which is a fully identified structure -- that is, a model in which the parameters of the AR are one-to-one functions of the parameters of a structural model, as in conventional econometric work.

Recapitulating the preceding paragraphs, it is suggested that one begin by fitting an unstructured AR, attempting an interpretation of it via the MAR and the "natural" interpretation of the innovations. In doing this one tries various triangular orthogonalizations of the innovations, which amounts to trying various Wold causal chain forms for the model. When results are sensitive to the ordering of variables in the orthogonalization, one may make some progress by using a priori hypotheses about the structure. At this final stage one might end up doing something quite similar to estimating overidentified structural models by usual econometric methods.

One might ask where the main differences between this and standard methodology lie -- even whether there are important differences. There are two main differences. The first is that the unstructured MAR is estimated first, and used as a standard. The conventional approach (in principle) treats the unrestricted reduced form as a corresponding standard. The "unrestricted" reduced form in fact embodies restrictions excluding lagged values of some endogenous variables from the system, and is likely to include strictly exogenous variables whose strict exogeneity is not subject to test. Also, if residuals from the unrestricted reduced form are serially correlated the conventional approach is to "correct" for the serial correlation, rather than to insist that all the serial correlation be explained by the model. The second main difference is that the approach suggested here does not presume that a unique, unambiguous structural interpretation exists. Even if, at the final stage of the analysis, an overidentified model is estimated, it is to be expected that more than one such model may be tried, their relative merits in capturing the form of the unrestricted MAR to be compared.

Besides the work already cited by Gordon, there is recent work by Hall [1978], Sargent [1977], and Taylor [1978], which proceeds somewhat in the spirit I am suggesting. None of these three experiments with Wold chain interpretations as Gordon and I do, but all three insist that their models account for a complete multivariate serial correlation sturcture. Sargent and Taylor make direct comparisons of their overidentified models with unrestricted AR alternatives. Hall entertains multiple structural interpretations of the same estimated pattern of serial correlations.

Innovation-accounting, Granger Causality, and their Pitfalls.

Each element of the innovation vector e is by construction causally prior in Granger's sense[8/] to all the variables in the estimated AR. The natural, direct, interpretation of the innovation-accounting scheme amounts to giving the natural interpretation to a Granger causal ordering. All the debate and confusion in the econometric literature about interpretation of Granger orderings is thus relevant to innovation accounting as well. I will summarize here some of the main pitfalls of interpreting Granger orderings which I laid out in an earlier paper [1976].

Prices for durable goods set in auction markets are likely to appear causally prior to all other variables. This is an implication of the "perfect market" hypothesis. Thus in a system with, say, a stock-price index included, we should expect that when the innovations are orthogonalized with stock-prices low in the ordering, innovations in other variables will explain little of stock prices. We should not interpret this to mean that stock prices do not depend on the rest of the economy, unless the result persists when other orderings of the variables are used in the triangular orthogonalization.

If one variable in a system of closely related variables is much more heavily affected by measurement error than other variables in the system, it will tend to appear passive. That is, innovations in

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[8] Granger [1969] did not use the term "causally prior". I use that term to summarize a situation where the "prior" variable causes the other variable and is not caused by it, in Granger's sense of the verb "cause".

a variable measured with great error, since they will consist largely of measurement error, will not explain much of the subsequent evolution of other variables. Also, in a system which, with correctly measured variables, would show sharp causal orderings, measurement error will tend to obscure those orderings.

Optimal control of a policy variable to minimize variance in a target variable may lead the policy variable to appear passive. This is not in itself misleading in the context of innovation accounting, except that in this situation the target variable innovations will in general be contaminated by any uncontrollable shocks in policy which may occur.

In all these three examples -- perfect markets, measurement error, and optimal control -- possibilities for error are somewhat more serious in innovation accounting than in testing formally for a causal ordering. As explained in the earlier paper, these three mechanisms cannot produce a Granger ordering without special, rather restrictive assumptions. It is a special form of optimal control, an extreme version of the perfect markets hypothesis, a special pattern of measurement error contamination, which leads to a causal ordering. Thus though these mechanisms could easily obscure a causal ordering which otherwise would be apparent, they cannot easily do the reverse -- make an ordering appear where there otherwise would be none. Sometimes in innovation-accounting we do reach conclusions about Granger-orderings -- e.g., "price and wage innovations have negligible effects on the real sector". But more often we are discussing less absolute conclusions -- "e.g., only 21% of the variance of GNP is accounted for by wage innovations".

To get a spurious conclusion that wage-innovations account for no GNP variance would take an extreme pattern of measurement errors. To get a substantial downward bias in the percentage of variance in GNP accounted for in those innovations would take a less extreme pattern.

Omitted variables can easily obscure a Granger ordering. It is easy to verify that if x_1 and x_2 are jointly Granger-prior to x_3, x_1 is not likely to be Granger-prior to x_3 in the bivariate system consisting of x_1 and x_3 unless x_1 and x_2 are unrelated. This amounts to saying that, whereas in a trivariated system x_3-innovations would explain no x_1-variance, in a bivariate system they might explain quite a bit. Thus, e.g., if money responds passively to investment and government expenditure for the most part, a system including only money and aggregate GNP might nonetheless show money innovations accounting for a considerable fraction of GNP variance. This could happen if money reflected investment demand with a shorter delay than the delay between investment and its multiplier effects on other components of expenditure. A Keynesian who believes that the business cycle originates in autonomous shifts in demand, not in monetary policy, can rightly object to conclusions that much of the variance of GNP is accounted for by monetary policy shifts if it is based on innovation accounting for a system without measures of the most volatile components of expenditure.

Nine-Variable Innovation Accounting

In the earlier paper [1977b] I showed results for an innovation-accounting for the U.S. and Germany of six variables: Money (M), Real GNP (Y), Unemployment rate (U), Wage (W), Price (P), and Import Price (PM).[9/] These variables were chosen to allow a comparison of simple monetarist and simple Keynesian Phillips-curve analyses of inflation. The Keynesian Phillips curve view[10/] that money has no effect on inflation except through its effect on real variables like labor market tightness or capacity utilization does not seem compatible with the data for either country. A "monetarist Phillips curve" -- money shocks generating temporary inflation, output increases, and unemployment decrease -- emerges in both countries in roughly similar form. Since these conclusions have a monetarist ring, the additional work reported here has expanded the list of variables in the direction of better articulating "autonomous expenditure" and including the interest rate. To the earlier list of variables has been added government expenditures (G), investment (I), and the interest rate (r).

Each country has one "target" variable (GNP for the US, P for Germany) whose variance is accounted for largely by its own innovation. Since the 6-variable model provides no explanation for such a variable, it was hoped that an expansion of the variable list might provide more insight into the sources of variance in these unexplained variables.

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[9]Y, I, and G are all deflated by P.

[10]Keynes certainly never put forth views of this sort. I use the adjective "Keynesian" here only to distinguish this view from the monetarist Phillips curve.

In the earlier paper, the AR was estimated as an unrestricted fourth-order autoregression. Each equation could therefore be consistently estimated by OLS; each contained 24 free coefficients (6 variables, 4 lags on each). The 9 variable system has 36 free coefficients in each equation in unconstrained form, and the same number of observations (about 108) per equation. While estimating 24 coefficients without constraints proved (to my surprise) to give reasonable results, estimating 36 per equation did not give reasonable results. The AR's in this paper are, therefore, estimated using what I have called observable "index models". These models limit the number of parameters in the AR by requiring that all cross-variable relations in the AR be expressible as common dependence of the m variables on k "indexes", which are themselves linear combinations of past values of the variables in the system.

The use of index models here raises questions concerning the extent to which the index restrictions fit and the extent to which results are affected by the index restrictions. In part because my efforts along these lines are still rather limited, and in part because discussing how well the index models perform would change the focus and extend the length of this paper, these issues are treated only in an appendix. The discussion proceeds from here on the maintained hypothesis that the index-model form, like the lag length chosen, is flexible enough not to distort the prominent relationships which appear in the data.[11]

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[11] The final version of the paper will discuss results for a nine-variable model fit to German data which are not now complete. In this version there may therefore be skipped table numbers or undiscussed tables.

Table 1 reproduces from Sims [1977b] the allocation of forecast error variance to innovations from the U.S. 6-variable MAR, and Table 3 reproduces the corresponding accounting from the 9-variable MAR. Both tables refer to triangularly orthogonalized innovations, with the order of the variables as given in the tables. Tables 1 and 3 are in many important respects similar. Wages, prices, and import prices all have most of their variance explained by money innovations over intermediate to long horizons. This result emerges even more strongly in the 9-variable system. Over intermediate horizons, much of GNP and unemployment variance is accounted for by GNP innovations. Money variance is largely accounted for by money innovations. Looking at Charts 1-9, one can see that the nature of the response to money innovations is broadly similar to that found in the earlier paper: wages and prices respond, after a lag, roughly proportionately to money, which itself moves to a new level and persists there. Price of imports also responds positively to the money innovation, though more than proportionately as before. A monetarist Phillips curve is still present, with output rising, unemployment falling, and the real wage rising temporarily after a money innovation. The Keynesian Phillips curve hypothesis that quantity variables as a block are exogenous relative to money and price variables is again rejected (with I and h added to the list of quantity variables), with a $\chi^2(24) = 47.7$, corresponding to a marginal significance level of about .002 and an F of 1.99.[12]

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[12] The latter part of the appendix on methodology discusses the interpretation of these statistics.

There are important differences, however, and they point for the most part in the same direction: This larger system is less compatible with the naive monetarist view that money is a powerful policy instrument and that erratic monetary policy is a major source of instability in the economy. Most obviously, the proportion of variance in GNP and unemployment accounted for in the long run by money innovations drops from .28 to .12 and from .34 to .15, respectively. It is true that a substantial proportion of variance in these variables is now attributed to interest rate innovations (.20 and .10, respectively), but when we examine the nature of the system's response to money and interest innovations, it appears unreasonable to treat money and interest innovations as dominated by shifts in monetary policy.

The GNP (Chart 2) and unemployment (Chart 3) response to a money innovation is smaller in magnitude than in the 6-variable system initially, and then reverses sign to produce a negative movement in output as large or larger than the initial positive movement. (In these respects the response to a money innovation in this larger system is closer to what was observed for the German data in the smaller system, though the duration of the positive effects on output is shorter in the 9-variable U.S. system than in the 6-variable German system.) This pattern of oscillating response to a money shock could be rationalized by a rational expectations theory along the lines of the Lucas supply curve[13] -- when producers are fooled into temporary expansions of monetary policy surprises, the resulting inefficiency reduces future output by as much as the initial gain in output -- but the required

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[13] As sketched out in Lucas [1973].

degree of intertemporal substitutability seems somewhat implausible to me.[14]

If interest-rate innovations are to be attributed to monetary policy shocks to bring the total proportion of variance accounted for by monetary policy shocks up to a level comparable with that in the smaller system, more serious difficulties arise for a monetarist interpretation. If injections of reserves affect deposits only with a delay, or if shifts in policy-makers' notions of the appropriate size of the money stock are anticipated by investors, we might find monetary policy shocks showing up first in the interest rate. A surprise rise in the interest rate would then be expected to be followed by a contraction in the money supply, deflation, and a fall in output. As can be seen from Chart 2, surprise rises in the interest rate not associated with contemporaneous surprises in the money supply do, after considerable delay, result in declines in output -- larger declines in output than those following negative innovations in the money supply. However, as can be seen from Chart 1, they are not followed by any substantial drop in the money supply; as can be seen from Chart 3 unemployment decreases in the first year following a positive interest rate innovation; and as can be seen from Charts 4, 5, and 6 inflation follows an interest rate innovation, for more than a year.

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[14] I do not claim, however, that any of the arguments given here refute the view that, because markets clear, prices are flexible, and expectations rational, the business cycle is an equilibrium phenomenon on which monetary policy can have no good effect. What is being argued against is the view, sometimes held by the same people, that monetary policy is the source of most of, or much of, macroeconomic instability.

These patterns themselves may simply reflect sampling error or distortions due to the index-model form. It might be best therefore to stop here, having asserted that these patterns clearly are not characteristic of a shift in money-supply behavior. Nonetheless, I offer some further speculation. One way to rationalize the signs of the responses is by comparative statics analysis in a textbook IS-LM framework, with the disturbing element a downward shift in the full-employment level of output, money stock held constant. This should, in the usual framework, lead to a new equilibrium with higher interest rate, lower output, and higher price level. The difficulty with this explanation is that it leaves the timing a puzzle -- the interest rate movement comes first, with the drop in output much delayed. To explain the timing we might suppose that the initial shock is a reduction in supply of durable raw materials. The initial shock would then first push up the prices of these goods without a sudden effect on production because of the existence of inventories and costs of rapidly adjusting employment and output. The speculative activity associated with the rise in raw materials prices would drive up interest rates immediately. Later, the volatile raw materials prices feed their way into general price indices and wages, and the higher input prices reduce output. The s-shaped response of unemployment to such a shock might be explained as the combined effect of complementarity of raw materials and labor, combined with speculative demand for labor in the short run when it becomes apparent that labor costs will be rising.

This highly speculative explanation makes the U.S. results more compatible with the 6-variable German results. In Germany, substantial feedback from prices to money appeared, and this does not show up in the U.S. Since the German economy is more open, its general price index might be more sensitive to raw materials prices. Also, the fact that the U.S. price index is the deflator for non-farm business product, while the German deflator is that for all of GNP, might be important. Since farm prices are highly volatile, they might contribute substantially to innovations in the GNP deflator and make its innovations more sensitive indicators of raw materials supply shocks. This explanation is also compatible with some of my own ongoing research on use of index-models for forecasting the U.S. economy. In this work, models with a roughly similar list of 9 variables, but with the GNP deflator replacing the business product deflator, show much more feedback of prices into real variables than appears in the U.S. 9-variable system of this paper.

These speculations point to an agenda for further research. An index of raw materials prices should be added to the system; and if it behaves according to these speculations, a check should be made to see whether the effect arises entirely from the 1973-74 commodity price boom or whether instead it fits other postwar episodes as well.

Regardless of how plausible one thinks the foregoing interpretation is, it remains true that most of variance in output is accounted for by output innovations. The conclusion that monetary policy disturbances do not account for much variance in output seems clear.

This does not mean that monetary policy is necessarily a weak instrument. Indeed, if one accepts the idea that some of the observed interest rate innovations are due to supply shocks, it appears likely that the effects of money innovations estimated in this system yield underestimates of the expansionary effects of monetary policy shocks. Money innovations are negatively correlated with interest rate innovations in this system. If in this system money innovations are mixtures of policy shocks, which would have negatively associated movements in M and r, with supply shocks, which would yield positively associated movements in M and r, we would expect that if we could isolate the effects of monetary policy shocks we would find them more expansionary than is the response to a money innovation in this system.

