A MODEL OF GRAIN STORAGE AND HEDGING BY FARMERS

by

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1. Introduction

It is hoped that this paper will prove a useful start in developing expected utility analysis of the choices faced by various participants in futures markets and eventually some reconsideration of theories of futures markets from this point of view.

The model analyzed represents circumstances as faced by a grain farmer when his harvest is known and he is making storage and hedging decisions. The scope of the analysis is limited in several respects. A one-period model is employed; only a limited number of options are recognized; and interrelations between the grain enterprise and other economic activities of the decision maker are neglected.

Within this limited framework an effort has been made to chooose relevant options and to provide a reasonably comprehensive qualitative analysis. This should aid in the development of needed extensions of the present model. To furnish help with practical decisions, two important extensions would seem to be the incorporation of marketing and hedging decisions made during the growing season, and consideration of a variety of circumstances regarding the availability of credit to the decision maker (see [8]). In the present analysis receipts at different dates are compared by simply applying a known interest charge.

Results reported here are obtained using only very general assumptions (e.g., risk aversion) about the decision makers preferences (utility function) and beliefs (personal probabilities). This seems a desirable way to start in order to know to what extent later conclusions based on more specific assumptions depend on the more specific assumptions.

A more complete description of the circumstances envisaged in the present model is given in Section 2 along with an informal statement of some results. These results and some extensions are established in Sections 3 and 4. Section 5 contains some discussion of the results, and is followed by an appendix which gives proofs of several propositions needed in Sections 3, 4.

2. Circumstances and Main Results

A grain farmer has just harvested and has n bushels on hand. He must decide how much to sell now, how much to store, how much to constract for future delivery, and what position, if any, to take on the futures market.

Suppose that time τ some months ahead is the time of year when this grain usually attains its seasonal peak price and that his opportunity to sell forward would involve delivery at τ.[1] Let a represent his current cash price and c (known) the price at which he can contract for τ-delivery. Let m $(0 \le m \le n)$ be the amount he decides to store and g $(0 \le g \le m)$ be the quantity he decides to sell forward.

He can also take a hedging (short) position on the futures market. Let b be the current price (per bushel) for futures contracts maturing at τ. Assume that any physical grain stored and not covered by a forward contract will be sold at τ and any short futures position will be closed at τ.[2] His return will then be -

$$\pi = r[a(n-m) - fs] + cg - dm + A(m-g) + (b + f - q - B)s$$

where:

r: cumulation factor converting current dollars to dollars at time τ ; equals $1 + (\frac{j}{12}$ times interest rate) where j is number of months until time τ .

f: margin requirement per bushel for futures transactions.

d: marginal cost of storage.

A: a random variable, unknown price to be realized for local cash grain at τ .

B: a random variable, unknown price of maturing futures contracts at time τ .

q: commission on futures contracts.

s: size (bushel) of short position in futures market $(0 \leq s)$.

m, g, s are the decision variables. Rewriting -

$$\pi = ran + (A - ra-d)m + (b - (r-1)f - q - B)s + (c-A)g$$

$$= k_0 + (A-k_1)m + (k_2-B)s + (k_3 - A)g \quad .$$

The k_i are known when m, s, g must be decided. In the formal analysis which follows, it is assumed that the farmer acts as though he has a subjective probability distribution of unknown A and B and acts to maximize expected utility of return or gain with respect to that subjective distribution. It is also assumed that he is risk averse (would demand favorable odds to participate in a pure game of chance; has concave utility function) and that his utility function and personal probabilities satisfy certain mathematical regularity conditions. These assumptions are sufficient to determine a number of conclusions that will be summarized after a little additional terminology is noted.

Call $(A-k_1)$, (k_2-B), (k_3-A) the respective returns to storing, "futuring," and "forwarding." Futuring will be a brief synonym for "taking a short position on the futures market;" forwarding will mean "contracting for delivery at time τ of grain already stored."

Let $A - B = H$, the farmer's basis at time τ (see [1] for a discussion of basis). $(A-k_1) + (k_2-B) = k_2 + A - B - k_1 = k_2 + H - k_1$ will be called the return to futured storage. It represents the effect on final return of simultaneously placing a bushel in storage and increasing one's short futures position by a bushel. $(A-k_1) + (k_3-A) = k_3 - k_1$ represents the effect of simultaneously adding a bushel to storage and selling an additional bushel forward, and will sometimes be called the return to forwarded storage.

In Section 3 it is assumed that the basis H and the price B of a futures contract at maturity are statistically independent. It seems likely to me that this assumption will prove to be a good approximation to reality in some grain marketing situations but not in others.[3] Therefore, the effects of relaxing this assumption are examined in Section 4.

If X is any quantity unknown when decisions are made, let EX be the expected or mean value of X computed from the decision maker's subjective probability distribution. Thus EA is his expected cash price at τ, $EA - k_1$ is his expected return to storing, $k_2 + EH - k_1$ is his expected return to futured storage, etc. We shall say the farmer is over, fully, or under hedged according to whether $s + g > m$, $= m$, or $< m$.

Assuming B and H independent, the principal results of the next section are -

Storage: Some grain should be stored if and only if at least one of the three returns: return to forwarded storage $(k_3 - k_1)$, expected return to futured storage $(k_2 + EH - k_1)$, expected return to storing $(EA - k_1)$, is positive.[4] If return to forwarded storage is positive, the entire supply should be stored.

Total Hedging: The farmer should overhedge if and only if expected return to futuring $(k_2 - EB)$ is positive.[4] He should fully hedge if $k_2 - EB = 0$, or if $(k_2 - EB) < 0$ and expected return to forwarding $(k_3 - EA)$ is nonnegative. He should underhedge if some grain is stored and both $(k_2 - EB)$ and $(k_3 - EA)$ are negative.

Forwarding: There should be no forward sales unless the return to forwarded storage $(k_3 - k_1)$ is positive. If $k_3 - k_1 > 0$ and if expected return to forwarding $(k_3 - EA)$ is greater than or equal to expected return to futuring $(k_2 - EB)$, i.e., if $k_3 - EH - k_2 \geq 0$, then the entire supply should be forwarded. If $(k_3 - EH - k_2) < 0$ then an amount less than the entire supply should be forwarded (possibly none).

<u>Futuring</u>: If expected return to futuring $(k_2 - EB)$ is positive, a short position in excess of the stored grain uncovered by forward sales should be taken. If $k_2 - EB < 0$, any short position taken should be less than the physical quantity stored and no short position should be taken unless two conditions hold - (a) expected return to forwarding is less than expected return to futuring and (b) expected return to futured storage is positive, i.e., $(EA - k_1) + (k_2 - EB) = (k_2 + EH - k_1) > 0$. If $k_2 - EB = 0$, any stored grain should be fully hedged; the hedging should be entirely by futuring if the return to forwarded storage $(k_3 - k_1)$ is negative; entirely by forwarding if expected return to forwarding $(k_3 - EA)$ is nonnegative; and by some of each if return to forwarded storage is positive while expected return to forwarding is negative.

Modifications of these rules required by possible dependence between B and H are indicated in Section 4.

3. Derivation of Results Under Independence

Mathematically, the decision maker's problem is to find values $\hat{m}$, $\hat{s}$, $\hat{g}$ which maximize the function

$$\eta(m,s,g) = E\psi[(A-k_1)m + (k_2-B)s + (k_3-A)g] \tag{1}$$

subject to $0 \leq m \leq n$, $0 \leq s$, $0 \leq g \leq m$.

ψ is the decision maker's utility function for gain,[5] η is his expected utility function. The other symbols were defined in the previous section. The task for this section is to relate the maximizers $\hat{m}$, $\hat{s}$, $\hat{g}$ to some circumstances and expectations of the farmer. Except when the contrary is stated, the following conditions are assumed -

(a) $\psi' > 0$, $\psi'' < 0$, $\lim_{x\to\infty} \psi'(x) = 0$

(b) A, B have finite means and variances;

any linear combination of A and B is

nontrivial

(c) $E|\psi[(A-k_1)m + (k_2-B)s + (k_3-A)g]| < \infty$ and

$E|Y\psi'[(A-k_1)m + (k_2-B)s + (k_3-A)g]| < \infty$

for all m, s, g in R^3

(d) $P(k_2-B \geq 0) < 1$

(e) $k_3 \neq k_1$

(f) $H = A - B$ is statistically independent of B

(a) is a standard assumption in expected utility theory. $\psi' > 0$ means that larger gains are preferred. $\psi'' < 0$ implies risk aversion. $\lim_{x\to\infty} \psi'(x) = 0$ is a weaker condition than bounded utility which has sometimes been assumed. (b), (c) are mathematical regularities which seem plausible. A trivial random variable is one that is constant with probability one. If a linear combination of A and B were trivial, one could be written as a linear function of the other and eliminated from the problem. (d) says that futuring is not a sure thing, i.e., it does not offer positive probability of gain with zero probability of loss. Inspection of grain market data (examples are offered in Section 5, page 33) suggests that, typically, $P(k_2 - B \geq 0)$ should be less than one-half. (e) is initially assumed for convenience. $k_3 = k_1$ is highly unlikely and will be seen to cause no difficulty if it should occur. However, the development is simplified by deferring this case to the end of the section. (f) has been discussed and is relaxed in the next section.

