A REMARK ON MICROECONOMIC MODELS OF AN ECONOMY AND ON A GAME THEORETIC INTERPRETATION OF WALRAS EQUILIBRIA*

by

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ABSTRACT

The main purpose of this note is to describe rigorously a game in strategic form whose Nash equilibria coincide with the Walras equilibria of the underlying economy. Consequently the Nash outcomes are Pareto efficient.

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Introduction

We start with a short discussion of Walras models and their microeconomic content.

The Walras macroeconomic model consists of a function, say f, such that its domain is the set of positive normalized prices, S, and its range is the set of excess demand vectors in R^{ℓ}. Most of the (mathematically) technical questions related to this model deal with existence, computability, local and global uniqueness, and local and global stability of the Walras equilibrium. By a Walras equilibrium (WE for short) we mean here a price p^* in S so that $f(p^*) = 0$.

A microeconomic primitive of this model is one where the set of economic agents, say T, is introduced explicitely and the excess demand functions, $f_t : S \rightarrow R^{\ell}_{+}$, for each t in T are postulated. The previous model is derived from this by defining $f = \Sigma_{t \in T} f_t$. A microeconomic flavor is added to the second model through axioms of revealed preference.

Further steps toward a more primitive model bring us to the neoclassical economic agent. The consumer, t in T, is characterized by his initial endowment, w_t in R^{ℓ}_{+}, and his preferences $\succsim_t$

It is postulated, for each t in T, that $\succsim_t$ is a monotonic, strictly quasiconcave, reflexive, transitive and total binary relation on R_+^ℓ. (The producers are excluded from explicit treatment in order to maintain a concise presentation.)

To sum up a neoclassical model of (pure exchange) economy (or of economic environment) is a list $E = (T, R_+^\ell, (w_t)_{t\in T}, \succsim_t)_{t\in T})$. The main application of the last model was to show the relation between WE and Pareto efficiency (PE for short). The derivation of the previous model from E is done by defining $f_t(p)$, for all t in T and for all p in S, as follows. $f_t(p)$ is the unique vector in R^ℓ which satisfies: $pf_t(p) = 0$, $f_t(p) + w_t \in R_+^\ell$ and, $y \succ_t f_t(p) + w_t$ implies $py > pw_t$. (It is well known that the conditions imposed on E guarantee the existence and uniqueness of $f_t(p)$ defined above.) A WE for E is a price p in S together with a T-list $(x_t)_{t\in T}$ of vectors in R_+^ℓ s.t. $\Sigma_{t\in T} x_t = \Sigma_{t\in T} w_t$ and for all t in T, $x_t - w_t = f_t(p)$.

However the designation "microeconomic model" leads one to expect something more than a consumer reacting mechanically to prices (prices the origin of which is still a mystery). Having at its disposal the most recent tools of (also statistical) decision theory, the microeconomic unit is expected to use them. Since a rigorous presentation of Walras neoclassical model by Arrow and Debreu appeared after the publication of von Neumann-Morgenstern book and Nash's paper, a minimal requirement of the economic agent would be to behave as smartly as a player in a noncooperative game. Indeed Debreu tried to apply this rationale in his early proof of existence of WE. But the game which he defined was not in strategic form, (strategies of one player depend on the strategies of other players) and the idea was abandoned (until recently). One can think of several reasons why those preoccupied with the extensions of Walras, Arrow-Debreu model did not improve its intrinsically microeconomic performance. For some, the interest in microeconomic model exists only to the extent that it supports their preconceived macroeconomic views. Another point of view is that the cooperative games theory (core) supplies a sufficient rationale for a WE.

A new interest in noncooperative-games-like models for economies originated recently from two independent sources. One of them is the first example of such models presented by Shapley-Shubik. Another source of interest are the remarkable results of Groves-Ledyard, followed up by Hurwicz that suggested an alternative to WE solution concept for neoclassical economy E. Since this model is not backed up by an economic mythology it has to be more consistent from the point of view of rational economic agents. Hurwicz suggested that the rules of the game (the mechanism) will be so defined that the equilibrium will be feasible (but nonequilibrium behavior may lead to nonfeasible or undetermined outcomes). Such a model whose Nash equilibria coincide with WE is introduced in the sequel. A related model was suggested recently by R. Wilson. A distinguishing characteristic of this model is that no additional artificial player is added to the group of economic agents and every agent controls prices as well as net trades as his strategic variables.

A Nash-Walras Game

For a given economy $E = (T, R_+^\ell, (w_t)_{t \in T}, (\succ_t)_{t \in T})$, a game in strategic form is defined. For each t in T, $S_t = \{(p,z) \in S \times R^\ell | p \cdot z = 0\}$ is the set of strategies of agent t. The outcome function, g, maps T-lists of strategies to T-lists of net-trades, $g: X_{t \in T} S_t \rightarrow (R^\ell)^T$ s.t. $\Sigma_{t \in T} g_t((p_t, z_t)_{t \in T}) = 0$ where g_t is the net-trade of agent t in T. The function g is defined as follows. Given a T-list of strategies $(p_t, z_t)_{t \in T}$ we first partition T to sets s.t. members of the same set announced the same price. More precisely let $T_o, T_1, \ldots, T_k$ be a partition of T, where t and t', $t \neq t'$, belong to the same T_i for some $i \geq 1$ iff $p_t = p_t'$ and where $T_o = T \backslash (\cup_{i=1}^k T_i)$. For h in T_i define $g_h((p_t, z_t)_{t \in T}) = z_h - (\Sigma_{t \in T_i} z_t)/\#T_i$. This definition guarantees that $\Sigma_{h \in T_i} g_h((p_t, z_t)_{t \in T}) = 0$ for $i=1,\ldots,k$. A Nash allocation is an allocation induced by a NE of the game described above.