Another difference between the 9 and 6 variable systems which deserves comment is the absence of forecasting power for unemployment innovations in the larger system. In the smaller system, a positive unemployment innovation was followed by an expansion of the money supply and a rise in output and the real wage, with some inflation eventually. This looked like use of money for demand management. If this interpretation were correct, as pointed out in the earlier paper, it would conflict with the view that anticipated movements in the money supply should have no effects on real variables. In the smaller system feedback from unemployment to money supply was statistically significant, so that the null hypothesis that money is exogenous was rejected at the .05 level -- though this hypothesis was rejected less strongly than any of the others tested and rejected in that paper. In the larger system, the hypothesis that money is exogenous yields a $\chi^2(6) = 4.7$, so exogeneity is easily accepted. A policy variable which did not contribute much to explaining variance in GNP might still be used in an important way as a stabilizing tool. Its low explanatory power could simply reflect its not being used in an erratic way. The fact that in this system no patterns appear which suggest stabilizing responses of monetary policy to shocks thus further weakens the case for the importance of monetary policy as a determinant of the behavior of real variables.

Is there any indication from this larger system that a Keynesian view of the economy makes sense? By "Keynesian view" here I mean the notion that variations in output and employment originate in the private sector, probably mostly in autonomous shifts in investment demand, and that government policy measures, fiscal as well as monetary, have an important role to play in moderating the fluctuations arising this way. In the monetarist ordering of variables, Table 3 makes it clear that investment can easily be taken as passive. The only variable to which investment innovations contribute noticeable variance is government expenditure. The test for the hypothesis that investment is completely passive yields a $\chi^2(6) = 13.7$, significant at between .05 and .02 levels with an F of 2.28, but this is obviously due to investment's tendency to induce decreases (see Chart 8) in government expenditures. Government expenditure innovations also contribute little to variance in other variables.

This result requires qualification, however, because investment and GNP innovations are very strongly correlated. Charts 10-18 and Table 4 present the MAR and decomposition of variance which emerge when the triangular orthogonalization is done in the more Keynesian order listed. Here the contemporaneous correlation of GNP and investment innovations is allocated to investment, and investment innovations become extremely important sources of variation in GNP and unemployment. Government expenditure still has no substantial explanatory power, however, despite being placed first in the ordering.

The nature of the response to an investment innovation as displayed in the second line of Charts 10-18 is consistent with these innovations representing shifts in investment demand: money and interest rate rise, wages rise, output rises, unemployment falls. However the implied multiplier-accelerator mechanism is of modest power. The peak response of output occurs in the first quarter after innovation in investment, coincident with the peak response of investment. At this point, the absolute deviation of investment from trend is roughly equal to the absolute deviation in GNP, so by this measure the multiplier is about one. (The log deviation of investment at the peak is .053, and of GNP is .0088. Their ratio, .17, is about the ratio of investment to GNP in the sample period.) Since the duration of investment's deviation from trend is short, and shorter than the duration of GNP's deviation from trend, a more reasonable measure is the ratio of the integral of GNP's deviation from trend to the integral of output's deviation from trend. Integrating over the first 8 quarters, which is the period in which the output deviation remains positive, yields a multiplier of about 1.6. Thus a Keynesian interpretation does not find bursts of investment demand generating demand increases in other sectors even larger than the burst in investment demand, but it does find short bursts of investment demand increasing GNP without decreasing other components of demand, and the increase is found to persist for a few quarters after the investment increase has dissipated. While this is not a picture of an inherently unstable multiplier-accelerator stabilized only by countercyclical policy, it does leave the view that much of macro-economic fluctuation originates in shifts in investment demand a more respectable one that the view that much of such fluctuations originate in fluctuations in monetary policy.

The Keynesian policy variable in this system, government expenditure, shows little evidence of effectiveness. The lack of explanatory power for its innovations could be due to the absence of erratic elements in expenditure policy -- were it not that 40% of variance in G is accounted for by its own innovations (Table 4). It responds positively to investment and output innovations, so is not apparently playing a strong stabilizing role.

It is interesting to note that, while the interest rate innovation generates a response in this orthogonalization similar to that in the Table 3 orthogonalization, the money innovation in this orthogonalization no longer has any expansionary effect on output. Despite this, it still accounts for the majority of variance in all the price level variables -- W, PM, and P. This result can be interpreted in the same way as, earlier in this paper, we interpreted interest rate innovations. In this orthogonalization, money innovations are not accompanied by corresponding interest rate movements, and it is therefore plausible to interpret them as passive responses, generated by the attempt to maintain constant interest rates in the face of an unforeseen speculative demand for credit as raw materials prices jump. If this interpretation is correct, then reducing variance in the price level is not a matter of reducing erratic shifts in monetary policy but rather of reducing the extent of accomodating response to supply shocks. It is clearly more problematical whether the latter change in monetary policy would be desirable.

As a final interpretive exercise, we will consider "wage-push". Wage innovations account for little variance in quantity variables in either orthogonalization, but in both they account for a small but non-negligible proportion of price variance over horizons of about two years. This is consistent with the existence of some wage-push effect. However, there is strong contemporaneous correlation of wage and price innovations, and all the responses of wages and prices to wage and price innovations are approximately flat. This can be shown with a little algebra to imply that if we were to invert the ordering of wages and prices in the orthogonalization, price innovations would appear to have about the same explanatory power for wage variance which wage innovations have for price variance in this orthogonalization. Thus, though there is some variation in prices and wages which originates within the wage-price pair, this system provides no evidence that one or the other of the wage-price pair has causal priority.

Conclusion

To the extent that monetarism is identified with the notion that instability is mainly generated by misguided attempts at policy intervention, the expansion of the U.S. model from 6 to 9 variables has made this position look less plausible. To the extent that Keynesianism is identified with the notion that instability originates in the private sector, especially in investment, the expansion has made Keynesianism look more plausible. On the other hand, the real quantity variables in the system do not behave in an unstable way, and there is little evidence of countercyclical movement in policy variables. Government expenditure plays a minor role in the system. Thus a sophisticated modern laissez-faire position along the lines of Sargent's recent work [1977] appears consistent with the results in this system. In fact, the one aspect of the smaller system which seemed to contradict that position (the response to unemployment innovations) fails to persist in the larger system.

Obviously a great deal remains to be done before these conclusions are firmly established. Some of the speculations I have introduced could be checked by expansions or modifications of the model. The system is complicated enough that other economists may well develop interpretations of the results which have not occured to me. The robustness of results to the index-model constraints, to small changes in the variable definitions, and to variations in time period or country needs to be checked.

I hope pursuit of this additional work proves to be as useful as it is interesting.

APPENDIX

The Index-Model Form

Sargent and I [1977] introduced two related classes of models for multivariate stochastic processes which have the property that the number of parameters in them grows linearly with the length of variable list, instead of quadratically as in an unrestricted vector AR. We called the models "observable index" and "unobservable index" models. Since the latter class of models has come to be called (for good heuristic reasons) "frequency domain factor analysis", I will call the former class simply "index models" henceforth in this appendix. Index models have an AR which can be written as

$$\text{A1)} \qquad y(t) = D*y(t) + a*c*y(t) + u(t) \ ,$$

where "*" indicates convolution (i.e. $a*c(t) = \sum_{s=-\infty}^{\infty} a(s)c(t-s))$, $D(s)$ is an $m \times m$ matrix-valued function which is non-zero only on the main diagonal and has $D(s) = 0$, $s < 1$, a is $m \times k$, c is $k \times m$, $a(s) = 0$ for $s \leq 0$, $c(s) = 0$ for $s < 0$. To keep the number of parameters in this system substantially smaller than the number in an unrestricted AR , one takes $k << m$.[A1/] In my own work to this point $Var(u(t))$ has been left unrestricted.[A2/] This

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[A1] There is no need to require that $a*c(0) = 0$ as we do here, but a model with $a*c(0) \neq 0$ leads to a complicated likelihood function and has in practice proven very difficult to fit.

[A2] However to make the number of parameters keep on growing linearly, one must start restricting this covariance matrix as size increases. A natural procedure would be to impose a factor analytic structure. This could be done without greatly affecting computation time.

specification has the interpretation that all cross-variable effects in the AR are accounted for by common dependence of the y's on the k "indexes" defined by $z(t) = c*y(t)$. Block Granger causal orderings can be imposed on an index model by linear constraints on the parameters. The earlier joint paper with Sargent [1977] discusses how index structure could arise from a number of simple theoretical macro-models.

In estimation, one must eliminate a redundancy in the parameterization. Obviously we can take any $k \times k$ matrix-valued function b such that $b(s) = 0$ for $s < 0$ (b is one-sided) and such that b has a one-sided inverse under convolution, and we can replace $a*c$ by $a*b^{-1}*b*c$ without changing the AR. If a or c has a $k \times k$ submatrix with a one-sided inverse under convolution, we can normalize that submatrix at the identity, and eliminate the redundancy. However not every $k \times k$ submatrix will in general be invertible -- in fact none need be -- and in practice it seems to turn out that only a few of the possible normalizations give good results.[A3/]

Estimation is carried out by maximum likelihood conditional on the initial observations. The log likelihood is $-.5\ T \log \det \mathrm{Var}(u)$, where Var(u) is the estimated variance matrix of u and T is sample size. In carrying out tests based on likelihood ratios, I have replaced T by $T-(Q/m)$, where Q is the total number of free parameters in

[A3]As a guide to practice, it should be noted that one should not normalize on a submatrix of c corresponding to variables which do not affect other variables or which do so only with substantial delay. One should not normalize on a submatrix of a corresponding to variables which do not respond to other variables or which do so only with a substantial delay.

D, a and c and m is the number of variables. This gives results a little more closely comparable to F ratios in small samples.

The starting point of estimation is important, because initial points with many zeroes in a and c can inadvertently introduce singularities in the second derivative matrix. In fact, if a and c are initially set at zero, the gradient of the likelihood is zero with respect to both a and c, so that most estimation routines will simply find the best-fitting array of univariate autoregressions, holding a and c both at zero.

A starting routine which works more often than not is to begin by holding a at zero until good D values are reached. This takes little time since the AR is linear in D. Then c is fixed at some plausible value, usually just ones and zeroes in various locations (though a singularity arises due to exact collinearity of a's with some D's if each c is given only one non-zero element) and the likelihood maximized with respect to a and D. This again takes little time because the AR is linear in these parameters. Finally, all constraints except the normalizations are released.

In this paper, all results reported are for a 9-variable ($m=9$), 2-index ($k=2$) system in which a, c, and D are all of length 3. This yields $3[2km + m] = 135$ parameters subject to 12 normalizing restrictions. The maximum lag length is 5.

It has happened often in my work that the highest function values are achieved not from direct maximization of the unconstrained likelihood, but from releasing the parameters after imposition of shrewdly chosen constraints. E. g., in the 9-variable 2-index U.S. system of this paper a substantial improvement in fit was obtained in releasing the system after imposition of the "investment passive" constraints, even though those constraints were rejected.

Assuming the normalization has been chosen well and one is otherwise lucky enough not to hit a bad patch in the likelihood surface, a converged solution for a 9×2 model starting from scratch takes about 300 seconds of CDC Cyber 74 time, costing (at the University of Minnesota subsidized rates) about $45, though bad luck and experimentation obviously make the costs of fitting a system of variables with which one has no previous experience many times this figure. The program I use estimates the second derivative matrix from the cross products of the gradients, so the 300 seconds comprises some 15 iterations on the 123 free parameters.

Measures of Fit

In smaller systems it is practical to compare the likelihood for an unconstrained AR to that for the estimated index model, using the asymptotic χ^2 distribution for the log likelihood ratio. In larger systems fitting the unconstrained system may be impossible, and in any case a more powerful test is likely to be a comparison of an index model with low k against one with higher k. Unfortunately such a test cannot be executed mechanically, because on the null hypothesis that the low - k model is correct, the second derivative matrix of the likelihood is singular for the high - k model.[A4/] The

[A4] John Geweke and Kenneth Singleton have pointed out [1978] that a similar problem arises in frequency-domain factor analysis, invalidating some of the tests of fit appearing in my joint paper with Sargent [1977].

singularity is easily avoided, by fixing the coefficients in the components of c with subscript higher than k -- i.e. by fixing the form of the indexes to be added to the model. One can of course make an asymptotically valid test by starting at the converged estimate for the smaller model and taking one Gauss-Newton step. This, together with the need to fix c, makes testing a k-index model against a $k+1$ index model much easier computationally than fitting a $k+1$ index model. On the other hand, the power of such a test of fit depends on the researcher's shrewdness and intellectual honesty in picking the form for the additional indexes.

Only one fit test has been tried with this paper's model at this time. It consisted of fitting a 3-index model in which the third index was the interest rate. (This required constraining the component of D corresponding to the interest rate at its 2-index converged value, to avoid singularity.) The choice was based on the important role played by interest rates in some smaller, unconstrained estimated forecasting models. Sadly, it must be reported that the test results in a $\chi^2(18) = 30.7$, which corresponds to a marginal significance level of between .05 and .02 and an F of 1.71. The variances of innovations in M, Y, and I are all reduced by about 10% by the addition of the third index. For each of these variables, there is a negative coefficient on the first lagged value of the interest rate in the corresponding AR equation.

Though this rejection of the 2-index model is not as strong as that for some other restrictions we have examined, it is strong enough that some follow-up action is clearly required. The first step, not yet complete at the time of this writing, is to see if the qualitative conclusions from the 2-index model change substantially when a 3-index MAR is used. This check should be complete by the time the final draft of this paper is circulated. If a full-blown three-index model, with only normalization constraints, is estimated, it involves 162 free parameters. This will require substantially more expensive computations, so it is to be hoped that a reasonable fit can be found without such a large expansion of the model.

We can also ask whether the expansion to nine variables improves the fit to the original six variables. The unconstrained six-variable AR is not nested in the 9-variable index model or vice versa, so no formal test is possible. The difference in log determinants of residual covariance matrices between an unconstrained third order 6-variable system and the 9-variable 2-index system's corresponding 6×6 submatrix is, however, about 29. Since the difference in numbers of parameters is also about 26, this difference does not appear large.[A5/] Nonetheless it does not seem that the additional three variables are helping to predict the first six.

It is worth commenting on the philosophy of tests of fit for systems like these -- with many degrees of freedom and many parameters.

[A5] There are $6 \times 6 \times 3 = 108$ parameters in the 6-variable unconstrained system. In the 9-variable system there are 123 free. Allocating 6/9 of these to the first six variables yields 82 parameters to fit the first six variables. If the models were nested we could use an asymptotic $\chi^2(26)$ distribution for the difference in log determinants of 29, and by this standard the difference is not "significant". However, this is obviously only a crude rule of thumb for deciding what is a big difference in a log likelihood.