It is shown in the Appendix that

(i) (a) and (c) imply that η has continuous partial derivatives which may be obtained by differentiation under the expectation.

(ii) (a) through (d) imply that η is strictly concave and has a unique maximum over the admissible set.

(i) follows from a proposition proved in [3, page 3] and (ii) is essentially due to Leland [7]. In both cases, there are minor differences in context which probably justify restatement of proofs.

Note that if we consider a simpler special case (discussed in [2]) where expected utility is a function of a single decision variable, say

$$\beta(\alpha) = E\psi(X + \alpha Y) \tag{2}$$

then if ψ satisfies (a); X, Y satisfy (b); $P(Y > 0) > 0$, $P(Y < 0) > 0$; and $E|\psi(X + \alpha Y)|$, $E|Y\psi'(X + \alpha Y)|$ exist for all $\alpha \in R$, then the propositions from the appendix assure that η is strictly concave, continuously differentiable, and has a unique maximum for $\alpha \in R$, say $\beta(\alpha_u)$. In this special case, it is easy to see that, for any given α, $\beta'(\alpha) \gtreqless 0 \Leftrightarrow \alpha_u \gtreqless \alpha$ (see Figure 1) or, equivalently,

(iii) $\beta'(\alpha) \stackrel{s}{=} \alpha_u - \alpha$

where "$\stackrel{s}{=}$" reads "agrees in sign with."

Now suppose α is confined to an admissible set $\mathcal{A} \subset R$. If $\mathcal{A}$ is closed and convex, β also has a unique maximum for $\alpha \varepsilon \mathcal{A}$, call it $\beta(\hat{\alpha})$. Clearly

$$\text{(iv)} \quad \mathcal{A} > \alpha_u \Rightarrow \hat{\alpha} = \min \{\alpha : \alpha \varepsilon \mathcal{A}\}$$
$$\mathcal{A} < \alpha_u \Rightarrow \hat{\alpha} = \max \{\alpha : \alpha \varepsilon \mathcal{A}\}$$

In what follows, we shall consider several circumstances under which our somewhat more complex problem reduces to the form of (2) and Propositions (iii) and (iv) are useful. This special case is illustrated in Figure 1.

We have $\beta'(\alpha_i) > 0$ for $i = 1$ and by (iii), $\alpha_u > \alpha_1$. $\beta'(\alpha_i) < 0$ for $i = 2, 3, 4$ so $\alpha_u < \alpha_i$ for $i = 2, 3, 4$. If $\mathcal{A} = \{\alpha : \alpha \leq \alpha_1\}$ then, by (iv), $\hat{\alpha} = \alpha_1$. If $\mathcal{A} = \{\alpha : \alpha_2 \leq \alpha \leq \alpha_4\}$ then $\hat{\alpha} = \alpha_2$.

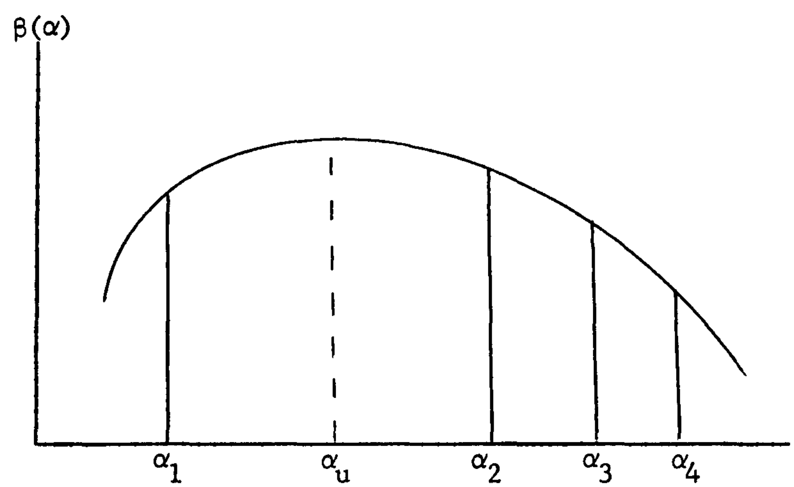

Figure 1

One other proposition from earlier work will be useful. We shall repeatedly want to determine the sign of a product of random variables of the form -

$$(3) \qquad E[Y\theta(X)] = (EY)(E\theta(X)) + \mathrm{Cov}\,(Y, \theta(X)) = \mathcal{E}\mathcal{X} + \mathcal{COV}$$

where X, Y, $\theta(X)$ have finite means and variances and θ is a positive, strictly decreasing function. $\mathcal{E}\mathcal{X} + \mathcal{COV}$ is just a short way of indicating the first and second terms. θ positive means

$$\mathcal{E}\mathcal{X} \overset{s}{=} EY$$

The following result [4, page 6] will sometimes determine $\mathcal{COV}$.

(v) **Let X, Y be random variables with finite means and variances. Suppose $Y = f(W, V)$, $X = g(W, Z)$ where f and g are strictly monotonic in their respective first arguments; W is nontrivial; and V, Z, W are independent.**

Let θ be strictly decreasing and such that $\theta(X)$ has finite mean and variance. Then $\mathrm{Cov}\,(Y, \theta(X))$ is negative if f, g are of the same monotonicity (both increasing or both decreasing) and $\mathrm{Cov}\,(Y, \theta(X))$ is positive if f, g are of opposite monotonicity.

Now to justify the conclusions on optimal choice that were stated in Section 2, recall that we want to choose $\hat{m}$, $\hat{s}$, $\hat{g}$ to maximize $\eta(m, s, g) = E\psi[(A - k_1)m + (k_2 - B)s + (k_3 - A)g]$ subject to $0 \le m \le n$, $0 \le s$, $0 \le g \le m$. A simple transformation of variables will prove helpful.

Let[7] $w = m - g$ and let

$$\gamma(m,s,w) = \eta(m,s,m-w) = E\psi[(k_3-k_1)m + (k_2-B)s + (A-k_3)w]$$

where $0 \leq m \leq n$, $0 \leq s$, $0 \leq w \leq m$.

Since the transformation is 1 - 1 onto, $\hat{m}$, $\hat{s}$, $\hat{m}-\hat{w}$ maximizes η if and only if $\hat{m}$, $\hat{s}$, $\hat{w}$ maximizes γ . Suppose $\hat{s}$, $\hat{w}$ were known, consider -

$$(4) \qquad \gamma'_m(m,\hat{s},\hat{w}) = E(k_3-k_1)\ \psi'[(k_3-k_1)m + (k_2-B)\hat{s} + (A-k_3)\hat{w}]$$

$$= (k_3-k_1)\ E\psi'[(k_3-k_1)m + (k_2-B)\hat{s} + (A-k_3)\hat{w}] \ .$$

Since $\psi' > 0$, $E\psi' > 0$ and $\gamma'_m \overset{s}{=} (k_3-k_1)$ regardless of $\hat{s}$, $\hat{w}$. Thus $k_3 - k_1 > 0$ implies that γ can be maximized by assigning m its highest admissible value (n) and $k_3 - k_1 < 0$ indicates that γ can be maximized by assigning m its least admissible value $(\hat{w})$. Hence -

$$(5) \qquad (k_3 - k_1) > 0 \Rightarrow \hat{m} = n, \ (k_3 - k_1) < 0 \Rightarrow \hat{m} = \hat{w} \ .$$

These two cases are examined separately. Consideration of the unlikely case that $k_3 = k_1$ is deferred to the end of the section.

