POOLING CROSS-SECTION AND TIME SERIES DATA IN THE ESTIMATION OF STOCHASTIC FRONTIER PRODUCTION FUNCTION MODELS

by

Lung-Fei Lee and Mark M. Pitt

Discussion Paper No. 78-98, March 1978

Center for Economic Research
Department of Economics
University of Minnesota
Minneapolis, Minnesota 55455

Pooling Cross-Section and Time Series Data in the Estimation of Stochastic Frontier Production Function Models*

Lung-Fei Lee and Mark M. Pitt

1. Introduction

The estimation of empirical production functions has long been of interest in quantitative economics. Typically, an "average" production function has been estimated. However, this estimated production function is not comparable with its conceptual definition as the maximum quantity of output attainable from given inputs. This discrepancy has been recognized, and attempts to deal with it began with the seminal work of Farrell (1957) and subsequently, Aigner and Chu (1968), Afriat (1972) and Richmond (1974), all of whom treated the production frontier as the maximum possible output given inputs. They estimated the frontier using linear and quadratic programming techniques. There are several disadvantages to their approach. The most important problem is that it does not allow for measurement errors in the data and random shocks in the production process which are outside the firms control. As a consequence, a few extreme measured observations determine the frontier and exaggerate the maximum possible output given inputs.

---

*This work was supported in part by a Graduate School Grant-in-Aid from the University of Minnesota. The assistance of the Central Bureau of Statistics of the Republic of Indonesia in obtaining the data is gratefully acknowledged.

Recognizing this problem, Timmer (1971) eliminated a certain percentage of the total observations. Such a selection procedure, however, is arbitrary and has no basis in statistical analyses. Recently, Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977) handle this problem with a more satisfactory conceptual basis. However with the exceptions of Meeusen and van den Broeck (1977) and Lee and Tyler (1978), empirical investigation utilizing these new techniques is limited and not entirely satisfactory.

In this paper, the approach of Aigner et al will be generalized to handle pooled data. A variance components model with non-normal disturbances for use with pooled time series of cross sections is introduced. Estimation of average production functions using a variance components model was the topic of Nerlove's (1965) classical work. Our model estimates frontier production functions rather than average functions and provides a measure of average inefficiency. To derive efficient estimates, maximum likelihood methods are proposed. The model is then applied to pooled micro data obtained from individual Indonesian weaving firms. Frontier production functions estimated by maximum likelihood methods are then compared with average production functions estimated by OLS and GLS. Sources of inefficiency among firms are then investigated and the policy implications of the results discussed.

2. A variance component-stochastic frontier production function model and efficiency measure

Consider the production function model with multiplicative disturbances

$$z = f(x, \beta) e^{\varepsilon} \qquad (1)$$

where x is a $1 \times K$ row vector of inputs, $f(x, \beta)$ is the theoretical maximum output, z is the observed output and e^{ε} is the stochastic error term. The stochastic frontier specification of Aigner et al (1977) and Meeusen and van den Broeck(1977) differs from previous studies in that the error, ε , is composed of two different types of disturbances

$$\varepsilon = u + v \tag{2}$$

where u is one-sided distributed, $u \leqq 0$, which represents technical inefficiency and v is a stochastic variable which represents measurement errors and uncontrolled random shocks in the production process. The nonpositive disturbance u reflects the fact that output must lie on or below its frontier $f(x,\beta)e^{v}$, since e^{u} has a value between zero and one. The frontier $f(x,\beta)e^{v}$ is stochastic as v consists of random factors beyond the firms control.

To simplify the estimation, a log-linear model will be considered. After a logarithmic transformation, (1) is simply

$$y = x\beta + \varepsilon \tag{3}$$

where $y = \ln z$. To generalize the model (3) to handle cross section and time series data, we consider the following variance components model:

$$y_{it} = x_{it}\beta + u_i + v_{it}, \qquad i = 1, \ldots, N; \quad t = 1, \ldots, T \tag{4}$$

where i represents the ith production unit, t the tth time period, x_{it} is a $1xK$ input vector and β is a $Kx1$ vector of parameters. The two different erors are assumed to be independent; $\{v_{it}\}$ and $\{u_i\}$ are independently and identically distributed. This model is similar to models studied by Nerlove (1965), Wallace and Hussain (1969) and others except that u_i is one-side distributed.

With the specification (1), a measure of each unit's efficiency can be defined as

$$z_{it}/f(x_{it},\beta)e^{v_{it}} \tag{5}$$

for the ith unit in the tth time period. As v_{it} is unobservable, (5) is not estimable. However, mean efficiency, defined as the expected value of the ratio in (5), is estimable and is a useful index. The mean efficiency measure is simply $E(e^u)$, the moment generating function $\phi(\lambda)$ evaluated at $\lambda = 1$.