As Schwarz [1978] and Leamer [1978] have pointed out, mechanical use of significance tests does not lead to consistent decisions as sample size increases. A Bayesian approach to model selection leads to a criterion which compares the asymptotic $\chi^2(n)$ statistic to $n \log T$ (where T is sample size) instead of to the .05 level of the $\chi^2(n)$ distribution. This criterion is only asymptotically correct however, and because $\log T$ grows slowly with T, a complete Bayesian analysis will probably give results quite different from the $n \log T$ criterion in samples of the size we are considering in this paper. Further, in contrast to the .05 significance level, which rejects most restrictions, the $n \log T$ criterion accepts most restrictions. This brings out an important characteristic of the criterion: it aims at providing a good rule for comparing nested hypotheses, or at best hypotheses of different dimension. In this work it is probably more interesting to compare the relative plausibility of various restrictions of roughly similar dimension. A criterion which accepts or rejects them all is of little help.

For these purposes we might just rely on the marginal significance level of the χ^2 statistic, distinguishing very strongly rejected from weakly rejected hypotheses. Even this idea has difficulties, however. The relevant distribution theory is asymptotic only, and requires that the number of degrees of freedom in the χ^2 statistic be small relative to the number of degrees of freedom in the data (total data points minus number of parameters). It is known, for example, (see Miller [1968]), that when there are many degrees of freedom in both numerator and denominator of an F statistic and normality assumptions are violated in the direction of fat tails, then the F distribution will lead to rejection of the null hypothesis more often than indicated by the nominal significance level. This kind of effect might apply here, since we are in effect testing with many degrees of freedom in the numerator.

The conclusion should be that "χ^2-tests" like those reported in this paper ought to be regarded as descriptive statistics, helping to characterize the form of the likelihood function, rather than as yielding decisions about the correct form of the model. To this end, the paper reports not only acceptances and rejections, but also the approximate marginal significance level and, to aid in comparison to the $n \log T$ criterion, the ratio of the chi-squared statistic to its degrees of freedom (a pseudo-F-statistic which we call simply F). For the sample size we are considering, the $n \log T$ criterion would call for rejecting only hypotheses for which F exceeds 4.55. [A6]

[A6] Note that the Akaike criterion (which is described by Schwarz [1978]) can be expressed as calling for rejecting restrictions for which the F exceeds 2.0. The Akaike criterion has at best a heuristic justification, apparently, and it is, like mechanical hypothesis testing, not consistent as sample size increases.

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Data Notes

Money: In the U.S., this is M1, seasonally adjusted, as prepared by the Board of Governors of the Federal Reserve System and published in Business Statistics and the Survey of Current Business by the Department of Commerce.

Real GNP: In the U.S., this is the series published in the same sources listed above for M1 and prepared by the Department of Commerce. It is seasonally adjusted.

Unemployment rate: In the U.S., this is the rate for all civilian workers, seasonally adjusted, prepared by the Bureau of Labor Statistics and published in the sources already cited for the U.S.

Wages: For the U.S., this is a seasonally adjusted index of average hourly compensation of all private nonfarm employees, prepared by the Bureau of Labor Statistics and published in Business Conditions Digest by the Department of Commerce.

Prices: For the U.S. this is a seasonally adjusted price deflator of Gross National Product of the non-farm business sector, as prepared by the Department of Commerce and published in the Survey of Current Business.

Import Prices: For the U.S., this is the Unit Value of General Imports as published by the Department of Commerce in the Survey of Current Business.

Investment: For the U.S., this is seasonally adjusted Gross Private Domestic Investment from the National income accounts as published in Business Statistics and the Survey of Current Business.

Government Expenditures: For the U.S., this is seasonally adjusted federal government expenditures on goods and services from the national income accounts.

Interest Rate: For the U.S., this is the rate on 90-day Treasury bills, as published in the Survey of Current Business and Business Statistics.

The data series for the U.S. were all originally for the period 1948-75, quarterly. All were logged, then detrended by linear regression, before the index model was fit. Because the index model generated a 5th order AR, five periods at the beginning of the sample were dropped to yield a period of fit of 1949.2-1975.4.

Table 1

PROPORTIONS OF FORECAST ERROR k QUARTERS AHEAD
PRODUCED BY EACH INNOVATION: U.S. 1949-1975

Triangularized innovation in:

Forecast error in:	k	M	Y/P	U	W	P	PM
M	1	1.00	0	0	0	0	0
	3	.96	0	.03	0	0	0
	9	.73	0	.24	.02	0	0
	33	.54	0	.27	.09	0	.09
Y/P	1	.15	.85	0	0	0	0
	3	.35	.59	.04	.01	.01	0
	9	.30	.18	.37	.13	.00	.02
	33	.28	.15	.33	.16	.02	.06
U	1	.02	.35	.63	0	0	0
	3	.14	.49	.32	0	.03	0
	9	.26	.20	.41	.09	.02	.02
	33	.34	.14	.34	.13	.03	.03
W	1	.08	.05	.04	.84	0	0
	3	.17	.06	.07	.55	.09	.06
	9	.45	.02	.05	.25	.08	.16
	33	.64	.02	.19	.07	.02	.07
P	1	.0	.04	.15	.24	.56	0
	3	.04	.01	.14	.36	.33	.12
	9	.14	.02	.12	.25	.11	.36
	33	.60	.02	.20	.07	.02	.09
PM	1	0	0	.06	.05	.08	.81
	3	.01	.01	.02	.13	.10	.75
	9	.06	.02	.13	.08	.03	.68
	33	.54	.03	.20	.04	.01	.18

Table 2

PERCENTAGES OF FORECAST ERROR k QUARTERS AHEAD
PRODUCED BY EACH INNOVATION: WEST GERMANY 1958-1976

Triangularized innovation in:

Forecast error in:	k	M	Y/P	U	W	P	PM
M	1	1.00	0	0	0	0	0
	3	.84	.04	.05	.01	.04	.02
	9	.53	.04	.14	.08	.20	.01
	33	.39	.05	.13	.07	.27	.09
Y/P	1	.07	.93	0	0	0	0
	3	.14	.79	.01	.05	0	0
	9	.15	.47	.03	.06	.03	.25
	33	.13	.22	.05	.04	.42	.14
U	1	0	.03	.97	0	0	0
	3	.19	.09	.67	.03	.02	0
	9	.15	.10	.37	.02	.08	.29
	33	.09	.11	.15	.02	.50	.14
W	1	0	.03	.01	.96	0	0
	3	.11	.18	.01	.59	.03	.09
	9	.23	.23	.02	.23	.24	.05
	33	.21	.13	.08	.15	.31	.12
P	1	.02	.02	0	.10	.86	0
	3	.03	.06	.05	.09	.76	0
	9	.05	.13	.03	.05	.68	.06
	33	.08	.10	.04	.05	.67	.06
PM	1	.06	0	.02	0	.02	.89
	3	.04	0	.02	.01	.08	.85
	9	.10	.04	.09	0	.16	.61
	33	.06	.08	.04	.02	.57	.23

Table 3

Proportions of Forecast Error Variance k Quarters Ahead Produced by Each Innovation:

U.S., 1949.2-75.4, 9 Variables, "Monetarist" Ordering.

Forecast error in:	k	M	Y	U	W	P	PM	I	G	r
M	1	1.00	0	0	0	0	0	0	0	0
	3	.98	.01	0	0	0	0	0	0	.01
	9	.97	.01	0	0	0	0	0	0	.01
	33	.96	.02	0	0	0	0	0	0	.02
Y	1	.13	.87	0	0	0	0	0	0	0
	3	.13	.86	0	0	0	0	0	0	0
	9	.09	.69	0	.01	.02	0	0	.01	.18
	33	.12	.56	.01	.05	.02	.01	.01	.01	.20
U	1	.06	.39	.55	0	0	0	0	0	0
	3	.05	.62	.28	0	.01	0	0	0	.04
	9	.04	.75	.13	0	.01	.01	0	0	.06
	33	.15	.59	.10	.01	.02	.01	0	.01	.10
W	1	.14	.08	0	.78	0	0	0	0	0
	3	.20	.11	.01	.59	.01	.01	0	.01	.07
	9	.52	.03	0	.20	.01	.03	0	.01	.19
	33	.85	.02	0	.05	0	.01	0	0	.05
P	1	0	.12	.10	.12	.67	0	0	0	0
	3	.10	.06	.02	.15	.48	.04	0	.02	.14
	9	.44	.16	0	.03	.09	.04	0	.01	.23
	33	.75	.09	0	.01	.03	.02	0	.01	.08
PM	1	.02	0	.03	.01	.06	.88	0	0	0
	3	.07	0	.01	.03	.09	.76	0	.01	.04
	9	.36	.08	.01	.02	.04	.33	0	.01	.16
	33	.70	.08	0	.01	.02	.12	0	.01	.07
I	1	.06	.56	0	0	.01	.01	.36	0	0
	3	.08	.55	0	.01	.03	.01	.28	0	.03
	9	.11	.49	.01	.02	.03	.02	.25	0	.07
	33	.15	.42	.01	.02	.03	.03	.22	.01	.11
G	1	0	.03	.01	0	.08	.04	.18	.65	0
	3	.01	.16	.02	.01	.07	.07	.15	.52	0
	9	.01	.46	.01	.01	.04	.05	.08	.34	0
	33	.02	.46	.01	.01	.04	.05	.08	.32	.01
r	1	.02	.15	.03	.03	.05	.02	0	.08	.61
	3	.02	.20	.03	.02	.05	.03	.01	.07	.56
	9	.02	.26	.03	.02	.05	.03	.01	.06	.52
	33	.07	.25	.03	.02	.05	.03	.01	.06	.49

Table 4

Proportions of Forecast Error Variance k Quarters
Ahead Produced By Each Innovation:

U.S., 1949.2-75.4, 9 Variables, "Keynesian" Ordering

Forecast error in:	k	G	I	W	PM	U	Y	r	M	P
G	1	1.00	0	0	0	0	0	0	0	0
	3	.91	.03	.01	.01	.01	0	0	.02	0
	9	.64	.19	0	.02	.02	.04	0	.08	0
	33	.60	.20	.01	.02	.02	.05	0	.10	0
I	1	.01	.99	0	0	0	0	0	0	0
	3	.01	.95	0	0	0	0	.02	.01	0
	9	.01	.84	.01	.02	.01	0	.06	.04	.01
	33	.01	.73	.01	.03	.02	.01	.09	.09	.01
W	1	0	.13	.87	0	0	0	0	0	0
	3	.01	.19	.69	.01	.01	.01	.05	.03	.01
	9	0	.07	.35	.06	0	0	.05	.44	.02
	33	0	.08	.20	.02	.01	.04	.07	.57	.01
PM	1	.01	.02	.01	.97	.0	0	0	0	0
	3	0	.04	.04	.86	.01	0	.02	.03	0
	9	0	.02	.06	.43	0	0	.04	.43	.02
	33	0	.05	.09	.15	.01	.03	.07	.59	.01
u	1	.04	.28	.01	.01	.67	0	0	0	0
	3	.05	.47	.01	0	.40	.02	.05	0	0
	9	.04	.51	0	.02	.23	.09	.04	.07	0
	33	.03	.39	.03	.03	.17	.08	.06	.21	.01
Y	1	.02	.65	.01	0	.06	.24	0	0	0
	3	.03	.64	.01	0	.07	.25	0	0	0
	9	.01	.45	.01	.02	.07	.24	.12	.07	.02
	33	.01	.36	.05	.03	.06	.19	.13	.13	.03
r	1	.01	.10	.01	.01	.06	0	.82	0	0
	3	0	.13	.01	.02	.07	0	.77	0	0
	9	0	.18	.01	.02	.07	.01	.70	.01	0
	33	.01	.18	.01	.03	.07	.01	.63	.06	0
M	1	0	.06	.09	0	.01	.05	.08	.70	0
	3	0	.03	.08	0	.02	.04	.14	.69	0
	9	0	.03	.08	0	.01	.05	.15	.68	0
	33	0	.09	.08	0	.02	.08	.15	.58	0
P	1	.04	.01	.08	.09	0	.27	.06	.03	.42
	3	.05	.01	.16	.15	.03	.11	.17	.17	.15
	9	0	.01	.09	.13	.01	.02	.07	.65	.02
	33	0	.06	.10	.04	.02	.04	.09	.65	.01

APPENDIX

RESULTS FOR GERMAN DATA

In certain broad respects, expansion of the German system to nine variables has yielded conclusions similar to those arising from the expansion of the U.S. system. The real effects of money-innovations now appear smaller, investment and GNP innovations are strongly correlated, with investment "explaining" much of GNP variance in a multiplier-like fashion when investment is put early in the triangularization. Though from the variance decompositions in Table 2 and 5 it might appear that the German data now give a larger role to money-innovations in determining prices, detailed examination of the plotted responses shows that this result arises from a pattern of response quite different from that observed in the U.S. We proceed now to examine these conclusions more carefully.

Though it is not evident from the charts, the responses of the U.S. system in the very long run are clearly nonsensical -- if 64-quarter responses rather than 32-quarter responses are plotted, it becomes evident that the system possesses slowly damped complex roots which generate long, wide, oscillations. Further, these oscillations are not synchronized, so that the tendency of prices, wages, and money to move together over 32 quarters disappears as these oscillations set in. This result is not unexpected. In a long enough horizon, the system's largest root must eventually dominate the simulated response. It can be shown [1/] that the least squares fit

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[1/] I have done so in a still-unpublished paper.

of an autoregressive system can be made arbitrarily good while any finite number of roots of the system are fixed arbitrarily. Also, any finite number of roots can be required to be unstable, [2/] even when the true system is stable, and the fit can then be made arbitrarily good. These facts are symptoms of the tenuous connection between the values of individual roots in the autoregressive system and its least squares fit. But since individual roots determine the nature of long-run responses, it is clear that least-squares fits are tenuously connected to long-run responses. In common-sense terms, it is apparent that the responses of the system over time horizons longer than the sample period cannot be very well determined by the sample period fit. The six-variable German system had fewer parameters and a longer sample period than the current nine-variable system. Already with the six-variable system responses plotted for the German data appeared less stable than for the U.S. data. With this nine-variable system the tendency is even stronger. Accordingly, Tables 5 and 6 are extended out only to 17 quarters, and little attention will be given to analysis of the way the system behaves over horizons longer than 16 quarters.

At the 17-quarter horizon [3/] money-innovations account for eight per cent of variance in real GNP in Table 5, about half as much as in Table 2. Even this eight per cent is suspect, however, as it arises almost entirely from a positive response beginning after the

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[2/] Stable roots can be fixed at arbitrary values. To make the fit arbitrarily good, unstable arbitrary roots must in general be moved arbitrarily close to the unit circle as the approximation is improved.

