A DYNAMIC DOMINANT-FIRM MODEL OF INDUSTRY STRUCTURE*

by

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1. Introduction

Some of the stylized facts of industry structure that an adequate theory should account for, are:

1. Many manufacturing industries are characterized by highly unequal market shares. A common pattern is for the largest producer to be about twice the size of the second largest, who is substantially larger than the third.
2. The rankings of firms in industries according to their market shares, to the extent that these shares are significantly different, are fairly stable over time.
3. The market share of the dominant firm typically declines over time.

Evidence of this, in particular on the last point, can be found in Burns [3, pp. 77-140], who describes the histories of several industries for 3 or 4 decades. On page 142 he says: "It appears to be the common fate of leaders to suffer a decline in their proportion of the total business in the market." Scherer [27, pp. 217-218] also provides evidence to substantiate this claim. For example, the U.S. Steel production of ingots and castings[1] declined from 65 percent of total U.S. production in 1902 to 52 percent in 1915, 39 percent in 1931, 28 percent in 1961, and 21 percent in 1968. In 1919, the American Viscose Company controlled 100 percent of the domestic rayon market. Once key patents expired, new entry caused its share to fall to 42 percent in 1930 and to 26 percent in 1949. American Can controlled 90 percent of all tin can output in 1901. Its high pricing policy encouraged new entrants, and its market share fell to 63 percent in 1913 and to 40 percent by 1960.

Similar histories are observed in corn products refining, farm implements, synthetic fibers, aluminum extrusions, and, on the regional level, in the gasoline industry.

Nevertheless, in spite of the decline in market shares, a few firms have retained dominant positions in their markets for decades. Burns [3] found substantial evidence of price-leadership behavior among some of these firms. Dominant-firm or price-leadership models attracted a lot of attention in the fifties and before. However, as Cyert and March [6] pointed out, the fact that the market share of the dominant firm has shown a steady downward trend is difficult to explain on the basis of the traditional price-leadership model. Worcester [30], who, according to Scherer [27, p. 216], provided the first complete and still definitive analysis of the dynamics of dominant-firm pricing, concluded that the dominant-firm case is a short-run phenomenon that will break down in the long run. If this is the case, it is hard to explain why this process would take three-fourths of a century or more as is indicated by the examples above.

This paper represents an attempt to re-examine the dominant-firm case in a model which is inherently dynamic in the sense that there are structural interconnections over time. This contrasts with the static models or sequences of static models which have dominated the industrial organization literature up to now.[2] The main dynamic aspect has typically been the description of how expectations are formed, and even this often in the most naive way.[3] Focusing on how firms may behave out of equilibrium may tell something about stability, but it is difficult to derive testable hypotheses of any interest from such analyses. Besides, even in equilibrium, firms are confronted with an inherently dynamic situation for which the static models may not be rich enough to provide the insights needed.

In our model, the capital stock at the start of each period uniquely determines output (and sales) in that period. We assume increasing cost of adjustment in changing capacity from one period to the next. This is a well-known theoretical explanation, in an optimization framework, of the empirical fact that firms do not immediately adjust their capital stock to the desired levels.[4] We also let the industry demand curve shift according to an autoregressive process, thus facing the firms with a situation similar to a business cycle.

The basic behavioral assumption will be noncooperative. However, for purely noncooperative behavior to explain observed consistent differences in market shares one may typically resort to different cost structures, which is unsatisfactory without an explanation of how these cost differences came about.[5] We shall instead assume that one firm, for instance the original monopoly firm in the industry, is dominant in the sense that it takes into account how the other firms will react to its decisions. In the existing dominant-firm literature it was often assumed that the other firms in the industry behaved competitively. Since the number of rivals is not necessarily very large, we assume that they behave noncooperatively among themselves. We also indicate how this structure can be generalized to allow for more general distributions of market shares.

Simple static models generally do not give a satisfactory explanation of why there is only a limited number of firms in many industries. Several "barriers to entry" have been suggested, in particular in the empirical literature.[6] Barriers that are often mentioned, are limit pricing, which has received a lot of theoretical attention,[7] economies of scale, and capital requirements.

Although limit pricing might be relevant for a dominant-firm model, we shall ignore the issue in this paper. Regarding the second barrier, we are assuming constant returns to scale in the long run in order to be consistent with the large body of evidence supporting this hypothesis in many industries.

This leaves the capital-requirements barrier, which will turn out to be important in our dynamic model as a determinant of the number of firms in the industry. When entering an industry, a firm has to make initial investments which imply negative cash inflows for a number of periods. With a positive interest rate these initial outlays will count heavily when the total sum over the horizon of the net discounted cash inflows is computed. This sum will become small compared to the corresponding sum for a firm already in the industry, perhaps even negative, in which case it clearly does not pay to enter. Thus it becomes important to analyze not only stationary points, but also the equilibrium paths toward these points.