3. Maximum Likelihood Methods

Since u is one-side distributed it has nonzero mean, which cannot be identified from the intercept in (4) without knowledge of its specific distribution. Following Aigner et al, we consider the case of u_i as truncated normal[1] and $v_i \sim N(0,\sigma_v^2)$. It is well-known that in a variance components model under the assumption that both u and v are normal with zero mean, the generalized least squares method is asymptotically efficient in the estimation of β (see, e.g. Maddala (1971)). This is not the case in our specification since generalized least squares does not utilize the information of u's truncation. To find efficient estimates, maximum likelihood procedures are necessary.

To derive the likelihood function for the cross section of time

[1]The density function of the truncated normal variable u is

$$h(u) = \frac{2}{\sqrt{2\pi}\,\sigma_u} \exp\left\{-\frac{u^2}{2\sigma_u^2}\right\} \qquad u \le 0$$

series data, consider the T vector ε_i for the ith unit. With the convolution formulae, it is straightforward to show that the joint density function of ε_i is

$$g(\varepsilon) = \frac{2}{(2\pi)^{T/2}\,\sigma_v^{T-1}(\sigma_v^2 + T\sigma_u^2)} \exp\left\{ -\frac{1}{2\sigma_v^2}\varepsilon'\left(I - \frac{\sigma_u^2}{\sigma_v^2 + T\sigma_u^2}\ell\ell'\right)\varepsilon\right\}.$$

$$\left[1 - \Phi\left(\frac{\sigma_u}{\sigma_v(\sigma_v^2 + T\sigma_u^2)^{\frac{1}{2}}}\ell'\varepsilon\right)\right] \qquad (6)$$

where I is a $T \times T$ identity matrix, ℓ is a T column vector with all elements equal to unity and Φ is the standard normal c.d.f. The relevant log-likelihood function for the pooled data can then be derived, which is (up to an additive constant)

$$\ln \mathcal{L} = -\frac{N(T-1)}{2}\ln\sigma_v^2 - \frac{N}{2}\ln(\sigma_v^2 + T\sigma_u^2)$$

$$-\frac{T}{2\sigma_v^2}\Sigma_{i=1}^N\left(\frac{y_i'y_i}{T} - 2\frac{y_i'X_i}{T}\beta + \beta'\frac{X_i'X_i}{T}\beta\right) +$$

$$\frac{T^2\sigma_u^2}{2\sigma_v^2(\sigma_v^2 + T\sigma_u^2)}\Sigma_{i=1}^N(\bar{y}_i - \bar{X}_i\beta)^2 +$$

$$\Sigma_{i=1}^N \ln\left[1 - \Phi\left(\frac{T\sigma_u}{\sigma_v(\sigma_v^2 + T\sigma_u^2)^{\frac{1}{2}}}(\bar{y}_i - \bar{X}_i\beta)\right)\right] \qquad (7)$$

where X_i is a $T \times K$ matrix, y_i is a $T \times 1$ vector and $\bar{y}_i$, $\bar{X}_i$ are the sample means of y and x for the ith unit. It is interesting to observe from (7) that the sample means and second moments of (y_i, X_i) for i are sufficient statistics[2] for our model.

Taking first derivatives,

$$\frac{\partial \ln \mathcal{L}}{\partial \beta'} = \frac{T}{\sigma_v^2} \Sigma_{i=1}^{N} \left(\frac{y_i' X_i}{T} - \beta' \frac{X_i' X_i}{T}\right) - \frac{T^2 \sigma_u^2}{\sigma_v^2(\sigma_v^2 + T\sigma_u^2)}$$

$$\Sigma_{i=1}^{N} (\bar{y}_i - \bar{X}_i \beta)\bar{X}_i + \frac{T\sigma_u}{\sigma_v(\sigma_v^2 + T\sigma_u^2)^{\frac{1}{2}}} \Sigma_{i=1}^{N} \frac{\phi(\xi_i)}{1 - \Phi(\xi_i)} \bar{X}_i \quad (8)$$

$$\frac{\partial \ln \mathcal{L}}{\partial \sigma_u^2} = - \frac{NT}{2(\sigma_v^2 + T\sigma_u^2)} + \frac{T^2}{2(\sigma_v^2 + T\sigma_u^2)^2} \Sigma_{i=1}^{N} (\bar{y}_i - \bar{X}_i \beta)^2$$

$$- \frac{T\sigma_v}{2\sigma_u(\sigma_v^2 + T\sigma_u^2)^{3/2}} \Sigma_{i=1}^{N} \frac{\phi(\xi_i)}{1-\Phi(\xi_i)} (\bar{y}_i - \bar{X}_i \beta) \quad (9)$$

$$\frac{\partial \ln \mathcal{L}}{\partial \sigma_v^2} = - \frac{N(T-1)}{2\sigma_v^2} - \frac{N}{2(\sigma_v^2 + T\sigma_u^2)} + \frac{T}{2\sigma_v^4} \Sigma_{i=1}^{N} \left(\frac{y_i' y_i}{T} - \right.$$

