CONVERGENT NON-TATONNEMENT RESOURCE ALLOCATION PROCESSES FOR NON-CLASSICAL ENVIRONMENTS

by

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ABSTRACT

Two classes of decentralized non-tatonnement resource allocation processes, one for non-decomposable environments and the other for decomposable environments, are presented which are non-wasteful and unbiased and have certain stability properties. Non-convexities and indivisibilities are admitted, though the purely indivisible case is ruled out. The essential environmental requirements are local non-satiation of preference relations and continuity of preference relations and technology.

1. INTRODUCTION

The basic theorems of welfare economics state that, in "classical economic environments," every equilibrium allocation of the perfectly competitive resource allocation process is a Pareto-optimum and every Pareto-optimum can be attained as an equilibrium allocation of that process with the aid of some redistribution of initial resource endowments (cf. Arrow [1, Theorems 4 and 5] and Debreu [4, Theorem (1) of 6.3 and Theorem (1) of 6.4], for instance). According to the terminology introduced by Hurwicz [5], the perfectly competitive resource allocation process is non-wasteful and unbiased for classical economic environments, where, by "classical economic environments," we mean economic environments free of externalities, non-convexities, and discontinuities. Since the perfectly competitive resource allocation process is neither non-wasteful nor unbiased for non-classical economic environments, it is natural to ask whether or not there exists a resource allocation process which is non-wasteful and unbiased for those environments. In particular, it is of interest to ask whether we can find such a process which is informationally decentralized in some sense. The "greed process" developed by Hurwicz [5] is an example of such processes. This process is non-wasteful and unbiased for every economic environment free of externalities, but fails to be dynamically stable.[1] By relaxing Hurwicz' definition of informational decentralization, Camacho [3] presented a process, called "D process," which is non-wasteful and unbiased for every economic environment. This process is not dynamically stable, either.

Dynamically stable processes have been proposed by Kanemitsu [7], Ledyard [8], and Hurwicz, Radner, and Reiter [6]. The "inertia-greed process" of Kanemitsu [7] is of the tatonnement type, while the "P process" of Ledyard [8] and the "B process" of Hurwicz, Radner, and Reiter [6] are of the non-tatonnement type. The inertia-greed process is designed for economic environments free of externalities and indivisibilities, and the B process is designed for economic environments which are free of externalities and contain either indivisible commodities only or divisible commodities only. The P process is designed for economic environments which admit externalities but not non-convexities. Thus the economic environments which are free of externalities and contain both divisible and indivisible commodities are covered by neither the inertia-greed process nor the B process, while the economic environments with non-convexities are not covered by the P process.

The purpose of this paper is to fill these gaps in environmental coverage. We shall present two classes of non-stochastic, non-tatonnement resource allocation processes, one for economies with externalities and the other for economies without externalities. The former covers environments much broader than those covered by the P process. But, our processes will use set-valued messages, while the P process uses point-valued messages. So, information processing should be much more complicated in our processes than in the P process. This is a price for obtaining broader environmental coverage. Indeed, use of point-valued messages in the P process is made possible by convexity assumptions on the environments. The situation suggests a trade-off relation between environmental coverage and informational requirements.[2] Our processes for economies without externalities extend the environmental coverage of the inertia-greed process but not that of the B process. For, our processes do not cover the purely indivisible case. The fact that the purely indivisible case is covered by the B process seems to be supported by the stochastic nature of the process. So, whether or not there exists a dynamically stable non-stochastic resource allocation process for the purely indivisible environments is an open problem of interest.

The stability properties shared by our processes are similar to those shared by the P process and the B process. First of all, if the sequence of allocations generated by these processes has a cluster point then this cluster point is necessarily Pareto-optimal. But it is not always the case that the sequence has a limit point. Thus the processes are not stable in the allocation space. On the other hand, if the preference relations of agents are representable by utility functions then the sequence of utility allocations associated with the sequence of allocations is convergent. This kind of stability relies heavily on the monotonicity of the processes with respect preference relations.

In our processes, transactions take place even if an equilibrium has not yet been attained. This is the reason why the name "non-tatonnement" is attached to our processes. It is assumed, however, that no consumptions (nor productions) are carried out until some equilibrium obtains. So it is natural to interpret our models in short-run context.[3]

In Section 2, the assumptions on environments will be postulated, and some preliminary results on environmental properties will be stated. In Section 3, the resource allocation processes will be defined, and the main results will be stated. All the results are proved in Section 4.