We first discuss in Section 2 a simple static model which will turn out to be related to the dynamic model in a certain way. In Section 3 the dynamic model is presented, and the equilibrium concept used in this paper is discussed. In Section 4 some equilibrium solutions are computed, and certain characteristics are emphasized which appear to be consistent with empirical facts of many industries.

Our equilibrium is defined in policy or decision rule space. In order for an equilibrium concept to be of any interest, there should be a tendency in this economy toward the equilibrium. This issue is clarified in Section 5, and it is argued that our equilibrium concept is superior to possible alternative concepts in this regard. However, one possible source of instability is pointed out. Some concluding comments are provided in Section 6.

2. A Simple Static Model

In most textbooks on microeconomics, game theory is listed as one of many approaches to oligopoly theory. The game-theoretic solution usually referred to is the minimax solution.[8] However, some of the other solutions, such as the Cournot solution and the Stackelberg or dominant-player solution (see [5] and [28]), are also really game-theoretic solutions with different behavioral assumptions. What is special about the Cournot solution, for instance, is the attention given to the question of stability. This solution assumes that firms have static expectations, that is, each firm expects the other firms to produce the same output next period as they did this period.[9]

As a simple illustration, consider the following model. Assume that the inverse demand function is given by

$$p = a - b \sum_{j=1}^{n} y_j \quad ,$$

where p is the price, y_j is output of the jth firm, and a and b are given parameters. Assuming no costs of production, a profit maximizing firm i would attempt to maximize py_i . In the Cournot model each firm would take y_j , $j \neq i$, as given and determine the optimal output. If expected and realized output are different, each firm will revise its expectations and think that the output of the rivals will remain the same in the next period as in the last period. If the system is stable, one will eventually get arbitrarily close to an equilibrium such that there are no surprises with regard to the actual outputs of the other firms.[10] This solution is the non-cooperative solution in which each firm has perfect foresight in the sense that the expected outputs turn out to be the actual ones.

For our particular model the first-order conditions for a maximum for each firm are

$$a - 2by_i - b \sum_{\substack{j=1 \\ j\neq i}}^{n} y_j = 0 \;, \qquad i = 1, \ldots, n \quad .$$

The noncooperative equilibrium can be found by solving the following system of equations:

$$\begin{bmatrix} 2b & b \ldots & b \\ b & 2b \ldots & b \\ & \cdot \;\; \cdot \;\; \cdot & \\ b & b \ldots & 2b \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} a \\ a \\ \vdots \\ a \end{bmatrix} \quad .$$

The inverse of the coefficient matrix above can be written as $1/b[I_n - \frac{1}{n+1} J_n]$, where I_n is the n-dimensional identity matrix, and J_n is the $(n \times n)$-matrix with each element equal to one.[11] Using this result we easily obtain the solution

$$y_i = \frac{a}{b(n + 1)} \;, \qquad i = 1, \ldots, n \quad .$$

Leader-follower type models have a long tradition in the theory of duopoly, dating back at least to the work of Stackelberg [28]. In this paper we make the generalization of allowing more than one follower. The dominant firm announces its decision first, or, for some reason or other, its decision is taken as given by the rivals who behave noncooperatively among themselves. This enables the dominant firm to take into account the rival firms' reactions to its decisions. For our static model, with firm n being the dominant firm, the decisions of the rivals can be written as

$$y_i = \frac{a}{bn} - \frac{1}{n} y_n , \qquad i = 1, \ldots, n-1 ,$$

using the result above for the purely noncooperative case. The first-order condition for the dominant firm is

$$a - b \sum_{j=1}^{n} y_j - b y_n \left(1 + \sum_{j=1}^{n-1} \frac{\partial y_j}{\partial y_n}\right) = 0 .$$

But $\partial y_j / \partial y_n = -1/n$, $j = 1, \ldots, n-1$, which means that we can write the condition as

$$a - \frac{n+1}{n} b y_n - b \sum_{j=1}^{n-1} y_j = 0 .$$

Substituting the solutions for the rivals, we obtain

$$y_n = \frac{a}{2b} ,$$

and the outputs for the rivals can now be written as

$$y_i = \frac{a}{2bn} , \qquad i = 1, \ldots, n-1 .$$

We thus see that the equilibrium output of the dominant firm remains unchanged as more firms enter the industry, although its market share will decline somewhat. The dominant-firm market share is given by

$$s_n = \frac{y_n}{\sum_{j=1}^{n} y_j} = \frac{\frac{a}{2b}}{\frac{a}{2b} + \frac{(n-1)a}{2bn}} = \frac{n}{2n-1} ,$$

which approaches 0.5 as the number of firms increases. We note that the parameters of the demand curve do not affect the market share.