[3/] 17 "quarters ahead" in Tables 1-6 corresponds to the ordinate labeled in "16" in the plots.

twelfth quarter. Whether or not this delayed response is entirely spurious, it is hard to rationalize it, from either Keynesian or Monetarist perspectives, as a direct response to policy shifts. There is a negative response of output to interest rate innovations, but it is much smaller than in the U.S. This might be explained by the fact that interest rate shocks themselves are about half as large and persist for only four instead of six quarters. The overall conclusion is that monetary sector shocks can account for only a very small portion of real GNP variance in Germany, and the similarity of the responses to monetary shocks in the U.S. and Germany has largely disappeared in this expanded system.

In the six-variable system it was already apparent that money innovations did not play the central role in generating inflation in Germany that they did in the U.S. As was pointed out in the earlier paper, that result was probably due to the fact that money innovations do not generate extended excursions of the money stock away from the trend line in the German system the way they do in the U.S. In the expanded German system the same pattern emerges -- money innovations do not persist as much in the U.S., and do not produce parallel movements in prices. However in the expanded German system, with the shorter sample, the neutrality property seems to have dwindled away. In the earlier work with German data, most inflation was accounted for by price-innovations, which produced roughly parallel movements of prices, wages and money stock. In the larger system, a money innovation produces, with a long lag, steady deflation of prices, while wages initially rise then steadily deflate in parallel with prices. Money supply itself steadily drops back to the trend line after the initial shock. This pattern results in considerable explanatory power being attributed to money innovations in Table 5 in the price and wage rows at long horizons. The fact that these results violate long-run neutrality of money and that they seem to depend on large roots of the system which have little impact over the first eight quarters or so of response makes me suspicious of them. The long delay in the response of price to money innovations is similar to what is observed in the U.S. system, but there the system shows rough long-run neutrality. It may be that with a longer sample the German system will start to show a more easily interpretable pattern.

The proportion of GNP variance accounted for by GNP innovations is in Table 5 substantially larger than the corresponding proportion in Table 2, for the six-variable system. In fact, it appears likely that the restriction that real GNP be exogenous would be rejected weakly, if at all, though a formal test of that hypothesis has not been prepared. (The hypothesis that Y, I, G and U are jointly exogenous is rejected, with $\chi^2(24) = 67.7$, $F = 2.82$.) Thus, as with the U.S. data, the naive monetarist position that most business cycle fluctuations arise from shocks to monetary policy appears inconsistent with the data.

On the other hand, with a "Keynesian" orthogonalization the German data tell the same sort of story as the U.S. data: most of GNP variance is accounted for by variations in autonomous expenditure. A notable difference is that in Germany the contribution of government expenditure innovations is non-negligible, so that the contribution of the first two columns of Table 6 in the GNP row is, combined, about that of the second column alone in Table 4. It will be observed from the charts that GNP responds in very similar fashion to government expenditure innovations and to investment innovations. Rough calculations (in this case based on 1960 ratios of G and I to GNP and subject to some uncertainty as to whether even these ratios are correct because, under time pressure, I could not use the original data source) suggest a G-multiplier of about two and an I-multiplier of about one. However this apparent greater power of G-innovations arises in good part because, with G first in the orthogonalization, a G-innovation generates a strong response in I, while the response of G to I is weaker. It should be noted that the "G" variable in Germany is total public consumption, while in the U.S. it is the more limited quantity federal government purchases of goods and services.

Table 5

Proportions of Forecast Error Variance k Quarters Ahead Produced by Each Innovation:

West Germany 1961.2 - 1976.4

Forecast error in:	k	M	Y	U	W	P	PM	I	G	R
M	1	1.00	0	0	0	0	0	0	0	0
	3	.88	.04	.01	.05	0	0	0	.01	0
	9	.75	.04	.05	.07	.01	.01	0	.02	.04
	17	.58	.16	.05	.09	.03	.02	.02	.02	.04
Y	1	0	1.00	0	0	0	0	0	0	0
	3	.01	.97	.01	.01	0	0	0	0	0
	9	.02	.89	.01	.03	0	.02	.01	.01	.01
	17	.08	.60	.01	.07	.07	.03	.04	.06	.02
U	1	.05	.01	.94	0	0	0	0	0	0
	3	.04	.04	.89	.02	0	0	0	0	0
	9	.05	.05	.84	.04	0	.01	.01	0	0
	17	.05	.13	.70	.07	0	.02	.02	.02	0
W	1	.19	.08	.02	.72	0	0	0	0	0
	3	.22	.08	.05	.01	0	.02	0	.01	.01
	9	.07	.44	.05	.24	.01	.07	.07	.04	.02
	17	.20	.35	.04	.13	.08	.03	.05	.10	.02
P	1	0	.41	.12	.11	.36	0	0	0	0
	3	.06	.24	.07	.07	.50	.05	.01	0	0
	9	.21	.19	.02	.09	.24	.05	.05	.11	.05
	17	.39	.11	.03	.05	.22	.01	.02	.05	.04
PM	1	.03	0	.05	.09	.13	.70	.0	0	0
	3	.03	C	.06	.11	.12	.68	0	0	0
	9	.02	.06	.06	.15	.09	.61	.01	0	0
	17	.06	.20	.06	.16	.05	.39	.03	.04	.01
I	1	.01	.58	0	.03	.01	.01	.37	0	0
	3	0	.60	0	.03	.01	0	.35	0	0
	9	0	.55	.01	.06	.01	.05	.31	0	.01
	17	.10	.38	.02	.13	.08	.05	.14	.08	.02
G	1	.06	.13	.04	.01	.02	.07	0	.68	0
	3	.05	.17	.03	.01	.01	.08	.01	.64	0
	9	.04	.30	.03	.05	.01	.09	.03	.45	.01
	17	.10	.32	.03	.06	.04	.06	.04	.34	.01
R	1	.18	0	.01	.11	0	.01	0	0	.68
	3	.26	.06	0	.06	0	.03	0	0	.58
	9	.12	.33	.05	.15	.01	.04	.03	.03	.25
	17	.10	.32	.04	.16	.02	.06	.03	.04	.22

Table 6

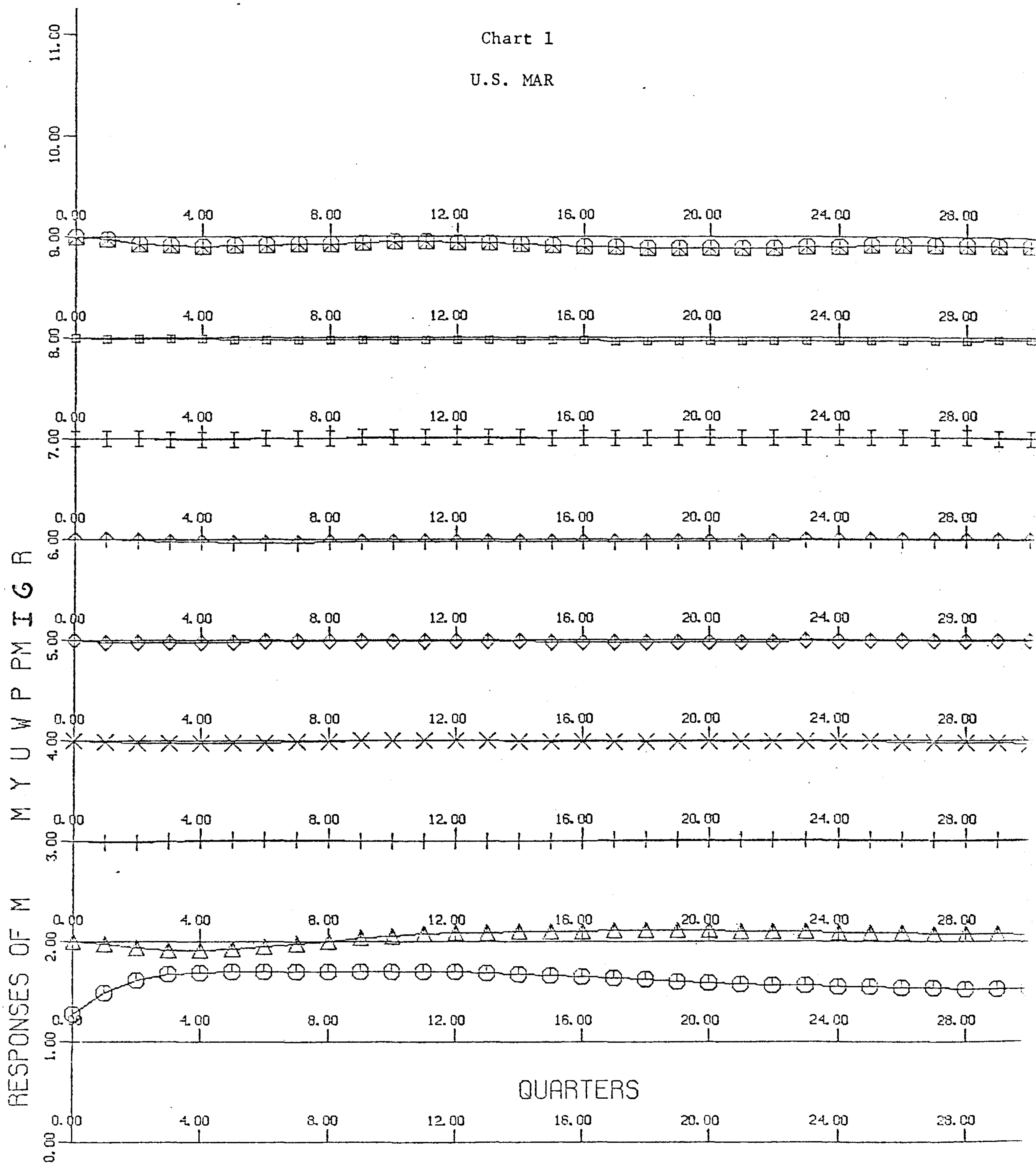
Proportions of Forecast Error Variance k Quarters
Ahead Produced by Each Innovation:

West Germany 1961.2 - 1976.4

Forecast error in:	k	G	I	W	PM	U	Y	R	M	P
G	1	1.00	0	0	0	0	0	0	0	0
	3	.96	.01	.01	.01	0	0	.01	0	0
	9	.75	.08	.06	.04	.02	.03	.01	.01	.01
	17	.54	.10	.13	.03	.08	.04	.01	.04	.02
I	1	.10	.90	0	0	0	0	0	0	0
	3	.10	.89	0	0	0	0	0	0	0
	9	.10	.80	.02	.03	.01	.01	0	.02	.01
	17	.15	.35	.21	.03	.09	.05	.01	.05	.07
W	1	.07	.10	.83	0	0	0	0	0	0
	3	.08	.09	.76	.04	.01	0	0	.02	0
	9	.19	.25	.31	.09	.05	.06	.02	.02	.02
	17	.16	.16	.27	.04	.04	.15	.06	.01	.10
PM	1	0	.02	.09	.88	0	0	0	0	0
	3	0	.01	.11	.87	0	0	0	0	0
	9	.03	.02	.14	.78	.01	.01	0	0	0
	17	.09	.06	.22	.49	.06	.04	0	.02	.02
U	1	.08	.01	.01	.09	.81	0	0	0	0
	3	.10	.01	.05	.06	.77	.01	0	0	0
	9	.09	.03	.05	.11	.69	.01	0	.01	0
	17	.07	.07	.08	.12	.61	.03	.01	.01	0
Y	1	.14	.46	0	0	0	.40	0	0	0
	3	.12	.44	.02	.02	.01	.38	0	0	0
	9	.13	.39	.05	.03	.03	.35	.01	.01	.01
	17	.16	.25	.16	.02	.06	.21	.01	.05	.07
R	1	0	.01	.01	.01	0	0	.97	0	0
	3	.03	.04	.02	.04	0	.01	.88	0	0
	9	.08	.14	.16	.08	.07	.08	.37	0	0
	17	.11	.14	.18	.09	.06	.07	.33	.01	.02
M	1	.06	0	.16	0	.04	.01	.22	.51	0
	3	.12	.02	.15	0	.03	.01	.22	.45	0
	9	.10	.02	.12	.02	.04	.02	.23	.44	.01
	17	.10	.11	.10	.05	.03	.03	.19	.37	.01
P	1	0	.22	.06	.16	.06	.18	.02	.03	.27
	3	0	.16	.03	.07	.03	.11	.06	.07	.48
	9	.16	.04	.28	.01	.14	.04	0	.12	.22
	17	.10	.01	.24	.01	.24	.04	.01	.23	.12

Chart 1

U.S. MAR



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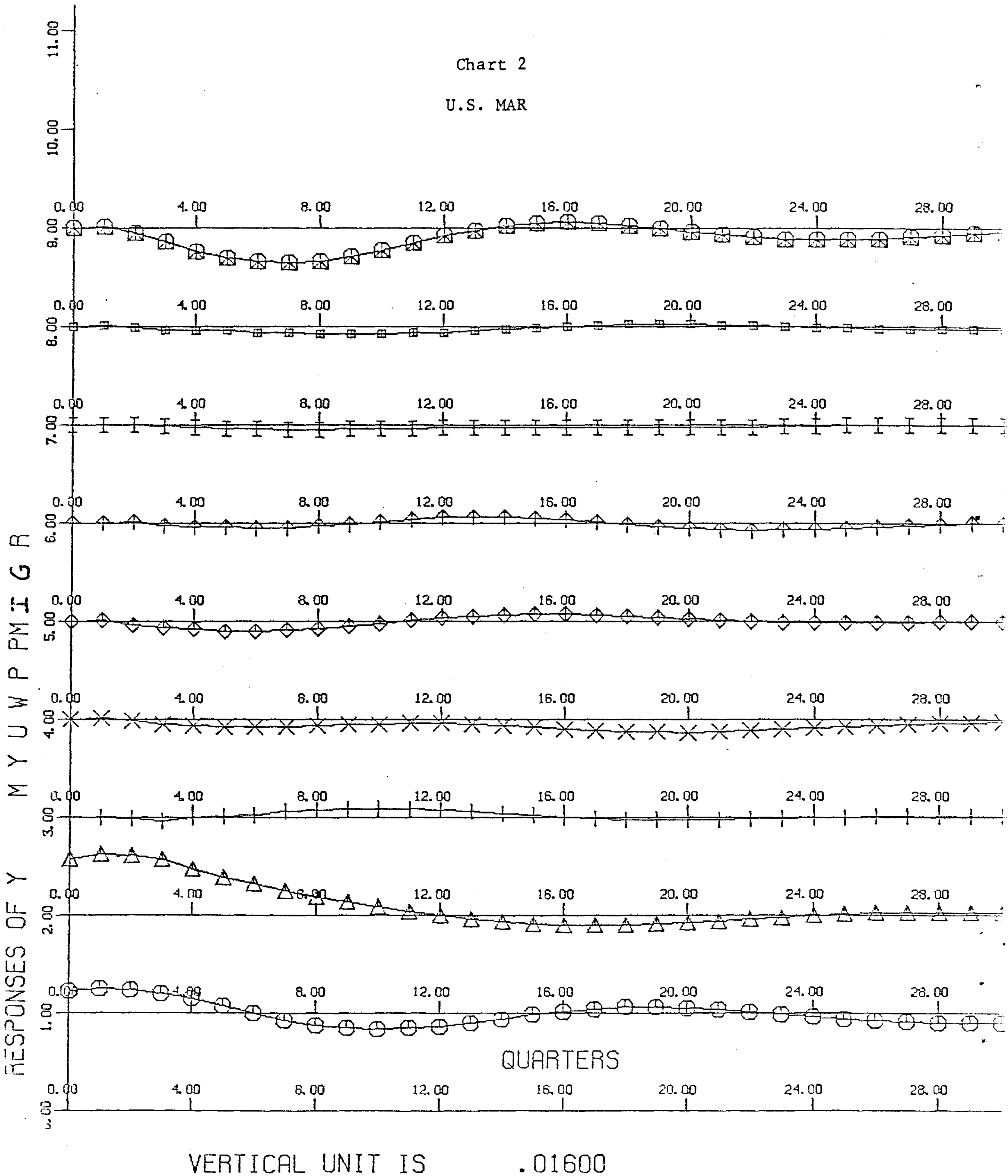