$$\left. 2\frac{y_i' X_i}{T}\beta + \beta' \frac{X_i' X_i}{T}\beta\right) - \frac{T^2 \sigma_u^2 (2\sigma_v^2 + T\sigma_u^2)}{2\sigma_v^4(\sigma_v^2 + T\sigma_u^2)^2} \Sigma_{i=1}^{N} (\bar{y}_i - \bar{X}_i \beta)^2 +$$

$$\frac{T\sigma_u(2\sigma_v^2 + T\sigma_u^2)}{2\sigma_v^3(\sigma_v^2 + T\sigma_u^2)^{3/2}} \Sigma_{i=1}^{N} \frac{\phi(\xi_i)}{1-\Phi(\xi_i)} (\bar{y}_i - \bar{X}_i \beta) \quad (10)$$

where ϕ and Φ are standard normal density and distribution functions evaluated at ξ_i with $\xi_i = \frac{T\sigma_u}{\sigma_v(\sigma_v^2 + T\sigma_u^2)^{\frac{1}{2}}}(\bar{y}_i - \bar{X}_i \beta)$.

---

[2]From (7), it is obvious that the maximum likelihood procedure can be applied to inbalanced models without further complications.

The second derivatives can also be derived but they are relatively complicated. To find the maximum likelihood estimates, various numerical algorithms which require only first derivatives, such as the Davidon-Fletcher-Powell (DFP) algorithm, can be used. Asymptotic standard errors can either be numerically approximated or constructed from the first derivatives upon convergence.

To compute the maximum likelihood estimates, it is desirable to have good initial estimates. In our model, OLS estimates and analysis of variance estimates can be easily derived which can be used as initial estimates. Let $X = \begin{bmatrix} X_1 \\ \vdots \\ X_n \end{bmatrix}$ be a NT x K matrix and $Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$ be a NT x 1 vector. The OLS estimates have the following variance matrix

$$(X'X)^{-1}X'(\sigma_v^2 I_n \otimes I_T + \sigma_u^2 I_n \otimes \ell\ell')X(X'X)^{-1} \tag{11}$$

which can be used to compare its efficiency with other estimators. The analysis of variance estimates for σ_u^2 and σ_v^2 are (see Graybill (1961), Wallace and Hussain (1969))

$$\hat{\sigma}_v^2 = \frac{1}{N(T-1)} \Sigma_{i=1}^{N} \Sigma_{t=1}^{T} \left(\tilde{\varepsilon}_{it} - \frac{1}{T} \Sigma_{t=1}^{T} \tilde{\varepsilon}_{it} \right)^2 \tag{12}$$

$$\hat{\sigma}_u^2 = \frac{\Sigma_{i=1}^{N} \left(\frac{1}{T} \Sigma_{t=1}^{T} \tilde{\varepsilon}_{it} - \frac{1}{NT} \Sigma_{i=1}^{N} \Sigma_{t=1}^{T} \tilde{\varepsilon}_{it} \right)^2}{N-1} - \frac{\hat{\sigma}_v^2}{T} \tag{13}$$

where $\tilde{\varepsilon}_{it}$ is the estimated residuals. As argued in Amemiya (1971), more efficient estimates of σ_u^2 and σ_v^2 than the estimates derived from least squares residuals can be derived from generalized least squares residuals. The generalized least squares estimates for our model are

$$\hat{\beta}_G = [X'(I_n \otimes I_T - \frac{\tilde{\sigma}_u^2}{\tilde{\sigma}_v^2 + T\tilde{\sigma}_u^2}(I_n \otimes \ell\ell'))X]^{-1} X'(I_n \otimes I_T - \frac{\tilde{\sigma}_u^2}{\tilde{\sigma}_v^2 + T\tilde{\sigma}_u^2} I_n \otimes \ell\ell')Y \tag{14}$$

where $\tilde{\sigma}_u^2$ and $\tilde{\sigma}_v^2$ are consistent esti ıates of σ_u^2 and σ_v^2. The GLS estimates have the variance matrix

$$[X'(I_n \otimes I_T - \frac{\tilde{\sigma}_u^2}{\tilde{\sigma}_v^2 + T\tilde{\sigma}_u^2}(I_n \otimes \ell\ell'))X]^{-1}. \tag{15}$$

It is well known (Wallace and Hussain (1969)) that GLS estimates are more efficient than OLS estimates and that OLS estimates have zero efficiency in the limit. Since u is not normal, maximum likelihood estimates (MLE) will be more efficient than GLS estimates.