3. A Dynamic Model of Oligopoly

We assume that the industry produces a homogeneous commodity, and that the inverse demand curve can be written on the form

$$p_t = a_t - b \sum_{j=1}^{n} y_{jt} \quad ,$$

where p_t is the price, a_t is a stochastic demand shift variable, b is a fixed parameter, y_{jt} is output (equal to sales) by firm j in period t, and n is the number of firms in the industry. The price can be thought of as being measured net of any constant unit production cost.

Output per period is assumed to depend on the capital stock at the start of the period. Without loss of generality we can choose units so that maximum output is equal to the capital stock. Assuming also that each firm always uses all of its capacity, we can write

$$(1) \qquad y_{i,t+1} = (1-\delta)y_{it} + x_{it} \ , \qquad i = 1,\ldots,n \ ,$$

where x_{it} is investment by firm i in period t, and δ is the depreciation rate.

The unit cost of investment is assumed to be q as long as the capital stock is just maintained. For deviations from this investment rate δy_{it} we assume a quadratic cost of adjustment $c(x_{it}-\delta y_{it})^2$. This insures that we have constant returns to scale in the long run. The fact that the cost of adjustment depends only on the individual firm's own change in capacity means that these costs essentially can be thought of as being internal to the firm. For instance, it may require more resources to increase capacity fast in one period than if the expansion is spread across several periods. Another explanation is that a given quantity of machines makes a better plant in the sense of yielding more productive services the longer the period is over which it is assembled. One could easily have used an alternative (or additional) assumption with cost of adjustment depending on the deviation of total industry investment from what is needed to maintain the industry capital stock. In any case, the cost structures of the firms are assumed to be the same. The problem formulations of each firm are therefore symmetrical in some sense.

Each firm is assumed to maximize the expected sum of net discounted cash inflows over the horizon. For a T-period horizon (T possibly being infinite) we can write the objective functions as

$$(2) \qquad E\{\sum_{t=1}^{T} \beta^{t-1} w_i(y_t, a_t, x_{it})\} \quad ,$$

where

$$w_i(y_t, a_t, x_{it}) = (a_t - b\sum_{j=1}^{n} y_{jt})y_{it} - qx_{it} - c(x_{it} - \delta y_{it})^2 \quad ,$$

and $\beta = 1/(1+r)$, where r is the interest rate.

Randomness enters the model through the parameter a_t of the demand function. We assume that a_t is subject to a first-order autoregressive process given by

(3) $a_t = \rho a_{t-1} + \mu + \varepsilon_t$,

where $-1 < \rho < 1$, $\mu > 0$, and ε_t are random disturbances uncorrelated over time with mean zero and finite variance σ_ε^2 .

In this paper we assume that the nth firm is dominant, while the other n - 1 firms behave noncooperatively, given what the dominant firm does. This still leaves three possible solutions which may be candidates for an equilibrium concept, namely the open-loop, closed-loop, and feedback solutions. These three solutions will in general all lead to different time paths of the variables, even in the absence of uncertainty.

Consider first the case in which each firm would determine a sequence of values for its decision variable over the entire horizon, given the initial state. This corresponds to the open-loop solution, and the decisions can be written as

$$x_{it} = X^o_{it}(y_1, a_0, x_{n1}, \ldots, x_{nT}, \varepsilon_1, \ldots, \varepsilon_{t-1}), \qquad i = 1, \ldots, n-1 ,$$

$$x_{nt} = X^o_{nt}(y_1, a_0, \varepsilon_1, \ldots, \varepsilon_{t-1}), \qquad t = 1, \ldots, T .$$

Alternatively, the solutions could be given in terms of policy or decision rules, that is, functions of the current state. For our case this alternative could be written as

$$x_{it} = X_{it}(y_t, a_{t-1}, x_{nt}), \qquad i = 1, \ldots, n-1 ,$$

$$x_{nt} = X_{nt}(y_t, a_{t-1}), \qquad t = 1, \ldots, T .$$

Both the closed-loop and feedback solutions can be characterized by decision rules of this form. The closed-loop solution gives the highest possible value of the T-period objective function of the dominant firm over the space of feedback rules when taking into consideration the effects of present and future decision rules on the rivals' decisions. This solution represents a commitment for the entire horizon, but, after the first period, the original plan is no longer optimal for the remaining $T - 1$ periods. The reason is that, since the first period is now part of history, the effects of the decisions in the remaining $T - 1$ periods on the rivals' decisions in that period will no longer be taken into account. This leads to an inconsistency under replanning, a property which, by the way, is shared by the open-loop solution.[12] The feedback solution, on the other hand, is best at every time period of the horizon, given the state at that time, and given that decisions will be similarly selected in the future. In this case there is obviously no inconsistency under replanning.[13]