2. ENVIRONMENTS

We shall consider economies with m commodities and n agents. The set of commodities and the set of agents are denoted by $H = \{1, \ldots, m\}$ and $I = \{1, \ldots, n\}$, respectively. The m-dimensional Euclidean space R^m will be referred to as the commodity space. For each $i \in I$, his consumption x_i and production y_i are points of R^m. An n-tuple $x = (x_i)_{i\in I}$ of individual consumptions is called a consumption allocation, and an n-tuple $y = (y_i)_{i\in I}$ of individual productions is called a production allocation. The ordered pair (x, y) of a consumption allocation x and a production allocation y is called an allocation. To each $i \in I$, there corresponds (1) the set D_i of i-possible allocations which is a subset of R^{2mn}, (2) his preference relation Q_i which is a complete, transitive binary relation on D_i,[4] and (3) his initial resource endowment a_i which is a point of R^m. The ordered triple (D_i, Q_i, a_i) is called the environmental characteristic of agent i. The n-tuple $(D_i, Q_i, a_i)_{i\in I}$ is called an environment.

The set of possible allocations and the set of attainable allocations are defined by $D = \cap_{i\in I} D_i$ and $A = \{(x, y) \in D: \Sigma_{i\in I}(x_i - y_i - a_i) = 0\}$, respectively. An attainable allocation (x, y) is said to be Pareto-optimal if there exists no $(x', y') \in A$ such that $(x, y)\overline{Q_i}(x', y')$ for every $i \in I$, where $(x, y)\overline{Q_i}(x', y')$ means that $((x, y), (x', y')) \notin Q_i$. This definition of Pareto-optimality is somewhat broader than the usual one (cf. Arrow and Hahn [2, p. 91]).

An environment $(D_i, Q_i, a_i)_{i\in I}$ is said to be decomposable if (1) there exists an n-tuple $(D^i)_{i\in I}$ of subsets of R^{2m} such that $D = \prod_{i\in I} D^i$ and (2) for each $i \in I$ there exists a binary relation Q^i on D^i such that $Q_i =$

$\{((x, y), (x', y')) \in (\prod_{i\in I} D^i)^2\colon\ (x_i, y_i)Q^i(x_i', y_i')\}$. It is easy to see that, for every $i \in I$, Q^i is complete and transitive. In case $(D_i, Q_i, a_i)_{i\in I}$ is a decomposable environment, we shall denote it by $(D^i, Q^i, a_i)_{i\in I}$. Note that decomposability does not imply that, for each agent, his preference relation is independent of his production activities or his consumption (resp. production) possibility is independent of his production (resp. consumption) possibility.

For a decomposable environment $(D^i, Q^i, a_i)_{i\in I}$, it will be convenient to introduce some other notations. The <u>set of trades</u> is defined by $F = \{z \in R^{mn}\colon \Sigma_{i\in I} z_i = 0\}$. A point d of R^{mn} is called a <u>redistribution</u> if $d - a \in F$. The <u>set of attainable redistributions</u> is defined by $B = \{d \in R^{mn}\colon\ d - a \in F$ and there exists a production allocation y such that $(d_i + y_i, y_i) \in D^i$ for every $i \in I\}$. For each $i \in I$, his <u>production set</u> is defined by $Y_i = \{y_i \in R^m\colon (x_i, y_i) \in D^i$ for some $x_i \in R^m\}$. An attainable redistribution d is said to be <u>Pareto-optimal</u> if there exists $y \in \prod_{i\in I} Y_i$ such that (1) $(d_i + y_i, y_i) \in D^i$ for every $i \in I$ and (2) there exists no $(d', y') \in B \times \prod_{i\in I} Y_i$ such that $(d_i' + y_i', y_i') \in D^i$ and $(d_i + y_i, y_i)\overline{Q}^i(d_i' + y_i', y_i')$ for every $i \in I$.