<u>Case I</u>: $k_3 < k_1$

By the second part of (5), there are no forward sales if $k_3 < k_1$ $(\hat{m} = \hat{w} \Rightarrow \hat{g} = \hat{m} - \hat{w} = 0)$ so there are just two decisions to be made, namely m and s . Let, recalling $A = B + H$,

$$(6) \qquad \mu(m,s) = \gamma(m,s,m) = \eta(m,s,0) = E\psi[(B + H - k_1)m + (k_2-B)s]$$

$0 \leq s$, $0 \leq m \leq n$ be the expected utility function obtained by recognizing the equality of m and w. To investigate possible optimal values of the short futures position, note

$$\text{(7)} \qquad \mu_s' = E(k_2-B)\ \psi'[(B+H-k_1)m + (k_2-B)s]$$

The right side of (7) is of the same form as the left side of (3), page 10, if we let

$$Y = (k_2-B),\ X = (B+H-k_1)m + (k_2-B)s\ ,\quad \theta = \psi'$$

and $\qquad \mu_s' = \mathcal{E}\mathcal{X} + \mathcal{COV}$

where

$$\mathcal{E}\mathcal{X} = [E(k_2-B)]\ [E\psi'[(B+H-k_1)m + (k_2-B)s]]$$

$$\mathcal{COV} = \text{Cov}\ [(k_2-B),\ \psi'[(B+H-k_1)m + (k_2-B)s]]\ .$$

Since $\psi' > 0$, the second factor of $\mathcal{E}\mathcal{X}$ is positive and

$$\text{(8)} \qquad \mathcal{E}\mathcal{X} \stackrel{s}{=} k_2 - EB\ .$$

Recall that ψ' is strictly decreasing $(\psi'' < 0)$ and H is independent of B. Thus Proposition (v), page 10, applies and

$$\text{(9)} \qquad \mathcal{COV} \stackrel{s}{=} m - s\ .$$

Since $\hat{m}$ is known. Then $\mu(\hat{m},s)$ and $\mu_s'(\hat{m},s)$ are functions of a single decision variable s. From assumptions (a) through (d), page 7, $\mu(\hat{m},s)$ has the same properties as $\beta(\alpha)$ in Equation (2), page 8. $\mu(\hat{m},s)$ is a strictly concave, continuously differentiable function of s which has a unique unrestricted maximum s_u and, by (iii), $s_u - s \stackrel{s}{=} \mu_s'(\hat{m},s)$.

Thus (8) and (9) imply $\mu_s'(\hat{m},\hat{m}) \overset{s}{=} k_2 - EB$ and, for given $\hat{m}$,

(10) $$\hat{s}_u - \hat{m} \overset{s}{=} k_2 - EB \; .$$

Recognizing the restriction $0 \le s$ yields

(11) $$\hat{s} - \hat{m} \overset{s}{=} k_2 - EB \text{ except that } k_2 - EB < 0 , \quad \hat{m} = 0 \Rightarrow \hat{s} = 0 \; .$$

Thus, if something is stored and forward sales are not attractive $(k_3 < k_1)$, the decision maker will over, under, or fully hedge in futures according to whether expected return to a short position is positive, negative or zero. This partly covers the conclusions stated under Total Hedging and under Futuring in Section 1.

Now consider possible choices of m assuming $\hat{s}$ known $(0 \le \hat{s})$.

$$\mu_m' = E(B+H-k_1) \; \psi'[(B+H-k_1)m + (k_2-B)s] \; .$$

Again, this is of the form of Equation (3) with X as before and $Y = B + H - k_1$. Therefore,

$$\mathcal{E}\Upsilon = EB + EH - k_1 = EA - k_1$$

and if $m = 0$, $\mathcal{COV} \overset{s}{=} \hat{s}$ by (v), page 10. Also by (v), $\mathcal{COV} < 0$ when $\hat{m} = \hat{s} > 0$.

Accordingly, taking $\hat{s}$ as given,

$$\text{(12)} \qquad EA - k_1 > 0 \Rightarrow \mu_m'(0,\hat{s}) > 0 \Rightarrow \hat{m} > 0$$

$$EA - k_1 = 0,\ \hat{s} > 0 \Rightarrow \mu_m'(0,\hat{s}) > 0,\ \mu_m'(\hat{s},\hat{s}) < 0$$

$$\Rightarrow 0 < \hat{m} < \hat{s}$$

$$EA - k_1 \le 0,\ \hat{s} = 0 \Rightarrow \mu_m'(0,0) \le 0 \Rightarrow \hat{m} = 0 \quad .$$

Together (12) and (11) make a number of qualitative assertions about optimal storage and futures hedging for various circumstances regarding expected return to futuring and to storage. Before summarizing these, it seems useful to supplement them with a further result obtained by another change of variable.

Let $z = m - s$. Then z (which might be negative) represents unhedged storage. Rewrite expected utility -

$$\zeta(m,z) = \mu(m,m-z) = E\ \psi[(k_2 + H - k_1)m + (B-k_2)z]$$
$$0 \le m \le n,\ z \le m$$

If $\hat{z}$ is temporarily regarded as fixed, we may obtain

$$\zeta_m' = E(k_2 + H - k_1)\ \psi'[(k_2+H-k_1)m + (B-k_2)z]$$

$$\mathcal{E}\chi \overset{s}{=} k_2 + EH - k_1$$

$$\mathcal{COV} \overset{s}{=} -m$$

$$\hat{m}_u \overset{s}{=} k_2 + EH - k_1$$

(13) $$k_2 + EH - k_1 > 0 \Rightarrow \hat{m} > 0$$

$$k_2 + EH - k_1 \leq 0 \Rightarrow \hat{m} = \max\{\hat{z}, 0\} \quad .$$

The reader can readily verify that examining ζ_z' merely reproduces the results already obtained from μ_s'.

The implications of (11), (12), and (13) are summarized in Table 1. The conclusions regarding $\hat{m}$ given in rows 1 - 3 follow immediately from the first part of (13) and the corresponding conclusions about $\hat{s}$ follow immediately from (11). Consider row 4.

(I) $k_2 - EB > 0 \Rightarrow \hat{s} > \hat{m} \geq 0$

by (11) and the restriction $0 \leq m$

(II) $\hat{s} > 0 \Rightarrow \hat{z} < \hat{m}$

by definition of z, $z = m - s$

(III) $k_2 + EH - k_1 \leq 0 \Rightarrow \hat{m} = \max\{0, \hat{z}\} = 0$

where $\Rightarrow$ follows from (13) and the final equality from (II) above

Consider row 5.

(I) $k_2 - EB \leq 0 \Rightarrow \hat{s} \leq \hat{m} \Rightarrow \hat{z} \geq 0$

by (11) and definition of z

(II) $k_2 + EH - k_1 \leq 0$, $\hat{z} \geq 0 \Rightarrow \hat{m} = \hat{z} \Rightarrow \hat{s} = 0$

by (13) and definition of z

(III) $EA - k_1 \leq 0$, $\hat{s} = 0 \Rightarrow \hat{m} = 0$

by (12)

Table 1

NEGATIVE RETURN TO FORWARDED STORAGE $(k_1 > k_3,\ \hat{g} = 0)$

Circumstances of Expected Returns			Optimal Decisions	
1	2	3 (1-2)		
Futured Storage $k_2 + EH - k_1$	Futuring $k_2 - EB$	Storing $EA - k_1$	Amount Stored $\hat{m}$	Short Future $\hat{s}$
+	+		+	$>\hat{m}$
+	0	+	+	$=\hat{m}$
+	-	+	+	$<\hat{m}$
θ	+	-	0	+
θ	θ	θ	0	0
θ	θ	+	+	0

+, -, 0 indicate respectively a positive, negative, or zero value for the quantity specified at the top of the column. θ indicates a non-positive value.

The blank in the first row, third column indicates that expected return to storing need not be specified to obtain that $\hat{m}$ is positive and $\hat{s} > \hat{m}$.

Note that the third entries in the second, third and fourth rows are implied by the first two entries in these rows since $k_2 + EH - k_1 = (k_2 - EB) + (EA - k_1)$.

Consider row 6.

$$\text{(I)}\quad k_2 - EB \le 0 \Rightarrow \hat{s} \le \hat{m} \Rightarrow \hat{z} \ge 0$$

as in row 5

$$\text{(II)}\quad k_2 + EH - k_1 \le 0,\ \hat{z} \ge 0 \Rightarrow \hat{m} = \hat{z} \Rightarrow \hat{s} = 0$$

as in row 5

$$\text{(III)}\quad EA - k_1 > 0 \Rightarrow \hat{m} > 0$$

by (12)

<u>Case II</u>: $k_3 > k_1$

The procedure follows the pattern of Case I and will be presented more briefly. Case II assumes $k_3 > k_1$ which implies $\hat{m} = n$. Rewriting (1)

$$\eta(n,s,g) = E\psi[(A-k_1)n + (k_2-B)s + (k_3-A)g]$$

$$0 \le s, \quad 0 \le g \le n$$

$$\eta_s' = E(k_2-B)\ \psi'[(A-k_1)n + (k_2-B)s + (k_3-A)g]$$

$$\mathcal{E}\mathcal{X} \overset{s}{=} k_2 - EB$$

$$\mathcal{C}\mathcal{O}\mathcal{V} \overset{s}{=} n - s - g$$

Temporarily taking $\hat{g}$ as given,

$$(15)\qquad \hat{s} + \hat{g} - n \overset{s}{=} k_2 - EB \quad \text{except that} \quad \hat{s} = 0 \quad \text{if} \quad k_2 - EB < 0,\ \hat{g} = n$$

$$\eta_g' = E(k_3-B-H)\ \psi'[(B+H-k_1)n + (k_2-B)s + (k_3-B-H)g]$$

$$\mathcal{E}\mathcal{X} \overset{s}{=} k_3 - EB - EH$$

$$n \ge \hat{s} + g \Rightarrow \mathcal{C}\mathcal{O}\mathcal{V} > 0 \quad \text{except} \quad n = g,\ \hat{s} = 0 \Rightarrow \mathcal{C}\mathcal{O}\mathcal{V} = 0$$