With the truncated normal distribution, the mean efficiency measure (Lee and Tyler (1978)) is

$$E(e^u) = 2e^{\sigma_u^2/2}(1 - \Phi(\sigma_u)) \tag{16}$$

Alternative one-side distributions for u rather than truncated normal distributions can also be used. Among those, Afriat (1972) and Richmond (1974) utilize the one-parameter Gamma distribution. Meeusen and van den Broeck (1977) and Aigner et al (1977) also utilize the exponential distribution. All of these distributions have similar theoretical properties, however, the truncated normal distribution is preferred from the computational point of view.

4. Data and Empirical Results

Cross-section and time-series data on Indonesian weaving establishments are pooled for the estimation of a stochastic frontier Cobb-Doublas production function.[3] Data on fifty Indonesian weaving firms for the years 1972, 1973 and 1975 were obtained from manufacturing surveys conducted by the Central Bureau of Statistics (Biro Pusat Statistik) of Indonesia. Output was measured by value-added, capital services by electricity consumption and labor inputs by the value of total wage payments. Other measures of capital services available include horse-power of installed machinery and the value of energy consumed. Previous research with similar Indonesian data (Pitt (1978)) found electricity consumption to be the preferred measure of capital inputs. It is felt that wage payments are a better measure of labor input than man-months because of the heterogeneous nature of the labor force in this sector. Value-added and wage payments were adjusted to constant units by deflation with appropriate price and wage indices. Information on other firm characteristics was available and used in analyzing the sources of inefficiency as described in Section 5. The variables used are:

---

[3]Alternative functional specifications are conceivable. The Cobb-Douglas specification is computationally easier and has been found applicable to Indonesian data in other studies in progress.

Capital	-	annual consumption of electricity in kilowatt-hours.
Labor	-	annual deflated wage payments (1972 base-year).
72D	-	time dummy variable; 72D = 1 for 1972, 0 otherwise.
73D	-	time dummy variable; 73D = 1 for 1973, 0 otherwise.
72L	-	interaction term of labor with 72D time dummy variable, i.e. 72L = 72D x Labor.
73L	-	interaction term of labor with 73D time dummy variable, i.e. 73L = 73D x Labor.
72K	-	interaction term of capital with 72D time dummy variable, i.e. 72K = 72D x Capital.
73K	-	interaction term of capital with 73D time dummy variable, i.e. 73K = 73D x Capital.
Year	-	year firm began production (in two digits).
Foreign	-	dummy variable for firm ownership; takes the value one if firm is foreign owned and zero otherwise. Firms are considered foreign-owned if foreign participation exceeds 50 percent.[4]
Size	-	firm size measured as total man-months (in thousands) of labor supplied over the three years observed.

It is interesting to note that the output and factor input variables have large variances across establishments and time

[4] The only firm in our sample that had foreign participation but was not considered "foreign" in the analysis was only 25 percent foreign-owned.

periods.

Table 1 reports the results of applying the variance components model to the pooled data. The OLS and GLS methods estimate the average production function, while the MLE method estimates the stochastic frontier production function. Since the average production function lies below the frontier production function in input-output space, its intercept term is smaller than that of the frontier production function.

The results of estimating the production functions without separate time intercepts is reported in columns (1), (3) and (5) of Table 1. Note that the intercept of the estimated frontier function is less than that of the average functions. The three estimates of capital and labor elasticities are all significant and are in fairly close agreement as expected,[5] although the GLS estimates are closer to the MLE estimates than are the OLS estimates. In all cases the elasticity of capital inputs is much smaller than that of labor and their sum is slightly less than one. This decreasing returns to scale satisfies a theoretical expected property for a competitive industry.

---

[5]Theoretically, the log-linear average function is only a downward shift of the log-linear frontier function. These production functions can be adjusted to frontier functions by correcting the intercept term by $E(-u) = 2\sigma_u / \sqrt{2\pi}$ under the assumption of a one-sided inefficiency distribution which is truncated normal. However, the estimates will not be as efficient as the MLE estimates.

Table 1

Average Cobb-Douglas Production Functions (OLS,GLS)

and Stochastic Frontier Cobb-Douglas Production Functions (MLE)

Estimated With Pooled Data*

	OLS		GLS		MLE	
Constant	-2.7482 (.5129)	-3.3422 (.5381)	-2.5824 (.5059)	-3.2190 (.5278)	-2.2142 (.4668)	-2.8056 (.5599)
1972D		.5404 (.1587)		.5175 (.1580)		.5249 (.1511)
1973D		.5017 (.1516)		.4918 (.1515)		.4950 (.1278)
Capital	.2105 (.0528)	.2516 (.0540)	.1823 (.0512)	.2277 (.0517)	.1918 (.0527)	.2357 (.0524)
Labor	.7708 (.0809)	.7364 (.0804)	.7950 (.0792)	.7606 (.0780)	.7855 (.0697)	.7512 (.0728)
σ_u^2	.1366 (-----)	.1530 (----)	.1473 (-----)	.1587 (-----)	.2623 (.2060)	.3030 (.1729)
σ_v^2	.6346 (-----)	.5610 (-----)	.6257 (-----)	.5564 (-----)	.6727 (.0924)	.6022 (.0796)
$E(e^u)$	.7620	.7510	.7547	.7473	.6940	.6772
$\bar{R}^2$	.7167	.7343				
Log-likelihood value					-88.3649	-81.9888

*The standard errors are in brackets. The standard errors in OLS and GLS are computed according to equations (11) and (15).