PROPOSITION 1: <u>Let</u> $(D^i, Q^i, a_i)_{i\in I}$ <u>be a decomposable environment</u>. <u>Then, for every</u> $d \in B$, d <u>is a Pareto-optimal redistribution if and only if there exists</u> $y \in \prod_{i\in I} Y_i$ <u>such that</u> $(d + y, y)$ <u>is a Pareto-optimal allocation</u>.

The class of non-decomposable environments to be considered in this paper satisfies the following four assumptions:

ASSUMPTION 1: <u>For every</u> $i \in I$, D_i <u>is closed in</u> R^{2mn}.

ASSUMPTION 2: <u>For every</u> $i \in I$, <u>the set</u> $\{(x', y') \in D_i: (x', y')Q_i(x, y)\}$ <u>is closed in</u> D_i <u>for every</u> $(x, y) \in D_i$.

ASSUMPTION 3: <u>For every</u> $i \in I$, <u>every</u> $(x, y) \in D_i$, <u>and every positive real number</u> ε <u>there exists</u> $(x', y') \in D_i$ <u>such that</u> $(x, y)\overline{Q_i}(x', y')$ <u>and</u> $d((x, y), (x', y')) < \varepsilon$.[5]

ASSUMPTION 4: $(a, 0) \in D$.

The class of decomposable environments to be considered satisfies the four assumptions below. To state these, we need a few more notations. For each $i \in I$, his <u>consumption set</u> is defined by $X_i = \{x_i \in R^m: (x_i, y_i) \in D^i \text{ for some } y_i \in R^m\}$. For each $i \in I$ and each $b \in R^m$, let $Y_i(b) = \{y_i \in Y_i: y_i \geqq b\}$.[6]

ASSUMPTION 1*: <u>For every</u> $i \in I$, (a) D^i <u>is closed in</u> R^{2m}, (b) X_i <u>is bounded from below, and</u> (c) $Y_i(b)$ <u>is bounded for every</u> $b \in R^m$.

Part (a) of Assumption 1* is equivalent to Assumption 1. The following is equivalent to Assumption 2.

ASSUMPTION 2*: <u>For every</u> $i \in I$, <u>the set</u> $\{(x_i', y_i') \in D^i: (x_i', y_i')Q^i(x_i, y_i)\}$ <u>is closed in</u> D^i <u>for every</u> $(x_i, y_i) \in D^i$.

Assumption 3 can be weakened as follows.

ASSUMPTION 3*: <u>For every</u> $i \in I$, <u>every</u> $(x_i, y_i) \in D^i$, <u>and every positive</u>

real number ε there exists $(x_i', y_i') \in D^i$ such that $(x_i, y_i)\overline{Q^i}(x_i', y_i')$ and $d((x_i, y_i), (x_i', y_i')) < \varepsilon$.

Assumption 4 can also be weakened as follows.