Now considering $\hat{s}$ as given,

(16) $k_3 - EA \geq 0 \Rightarrow \hat{g} > n - \hat{s}$ except that $k_3 - EA \geq 0,\ \hat{s} = 0 \Rightarrow \hat{g} = n$

$k_3 - EA < 0 \Rightarrow \hat{g} < n$

Again, the conclusions can be supplemented by a change of variable. Let $v = g + s$ and let

$$\zeta(v,g) = \eta(n,v-g,g) = E\psi[(A-k_1)n + (k_2-B)v + (k_3-H-k_2)g]$$
$$0 \leq g \leq n \qquad g \leq v$$

$$\zeta'_g = E(k_3-H-k_2)\ \psi'[(B+H-k_1)n + (k_2-B)v + (k_3-H-k_2)g]$$
$$Ex \overset{s}{=} k_3 - EH - k_2$$
$$Cov \overset{s}{=} n - g$$
$$\hat{g}_u - n \overset{s}{=} k_3 - EH - k_2$$

For given $\hat{v}$

(17) $k_3 - EH - k_2 \geq 0 \Rightarrow \hat{g} = \min\ \{\hat{v},\ n\}$

$k_3 - EH - k_2 < 0 \Rightarrow \hat{g} < n$

Note that $k_3 - H - k_2$ is the excess of return to forwarding over return to futuring. It is the effect on final return of simultaneously reducing the short future by a bushel while increasing forward sales by a bushel.

The implications of (15), (16), (17) are given in Table 2. The reasoning is sufficiently like that underlying Table 1 to not require repetition.

Now consider the case that $k_3 = k_1$. Write expected utility as -

$$(3') \qquad \gamma(m,s,w) = E\psi[(k_3-k_1)m + (k_2-B)s + (A-k_3)w]$$

$$= E\psi[(k_2-B)s + (A-k_3)w] = \theta(s,w)$$

$$0 \leq s \qquad 0 \leq w \leq n$$

Varying m while holding s and w constant does not affect the argument of ψ and therefore does not affect expected utility. Clearly, optimal choice is not generally unique if $k_3 = k_1$. Recall that $w = m - g$ and $0 \leq g \leq m$, so if $\hat{w} = n$, then $\hat{m} = n$, $\hat{g} = 0$. If $\hat{w} = 0$, then $\hat{m} = \hat{g} = 0$. However, if $0 < \hat{w} < n$, there is a range of variation for m, g (specifically $\hat{w} \leq m \leq n$ with $g = m - \hat{w}$) that corresponds to maximum expected utility.

Note that setting $m = n$ does not restrict the range of variation of s, w . Since, in this case, expected utility may be stated as a function of s, w; this means that no expected utility is lost if the decision maker sets $m = n$ and then proceeds as in Case II. Alternatively he could set $g = 0$ and proceed as in Case I. Thus Tables 1 and 2 are also relevant to $k_3 = k_1$ in that the decision maker can consult either table without loss of expected utility. The rules given on page 5 correspond to setting $g = 0$ when $k_3 = k_1$.

Table 2

POSITIVE RETURN TO FORWARDED STORAGE ($k_3 > k_1$, $\hat{m} = n$)

Circumstances of Expected Returns			Optimal Decisions		
1	2	3 (2+1)			
Forwarding Less Futuring $k_3 - EH - k_2$	Futuring $k_2 - EB$	Forwarding $k_3 - EA$	Forward Sales $\hat{g}$	Total Hedge $\hat{s} + \hat{g}$	Short Future $\hat{s}$
-	+		$< n$	$> n$	$> (n-\hat{g})$
-	0	-	$< n$	n	$n-\hat{g}$
-	-	-	$< n$	$< n$	$< (n-\hat{g})$
$\oplus$	+	+	n	$> n$	+
$\oplus$	θ	$\oplus$	n	n	0
$\oplus$	θ	-	$< n$	$= \hat{g}$	0

+, -, 0 indicate respectively positive, negative, or zero value for the variable specified at the top of the column. θ indicates nonpositive, $\oplus$ nonnegative.

The blank in row 1, column 3 indicates that, in this instance, the expected return to forwarding need not be specified to obtain the results given by the final three entries in that row. Note that the third entries in rows 2, 3, 4 are implied by the first two entries in these rows since $k_2 + EH - k_3 = (k_2 - EB) + (k_3 - EA)$.

4. Effects of Interdependence Between Price and Basis

Recall that the version of basis relevant to this analysis is the difference between the price a farmer can realize in his local market and the contemporaneous quotation for maturing or soon-to-mature futures contracts.[8] At the time storage and hedging decisions are made (harvest in the present model) these prices and therefore their difference, the basis, are random variables.

Factors that finally determine a particular producer's basis will vary from one farm situation to another but cost of transportation to the relevant terminal market; quality of the farmer's grain; quality premiums and discounts in that particular year; and current circumstances of local demand and supply are likely to be dominant factors in many cases. These seem sufficiently independent of the general national and often international supply and demand forces determining the futures quotation that the statistical independence assumed so far may often be a good first approximation.

However, it also seems plausible that some dependence typically exists - quality premiums or discounts might well have some tendency to increase when price is higher, local demand may be to some extent associated with total world demand although the association may typically be weak.

Pending careful empirical studies to determine the kind and importance of dependence, it seems useful to consider possible effects in a preliminary fashion. Again, let A be local cash price, B the concurrent futures quotation, and $H = A - B$ the basis. Instead of assuming H is independent of B, assume $H = \delta B + V$ with V independent of B. δ is then the regression coefficient of H on B. Then $A = \upsilon B + V$ with $\upsilon = 1 + \delta$ the regression coefficient of A on B.[9]

The observed fact that year to year fluctuations in a particular basis tend to be small relative to price fluctuations suggests $|\delta|$ should be quite a lot smaller than one and this is reinforced if, as I have presumed, H and B are not highly correlated. υ is then close to one. The special case $\upsilon = 1$ corresponds to independence between B and H.

Reviewing the development presented in Section 3, it is seen that the assumed independence of B and H was first used in obtaining Equation (9), page 12. To show the effects of changing this assumption, an entirely analogous development is sketched below using the same equation numbers as in Section 3 but with asterisks to denote that an assumption has been changed.

<u>Case I</u>: $k_3 < k_1$, $\hat{g} = 0$

(6*) $$\mu(m,s) = E\psi[(A - k_1)m + (k_2 - B)s] = E\psi[(\upsilon B + V - k_1)m + (k_2 - B)s]$$

(7*) $$\mu_s' = E(k_2 - B)\ \psi'[(\upsilon B + V - k_1)m + (k_2 - B)s]$$

(8*) $$\mathcal{E}\chi \overset{s}{=} k_2 - EB$$

Now, using (v), page 10 and the independence of B and V -

(9*) $$\mathcal{COV} \overset{s}{=} \upsilon m - s$$

and for given $\hat{m}$,

(10*) $\quad s_u - \upsilon\hat{m} \overset{s}{=} k_2 - EB$

Recognizing $0 \le s$ yields

(11*) $\quad \hat{s} - \upsilon\hat{m} \overset{s}{=} k_2 - EB$ except that $k_2 - EB < 0$, $\hat{m} = 0 \Rightarrow \hat{s} = 0$

Continuing,

$$\mu_m' = E(A - k_1)\ \psi'[(A - k_1)m + (k_2 - B)s] = E(\upsilon B+V-k_1)\ \psi'[(\upsilon B+V-k_1)m + (k_2-B)s]$$

whence

$\mathcal{E}\mathfrak{X} = EA - k_1$ and if $m = 0$, $\mathcal{COV} \overset{s}{=} \hat{s}$. Also $\upsilon\hat{m} = \hat{s} > 0 \Rightarrow \mathcal{COV} < 0$.