In order to catch any time effects, two time dummy variables, 72D and 73D, are introduced into the linear equations.[6] The results of the model including time dummies is reported in columns (2), (4) and (6) of Table 1. In every case, both time dummies are significant by the t test or likelihood ratio test at the 5 percent level of significance. With time dummies, the elasticities on labor are slightly lower and on capital are higher, while returns to scale are slightly underestimated.

Estimates of σ_u^2 and σ_v^2 are derived directly from the maximum likelihood procedure. To estimate the variances of the two error components under the OLS and GLS procedures, the modified "analysis of variance" method was used. That is, estimates of ε_{it} are made based on the estimated β's, and the variances are then computed according to expressions (12) and (13). In every case, the estimated σ_u^2 are not swamped by σ_v^2. The introduction of time dummies has in every case increased σ_u^2 slightly. Even though the coefficients estimated by the different methods are similar, there are striking differences in the estimates of the two variances σ_u^2 and σ_v^2 and hence the mean efficiency measures. The estimates of both σ_u^2 and σ_v^2 in the MLE are larger than the other estimates. The MLE estimate of σ_u^2 is nearly twice as large as the other estimates and the mean level of efficiency is seven percentage points below that obtained by the other methods of estimation. The mean efficiency of the Indonesian weaving industry is about 67.7 percent, which is comparable to the 62.5 percent average

[6] The introduction of time dummies into the model rather than error components simplifies the maximum likelihood approach, especially when the time series are short.

efficiency found for all Brazilian industry (Lee and Tyler (1978)) but lower than the 90.9 percent found for the French textile industry (Meeusen and van den Broeck (1977)).

The GLS estimates of β, σ_u^2 and σ_v^2 are known to be more efficient than the OLS estimates (Amemiya, (1971)), but less efficient than the MLE since the information of non-normal disturbances are not incorporated in the GLS method. Comparison of the standard error of the estimates reported in Table 1 confirms the efficiency of the MLE procedure.

To establish the legitimacy of pooling the time series of cross sections and to detect any possible change in technology, a model which allows different factor elasticities across time periods within the variance components framework is estimated. The estimated equation is:

$$\begin{aligned}\text{Output} = &\underset{(0.7327)}{-3.4859} + \underset{(1.0564)}{1.8587}\ 72D + \underset{(0.9275)}{1.1260}\ 73D \\ &+ \underset{(0.1841)}{0.1967}\ 72L + \underset{(0.1665)}{0.1200}\ 73L - \underset{(0.1405)}{0.2286}\ 72K \\ &- \underset{(0.1220)}{0.1201}\ 73K + \underset{(0.1060)}{0.3568}\ \text{Capital} \\ &+ \underset{(0.1427)}{0.6352}\ \text{Labor} \qquad \text{Log-Likelihood value} = -80.6113\end{aligned}$$

$$\sigma_u^2 = \underset{(0.1636)}{0.2931} \qquad \sigma_v^2 = \underset{(0.0842)}{0.5928} \qquad E(e^u) = 0.6811$$

All interaction terms are not significantly different from zero by the t test. A more powerful joint test is the log likelihood ratio comparing the MLE estimate above with the MLE estimate (with time dummies) of Table 1. The -2 log likelihood ratio of 2.755 is not significant at any conventional level based on the $\chi^2(4)$ distribution. Thus, for our data, it is legitimate to pool the time series of cross sections.

To shed more light on the performance of the variance components model and the maximum likelihood procedure, production functions were estimated separately for each cross-section by both the OLS and maximum likelihood procedures. The results are reported in Table 2. In two of the three cross-sections estimated with OLS, it was not possible to derive positive estimates of σ_u^2 or σ_v^2 from the estimated second and third moments of the least squares residuals as suggested by Aigner et al (1972). In these two cases the maximum likelihood procedure also does not perform altogether satisfactorily. The MLE σ_u^2 are very small and are swamped by σ_v^2. Furthermore, these estimates have huge variances as indicated by the standard errors on σ_u/σ_v and the estimated intercepts. The variances associated with the factor input coefficients in all separate cross-section estimations exceed those estimated from pooled data. These results demonstrate the superior performance of our approach.