A requirement of an equilibrium should be that it is stable in the sense that there is a tendency for the decision rules to move toward the equilibrium decisions. It seems reasonable, in particular in a stochastic environment, for an equilibrium to be characterized by solutions in policy or decision rule space rather than sequence space. A property making the feedback solution a reasonable candidate for an equilibrium is that if any one firm perfectly foresees the rival firms' decision rules and solves the control problem resulting when these rules are considered as constraints along with the relations (1) and (3), then its equilibrium decision rules turn out to be the optimal ones for this problem. Moreover, if the decision rules start out by being away from equilibrium, but are modified, for example according to an adaptive process, as more is learned about the other firms' behavior, then, under reasonable assumptions, this process will converge toward the feedback solution.[14]

We shall now define formally what we mean by equilibrium.

Definition: An equilibrium for each time period t, $t = 1,\ldots,T$, is a set of decision rules $x_{it} = \bar{X}_{it}(y_t, a_{t-1}, x_{nt})$, $i = 1,\ldots,n-1$, and $x_{nt} = \bar{X}_{nt}(y_t, a_{t-1})$ such that

$$\max_{x_{it}} E[w_i(y_t, a_t, x_{it}) + \beta v_{i,t+1}(y_{t+1}, a_t) \,|\, \bar{X}_{jt},\ j=1,\ldots,n,\ j \neq i]$$

$$= E[w_i(y_t, a_t, x_{it}) + \beta v_{i,t+1}(y_{t+1}, a_t) \,|\, \bar{X}_{jt},\ j=1,\ldots,n]\ ,\ i=1,\ldots,n\ ,$$

where

$$v_{i,t+1}(y_{t+1}, a_t) = E[\sum_{s=t+1}^{T} \beta^{s-t-1} w_i(y_s, a_s, x_{is}) \,|\, x_{is} = \bar{X}_{is}(y_s, a_{s-1}, x_{ns}),\ i=1,\ldots,n-1\ ,$$
$$x_{ns} = \bar{X}_{ns}(y_s, a_{s-1}),\ s=t+1,\ldots,T]$$

In the definition above $v_{i,t+1}$ gives the value to firm i of all firms following the equilibrium decision rules from period $t+1$ until the end of the horizon. In other words, each firm chooses the best decision rule for period t, given the last observed state variables y_t and a_{t-1}, the decision rules of the other firms, and that decisions will be similarly selected in periods $t+1,\ldots,T$.

The equilibrium decision rules can be computed by backward induction. As a first step in the computations for time period t we find

$$(4) \qquad V_{it}(y_t, a_{t-1}, x_t^{(i)}) = \max_{x_{it}} E\{w_i(y_t, a_t, x_{it}) + \beta v_{i,t+1}(y_{t+1}, a_t)\}$$

$$i = 1,\ldots,n-1\ ,$$

subject to the constraints (1) and (3). The solutions will be of the form

$$x_{it} = f(y_t, a_{t-1}, x_t^{(i)}) \ , \quad i = 1,\ldots,n-1 \ .$$

Assuming noncooperative behavior among these firms, we can solve the system of $n-1$ equations to get

$$(5) \qquad x_{it} = \bar{X}_{it}(y_t, a_{t-1}, x_{nt}) \ , \ i = 1,\ldots,n-1 \ .$$

The dominant firm can determine its decision rule at time t by solving

$$v_{nt}(y_t, a_{t-1}) = \max_{x_{nt}} E\{w_n(y_t, a_t, x_{nt}) + \beta v_{n,t+1}(y_{t+1}, a_t)\}$$

subject to (1), (3) and (5). The solution is of the form

$$(6) \qquad x_{nt} = \bar{\bar{X}}_{nt}(y_t, a_{t-1}) \ .$$

When we substitute the expression for x_{nt} from (6) into (4) we get

$$v_{it}(y_t, a_{t-1}) = V_{it}[y_t, a_{t-1}, \bar{\bar{X}}_{nt}(y_t, a_{t-1})] \ ,$$

which completes the computations for time period t .

In the case of infinite horizon the stationary decision rules would be of the form $x_i = X_i(y,a_{-1},x_n)$, $i = 1,\ldots,n-1$, and $x_n = X_n(y,a_{-1})$, which would satisfy a set of n functional equations, one for each firm. The computational procedure outlined above could then be the basis for successive approximations by value iterations, where the solution for a T-period horizon would be used to compute the solution for a (T+1)-period horizon, and so one. An alternative computational procedure for obtaining the stationary decision rules would be policy iterations rather than value iterations.

For our model, in which the objective functions are quadratic and the constraints linear with additive disturbances, the solutions are easily computable. The value functions are all quadratic, and the decision rules linear.[15] The results reported in Section 4 were computed with T large enough to make the difference between the first and second period decision rules very small.