Accordingly, taking $\hat{s}$ as given with $\hat{s} \ge 0$ and using (iii) and (iv), pages eight and nine,

(12*) $\quad EA - k_1 > 0 \Rightarrow \mu_m'(0, \hat{s}) > 0 \Rightarrow \hat{m} > 0$

$$EA - k_1 = 0,\ \hat{s} > 0 \Rightarrow \mu_m'(0, \hat{s}) > 0,\ \mu_m'(\upsilon\hat{s}, \hat{s}) < 0$$

$$\Rightarrow 0 < \upsilon\hat{m} < \hat{s}$$

$$EA - k_1 \le 0,\ \hat{s} = 0 \Rightarrow \mu_m'(0, 0) \le 0 \Rightarrow \hat{m} = 0$$

Let $r = \upsilon m - s$ and

$$\rho(m,r) = \mu(m,\upsilon m-r) = E\psi[(\upsilon k_2+A-\upsilon B-k_1)m + (B-k_2)r]$$

$$= E\psi[(\upsilon k_2+V-k_1)m + (B-k_2)r]$$

$0 \le m \le n \qquad r \le \upsilon m$

$$\rho_m' = E(\upsilon k_2+V-k_1)\ \psi'[(\upsilon k_2+V-k_1)m + (B-k_2)r]$$

$$\mathcal{E}\mathcal{I} \overset{s}{=} \upsilon k_2 + EV - k_1$$

$$\mathcal{C}\mathcal{O}\mathcal{V} \overset{s}{=} - m$$

whence

$$m_u \overset{s}{=} \upsilon k_2 + EV - k_1$$

and, treating $\hat{f}$ as given and taking account of restrictions,

$$(13^*) \qquad \upsilon k_2 + EV - k_1 > 0 \Rightarrow \hat{m} > 0$$

$$\upsilon k_2 + EV - k_2 \leq 0 \Rightarrow \hat{m} = \max\{0, \frac{\hat{f}}{\upsilon}\}$$

Taking υ to be positive $\upsilon\hat{m} \overset{s}{=} \hat{m}$ and (11*), (12*), (13*) may be obtained from (11), (12), (13) by replacing m with υm and $k_2 + H - k_1$ with $\upsilon k_2 + V - k_1$. Therefore, the implications of (11*), (12*), (13*) are those of the earlier equations with these same substitutions. The implications are summarized in Table 3.

Suppose it has been decided to store a bushel of grain. If nothing else is done the prospective return (above current sale) is $A - k_1 = \upsilon B + V - k_1$. Alternatively, if some quantity λ is simultaneously sold forward on the futures market, the prospective joint return is $\upsilon B + V - k_1 + \lambda(k_2 - B) = (\upsilon - \lambda)B + V + \lambda k_2 - k_1$. The variance of this return is $(\upsilon - \lambda)^2 \text{ Var } B + \text{Var } V$. If one wishes to determine λ to minimize the variance of the prospective return, clearly $\lambda = \upsilon$ is the minimizing value. If m bushels were stored, a short futures position of υm bushels would minimize the variance of the joint return.

For convenience, let us refer to a lot of grain covered by a short futures contract of size υ times (quantity of grain) as in "firmed"

Table 3

NEGATIVE RETURN TO FORWARDED STORAGE ($k_1 > k_3$, $\hat{g} = 0$)

Circumstances of Expected Returns			Optimal Decisions	
1	2	3 $(1-\upsilon 2)$		
Firmed Storage $\upsilon k_2 + EV - k_1$	Futuring $k_2 - EB$	Storing $EA - k_1$	Amount Stored	Short Future
+	+		+	$> \upsilon\hat{m}$
+	0	+	+	$= \upsilon\hat{m}$
+	-	+	+	$< \upsilon\hat{m}$
θ	+	-	0	+
θ	θ	θ	0	0
θ	θ	θ	+	0

storage (the prospective return is as firm in the sense of small variance as can be achieved by futuring) and $\upsilon k_2 + V - k_1$ as the return to firmed storage. $\upsilon(k_2 - B) = \upsilon$ times (return to futuring) will be called return to firming. As indicated by (11*) or by the last column of Table 3, if $k_1 > k_3$, then having stored grain exactly firmed is optimal if and only if the expected return to futuring is zero. Comparison of Table 3 with Table 1 confirms that, when interdependence is permitted, expected return to firmed storage plays a role entirely analogous to that of expected return to futured storage when independence between B and H is assumed. When $\upsilon = 1$, independence is implied and firmed storage and futured storage become identical.

For Case II $(k_3 > k_1, \hat{m} = n)$, the analogues to (15), (16), (17) are

For given $\hat{g}$

(15*) $\hat{s} - \upsilon(n - \hat{g}) \overset{s}{=} k_2 - EB$ except that $\hat{s} = 0$ if $k_2 - EB < 0$ and $\hat{g} = n$.

For given $\hat{s}$

(16*) $k_3 - EA \geq 0 \Rightarrow \hat{g} > n - \frac{\hat{s}}{\upsilon}$ except that $k_3 - EA \geq 0$, $\hat{s} = 0 \Rightarrow \hat{g} = n$

$$k_3 - EA < 0 \Rightarrow \hat{g} < n$$

Let $v = g + \frac{s}{\upsilon}$, then $0 \leq g \leq n$, $g \leq v$

$$\xi(v,g) = \eta(n,\upsilon(v-g),g) = E\psi[(A-k_1)n + \upsilon(k_2-B)v + (k_3-V-\upsilon k_2)g]$$

$$\xi_g = E(k_3-V-\upsilon k_2)\ \psi'[(\upsilon B+V-k_1)n + \upsilon(k_2-B)v + (k_3-V-\upsilon k_2)g]$$

$$\mathcal{E}\mathcal{X} \overset{s}{=} (k_3 - EV - \upsilon k_2)$$

$$\mathcal{C}\mathcal{O}\mathcal{V} \overset{s}{=} n - g$$

and from (iii) and (iv), pages eight and nine

$$g_u - n \overset{s}{=} k_3 - EV - \upsilon k_2$$

For given $\hat{v}$ and taking account of restrictions

(17*) $$k_3 - EV - \upsilon k_2 \geq 0 \Rightarrow \hat{g} = \min\{\hat{v}, n\}$$

$$k_3 - EV - \upsilon k_2 < 0 \Rightarrow \hat{g} < n$$

Implications of (15*), (16*), (17*) are summarized in Table 4.

One notes that the difference between expected return to forwarding and expected return to firming plays the same role in this section as expected return to forwarding less expected return to futuring played in Section 3, and that the two differences are identical if $\upsilon = 1$.

It seems reasonable to describe the decision maker as fully firmed if $g + \frac{s}{\upsilon} = m$, overfirmed if $g + \frac{s}{\upsilon} > m$, and underfirmed if $g + \frac{s}{\upsilon} < m$. With this convention, the informal statement of conclusions in Section 1, page 5 need only be changed as indicated below to allow for possible interdependence between basis and price.

<u>Storage</u>: no change

<u>Total Hedging</u>: change hedging, overhedge, fully hedge, underhedge, to firming, overfirm, fully firm, underfirm, respectively.

<u>Forwarding</u>: change fourth line to read, "expected return to firming $\upsilon(k_2 - EB)$, i.e., if $k_3 - EV - \upsilon k_2 \geq 0$, then" and change $(k_3 - EH - k_2)$ to $(k_3 - EV - \upsilon k_2)$ in line 5.

Table 4

POSITIVE RETURN TO FORWARDED STORAGE $(k_3 > k_1,\ \hat{m} = n)$

Circumstances of Expected Returns			Optimal Decisions		
1	2	3 $(1+\upsilon 2)$			
Forwarding Less Firming $k_3 - EV - \upsilon k_2$	Futuring $k_2 - EB$	Forwarding $k_3 - EA$	Forward Sales $\hat{g}$	Total Firmed $\hat{g} + \frac{\hat{s}}{\upsilon}$	Short Future $\hat{s}$
-	+		$< n$	$> n$	$>(n-\upsilon\hat{g})$
-	0	-	$< n$	$= n$	$=(n-\upsilon\hat{g})$
-	-	-	$< n$	$< n$	$<(n-\upsilon\hat{g})$
$\oplus$	+	+	n	$> n$	+
$\oplus$	θ	$\oplus$	n	n	0
$\oplus$	θ	-	$< n$	$= \hat{g}$	0

Futuring: substitute the following

Firming: If expected return to futuring $(k_2 - EB)$ is positive, a short position in excess of $\upsilon(m-g)$ should be taken. If $k_2 - EB < 0$, any short position should be less than υm and no short position should be taken unless both - (a) expected return to forwarding is less than expected return to firming and (b) expected return to firmed storage $(\upsilon k_2 + EV - k_1)$ is positive. If $k_2 - EB = 0$ any stored grain should be fully firmed; entirely by a short position if $k_3 < k_1$; entirely by forwarding if expected return to forwarding $(k_3 - EA)$ is non-negative; and by some of each if $k_3 > k_1$ while $k_3 - EA < 0$.

5. Some Discussion of Assumptions and Results

Tables 1 - 4 summarize qualitative conclusions under alternative assumptions about expected returns from various ways of marketing the crop. Which assumptions are more plausible or relevant? A precise answer will depend on much additional careful research on behavior of cash, forward, and futures prices for grains and on personal probability formation by farmers. However it seems reasonable to suppose that personal probabilities will often approximately reflect historical frequencies and will also be consistent with widely held theories.