THEOREM: The Nash allocations of an economy E coincide with the Walras allocations of E. (Hence there exists at least one Nash equilibrium for E and each Nash equilibrium is Pareto efficient.)

Proof: Let $(p_t, z_t)_{t \in T}$ be a Nash equilibrium and let $\{T_i\}_{i=0}^k$ be the corresponding partition of T with $p_i \equiv p_t$ for any t in T_i, $i = 1,2,\ldots,k$. For each h in T denote by x_h the resulting net trade, $g_h((p_t, z_t)_{t \in T})$ of agent h.

Clearly the Theorem holds if $\#T \leq 1$. From here on suppose then, that $\#T \geq 2$. By Claim (i), which follows, $T \setminus T_o \neq \emptyset$ and $k \geq 1$.

<u>Claim (i)</u> $\#T_o \leq 1$.

Otherwise, any agent h in T_o can obtain as a net trade any x in R^{ℓ} by playing (p_h, z) with

$$z = \frac{\#T_o}{\#(T_o - \{h\})} \; x + \Sigma_{t \in T - \{h\}} \; z_t / \#T_o .$$

Choosing $x + w_h$ preferred to $x_h + w_h$ by h yields a contradiction.

<u>Claim (ii)</u> For each h in T and each i, $i=1,\ldots,k$ $x_h + w_k \succsim_h f_h(p_i) + w_h$ where $f_h(p)$ is the excess demand of agent h for prices p in S.

This is clear since given the strategies of others, agent h can always obtain the net-trade $f_h(p_i)$ if he plays the strategy

$$(p_i, z) \quad \text{with} \quad z = \left(f_h(p_i) + \frac{\Sigma_{t \in T_i - \{h\}} z_t}{\#(T_i - \{h\}) + 1} \right) \frac{\#(T_i - \{h\}) + 1}{\#(T_i - \{h\})}$$

<u>Claim (iii)</u> For each h in T and each i, $i=1,\ldots,k$ $x_h = f_h(p_i)$.

By the definition of g we have for each i, $i = 1,\ldots,k$ and for each t in T_i, $p_i x_t = 0$, hence claim (ii) implies, $x_t = f_t(p_i)$. Suppose, per absurdum, that for some h in $T \setminus T_i$, $x_h \neq f_h(p_i)$. Then by claim (ii) $x_h + w_h \succ_h f_h(p_i) + w_h$ which implies $p_i x_h > 0$. But if $t \in T_j$, $\Sigma_{t \in T_j} x_t = 0$, which in turn implies, for $j \geq 1$, $p_i x_h{*} < 0$ for some h^* in T_j. The last inequality implies that $f_{h^*}(p_i) + w_{h^*} \succ_{h^*} x_{h^*} + w_{h^*}$, a contradiction to claim (ii). For the case $\{h\} = T_o$, $x_h = 0$ hence $p_i x_h > 0$ is a contradiction.

Claim (iii) implies that the Nash equilibrium net-trades $(x_t)_{t \in T}$ are obtainable as outcomes of the T - list of strategies $(p, f_t(p))_{t \in T}$ where p is any one of the p_i's , i=1,...,k. Hence the Nash outcome is a Walras allocation, which is of course, Pareto efficient. Clearly the opposite is also true; any Walras equilibrium gives rise to Nash equilibrium and the existence of Walras equilibrium for E is well known to be guaranteed. Q.E.D.

The heuristic defect of this model is the fact that for a t-list of strategies not in equilibrium the resulting outcome may not be feasible. Although in the aggregate the net trades are always feasible, their sum is zero, the net trade of the individual agent may not be feasible for him $(g_h((p_t, z_t)_{t \in T}) + w_h \notin R_+^{\ell})$. (From a practical point of view, the construction of such models is not totally unthinkable. Consider the following statement: If next year's budget is a the projected deficit is a', but if the budget is b, b > a, the projected deficit is b', b' < b; hence b is recommended. This recommendation is based on some implicit concept of equilibrium which if not sustained will result in nonfeasibility of b.)

The only closed models, or what we refer to alternatively, heuristically rigorous microeconomic models, are those of Shapley-Shubik and Pazner-Schmeidler. The Nash equilibria of these models are in general Pareto inefficient. However the Nash equilibria of the Pazner-Schmeidler model in the limiting case, i.e., a nonatomic continuum of agents, coincide with the Walras equilibria. Furthermore Postlewaite-Schmeidler showed that the Nash equilibria of sufficiently large economies are approximately efficient.

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