5. Sources of Inefficiency

We have found that the mean level of inefficiency in the Indonesian weaving industry is slightly greater than 32 percent. However, this measure evaluates the industry as a whole, and provides no information on the inefficiency of individual firms in the sample. From a policy point of view, it is of interest

Table 2

Average Cobb-Douglas Production Functions (OLS) and

Stochastic Frontier Production Functions (MLE) for each

Cross-Section*

	1972		1973		1975	
	OLS	MLE	OLS	MLE	OLS	MLE
Constant	-2.0529 (.9266)	-2.051 (8.4500)	-2.9462 (.6972)	-2.945 (5.473)	-4.000 (.7446)	-3.53 (.7077)
Capital	.1251 (.1205)	.1251 (.1168)	.2633 (.0648)	0.2633 (0.0629)	.3661 (.1001)	.3779 (.1028)
Labor	.8351 (.1594)	.8351 (.1545)	.7313 (.1007)	.7313 (.0976)	.6295 (.1399)	.6273 (.1399)
σ_u/σ_v	----	.0028 (10.86)	----	.0022 (8.8720)	1.2867 (-----)	1.297 (.9953)
$\sigma_u^2 + \sigma_v^2$	----	.9402 (.1916)	----	.5886 (.1187)	.9154 (-----)	.9178 (.4245)
$E(e^u)$	----	.9602	----	.9718	.5986	.6111
$\bar{R}^2$	.6414		.7815		.7899	

*The OLS consistent estimates of σ_u/σ_v, $\sigma_u^2 + \sigma_v^2$ are derived from the second and third least squares estimated residuals moments as described in Aigner et al to distinguish the inefficient firms from the efficient firms, and to determine whether inefficient firms share some common set of characteristics. The traditional approach to this problem has been based on the analysis of covariance, which includes separate intercept terms for each firm in the estimation of a production function from pooled data. Another approach, which has a more sound theoretical justification, is based on the variance components model (see Amemiya (1976)) with firm characteristics included as extra regressors.

The use of analysis of covariance techniques has a long history in econometric literature (Mundlak 1961; Hoch 1962). However, as shown by Maddala (1971), analysis of covariance estimates do not utilize any between group information. Nevertheless, the approach is appropriate if it is felt that firm inefficiency is correlated with labor and capital inputs.

To investigate the sources of inefficiency, the separate firm intercepts obtained from the analysis of covariance estimates (with time dummies) are regressed on a set of firm characteristics. Three firm characteristics, age, size and ownership, were found to be most important.[7]

---

[7] Other firm characteristics investigated included rural versus urban location, provincial location, value of new investment over the three years of observation, rate of growth of output over period of observation, presence of owner at production facility, and share of non-operative man-months of labor in total man-months of labor supplied. All these variables were statistically insignificant in the analysis and are not reported.

Firm age was measured as the year the firm began production, size was measured as total employment of labor and ownership distinguished between foreign and domestic ownership. The results of the analysis of covariance estimates derived from the pooled data and the regression of separate firm intercepts on these three firm characteristics are:

Analysis of Covariance

$$\text{Output} = \underset{(0.1676)}{0.4509}\ 72D + \underset{(0.1557)}{0.4615}\ 73D + \underset{(0.0677)}{0.1623}\ \text{Capital} + \underset{(0.1607)}{0.8024}\ \text{Labor} \qquad (17)$$

$$\bar{R}^2 = .7915$$

$$\text{Firm intercept} = -2.3870 - \underset{(0.3086)}{0.7157}\ \text{Foreign} + \underset{(0.0074)}{0.0312}\ \text{Year} + \underset{(0.0064)}{0.0169}\ \text{Size} \qquad (18)$$

$$\bar{R}^2 = .2559$$

where the separate firm dummy coefficients are in logarithmic form. All three independent variables are significant at the 5 percent level. The results indicate that larger firms are more efficient than smaller firms. Such a relationship has been noted by other investigators (see for example, Mellor (1976)) and is thought to represent scale economies with respect to organization and technical knowledge. The variable "Year", which represents the calendar year in which the firm commenced operation, has a positive sign indicating that older firms are less efficient. One would expect two opposing effects underlying this firm characteristic. Older firms have had more time to learn and become more experienced in their operations and thus be more efficient. Countering this effect, the durability and high replacement cost of capital result in the use of equipment by older firms which does not embody more recent technological advances. Younger firms are able to adopt the most efficient technologies available at the time of their conception. Apparently, in our sample, this effect greatly outweights the learning advantages of older firms.

Finally, it is found that foreign-owned firms, ceteris paribus, are less efficient than domestically owned firms. This may be due to foreign firms operation in unfamiliar circumstances. An alternative hypothesis, developed by Wells (1973) in the context of foreign investment in Indonesia, claims that foreign firms do not simply maximize output given inputs but are more concerned with the smoothness of operation and the engineering aesthetic.