By simple analysis, one expects return to storing, $A - k_1$, to typically be positive because, over time, storage services will only remain available on this basis. Futures markets offer holders of grain a way to reduce risks, the risks being assumed by speculators who anticipate gain from holding long positions. Futures prices represent an equilibrium between hedger holders or prospective holders of the commodity who are willing to sacrifice a little expected return for added security and speculators who expect a positive return for the accommodation they furnish. If either party is persistently disappointed he may be expected to leave the market so the market should typically show negative returns to futuring, $k_2 - EB$, and positive returns to futured storage, $k_2 + H - k_1$. Returns to storage are then divided between storer and speculator: $A - k_1 = (k_2 + H - k_1) + (B - k_2)$ with, on the average, both terms on the right positive.

To the extent that this is true and becomes reflected in farmers' personal probabilities the third rows of Tables 1 and 3 are most often relevant for farmers who do not have attractive opportunities to sell forward $(k_3 < k_1)$.

There is little current information on opportunities to sell forward. There is a widespread impression that forward sales have substantially increased in recent years. It seems likely that circumstances of a given farmer might often strongly affect his opportunities. If there are buyers for processors or marketing associations that engage in forward transactions in his area he is more likely to get a favorable offer, especially if he has a grade that processors anticipate will be needed. Note in Tables 2 and 4 that a forward opportunity better than harvest price is all that is required to justify storing the whole crop, but a forward opportunity at least as good as expected price at the usual seasonal peak is required to justify selling the entire crop forward. Furthermore, even the latter condition is not sufficient if expected return to futuring is sufficiently high (row 1 of each table).

Putting consideration of forward transactions aside, Table 5 contains some preliminary hints about some components of basis.

The table shows average prices of December wheat futures in the delivery months at Kansas City and Chicago for the crop years 1950 - 1975 along with average December cash prices for contract grade at various locations in the winter wheat area and the resulting basis for crop years for which quotations are available. This is the basis for a trader in deliverable wheat at the location of the cash quotation. It differs from a typical farmer's basis in that the latter also contains components for transportation to the location and for possible differences of grade. The principal components in the tabulated basis are probably transportation and differences in supply and demand at different locations.

The only random variable in the return to futured storage $(k_2 + H - k_1)$ is H, the basis. For this reason a producer who hedges in futures is said to be "gambling on the basis." The advantage cited is that "basis is more predictable than price." In an informal way this is confirmed by general experience and by comparisons of the standard deviations of price and basis in Table 5. Means and standard deviations are calculated for years prior to 1971 as well as for all available years. Calculations through 1971 are included because of the extreme fluctuations in grain markets since 1972.

Table 6 lists the historical returns to futuring, storing, and futured storage based on the price and basis data in Table 5. It is seen that returns have depended significantly on location and type.

Table 5

DECEMBER PRICES AND BASIS AT SEVERAL LOCATIONS

	Hard Red Winter Wheat					Soft Red Winter Wheat				
	Kansas City	Omaha		St. Louis		Chicago	Toledo		St. Louis	
	(Dec. Future)	Cash	Basis	Cash	Basis	(Dec. Future)	Cash	Basis	Cash	Basis
1950	230					234			238	4
1951	252					264			265	1
1952	239					232			234	2
1953	213					203			211	8
1954	237					227			233	6
1955	210					208			214	6
1956	229					239	238	- 1	242	3
1957	215					219	219	0	225	6
1958	192					194	188	- 6	200	6
1959	200					197	198	1	205	8
1960	199					205	199	- 6	210	5
1961	203					205	204	- 1	209	4
1962	218			229	11	208	210	2	215	7
1963	214			228	14	216	217	1	224	6
1964	162	161	-1	169	7	150	147	- 3	155	5
1965	156	158	2	169	13	165	169	4	170	5
1966	182	183	1	188	6	177	180	3	188	11
1967	151	154	3	159	8	145	144	- 1	150	5
1968	136	137	1	143	7	128	131	3	138	10
1969	141	145	4	150	9	144	145	1	150	6
1970	154	156	2	164	10	168	172	4	168	0
1971	152	155	3	157	5	173	157	-16	157	-16
1972	257	251	-6	254	- 3	260	264	4	259	- 1
1973	514	511	-3	524	10	534	550	16	546	8
1974	469	464	-5	457	-12	469	459	-10	457	-12
1975	346	346	0	336	-10	335	328	- 7	336	1
Thru 1971 -										
MEAN	195	156	2	176	9	196	182	- 1	200	4
ST. DEV.	35	13	2	30	3	35	33	5	35	5
Thru 1975 -										
MEAN	226	235	0	238	5	227	226	- 1	231	4
ST. DEV.	90	132	3	120	8	92	107	7	92	6

Sources: Futures prices are from the Kansas City Board of Trade and Chicago Board of Trade Statistical Annals. Cash prices are from the USDA: Grain Market News.

Table 6

HISTORICAL RETURNS TO UNHEDGED AND FUTURED STORAGE

Locations

Year	Kansas City $(k_2\text{-}B)$	Omaha $(A\text{-}k_1)$	St. Louis (Hard Winter) $(A\text{-}k_1)$	Omaha $(k_2\text{+}H\text{-}k_1)$	St. Louis (Hard Winter) $(k_2\text{+}H\text{-}k_1)$	Chicago $(k_2\text{-}B)$	Toledo $(A\text{-}k_1)$	St. Louis (Soft Red Winter) $(A\text{-}k_1)$	Toledo $(k_2\text{+}H\text{-}k_1)$	St. Louis (Soft Red Winter) $(k_2\text{+}H\text{-}k_1)$
1950	- 3.9					- 4.3		8.8		4.5
1951	- 17.6					- 25.4		27.8		2.4
1952	- 6.6					5.6		8.0		13.6
1953	- 2.1					- 0.4		13.3		12.9
1954	- 15.7					- 16.4		22.2		5.8
1955	7.9					- 4.4		6.6		2.2
1956	- 15.8					- 22.4	27.6	28.6	5.2	6.2
1957	- 2.3					0.5	7.1	2.9	7.6	3.4
1958	- 2.8					- 1.4	5.4	9.3	4.0	7.9
1959	- 7.6					- 2.5	8.5	8.3	6.0	5.8
1960	- 5.4					- 14.4	18.1	16.8	3.7	2.4
1961	- 2.2					- 3.4	10.4	8.2	7.0	4.8
1962	2.7		- 1.8		0.9	12.6	- 7.6	- 8.7	5.0	3.9
1963	- 18.3		22.5		4.3	- 28.4	34.0	32.8	5.6	4.4
1964	- 10.6	1.1	6.0	- 9.5	- 4.6	- 2.5	- 0.7	3.2	- 3.2	0.7
1965	- 10.3	2.9	8.8	- 7.4	- 1.5	- 15.6	18.0	16.0	2.4	0.4
1966	9.8	- 16.0	- 16.1	- 6.2	- 6.3	16.1	- 12.8	- 8.9	3.3	7.2
1967	10.8	- 13.7	- 11.8	- 2.9	- 1.0	14.2	- 8.3	- 5.4	5.9	8.8
1968	1.9	- 4.8	- 1.9	- 2.9	0	6.2	0.6	2.4	6.8	8.6
1969	- 13.1	8.1	11.0	- 5.0	- 2.1	- 11.9	11.2	12.1	- 0.7	0.2
1970	- 15.1	11.0	11.7	- 4.1	- 3.4	- 19.9	19.7	16.8	- 0.2	- 3.1
1971	- 4.3	- 1.9	- 2.0	- 6.2	- 6.3	- 17.8	3.1	5.2	-14.7	-12.6
1972	-100.7	88.5	94.5	-12.2	- 6.2	-101.7	113.8	105.7	2.1	4.0
1973	-242.1	207.0	209.6	-35.1	-32.5	-245.7	223.1	238.9	-22.6	- 6.8
1974	- 16.1	10.2	- 9.5	- 5.9	-25.6	- 17.4	2.0	- 6.3	-15.4	-23.7
1975	19.1	- 31.4	- 8.4	-12.3	10.7	27.6	- 14.4	- 8.4	13.2	19.2

Sources: Future prices: Kansas City and Chicago Board of Trade annual summaries
Cash prices: Grain Market News
Commissions, Margin Requirements, Storage Costs: Conversations with brokers and millers
Interest rates: 1% plus Prime Commercial Paper Rate, Survey of Current Business

For example, all 12 observed returns to futured storage in Omaha are negative while 22 of 26 observations for soft wheat in St. Louis are positive.

The frequency of various combinations of historical returns is shown in Table 7. If our price data were representative of farmer's circumstances, one might conjecture that expected returns would roughly reflect historical returns and use such a table as a loose indicator of the relevance of the various rows of Table 1. Although it cannot be taken very seriously, Table 7 to some extent supports the simple theoretical conclusions noted earlier (page 30). Careful empirical investigations are needed.