The results above can easily be extended to more complex hierarchical structures. At each level of dominance we could in general have several firms behaving noncooperatively among themselves while taking into account the reaction functions of the firms on lower levels, and taking as given the decisions of the higher-level firms. An interesting special case would have one firm on each hierarchical level of decreasing dominance. These firms would all end up with different market shares in the stationary equilibrium. Thus our model could be consistent with more general distributions of market shares that are observed in many industries.

4. Some Comparisons of Dynamic Equilibrium Solutions

In this section we choose values for the parameters of the model of Section 3, compute equilibrium solutions, and check the consistency of the results with empirical facts of industry structure. Some attention will be given to the question of what determines the number of firms in the industry. Finally, some conclusions will be drawn regarding the characteristics of the solutions when certain parameters change. These conclusions are tentative in the sense that they are based on simulations of numerical examples only.

We shall abstract from foreseen growth in demand, but shall sometimes investigate the effects of unforeseen jumps in demand which are then perceived by the firms as being permanent.

We assume that the firms in an industry with a given number of firms do not take into account the threat of entry when making their decisions. This assumption may be somewhat unrealistic, but is certainly less unsatisfactory the more firms there are in the industry. If a new firm actually does enter, the decision rules are then modified to the new equilibrium rules for an industry with one more firm.

We shall be interested in points which are stationary in the stochastic sense of the variables being at these points on the average while fluctuating around them as demand fluctuates. However, sometimes the path toward the stationary point will also be important, especially when a new firm enters, or when the demand changes permanently.

The basic example will have the following values for the parameters: $q = 2$, $c = 5$, $\delta = 0.1$, $r = 0.1$, $\mu = 0.2$, $\rho = 0.8$ and $\sigma_\varepsilon = 0.05$. The values for μ, ρ and σ_ε imply an average for a_t of one and a standard deviation of 0.083. The relative values of the per unit investment cost q and the cost of adjustment factor c should be reasonable. For instance, assume that a firm with capital stock of 0.2 wants to increase its capacity by 5 percent in one period. While the cost per unit of just maintaining its capital stock is 2, we now have to add to the total investment expenditures for this period the amount 0.0005. If we make this out in per unit terms, the total per unit cost is 2.017, or less than a one percent increase over the normal cost of maintaining the capital stock. This does not seem overly much considering the sizeable increase in capacity. With a 10 percent increase in capacity the per unit cost increase over normal cost would be 2.5 percent.

Some results for the numerical example are presented in Table 1. In addition to output (equal to capital stock) and price for stationary solutions, the expected present values of the firms are computed for two alternative starting points. The first assumes that the firms have already reached the stationary point (thus starting with capital stocks 0.2044 and 0.2348 in the case of duopoly), while the other computation is for the case of one firm just entering the industry (starting at 0 and 0.3 in the case of duopoly).

Bearing in mind the simple static model presented in Section 2, at least two things are striking about these results. While the static model predicts the output of the dominant firm to remain the same regardless of the number of firms entering and the market share to approach 50 percent,

Table 1

Number of firms in industry	2	3	4	5	6
Stationary solution:					
Output of dominant firm	.2348	.1969	.1736	.1582	.1473
Output per rival firm	.2044	.1511	.1188	.0974	.0823
Market share of dominant firm	.5345	.3945	.3276	.2888	.2635
Price of output	.5608	.5008	.4701	.4523	.4410
Expected value of dominant firm	.9330	.6524	.5166	.4395	.3908
Expected value per rival firm	.8126	.5009	.3535	.2708	.2187
Path toward stationary solution:					
Expected value of entering firm	.2482	.1067	.0561	.0335	.0218

we now see that the dominant firm output at the stationary level has decreased from 0.3 in a monopoly to 0.1473 when five additional firms have entered, and the market share has decreased to just over 1/4. This is even more striking since, as we pointed out, the cost of adjustment is quite small. We also see that the present value of what a firm can earn as it is entering the industry is a lot less than the present value after the stationary level of capital stock has been reached. The relative difference between the two present values becomes larger the larger is the number of firms already in the industry. For the sixth firm the present value is only 0.0218 compared to 0.2187 at the stationary level. Note that we have assumed that the investment made in any one period does not yield any productive services until the next period. In some industries longer lags are probably realistic, in which case these differences are likely to become even more dramatic.

The description in Burns [3] of several industries over 3 or 4 decades gives the impression that a typical pattern of development is that the output of dominant firms increases with increasing demand while at the same time experiencing a steady decline in market share. We shall see that such a development is possible without the dominant firm loosing its dominance as we have defined it.

For simplicity we assume that the situation described in Table 1 is disturbed by an unexpected jump in demand, which the firms then correctly perceive as being permanent. We assume that μ increases from 0.2 to 0.22, increasing the average intercept $\bar{a}$ with the price axis from 1 to 1.1, keeping the slope of the demand curve constant. Some results for this case are given in Table 2.