Chart 2
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12.00
16.00
20.00
24.00
28.00
1.00
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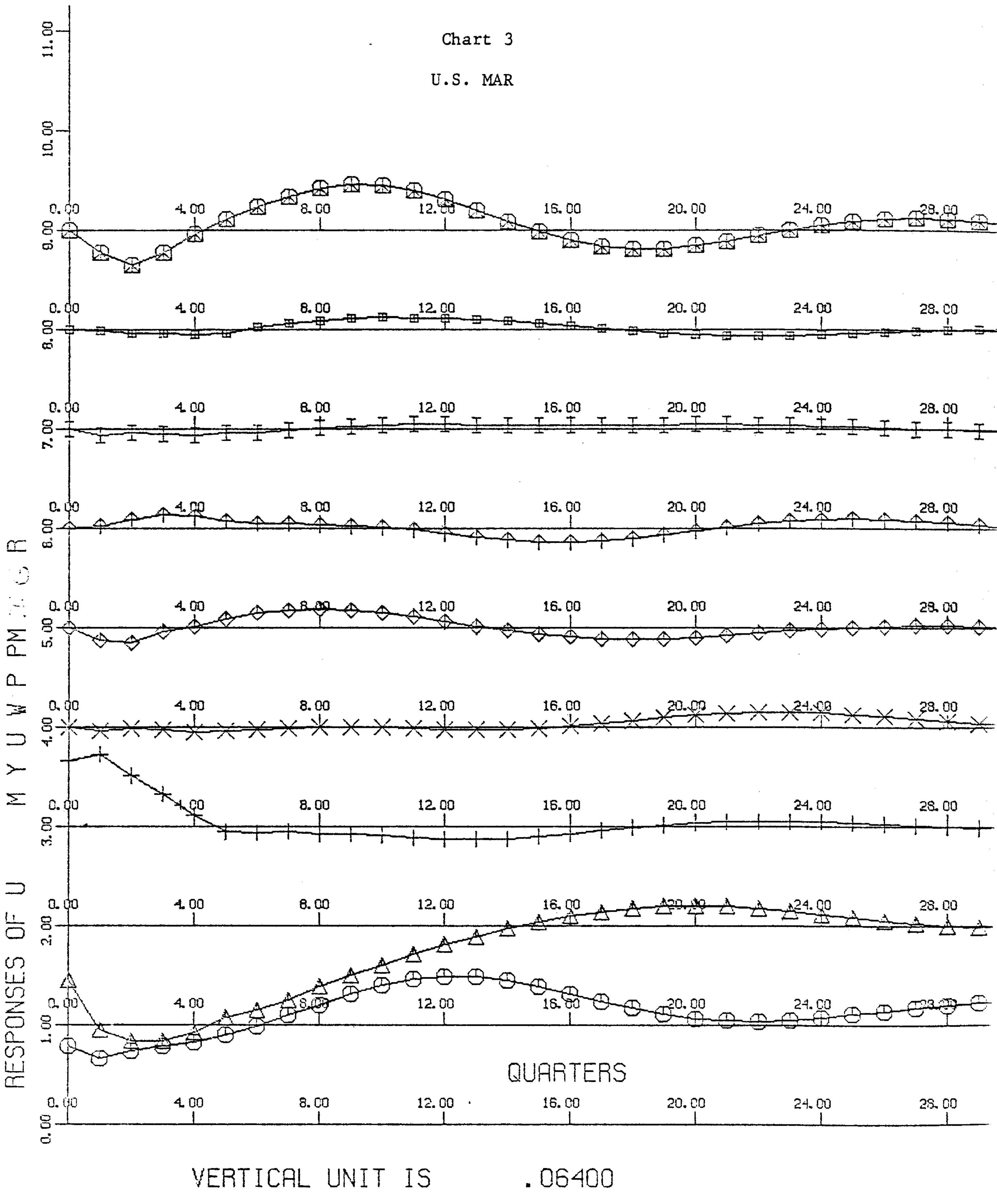
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M Y U W P PM
QUARTERS
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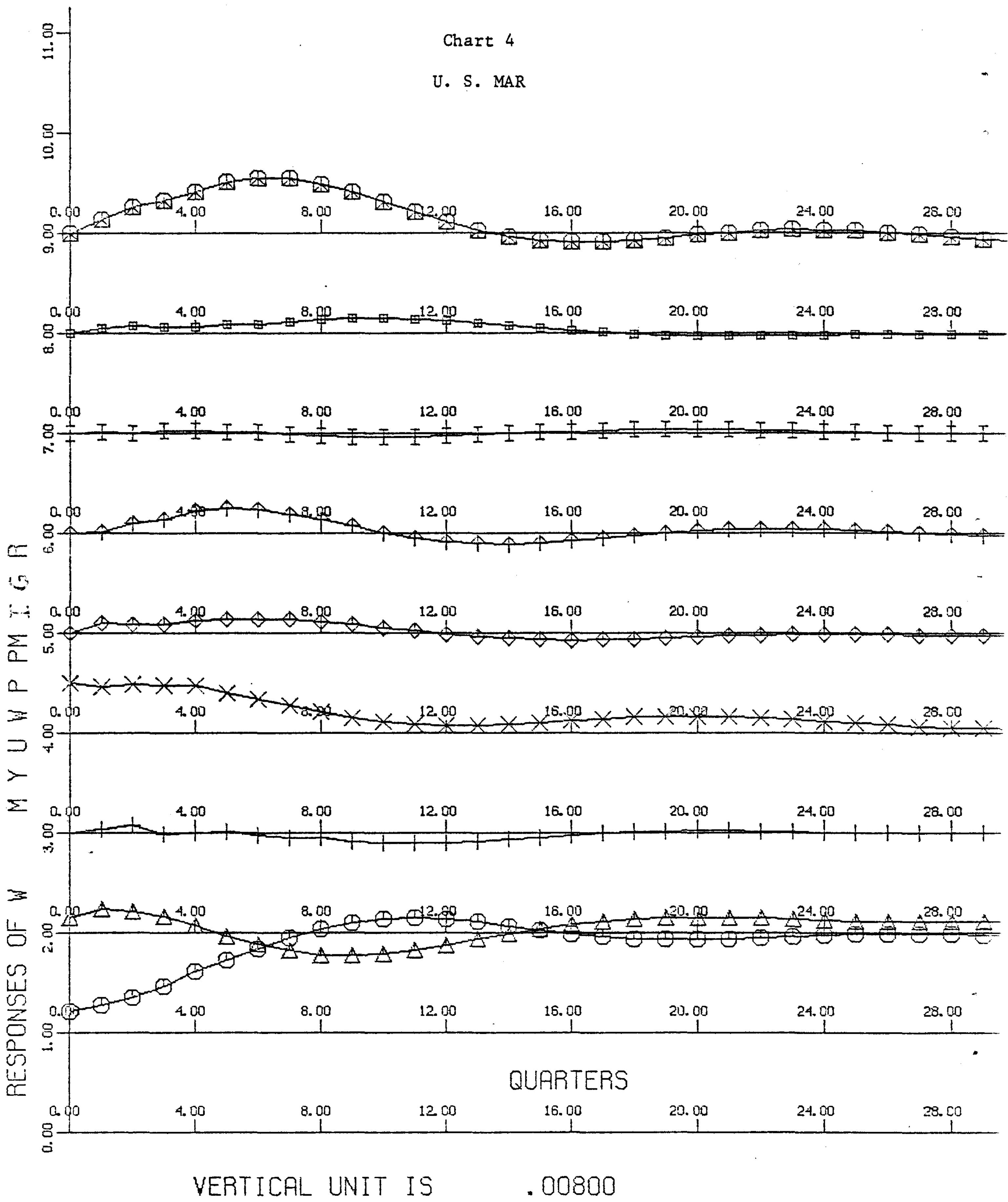
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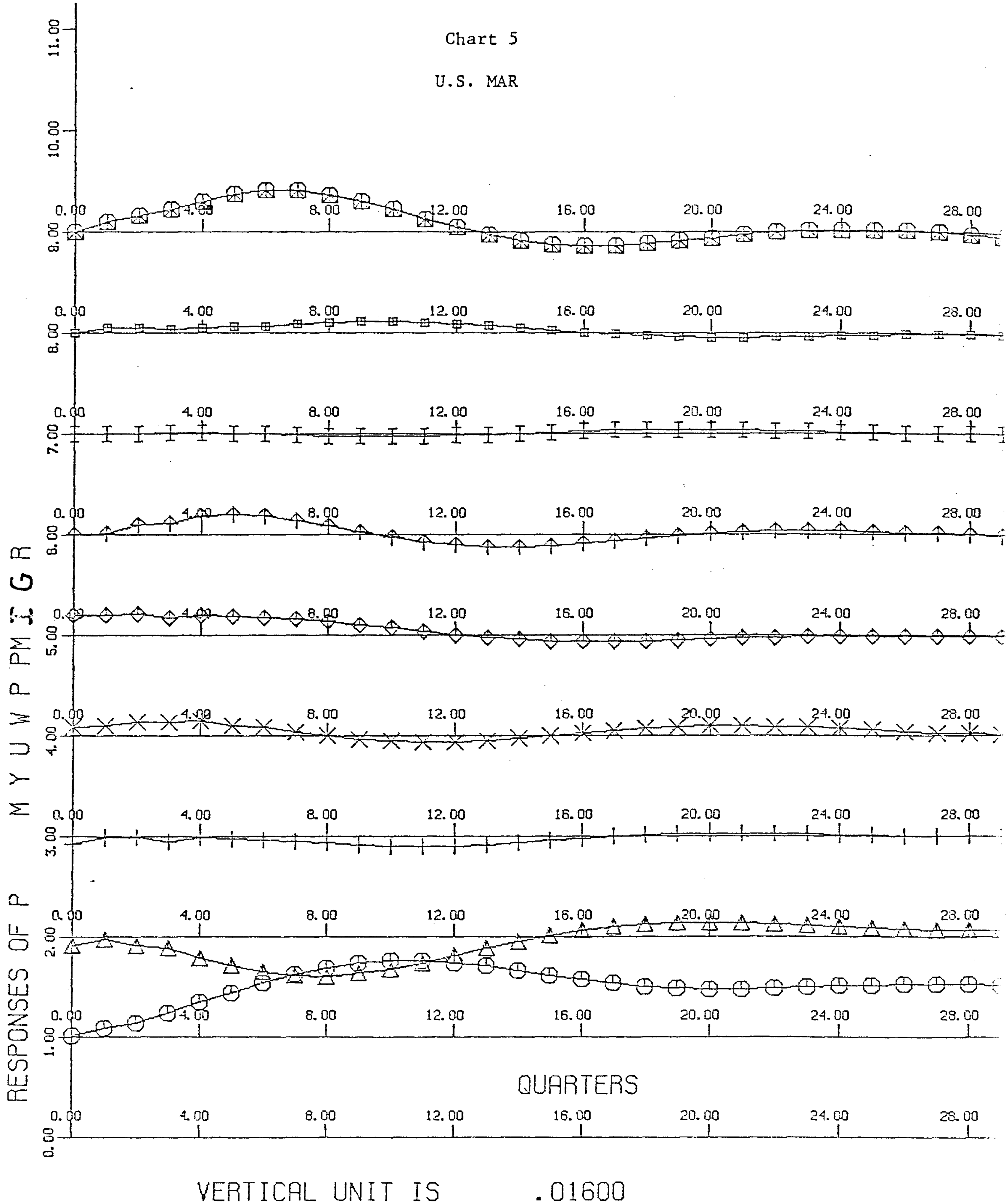
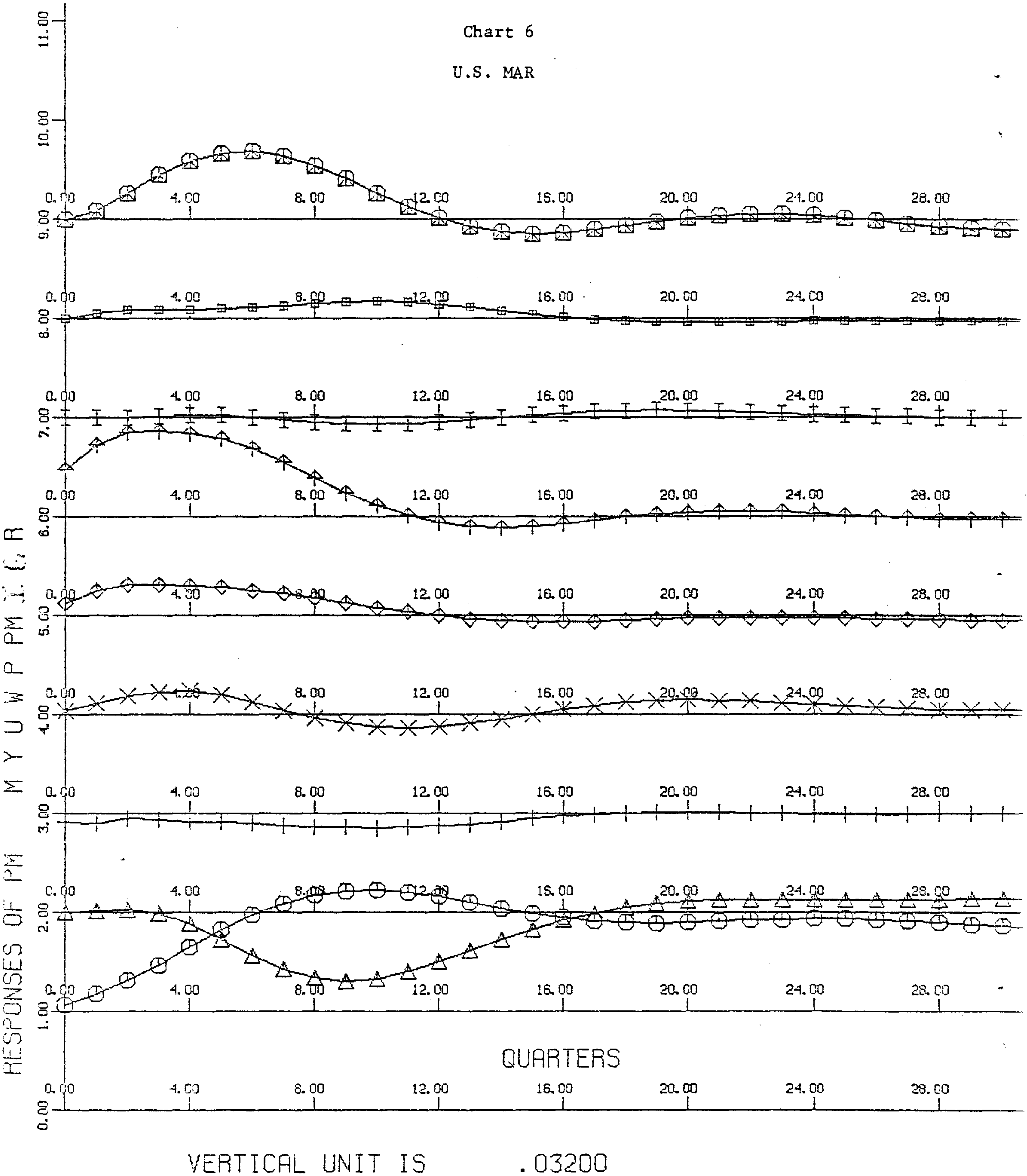
RESPONSES OF W
M Y U W P PM I G R
QUARTERS
VERTICAL UNIT IS .00800