The fact that return to short futures is usually negative checks with traditional futures theory that speculators who bear the risk of price fluctuations (by holding long positions) when much of the crop is unallocated require a normal premium. As a matter of incidental interest the historical premiums are shown in Table 8. The columns headed Kansas City and Chicago show net returns to long positions in these markets from July to December. Each entry is the negative of the corresponding entry in Table 6, less two commissions less twice the interest charge on required margin.

Just to see how the basis at various locations might be related to corresponding futures quotations, regressions were run with the results given in Table 9. Recall $H = \delta B + V$.

Table 7

FREQUENCY OF VARIOUS CIRCUMSTANCES

(k_2-B)	(k_2+H-k_1)	$(A-k_1)$	Number of Instances	Optimal Decisions $k_3 < k_1$ $\hat{m}$	$\hat{g}$	$\hat{s}$	$k_3 > k_1$ $\hat{m}$	$\hat{g}$	$\hat{s}$
+	+	+	5	+	0	$>\hat{m}$	n	$<n$	$>n-\hat{g}$
+	+	-	10	+	0	$>\hat{m}$	n	$<n$	$>n-\hat{g}$
+	-	-	6	0	0	+	n	n	+
-	+	+	24	+	0	$<\hat{m}$	n	$<n$	$<n-\hat{g}$
-	-	+	21	+	0	0	n	$<n$	0
-	-	-	5	0	0	0	n	n	0
+	0	-	1	0	0	+	n	n	+
			72						

INDIVIDUAL RETURNS

Return	Positive Instances	Negative Instances
$A-k_1$	50	22
k_2-B	22	50
k_2+H-k_1	39	32

Table 8

HISTORICAL RETURNS TO LONG FUTURES POSITIONS (JULY-DECEMBER)

Year	Kansas City	Chicago
1950	3.1	3.7
1951	16.8	24.6
1952	5.8	-6.4
1953	1.3	-.4
1954	14.9	15.6
1955	-8.7	3.6
1956	15.0	21.6
1957	1.3	-.5
1958	2.0	.6
1959	6.6	1.5
1960	4.6	13.6
1961	1.4	2.6
1962	-3.5	-13.4
1963	17.5	27.6
1964	9.6	1.5
1965	9.1	14.4
1966	-11.6	-18.0
1967	-12.4	-15.8
1968	-3.5	-7.8
1969	11.3	10.1
1970	13.3	18.1
1971	2.7	16.2
1972	99.3	100.3
1973	248.7	242.3
1974	11.3	12.6
1975	-16.3	-30.4

Table 9

REGRESSIONS OF BASIS ON FUTURES PRICE

Dependent Variable (Basis)	Independent Variable (Futures)	Regression Coefficient $\hat{\delta}$	t-value	R^2
St. Louis (Soft)	Chicago	-.017	-1.4	.07
Toledo	Chicago	.011	.75	.03
St. Louis (Hard)	Kansas City	-.034	-2.1	.27
Omaha	Kansas City	-.017	-3.2	.51

$\hat{\delta}$ was insignificant at the .1 level in the first two cases, significant at .05 in the third, and .01 in the fourth. While subject to all of the qualifications mentioned, the results are consistent with the possibility that allowing for dependence may not greatly alter optimal decisions.

The qualitative results of Sections 3 and 4 depend only on expected values of various returns and on quite general assumptions about utilities and personal probabilities. To obtain more specific results one would have to be more specific about the utility functions and probability distributions. Getting good empirical information to guide more specific assumptions promises to be a formidable task, but it could be highly rewarding in helping economists understand these decisions and in helping producers reach decisions more consistent with their objectives. The fact that it is possible to say quite a bit about optimal decisions on such general assumptions seems encouraging.

FOOTNOTES

1. For Minnesota crops, wheat and oats would typically be harvested in August with a peak price in January, the respective months would be November and June for corn, and October and June for soybeans. See Houck [5]. In the winter wheat belt, harvest falls in June or July and the typical peak price is in December or January.

2. If he stores uncontracted grain it is natural to contemplate selling at the time of the usual seasonal peak unless the farmer feels he has special knowledge that affects his expectation of the particular year's seasonal price pattern. Futures hedging contracts are usually closed simultaneously with the offsetting transaction in the physical commodity. A more complete model would permit the farmer to reconsider his uncontracted grain and futures position occasionally during the season.

3. Prices on futures markets are typically determined by national or international supply and demand. Basis is ordinarily determined by local transportation costs, quality of the particular lot of grain being considered, and sometimes local supply and demand. Thus, substantially independent movements are possible. Empirical investigation of this matter promises to be delicate since we are dealing with the producer's subjective distribution of basis and futures price at time τ given information available at harvest.

4. This requires a minor qualification. If return to forwarded storage is zero, the optimal decision is generally not unique (see page 19). There would never be loss of expected utility in proceeding as indicated here, but sometimes some storage exactly covered by a forward sale would yield the same final prospect and the same expected utility.

5. Using the utility function for gain implicitly assumes that the random variables affecting returns from the ventures explicitly considered are statistically independent of random components of return from other ventures. See Hildreth [2], pages 101-104.

6. Leland's assertion [7, footnote 3, page 38] that $E|W| < \infty$ implies $E|\psi(W)| < \infty$ for ψ as above is not correct. Let $\psi(x) = - e^{-x}$ be a utility function exhibiting constant absolute risk aversion and let $P(W = - n) = \frac{1}{2^n}$ for $n = 1, 2, \ldots$. Then $E|W| = 1$ while $E|\psi(W)| = \sum_1^\infty (\frac{e}{2})^n = \infty$. In economic contexts I think we typically want to assume $E|\psi| < \infty$ anyway, making his proof applicable.

7. w represents grain stored but not sold forward. Increasing m while holding s and w constant corresponds to simultaneously increasing storage and forward sales.

8. The discussion has been in terms of a maturing futures contract and it seems likely that most applications will involve a maturing contract or one for early delivery but the theory can be applied to a contract of whatever duration the farmer chooses as offering the most advantageous hedge.

9. Treating B as predetermined in the relation between A and B is consistent with the common practice of explicitly entering the current futures quotation and some elements of the basis in a calculation to determine and/or justify an offered local price.

10. (Return to futured storage) - (return to firmed storage) $= (1 - \upsilon)(k_2 - B)$. As noted on page 22, υ is expected to be close to one so the difference is small. For most farmers, H will be negative, it seems likely that in cases of interdependence $|H|$ increases with B so δ is negative and $1 - \upsilon$ positive. This would make return to firmed storage typically greater than return to futured storage ($k_2 - B$ is typically negative) and row 3 typically relevant in Table 3.

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APPENDIX

Let $\eta: R^N \to R$ be an expected utility function written

$$\eta(\alpha) = E\psi\left(\sum_1^N \alpha_n Y_n\right) = E\psi(\alpha Y)$$

where $Y_1 \cdot\cdot Y_N$ are random variables such that any linear combination of them is nontrivial and $\alpha_1 \cdot\cdot \alpha_N$ are decision variables.

Suppose

(I) $\mathcal{A} \subset R^N$ is closed, convex with $0 \in \mathcal{A}$

(II) $\psi' > 0$, $\psi'' < 0$, $\lim_{x\to\infty} \psi'(x) = 0$

(III) $E|\psi(\alpha Y)| < \infty$, $E|Y_n \psi'(\alpha Y)| < \infty$ for $n = 1 \cdot\cdot N$ and all $\alpha \in R^N$

(IV) $\exists L \ni [\alpha^o \in \mathcal{A},\ P(\alpha^o Y \geq 0) = 1 \Rightarrow \|\alpha^o\| < L]$

Then

(V) η is strictly concave

(VI) η has continuous partial derivatives which are obtained by differentiation under the expectation

(VII) η assumes a unique maximum on $\mathcal{A}$

<u>Proof</u>

(V) Let $(\Omega, \mathcal{F}, P)$ be the probability space of Y.

Let $0 < \lambda < 1$, $\lambda^* = 1 - \lambda$, $\beta \neq \alpha$. Then

$$\eta(\lambda\alpha+\lambda^*\beta) = E\psi(\lambda\alpha Y + \lambda^*\beta Y) = \int \psi(\lambda\alpha Y(\omega) + \lambda^*\beta Y(\omega))\, dP$$

$$> \int (\lambda\psi(\alpha Y(\omega)) + \lambda^*\psi(\beta Y(\omega))dP = \lambda E\psi(\alpha Y) + \lambda^* E\psi(\beta Y)$$

$$= \lambda\eta(\alpha) + \lambda^*\eta(\beta)$$

where the strict concavity of ψ (Assumption II) implies that

$$\psi(\lambda\alpha Y(\omega) + \lambda^{*}\beta Y(\omega)) > \lambda\psi(\alpha Y(\omega)) + \lambda^{*}\psi(\beta Y(\omega)) \quad \text{on} \quad \{\omega \mid \alpha Y(\omega) \neq \beta Y(\omega)\}$$

and nontriviality of linear combinations of components of Y guarantees that the latter set has positive probability.