<u>Table 2</u>

Number of firms in industry	3	4	5	6
Stationary solution:				
Output of dominant firm	.2298	.2025	.1845	.1718
Output per rival firm	.1763	.1386	.1136	.0961
Market share of dominant firm	.3945	.3276	.2888	.2635
Price of output	.5176	.4818	.4610	.4478
Path toward stationary solution:				
Expected value of entering firm	.1450	.0762	.0454	.0295

If the number of firms were to remain unchanged in the new situation, say at 5, all the firms' stationary outputs would have increased, with market shares being the same as before the demand shift. However, in the new situation the value of a sixth firm entering has increased from 0.0218 to 0.0295. This may now encourage a new firm to enter, in which case the new stationary equilibrium is characterized by the dominant firm producing 0.1718 compared to 0.1582 before the demand shift, but its market share has dropped from 0.2888 to 0.2635. We also see that at first the price will increase, for then eventually to drop below its original level.

Since the cost of adjustment associated with changes in capacity is the basic feature making our model dynamic, it may be of interest to know how sensitive our results are to changes in the cost of adjustment factor c. It turns out that if the cost of adjustment increases, ceteris paribus, the market share of the dominant firm becomes smaller. If, for instance, the parameter c is increased from 5 to 10, the stationary output of the dominant firm in a five-firm industry is 0.1430 instead of 0.1582, while the output of each of the four other firms is 0.1010 instead of 0.0974. We also found that higher costs of adjustment made the present values of the stationary solution for the nondominant firms higher. However, at the same time it became less profitable for a new firm to enter. With an increase in c from 5 to 10 the present values of the stationary solutions increased from 0.2708 to 0.2816 in a five-firm industry, while the present value of the entering firm decreased from 0.0335 to 0.0286. This shows that it can be important to know something about the equilibrium path toward the stationary level of capital stock when evaluating the capital requirements barrier to entry in the presence of cost of adjustment.

It has been suggested that industry demand typically becomes more elastic over time (see e.g., Scherer [27, pp. 213-216]). In the present model, in which the demand function is approximated by a linear curve, a reasonable way to investigate this possibility is to pick the stationary point on the demand curve for a given number of firms and then reduce both the slope parameter b and the average of the demand shift variable a_t so as to make the new demand curve go through the same point on average. This was done with a reduction in b from 1 to 0.9 for a number of combinations of the remaining parameters, and unlike the model in Section 2, the result was always a decrease in the market share of the dominant firm.

The present values for the entering firms in the examples presented so far have still been large enough for it to be profitable to enter. In reality there is probably a more or less fixed cost associated with entering an industry. It is likely to cost more for a firm starting from zero to increase its capacity by some amount than it would cost an established firm to increase its capacity by the same amount during the same period of time. Our present values can then be viewed as limits which must not be exceeded by this fixed cost in order for new firms to want to enter. The stochastic demand can work in two directions. Entering the industry in a period when demand happens to be high will increase expected profits. Also, expected present value of each firm is higher the larger is the variance in demand, although the contribution of this factor is very small in the examples above. However, the possibility of unfavorable drawings from the distribution of ε_t just after entry, resulting in losses which can be hard to make up for later due to the positive interest rate, may be a deterrent to prospective entrants. For example, the sixth firm entering the industry, the results of which are reported in Table 1, can expect a present value of 0.0218. However, there is a probability of approximately 0.05 of negative present value.

5. Stability of Equilibria

Stability in the context of this model can mean at least two different things. One can talk about whether the state will approach some stationary point (in a deterministic or stochastic sense) over time. Alternatively, one can refer to the question of whether the decision rules in the economy have a tendency toward some equilibrium rules, given shocks of some sort, or initial ignorance on the part of the agents. We are here interested in stability in the latter sense.

Assume that each firm, when solving its T-period maximization problem (for simplicity we think of T as being infinite), takes as given a set of decision rules expected to be used by the other firms throughout the horizon. If the industry is not in equilibrium, the new decision rule that is computed may be different from the one used in the past. This change leads the other firms to revise their expectations of what this firm's decision rule will be in the future, and so on.