Chart 5

U.S. MAR

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RESPONSES OF PM M Y U W P PM I G R
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24.00
28.00
0.00
1.00
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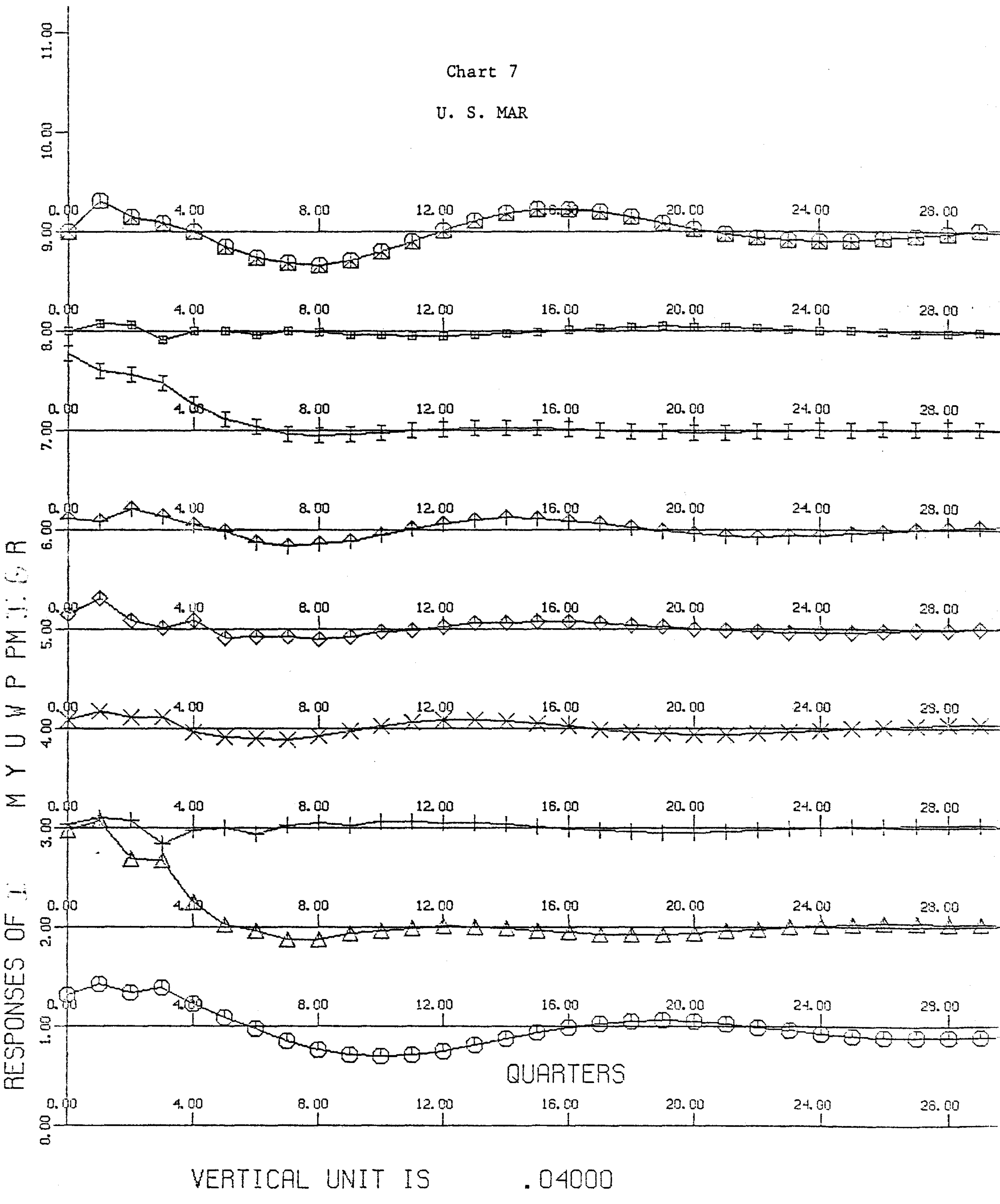
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U. S. MAR
QUARTERS
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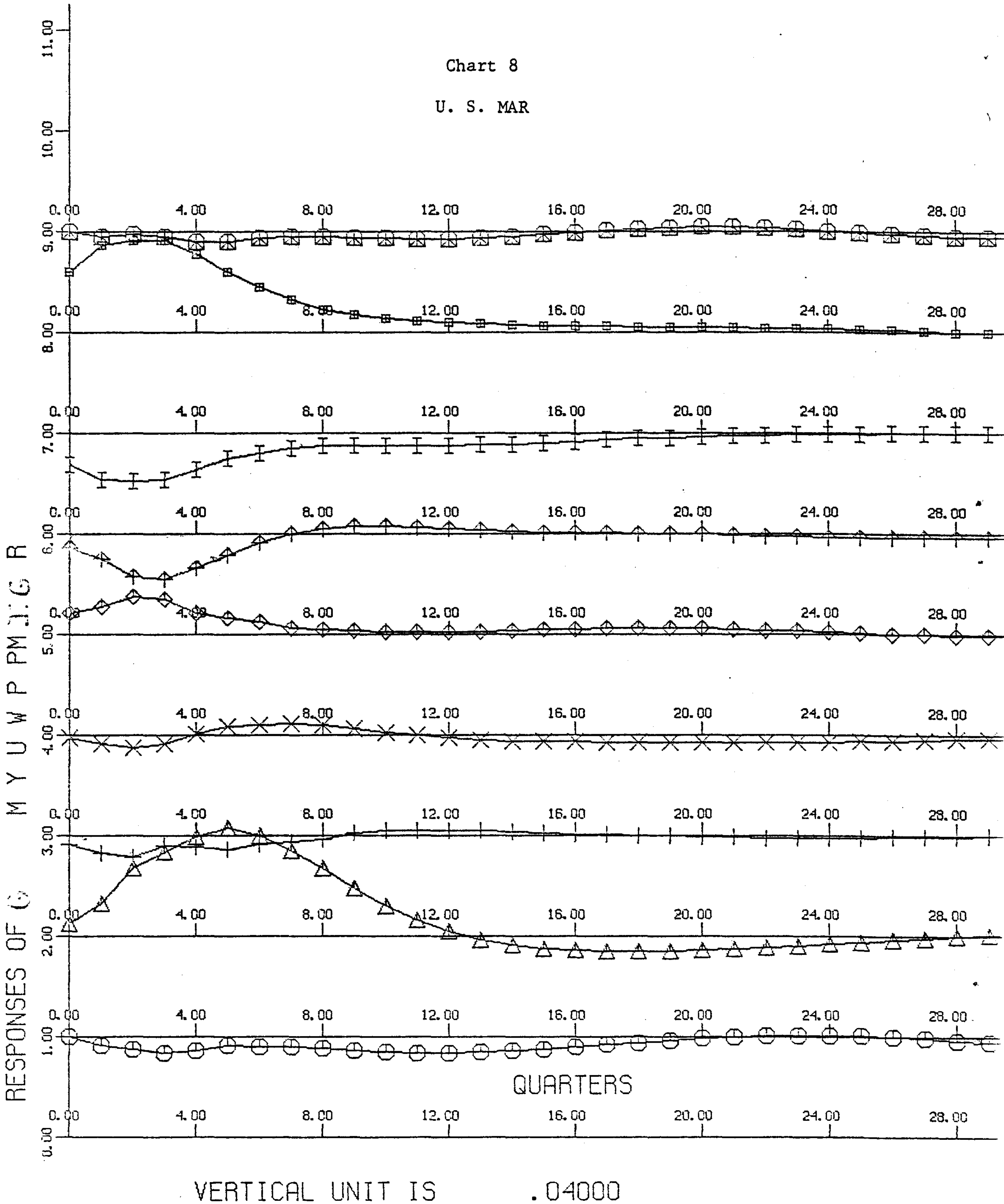

Chart 8
U. S. MAR
RESPONSES OF G
M Y U W P PM I G R
QUARTERS
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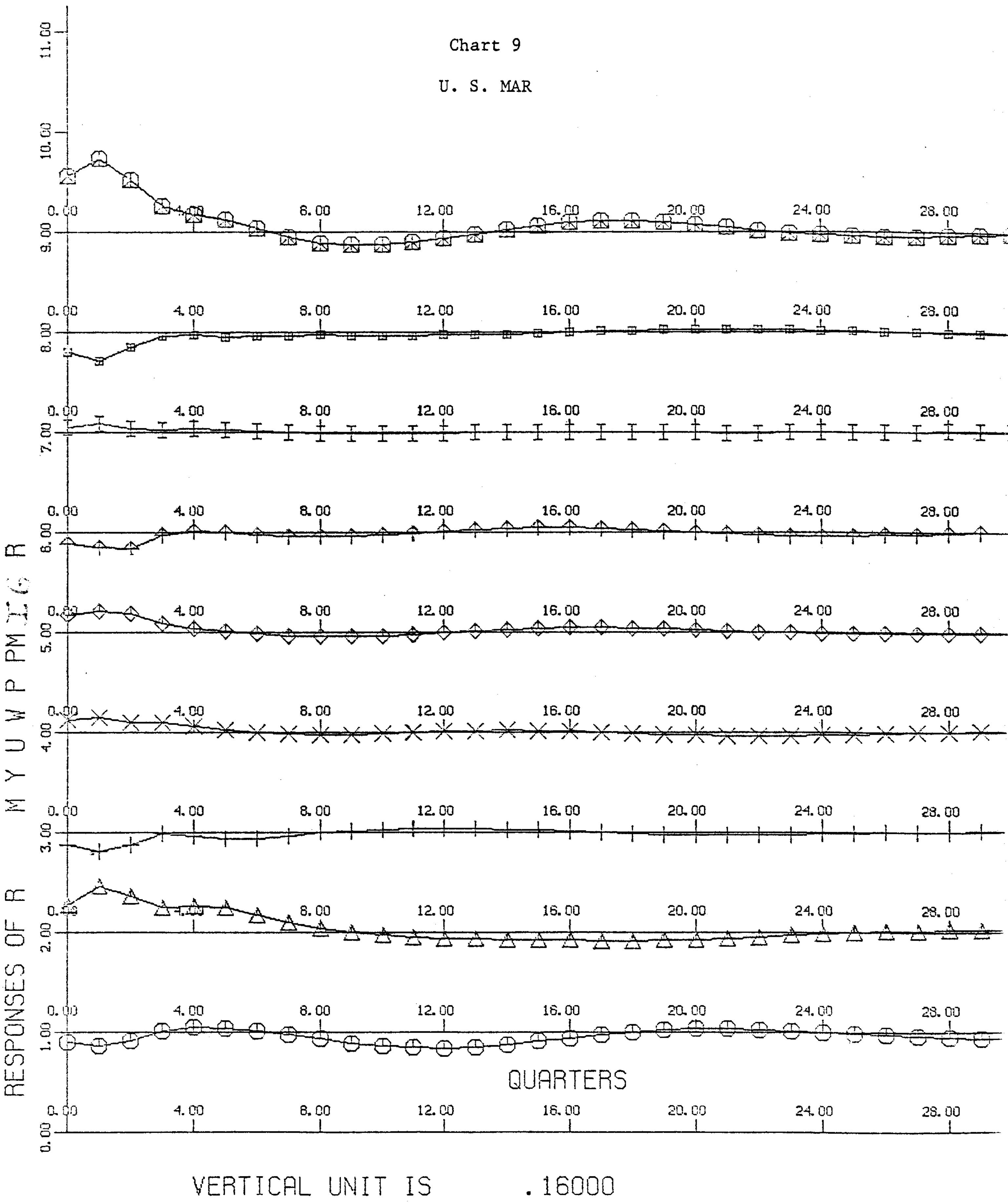