(VI) $$\frac{\partial\eta}{\partial\alpha_1} = \lim_{h\to 0} \frac{\eta(\alpha Y+hY_1) - \eta(\alpha Y)}{h} = \lim_{h\to 0} E \frac{\psi(\alpha Y+hY_1) - \psi(\alpha Y)}{h} .$$

By the Mean Value Theorem

$$\psi(\alpha Y + hY_1) = \psi(\alpha Y) + hY_1 \ \psi'(\alpha Y + hKY_1)$$

where K is a random variable $\ni 0 \leq K \leq 1$. Thus

$$\frac{\partial\eta}{\partial\alpha_1} = \lim_{h\to 0} E\, Y_1 \ \psi'(\alpha Y + hKY_1) \quad .$$

Let $Y_1^{+} = \max\{Y, 0\}$, $Y_1^{-} = \max\{-Y, 0\}$,

$$W^{+} = Y_1^{+} \ \psi'(\alpha Y - |h|Y_1), \ W^{-} = Y_1^{-} \ \psi'(\alpha Y - |h|Y_1) \quad .$$

Since ψ' is strictly decreasing

$|Y_1 \ \psi'(\alpha Y + hKY_1)| \leq W^{+} + W^{-} = \overline{W}$ and $\overline{W}$ is integrable since integrability of W^{+}, W^{-} is assured by (III). Hence, by Lebesgue's Convergence Theorem

$$\frac{\partial\eta}{\partial\alpha_1} = \lim_{h\to 0} EY_1\psi'(\alpha Y + hKY_1) = E \lim_{h\to 0} Y_1\psi'(\alpha Y + KY_1 h) = EY_1 \ \psi'(\alpha Y) \ .$$

By a result of Fenchel (see Katzner [6], page 198), called to my attention by M. Richter, a partial derivative of a concave function is continuous wherever it exists.

(VII) Let $C = \eta^{-1}([\eta(0), \infty))$; C is the inverse image of a closed set under a continuous function and therefore closed. From the strict concavity of η, C is strictly convex, and, by definition, C contains any maximizers of η.

Thus $C \cap \mathcal{A}$ is closed and convex. It suffices to show that $C \cap \mathcal{A}$ is bounded since compactness immediately follows and Weierstrass' Theorem assures a maximum. If there were two maximizers, the line segment joining them would lie in $C \cap \mathcal{A}$ and contain higher values of η (η strictly concave).

To show $C \cap \mathcal{A}$ bounded let $B_L = \{\alpha | \|\alpha\| \le L\}$ with boundary S_L and L the limit postulated in (IV). Define $\mathcal{S} = C \cap \mathcal{A} \cap S_L$. $\mathcal{S}$ is compact.

From $0 \in C \cap \mathcal{A}$, $C \cap \mathcal{A}$ convex it follows that if $\alpha \in C \cap \mathcal{A} \cap B_L^c$, then $\frac{L}{\|\alpha\|} \alpha \in \mathcal{S}$. If $\mathcal{S}$ is empty, $C \cap \mathcal{A} \subset B_L$ and we are through, so consider $\mathcal{S} \neq \phi$. Also, $C \cap \mathcal{A} \cap B_L^c \subset \text{Cone } \mathcal{S} = \{\alpha | \alpha = \lambda g,\ \lambda \ge 0,\ g \in \mathcal{S}\}$.

Choose any $g \in \mathcal{S}$ and for $\lambda \ge 0$, define $\mu(\lambda) = \eta(\lambda g) = E\psi(\lambda g Y) = E\psi(\lambda X)$. From (VI) μ is continuously differentiable and, from (V), strictly concave. Note

$$\mu' = EX\psi'(\lambda X) = EX^+\psi'(\lambda X^+) - EX^-\psi'(-\lambda X^-) = a(\lambda) - b(\lambda).$$

Since $\psi' > 0$, $a(\lambda) > 0$ and $b(\lambda) > 0$. From $\psi'' < 0$ and $\lim_{x \to \infty} \psi'(x) = 0$, one sees that $a(\lambda) \downarrow 0$ as $\lambda \to \infty$

while $b(\lambda)$ increases with λ. Thus $\mu'(\lambda)$ becomes and remains negative as λ increases so $\mu(\lambda)$ sometime returns to the value $\mu(0) = \eta(0)$ for a $\lambda \geq 1$, say $\lambda(g)$. Since $g \in \mathscr{G}$ was arbitrary we observe that $\forall\, g \in \mathscr{G}\ \exists\, \lambda(g) \geq 1 \ni \eta(\lambda(g)g) = \eta(0)$ and $\eta(\lambda g) < \eta(0)$ for $\lambda > \lambda(g)$.

Furthermore, since $\lambda(g)$ must solve $\eta(\lambda g) = \eta(0)$ and η is continuously differentiable with $\eta'_\lambda(\lambda(g)g) \neq 0$, $\lambda(g)$ is continuous (and indeed differentiable) by the Implicit Function Theorem. Therefore $\lambda(g)$ assumes a maximum, say λ^*, on compact $\mathscr{G}$. By this construction, $\alpha \in \text{Cone}\ \mathscr{G} \cap B^c_{\lambda^*} \Rightarrow \eta(\alpha) < \eta(0)$ or $C \cap \text{Cone}\ \mathscr{G} \cap B^c_{\lambda^*}$ is empty. It was observed above that $C \cap \mathcal{Q} \cap B^c_L \subset \text{Cone}\ \mathscr{G}$ so we conclude $C \cap \mathcal{Q} \cap B^c_L \subset B_{\lambda^*}$ bounded. The rest of $C \cap \mathcal{Q}$ is contained in B_L and hence bounded so $C \cap \mathcal{Q}$ is bounded.

In the storage-hedging problem,

$$\alpha = (m,\ s,\ g),\ Y = [(A - k_1),\ (k_2 - B),\ (k_3 - A)]\ ,$$

$$\mathcal{Q} = \{m,\ s,\ g \,|\, 0 \leq m \leq n,\ 0 \leq s,\ 0 \leq g \leq m\}\ .$$

Clearly (I) above is satisfied and Conditions (a) and (c) of Section 3, page 7 duplicate (II) and (III) of this Appendix. It remains to show that Condition (IV) is satisfied for the problem as specified in Section 3. Note that (IV) says that the admissible amount of any sure thing (a combination of random variables or ventures that can win but can't lose) is bounded. We must show that $\exists\, L$ such that

$$P[(A - k_1)m + (k_2 - B)s + (k_3 - A)g \geq 0] = 1 \Rightarrow (m^2 + s^2 + g^2)^{\frac{1}{2}} < L .$$

Since admissibility requires $m \leq n$, $g \leq n$ it suffices to show that s is bounded for any sure thing. Write

$$(A - k_1)m + (k_2 - B)s + (k_3 - A)g = Xw + Ys + kg$$

where

$$w = m - g,\ 0 \leq w \leq n,\ X = A - k_1,\ Y = k_2 - B,\ k = k_3 - k_1 .$$

Condition (d), page 7 requires that $P(Y < 0) > 0$. It follows via Lebesgue's Convergence Theorem that $\exists\ \varepsilon > 0 \ni P(Y < -\varepsilon) > 0$. Condition (b), page 7 requires that $E|X| < \infty$ which requires $\lim_{j\to\infty} P(X < j) = 1$. Thus if we choose a positive ε so that $P(Y < -\varepsilon) > 0$ and define $M_j = (X < j) \cap (Y < -\varepsilon)$, then $P(M_j) > 0$ must hold for sufficiently large j. Now if $Xw + Ys + kg \geq 0$ a.s. then, in particular, the inequality must hold on almost all of M_j. But $Xw + Ys + kg \geq 0$ for $\omega \in M_j \Rightarrow$ $jw - \varepsilon s + kg \geq 0 \Rightarrow \varepsilon s \leq jw + kg \Rightarrow s \leq \frac{(j + |k|)n}{\varepsilon}$. Hence

$$L = n\sqrt{2 + \frac{1}{\varepsilon^2}(j + |k|)^2}$$

is an upper bound for amounts of sure things. This completes the arguments that Conditions (a) - (e) of page 7 with $\mathcal{Q}$ as above imply Conditions (I) - (IV) of the Appendix.

It may be worth noting that if $k_3 = k_1$, η remains concave but not strictly concave. If $k_3 > k_1$ then $(A - k_1) + (k_3 - A)$ is a sure thing, but it is bounded by n. Whether or not there are other sure things cannot be said from the assumptions of the model, but the argument given assures that admissible amounts of any which might exist are bounded by L.