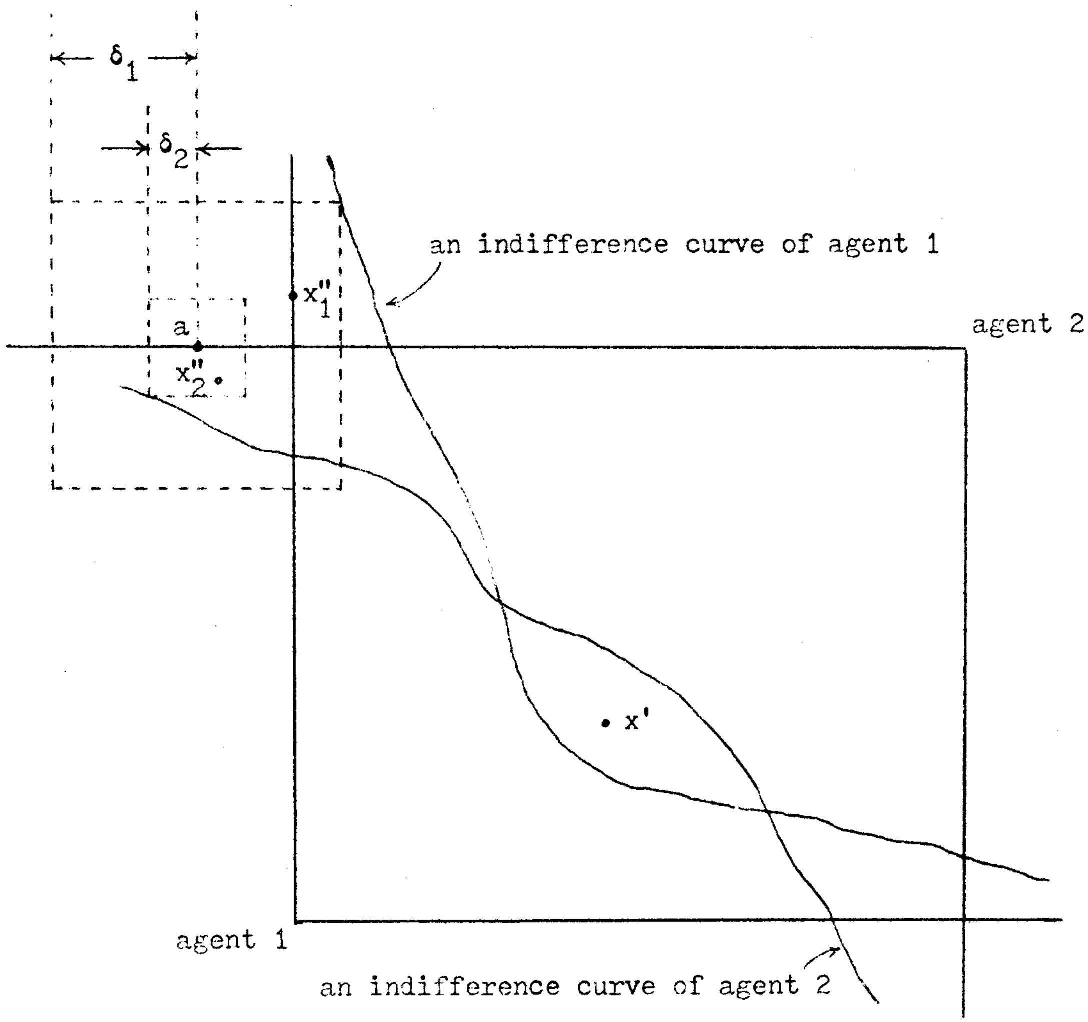
ASSUMPTION 4*: Either (a) there exists $y \in \prod_{i \in I} Y_i$ such that $(a_i + y_i, y_i) \in D^i$ for every $i \in I$ or (b) there exists $(x', y') \in A$, $(x'', y'') \in D$, and $\delta \in R^n$ such that, for every $i \in I$, $d(x_i'' - y_i'', a_i) < \delta_i$ and $(x_i', y_i')Q^i(x_i, y_i)$ for every $(x_i, y_i) \in D^i$ such that $d(x_i - y_i, a_i) < \delta_i$. (See Figure 1.)

This last assumption admits the possibility that some agents cannot survive with their initial resource endowments.

We note here that, among the assumptions above, the only one which has something to do with divisibility of commodities is that of local non-satiation of preference relations, i.e., Assumption 3 or Assumption 3*. Local non-satiation rules out the purely indivisible case, but does not require that there should exist at least one commodity which is divisible in the usual sense. All that Assumption 3 requires concerning divisibility is that for every $(x, y) \in \cup_{i \in I} D_i$ there should exist at least one commodity h which is divisible in some neighborhood of one point of the set $\{r \in R\colon\ r = x_{ih}$ or $r = y_{ih}$ for some $i \in I\}$. Such a commodity may be said to be locally divisible.