Comparing the analysis of covariance result with the MLE result found in Table 1, it is found that the capital coefficient is lower and the labor coefficient is higher with analysis of covariance estimation. Both coefficients are significant even though the standard errors of the analysis of covariance estimates are larger than the MLE estimates. On this basis, it is tempting to conclude that inefficiency is positively correlated with labor inputs and negatively correlated with capital inputs. Under these circumstances, as argued by Mundlak (1978), the labor and capital inputs should be explicitly included in the model in (18) to detect the correlation. However, when these two variables are included in (18), they are not significant. Hence, there is no evidence that such correlation exists in our sample when other firm characteristics are controlled.

Investigation of the sources of inefficiency can be performed within the variance components model by adding firm characteristics thought to be correlated with inefficiency as extra regressors in the estimated production functions. Results of such an estimation are reported in Table 3, where firm size, age and ownership characteristics are added as extra regressors.[8] The coefficients of these firms characteristics in all three estimations are of the same sign and magnitude as those found in equation (18). A joint test comparing the MLE specification of Table 3 with that of Table 1 finds that the addition of the three variables is highly significant with a -2 log likelihood ratio of 18.7126 and the χ^2 distribution with three degrees of freedom. In addition, the elasticities on both labor and capital become smaller and thus returns to scale fall. This is due to the inclusion of a measure of firm size which is correlated with efficiency in the regression. The inclusion of time interaction variables for labor and capital are also not significant in this specification.

Comparing the MLE specification of Table 1 and 3, note how the estimates of σ_v^2 do not differ in the OLS and GLS cases and are only slightly smaller in the MLE case when the extra regressors are added. On the other hand, estimates of σ_u^2 fall nearly 65 percent in the case of estimation with OLS and GLS and 57 percent with the MLE procedure. These three firm characteristics thus

[8]Other firm characteristics were also included but were not statistically significant, see footnote 7.

Table 3

Explanation of Technical Inefficiency by Firm's Characteristics

in the Variance Components Model

	OLS	GLS	MLE
Constant	-4.4612	-4.4693	-4.1981
	(.6892)	(.6868)	(.6820)
1972D	.4898	.4840	.4832
	(.1569)	(.1568)	(.1572)
1973D	.4740	.4718	.4713
	(.1509)	(.1509)	(.1514)
Capital	.2155	.2085	.2083
	(.0488)	(.0484)	(.0481)
Labor	.6713	.6840	.6809
	(.0935)	(.0928)	(.0915)
Foreign	-.5854	-.5973	-.6231
	(.3078)	(.3076)	(.3033)
Year	.0307	.0308	.0315
	(.0071)	(.0071)	(.0070)
Size	.0221	.0218	.0219
	(.0085)	(.0085)	(.0082)
σ_u^2	.0549	.0562	.1306
	(----)	(----)	(.1437)
σ_v^2	.5566	.5554	.5594
	(----)	(----)	(.0779)
$E(e^u)$	.8374	.8358	.7663
$\bar{R}^2$	.7672		
Log-Likelihood value			-71.7008

explain more than half the variance of the permanent component. In terms of the mean efficiency measure, they explain 34.7, 35.0 and 27.6 percent of the inefficiency with the OLS, GLS and MLE methods respectively.

In order to demonstrate the quantitative importance of the relationship between firm characteristics and inefficiency, the firms of our sample have been grouped into quintiles according to firm characteristics and the lower and upper quintiles compared. The results of this comparison are presented in Table 4. There it is seen that the youngest firms commenced production on average in the year 1971.55 while the mean first year of production for the ten oldest firms was 1942. The youngest of two firms of these vintages which were identical in every other respect would be expected to produce 254 percent of the output of the older firms based on the MLE estimates. If these representative firms had the mean characteristics of their quintiles, the youngest firm would produce 201 percent of the output of the older firm because the younger firms are smaller and have greater foreign participation. Similar efficiency differences hold for the quintiles grouped by size. In the case of ownership, a domestic firm is expected to produce 86 percent more output than an otherwise identical foreign firm. However, since foreign firms are typically newer and larger than domestic firms, this efficiency difference shrinks to 10 percent when these other characteristics are taken into account.

It is interesting to note that in two of three cases, the most efficient quintile of firms has a higher average capital-labor ratio than the less efficient quintile. Ownership is the exception, as the less efficient foreign owned firms have a capital-labor ratio

Table 4

Firm Characteristics and Efficiency
Differences in a Sample of 50
Indonesian Weaving Firms

Groups		Age	Size	Capital-Labor Ratio[e]	A	B
		Mean Value of Group Characteristics				
AGE	youngest quintile[a,b]	1971.55	16.382	474.21	254%	201%
	oldest quintile	1942.00	17.470	187.68		
SIZE	largest quintile[a]	1960.00	33.293	445.14	196%	169%
	smallest quintile	1958.10	2.606	175.27		
OWNERSHIP	domestic	1958.50	12.561	272.57	186%	110%
	foreign	1971.25	26.468	501.64		
EFFICIENCY[c]	most efficient quintile	1964.20	15.903	276.89	-	511%[d]
	least efficient quintile	1955.30	8.342	147.165		
AVERAGE		1959.52	13.673	362.62		

A: Output of more efficient quintile relative to less efficient quintile due only to distinguishing characteristic.