As an example, consider a duopoly with firm 2 as the dominant firm. Firm 2 and firm 1 initially expect the other firm to behave according to the rules $X^e_{10}(y,a_{-1},x_2)$ and $X^e_{20}(y,a_{-1})$, respectively. Assume now that each firm in turn revises its expectations, say according to an adaptive process in terms of decision rules:

$$X^e_{i,j+1} = X^e_{ij} + \lambda_i(X_{ij} - X^e_{ij}) \ , \quad i=1,2,; \ j=0,1,2,\ldots,$$

where X^e_{ij} is the decision rule that the other firm, at stage j of this process, expects firm i to follow in the future, X_{ij} is the decision rule that firm i actually uses at this stage, and λ_i is an adjustment coefficient such that $0 < \lambda_i \leq 1$. The special case of $\lambda_i = 1$ corresponds to static expectations in decision rule space. If this process converges, it will converge toward the equilibrium solution outlined in Section 3, thus indicating stability of the solution under this type of expectations formation. The presumption is not that this process provides a realistic description of behavior out of equilibrium. Rather, the convergence of such processes is just meant to indicate that the equilibrium concept is a reasonable one to use. The process outlined above, by the way, provides an alternative method of computing the stationary decision rules for an infinite horizon.

There is one possible source of instability remaining, namely the fact that in general an enlightened dominant firm can increase its profits by following a decision rule different from the equilibrium rule. If the dominant firm has perfect knowledge of how its future decisions will affect the form of the decision rules of the rival firms in earlier periods, it may realize that gains can be made by following a different decision rule.

To show that the potential gains can be quite substantial, consider the duopoly example in Section 4, the results of which are summarized in Table 1. For the purpose of this discussion, we shall abstract from randomness, which does not change the stationary point other than making it deterministic rather than stochastic. The equilibrium dominant firm decision rule then turns out to be

$$x_{2t} = -0.0829y_{1t} - 0.1881y_{2t} + 0.0846 .$$

If this is the decision rule followed when a new firm enters what was originally a monopoly, the present value of the profits from then on is 1.218 for the dominant firm and 0.2470 for the entering firm. Suppose instead that the dominant firm decides to use the rule

$$x_{2t} = 4.5y_{1t} + 0.12y_{2t} - 0.029 \quad ,$$

that is, any increases in the size of the entering firm will be met by much larger increases in its own size. If the entering firm takes this as given and selects the best possible decision rule, the stationary outputs will be 0.3 and 0.005 for the dominant and entering firm, respectively, and total present value of profits will be 1.621 and 0.017, which in the case of the dominant firm is quite near the present value of monopoly profits. In addition to showing the potential profit gains from deviating from the equilibrium rules, the above may be a reasonably good description of what a monopoly firm might try to do to prevent entry. Most likely the entering firm would refuse to take this decision rule as given and instead go ahead and increase capacity substantially, in which case the dominant-firm decision rule would no longer be so profitable. This means that not only is the closed-loop decision rule inconsistent under replanning, but it is also highly improbable that rivals would accept it as given.

Even when the duopoly is already at the stationary point of Table 1, there is a decision rule which, if followed by the dominant firm throughout the future and taken as given by the other firm, would increase the present value of the dominant firm from 0.933 to 1.23. Suppose the dominant firm attempted to do this. After one period the industry would have moved from point y_1 to y_2, and now, forgetting about period one, a different rule would be better from then on than the one used in period one, and similarly for the next periods. If the dominant firm could not resist the temptation to change the decision rule after the first period, the rival firm would eventually predict this to happen in the future as well, and this would affect its behavior from the very beginning.

For completeness we may point out that also the open-loop solution gives higher profits for the dominant firm than the feedback solution, although substantially less than the closed-loop solution. However, as noted in Section 3, there are certain problems with it from an equilibrium point of view.[16]

It is interesting to note that in a deterministic model (with $\mu = 1$, $\rho = 0$, and $c = 0$), the open-loop stationary point is the solution that can be obtained from a static model with user cost of capital equal to $q(\delta+r)$. The model in Section 2 is precisely this translation of the dynamic model in Section 3 if we let $a = \mu - q(r + \delta)$. One might think, then, that if the firms happened to start out at the stationary point, $y_1 = 0.15$ and $y_2 = 0.3$, then it would be best for them to continue at that point. This turns out not to be the case, which again is an example of inconsistency of the solution under replanning. In spite of the cost of adjustment, it would be optimal (in the open-loop solution) for the dominant firm to reduce the capital stock to 0.288 in the first period, and to 0.283 and 0.281 in the following two periods, with corresponding increases by its rival to 0.152, 0.154, and 0.156, for then slowly to move back toward the stationary point. Any time the dominant firm decided to replan, a similar disruption would be optimal from then on.

In order for a firm to want to be dominant, it obviously has to make higher profits than if it behaved noncooperatively along with the others. Given that such gains in profits are made, one might ask why the rivals would continue to be passive rather than trying to break up the dominance of the one firm. One possible answer is obtained by going back to the example in Table 1. Assume that there are 4 firms in the industry, and that the expected present value of the fifth firm is just low enough to keep it from entering. Now, if the nondominant firms were successful in breaking up the dominance of the dominant firm, turning the industry into a purely noncooperative one, a new stationary point would result. At this point the present value of a fifth firm would have increased from 0.0335 to 0.0422, which presumably would be sufficient to make it decide to enter the industry. In this new 5-firm industry the expected present value of each firm at the stationary point would be 0.3089, which is less than the original present value, 0.3535, of each of the three nondominant firms when the fourth one was still dominant and the fifth had not entered. We thus see that the dominance of one firm may be acceptable to the other existing firms as a barrier to entry, at least as long as the dominant firm does not attempt to follow the closed-loop rather than the feedback rule.