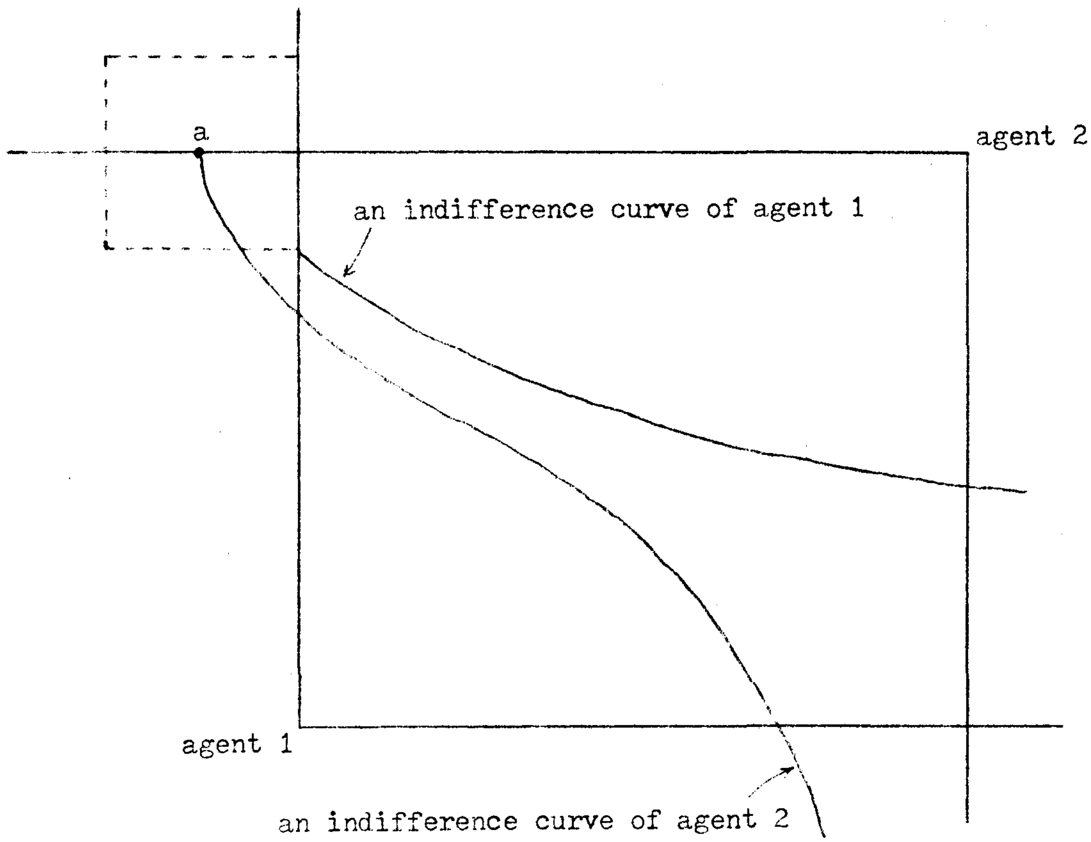
Under our assumptions, the decomposable environment $(D^i, Q^i, a_i)_{i \in I}$ can be regarded as a "pure-exchange" economy, by defining, for each $i \in I$, his "consumption set" by

$$X_i^* = \{x_i \in R^m\colon\ (x_i + y_i, y_i) \in D^i \text{ for some } y_i \in R^m\}$$



$$D^1 = D^2 = \{y \in R^2 : \; y \geqq 0\} \times \{0\}$$

Figure 1a: An example satisfying Assumption 4*



$$D^1 = D^2 = \{y \in R^2: \ y \geqq 0\} \times \{0\}$$

Figure 1b: An example violating Assumption 4* and his "preference relation" on X_i^* by

$$\succsim_i = \{(x_i, x_i') \in (X_i^*)^2 \text{: There exists } y_i \in Y_i \text{ such that } (x_i + y_i, y_i) \in D^i \text{ and } (x_i + y_i, y_i) Q^i (x_i' + y_i', y_i') \text{ for every } y_i' \in Y_i \text{ such that } (x_i' + y_i', y_i') \in D^i\}.$$

Then clearly $B = \{x \in \prod_{i \in I} X_i^*\colon\ x - a \in F\}$, which takes the usual form of the set of attainable allocations in a pure-exchange economy. We shall call $(X_i^*, \succsim_i, a_i)_{i \in I}$ the "pure-exchange" economy induced by the decomposable environment $(D^i, Q^i, a_i)_{i \in I}$. This has the following environmental properties.

PROPOSITION 2: _Under Assumption_ 1*, _for every_ $i \in I$, X_i^* _is closed in_ R^m.