Chart 9
U. S. MAR
RESPONSES OF R
M Y U W P PM IG R
QUARTERS
VERTICAL UNIT IS .16000

Chart 10

U. S. MAR

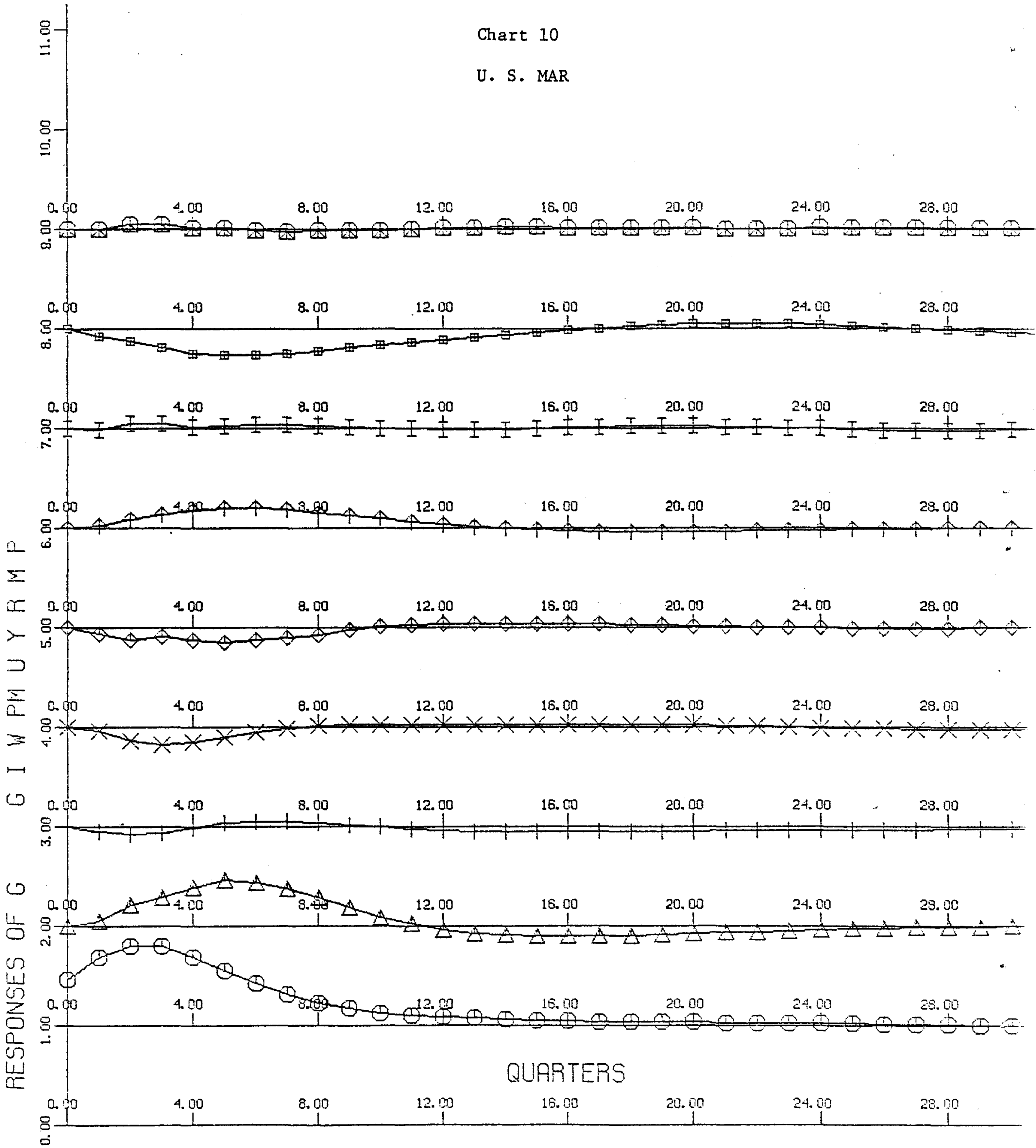

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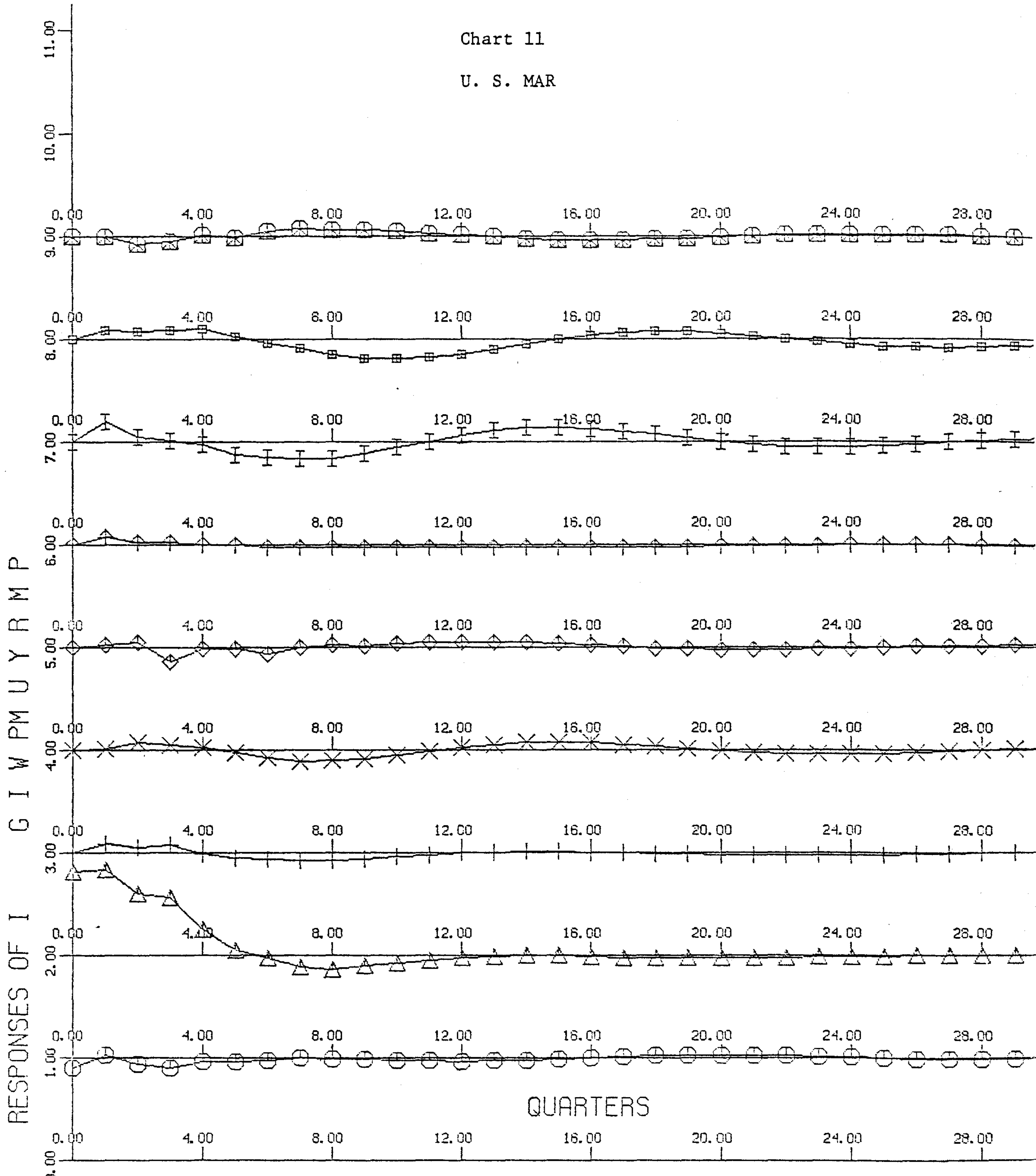
Chart 11

U. S. MAR



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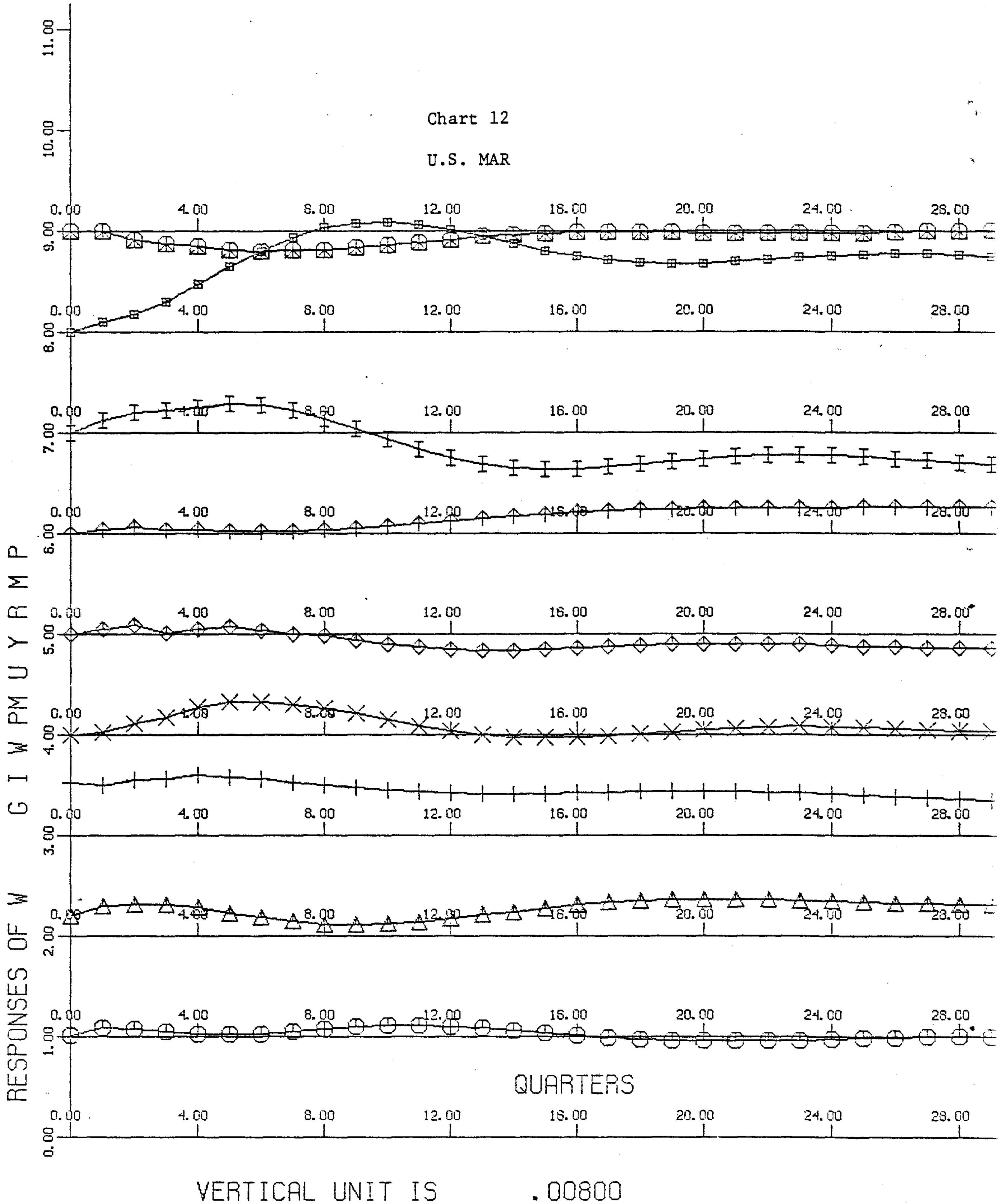
U.S. MAR