B: Output of more efficient quintile relative to less efficient quintile due to all characteristics.

---

a) Includes three foreign owned firms

b) Quintile has eleven firms.

c) Efficiency as determined by analysis of covariance

d) Total efficiency difference not just difference attributable to the three firm characteristics.

e) Measured as total electricity consumption over three years divided by total real wage bill over three years.

twice that of domestic firms. The non-monotonic relationship between capital-labor ratios and efficiency is further evidence by noting that both the ten most efficient and ten least efficient firms as determined by analysis of covariance have capital-labor ratios below the mean.

6. Conclusion

In this paper, a variance components model for the estimation of stochastic frontier production functions from a time series of cross sections is introduced. Maximum likelihood methods are discussed and used to estimate the model for the Cobb-Douglas case using pooled data from individual firms in the Indonesian weaving industry. The maximum likelihood procedure, which begins with the OLS estimates, performs satisfactorily. Utilizing the DFP algorithm, convergence is achieved rapidly in every case. Mean efficiency for the Indonesian weaving industry is estimated at 67.7 percent. An investigation of the sources of inefficiency finds three firm characteristics, age, size and ownership, important. These characteristics explain more than half of the variance of the inefficiency component and 26.7 percent of mean inefficiency. With these firm characteristics controlled, there is no evidence of a correlation between efficiency and the use of capital and labor.

References

Afriat, S. N., (1972), "Efficiency Estimation of Production Functions," International Economic Review 13, pp. 568-598.

Aigner, D. J. and S. F. Chu (1968), "On Estimating the Industry Production Function," American Economic Review 5-8, pp. 826-835.

Aigner, D. J. C.A.K. Lovell, and P. Schmidt (1977), "Formulation and Estimation of Stochastic Frontier Production Function Models," Journal of Econometrics 6, pp. 21-37.

Amemiya, T. (1971), "The Estimation of the Variances in a Variance Components Model," International Economic Review 12, No. 1, pp. 1-13.

Amemiya, T. (1976), "A Note on A Random Coefficients Model," Technical Reports No. 226, Institute for Mathematical Studies in the Social Sciences, Stanford University.

Farrell, M. J. (1957), "The Measurement of Productive Efficiency," Journal of the Royal Statistical Society (A,general)120,pt.3, pp. 253-281.

Graybill, F.A. (1961), An Introduction to Linear Statistical Models, McGraw-Hill Book Co. Inc., New York.

Hoch I. (1962), "Estimation of Production Function Parameters Combining Time Series and Cross Section Data," Econometrica, 30, pp. 556-578.

Lee, L. F. and W. G. Tyler (1978), "The Stochastic Frontier Production Function and Average Efficiency: An Empirical Analysis," Forthcoming in Journal of Econometrics.

Maddala, G. S. (1971), "The Use of Variance Components Models in Pooling Cross-Section and Time Series Data," Econometrica 39, pp. 341-358.

Meeusen, W., and J. van den Broeck (1977), "Efficiency Estimation from Cobb-Douglas Production Functions with Composed Error," International Economic Review 18, pp. 435-444.

Mellor, P. (1976), "Efficiency Frontiers for Industrial Establishments of Different Sizes," Explorations in Economic Research 3, No. 3, pp. 379-407

Mundlak, Y. (1961), "Empirical Production Function Free of Management Bias," Journal of Farm Economics 43(1961), pp. 44-56.

Mundlak, Y. (1978), "On the Pooling of Time Series and Cross Section Data," Econometrica, 46, pp. 69-85.

Nerlove, M. (1965), Estimation and Identification of Cobb-Douglas Production Functions, North-Holland Pub. Co., Amsterdam.

Pitt, M. M. (1978), "Indonesia" in Anne O. Krueger ed. Alternative Trade Strategies and Employment in Developing Countries, Vol. I, (National Bureau of Economic Research, forthcoming).

Richmond, J. (1974) "Estimating the Efficiency of Production," International Economic Review 15, June, pp. 515-521.

Timmer, C. P. (1971), "Using a Probabilistic Frontier Production Function to Measure Technical Efficiency," Journal of Political Economy 79, pp. 776-794.

Wallace, T. D., and Ashiq Hussain (1969), "The Use of Error Components Models in Combining Cross Section with Time Series Data," Econometrica 37, pp. 55-72.

Wells, L.T. (1973), "Economic Man and Engineering Man: Choice of Technology in a Low Wage Country," Public Policy 21, No. 3, pp. 319-342.