PROPOSITION 3: _Under Assumptions_ 1*, 2*, _and_ 3*, _for every_ $i \in I$, $\succsim_i$ _is a complete, transitive binary relation on_ X_i^* _such that_ (a) _the set_ $\{x_i' \in X_i^*\colon x_i' \succsim_i x_i\}$ _is closed in_ X_i^* _for every_ $x_i \in X_i^*$ _and_ (b) _for every_ $x_i \in X_i^*$ _and every positive real number_ ε _there exists_ $x_i' \in X_i^*$ _such that_ $x_i' \succ_i x_i$ _and_ $d(x_i, x_i') < \varepsilon$.[7]

PROPOSITION 4: _Under Assumption_ 4*, _either_ (a) $a \in \prod_{i \in I} X_i^*$ _or_ (b) _there exists_ $x' \in B$, $x'' \in \prod_{i \in I} X_i^*$, _and_ $\delta \in R^n$ _such that, for every_ $i \in I$, $d(x_i'', a_i) < \delta_i$ _and_ $x_i' \succsim_i x_i$ _for every_ $x_i \in X_i^*$ _such that_ $d(x_i, a_i) < \delta_i$.

In terms of the notation introduced for the "pure-exchange" economy, we can express the conditions for a Pareto-optimal redistribution in a more familiar way.

PROPOSITION 5: *Under Assumption* 1*, *for every* $x^* \in B$, x^* *is a Pareto-optimal redistribution if and only if there exists no* $x \in B$ *such that* $x_i \succ_i x_i^*$ *for every* $i \in I$.

3. RESOURCE ALLOCATION PROCESSES

Given two sets L and C, an ordered pair (T, h) is called a non-tatonnement resource allocation process (or simply, process) with language L and outcome space C if T is a function (called a response function) from $L^n \times C$ to L^n and h is a function (called an outcome function) from $L^n \times C$ to C. Let $K = (T, h)$. Then K is a function from $L^n \times C$ to itself. Clearly, a fixed point (M, c) of K is among the equilibrium concepts of the process we can think of. Once the fixed point has been attained, both of the message complex M and the outcome c remain constant. So, this fixed point (M, c) may be called a full equilibrium of the process (T, h), provided c satisfies certain attainability conditions. Furthermore, an attainable outcome c may be called a full equilibrium outcome if (M, c) is a full equilibrium for some $M \in L^n$. On the other hand, concentrating our attention on outcomes only, we may also think of the following two equilibrium concepts.

Given $c \in C$, we define a function $G_c\colon\ L^n \to L^n$ by $G_c(M) = T(M, c)$. Let N denote the set of natural numbers. Given $c \in C$ and $t \in N$, we define a function $G_c^t\colon\ L^n \to L^n$ by $G_c^t(M) = G_c(G_c^{t-1}(M))$, where $G_c^0(M) = M$. An attainable outcome c is called a weak equilibrium outcome if there exists $M \in L^n$ such that $c = h(G_c^{t-1}(M), c)$ for every $t \in N$. In this case, if the initial message complex M^0 is chosen equal to M, then $M^t = T(M^{t-1}, c)$ and $c = h(M^{t-1}, c)$ for every $t \in N$, so that the outcome c actually remains constant. On the other hand, an attainable outcome c is called a strong equilibrium outcome if $c = h(G_c^{t-1}(M), c)$ for every $M \in L^n$ and every $t \in N$. In this paper, we shall be concerned with the last two equilibrium concepts. We note here that both a strong equilibrium outcome and a full equilibrium outcome are a weak equilibrium outcome.

Our processes for the class of environments satisfying Assumptions 1, 2, 3, and 4 use as their language the set of all non-empty sets of allocations:

$$L_1 = \{U\colon\ U \text{ is a non-empty subset of } R^{2mn}\}.$$

Their outcome space is chosen to be the set of all allocations:

$$C_1 = R^{2mn}.$$

Given $i \in I$ and a positive real number r, we define a function $f_i^r\colon (L_1)^n \times C_1 \to R$, a function $S_i^r\colon (L_1)^n \times C_1 \to L_1$, and a function $T_i^r\colon (L_1)^n \times C_1 \to L_1 \cup \{\emptyset\}$ by

$$f_i^r(M, x, y) = \begin{cases} r & \text{if } d(M_i, (x, y)) = 0 \text{ or } A \cap (\cap_{j\in I} M_j) \neq \emptyset, \\ \dfrac{\Sigma_{j\in I-\{i\}} d(M_j, (x, y))}{\Sigma_{j\in I} d(M_j, (x, y))}\, d(M_i, (x, y)) & \text{otherwise,} \end{cases}$$

$$S_i^r(M, x, y) = \{(x', y') \in D_i\colon\ d((x', y'), (x, y)) \leqq \max\{d(D_i, (x, y)), f_i^r(M, x, y)\}\},$$

$$T_i^r(M, x, y) = \{(x^*, y^*) \in D_i\colon\ (x^*, y^*) Q_i (x', y') \text{ for every } (x', y') \in S_i^r(M, x, y)\},$$

respectively.