6. Concluding Comments

In this paper we have provided an equilibrium framework for industry structure consistent with certain observed persistent differences in market shares within industries, with the market share of the dominant firm typically declining slowly over time. Some essential features of the model are the assumption that one firm is dominant in the sense of taking into account the rival firms' reactions to its decisions, and the assumption of increasing costs associated with changes in capacity, thus introducing structural interconnections over time which make the model inherently dynamic. The cost structure was assumed to be the same for all firms. The predictions of this model turn out to be quite different from those of the corresponding static model.[17]

Determining an appropriate equilibrium concept for a dynamic dominant-firm model was shown to involve some rather unusual problems. An important consideration must be the stability of the equilibrium in the sense that there is a tendency in the economy toward this equilibrium, also when the equilibrium is characterized by dynamic decision rules as is the case in the present paper. We argued why our equilibria are likely to be stable in this sense, given that there is a dominant firm. We also showed that the very existence of a firm which is dominant rather than behaving noncooperatively along with the others, may provide a barrier to entry, thus making its dominance acceptable to the rivals. However, further research is needed on the stability of dynamic dominant-firm equilibria and on related issues, such as determining the conditions under which the dominance of one firm can be expected to persist.

Footnotes

1. This paragraph is based for the most part on Scherer [27] and references cited therein, and also on Burns [3].

2. Exceptions among noncooperative models are Clemhout et al. [4], Prescott [26], and Flaherty [11].

3. A typical example is the assumption of static expectations as in the Cournot model. More sophisticated reaction strategies are considered in Cyert and DeGroot [7, 8, 9], introducing learning over time, and in Friedman [12, 13].

4. See Eisner and Strotz [10], Lucas [24], and others.

5. Flaherty [11], in a recent paper, uses cost-reducing investment to explain cost differences. She is able to show that under certain assumptions stable noncooperative equilibria with unequal market shares are possible.

6. See for instance Bain [1] or Hall and Weiss [17].

7. See Kamien and Schwartz [19] and references cited therein.

8. See for instance Henderson and Quandt [18] or Scherer [27].

9. Examples of papers mainly concerned with the stability of the Cournot solution are McManus and Quandt [25], Hahn [16], and Hadar [15].

10. Telser [29] has shown that this process will not converge under static expectations when $n > 3$.

11. This result easily follows from Graybill [14, Thm. 8.3.4].

12. The same kind of inconsistency was shown by Kydland and Prescott [23] to exist in a model of economic stabilization with rational expectations.

13. Kydland [21] has a more detailed discussion of the three solution concepts. A simple illustration may be useful at this point. Assume that the horizon consists of two periods, $t-1$ and t , and that firm 2 wishes to maximize

$$w_{2,t-1}(x_{1,t-1},x_{2,t-1}) + w_{2t}(x_{1t},x_{2t})$$

subject to

$$x_{1,t-1} = X_{1,t-1}(x_{2,t-1},x_{2t})$$

and

$$x_{1t} = X_{1t}(x_{2,t-1},x_{2t}) \quad .$$

Taking $x_{1,t-1}$ as given and differentiating with respect to x_{2t} , we get:

$$\frac{\partial w_{2t}}{\partial x_{2t}} + \frac{\partial w_{2t}}{\partial x_{1t}} \frac{\partial X_{1t}}{\partial x_{2t}} = 0 \quad .$$

However, taking account of the effect of x_{2t} on firm 1's decision in period $t-1$, we get:

$$\frac{\partial w_{2t}}{\partial x_{2t}} + \frac{\partial w_{2t}}{\partial x_{1t}} \frac{\partial X_{1t}}{\partial x_{2t}} + \frac{\partial w_{2,t-1}}{\partial x_{1,t-1}} \frac{\partial X_{1,t-1}}{\partial x_{2t}} = 0 \quad .$$

Only if firm 2's decision in period t has no effect on firm 1's decision in period $t-1$ will the solution implied by the first equation be optimal for the two-period horizon as a whole.

14. Similar processes were used in [22] to investigate stability of a dynamic noncooperative model.

15. Computational details can be found in [20] and [21].

16. Brock [2] has studied open-loop solutions for a wide class of models.

17. As was pointed out, the static model can be viewed as the stationary-state version of the dynamic model when variational methods are used.

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