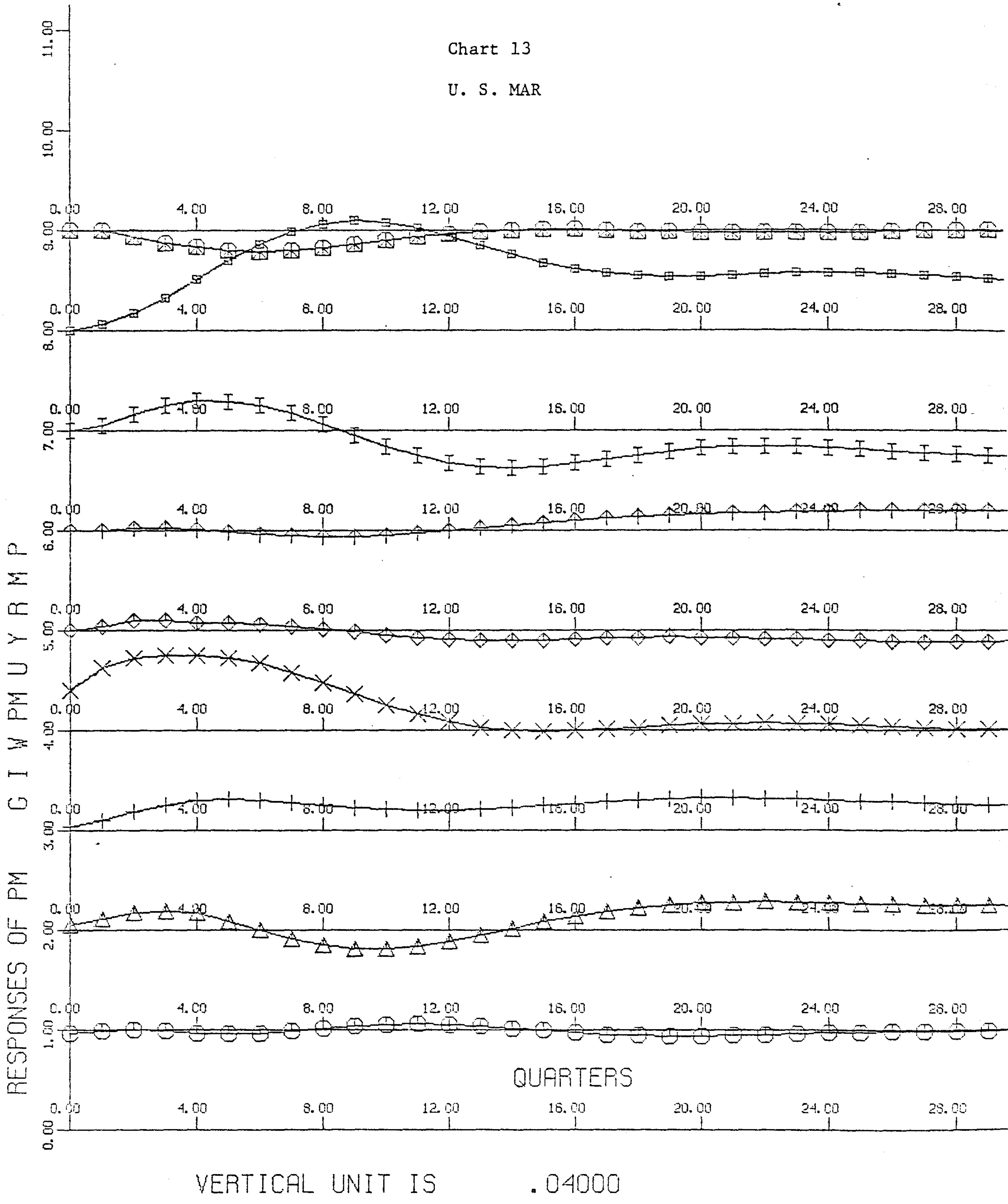


Chart 13
U. S. MAR
RESPONSES OF PM   G I W PM U Y R M P
QUARTERS
VERTICAL UNIT IS .04000

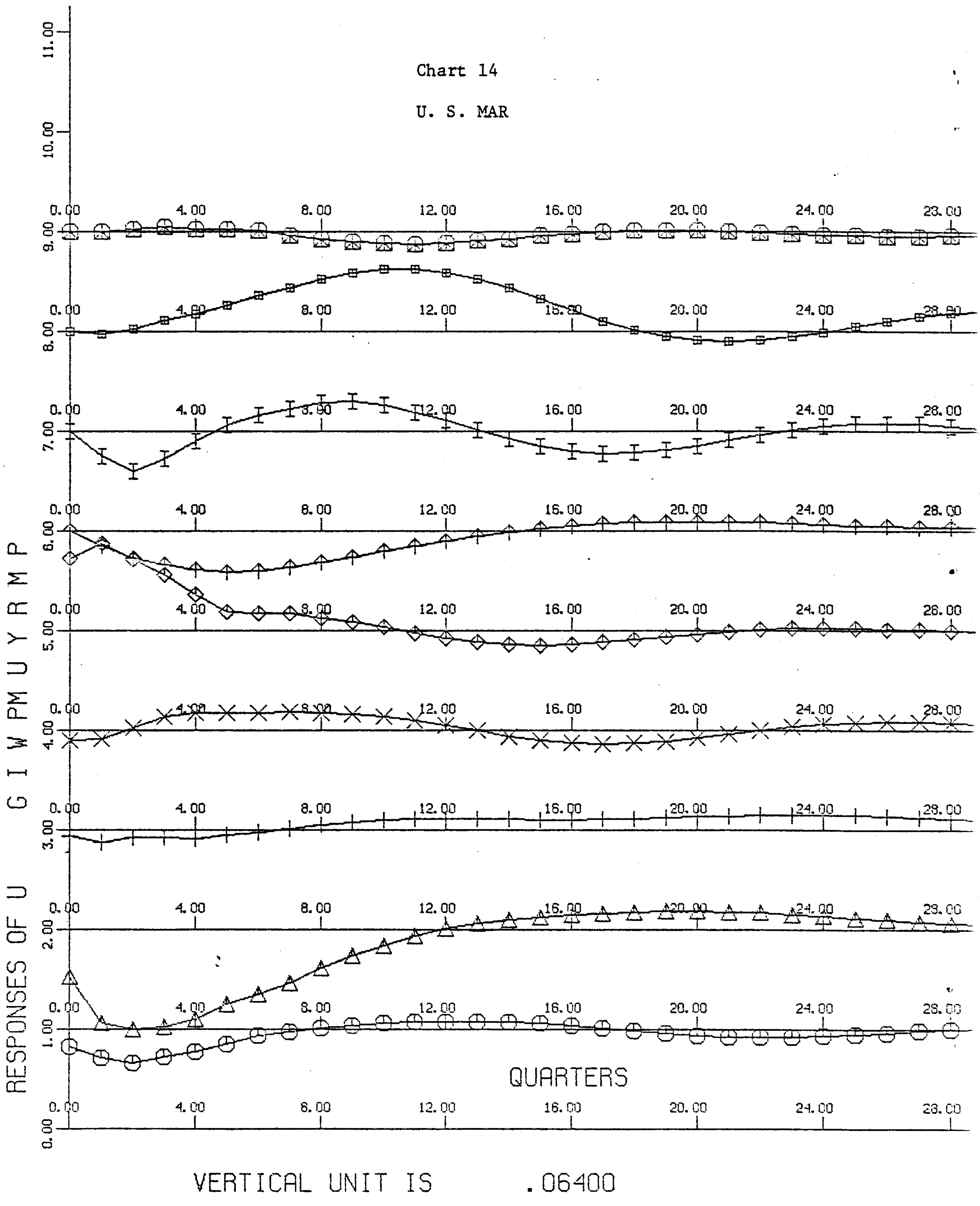
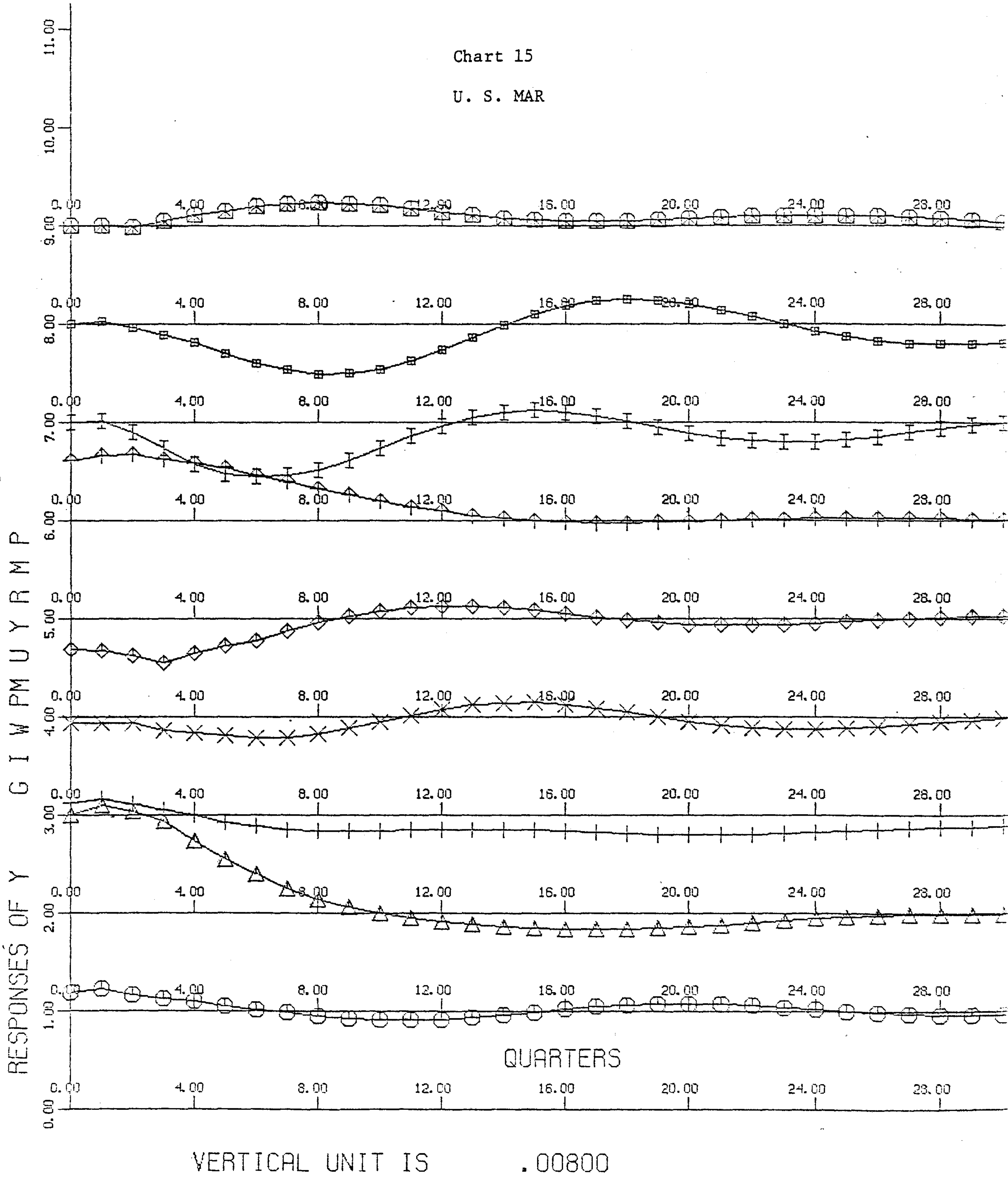


Chart 14
U. S. MAR
RESPONSES OF U G I W PM U Y R M P
QUARTERS
VERTICAL UNIT IS .06400

Chart 15

U. S. MAR



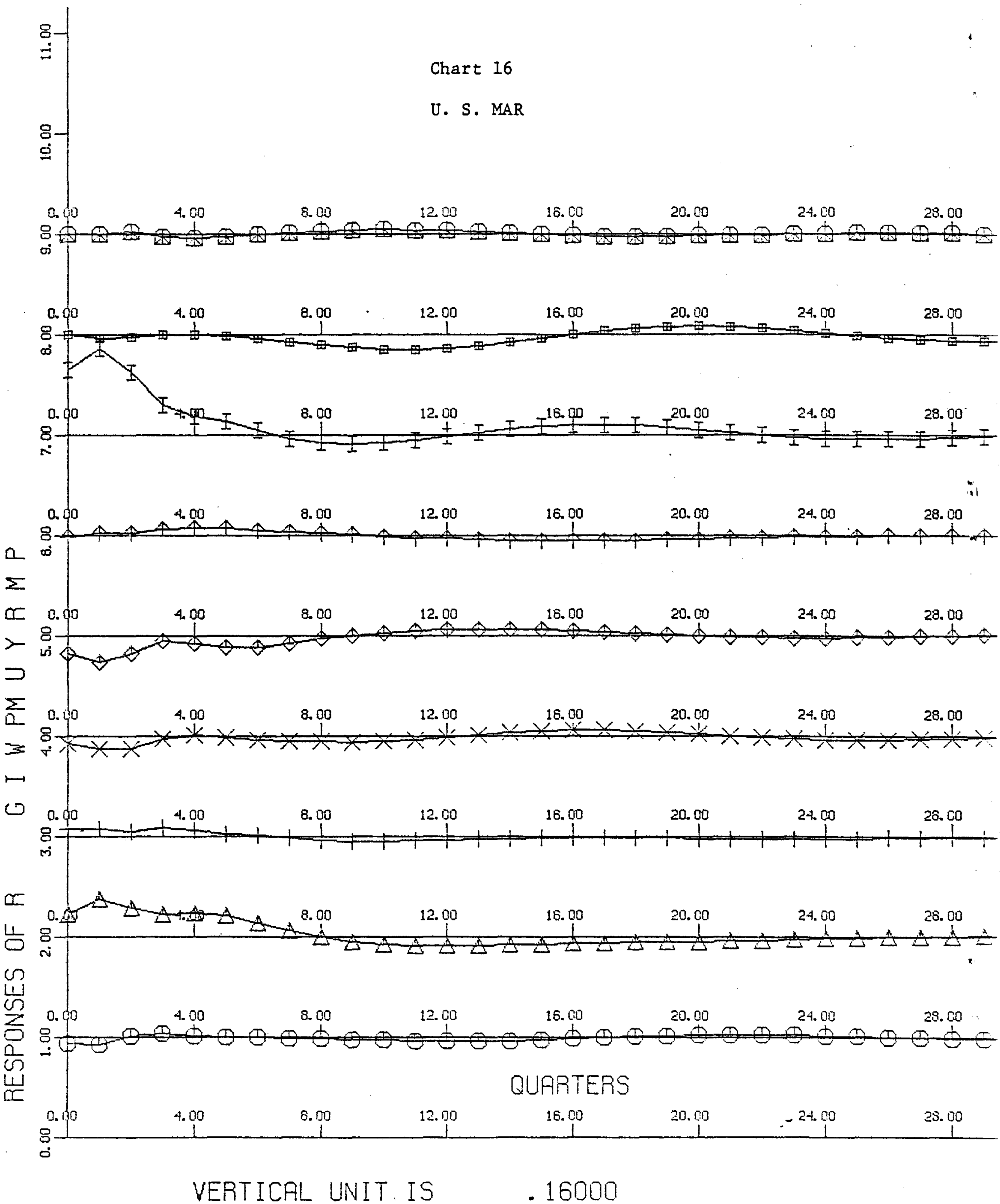


Chart 16
U. S. MAR
RESPONSES OF R
G I W PM U Y R M P
QUARTERS
VERTICAL UNIT IS .16000

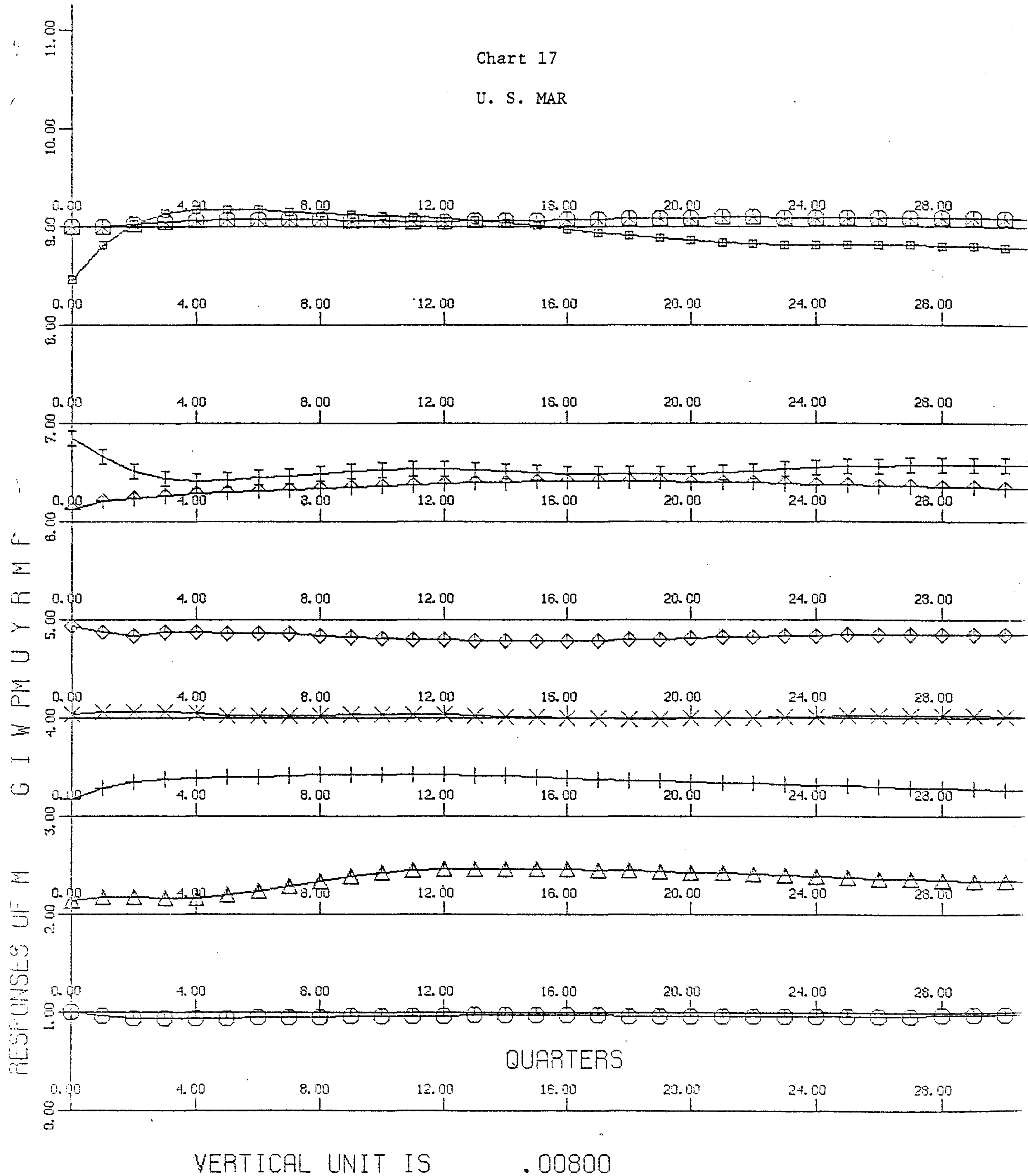


Chart 17
U. S. MAR
RESPONSES OF M
G I W PM U Y R M F
QUARTERS
VERTICAL UNIT IS .00800

Chart 18

U. S. MAR