Given a positive real number r, we define a function $T^r\colon (L_1)^n \times C_1 \to (L_1 \cup \{\emptyset\})^n$ by

$$T^r(M, x, y) = (T_i^r(M, x, y))_{i \in I}.$$

It will turn out that T^r is actually a function from $(L_1)^n \times C_1$ to $(L_1)^n$ (cf. Lemma 2 in Section 4). Thus T^r can be regarded as the response function of the process being constructed.

The set $T_i^r(M, x, y)$ consists of those i-possible allocations which are at least as desired as any element of $S_i^r(M, x, y)$. To use the terminology of Hurwicz [5], $T_i^r(M, x, y)$ may be said to represent the "greed" response to $S_i^r(M, x, y)$. The set $S_i^r(M, x, y)$ consists of those i-possible allocations whose distances from (x, y) do not exceed $\max \{d(D_i, (x, y)), f_i^r(M, x, y)\}$; therefore it may be regarded as the "ball" with center (x, y) and radius $\max \{d(D_i, (x, y)), f_i^r(M, x, y)\}$. Since the radius may be as large as the distance of D_i from (x, y) and, by Assumption 1, D_i is closed in R^{2mn}, it follows that $S_i^r(M, x, y)$ is always non-empty.

How much the radius of $S_i^r(M, x, y)$ may exceed $d(D_i, (x, y))$ depends on the message complex M. A message complex $M \in (L_1)^n$ is said to be <u>consistent</u> if $A \cap (\cap_{i \in I} M_i) \neq \emptyset$, and M is said to be <u>inconsistent</u> if it is not consistent. On the other hand, agent i's message $M_i \in L_1$ is said to be <u>ambitious</u> if $d(M_i, (x, y)) > 0$, and M_i is said to be <u>unambitious</u> if it is not ambitious. If $(x, y) \in A$, then the message complex M is consistent whenever, for every $i \in I$, M_i is unambitious and closed in R^{2m}. So, some agents being ambitious may often be a cause for message complexes to be inconsistent. If either the message complex M is consistent or agent i's message M_i is unambitious then the radius of $S_i^r(M, x, y)$ may be as large as r, while if M is inconsistent and M_i is ambitious then the radius of $S_i^r(M, x, y)$ cannot exceed a certain fraction of $d(M_i, (x, y))$, the multiplicative coefficient of $d(M_i, (x, y))$

being inversely related to the relative size of $d(M_i, (x, y))$ to the sum $\Sigma_{j\in I} d(M_j, (x, y))$ as specified in the definition of $f_i^r(M, x, y)$. In the latter case, agent i's new proposal $T_i^r(M, x, y)$ is required to contain an allocation which is less ambitious than any allocation belonging to the previous proposal M_i, i.e., $d(T_i^r(M, x, y), (x, y)) < d(M_i, (x, y))$. In this sense, $S_i^r(M, x, y)$ represents a sort of constraint. But, since $T_i^r(M, x, y)$ is not a subset of $S_i^r(M, x, y)$, this response rule does not correspond to "constrained optimization" in the usual sense.

Given a positive real number r, we define a correspondence $H^r: (L_1)^n \times C_1 \to C_1$ by

$$H^r(M, x, y) = \begin{cases} \{(x, y)\} & \text{if } A \cap (\cap_{i\in I} T_i^r(M, x, y)) = \emptyset, \\ A \cap (\cap_{i\in I} T_i^r(M, x, y)) & \text{otherwise.} \end{cases}$$

Since any (not necessarily continuous) selection h for H^r can be regarded as an outcome function of the process being constructed, we may call H^r an outcome correspondence. According to this outcome correspondence, no transactions should be made when the message complex $T^r(M, x, y)$ is inconsistent, while some new allocation should be chosen from the set $A \cap (\cap_{i\in I} T_i^r(M, x, y))$ when $T^r(M, x, y)$ is consistent.

Let

$$P_1(L_1, C_1) = \{(T, h): \text{There exists a positive real number } r \text{ such that } T = T^r \text{ and } h(M, x, y) \in H^r(M, x, y) \text{ for every } (M, x, y) \in (L_1)^n \times C_1\}.$$

Then any element (T, h) of $P_1(L_1, C_1)$ is a non-tatonnement resource allocation process with language L_1 and outcome space C_1. For every $i \in I$, T_i is independent of (D_j, Q_j) for every $j \in I - \{i\}$, but is dependent of consumptions and productions of other agents. At each stage of the process, all agents are assumed to be informed of the current allocation. In particular, they much know the initial allocation at the first stage of the process, which usually implies that each agent should know the initial resource endowments of other agents. In this sense, T is <u>not</u> privacy-preserving. It may be emphasized again, however, that the <u>preference fields</u> (D_j, Q_j) need not be communicated. These are far more difficult to communicate than the initial resource endowments. Since T is anonymous in the sense that any permutation of agents other than i does not affect T_i, the process (T, h) may still qualify as an informationally decentralized process in a very weak sense.

The <u>set of attainable outcomes</u> of $(T, h) \in P_1(L_1, C_1)$ is always chosen to be equal to A, the set of attainable allocations.

THEOREM 1: *Under Assumptions 1, 2, and 3, for every $(T, h) \in P_1(L_1, C_1)$, every weak equilibrium outcome of (T, h) is a Pareto-optimal allocation.*

THEOREM 2: *Under Assumptions 1, 2, and 3, for every $(T, h) \in P_1(L_1, C_1)$, every Pareto-optimal allocation is a strong equilibrium outcome of (T, h).*

THEOREM 3: *Suppose that Assumptions 1, 2, 3, and 4 are satisfied. Let $(T, h) \in P_1(L_1, C_1)$ and let $\{(x^{t-1}, y^{t-1})\}_{t\in N}$ be a sequence in R^{2mn} such that $(x^0, y^0) = (a, 0)$ and there exists a sequence $\{M^{t-1}\}_{t\in N}$ in $(L_1)^n$ such that $M^t = T(M^{t-1}, x^{t-1}, y^{t-1})$ and $(x^t, y^t) = h(M^{t-1}, x^{t-1}, y^{t-1})$ for every $t \in N$.*

Then every cluster point of $\{(x^{t-1}, y^{t-1})\}_{t\in N}$ is a Pareto-optimal allocation.

Theorems 1 and 2 say that every non-tatonnement resource allocation process in $P_1(L_1, C_1)$ is non-wasteful and unbiased. For this class of processes, every weak equilibrium outcome happens to be a strong equilibrium outcome. Theorem 3 concerns the stability properties of processes in $P_1(L_1, C_1)$. It says that if a sequence of allocations generated by a process in $P_1(L_1, C_1)$ converges to some allocation then the limit allocation is necessarily Pareto-optimal. In general, however, the sequences of allocations generated by the process are not convergent. Nevertheless, the following kind of stability properties are shared by our processes.

COROLLARY TO THEOREM 3: Suppose, in addition to Assumptions 1, 2, 3, and 4, that A is compact, and that, for each $i \in I$, Q_i can be represented by a utility function u_i. Let $\{(x^{t-1}, y^{t-1})\}_{t\in N}$ be as in Theorem 3. For each $t \in N$, let $u^t = (u_i(x^t, y^t))_{i\in I}$. Then the sequence $\{u^t\}_{t\in N}$ in R^n is convergent. If, furthermore, for each $i \in I$, u_i is lower semi-continuous, then $\lim_{t\to\infty} u^t = (u_i(x^*, y^*))_{i\in I}$ for some Pareto-optimal allocation (x^*, y^*).

That is, our processes are convergent in the "utility space." The B process of Hurwicz, Radner, and Reiter [6] has a similar property (cf. [6, Theorem 5.2]). Theorem 3 is also related to "global value stability" of Ledyard [8, Theorem 7].

Needless to say, the above results are valid also for decomposable environments, as long as Assumptions 1, 2, 3, and 4 are satisfied. But, for decomposable environments, it is possible to construct a process which is simpler and informationally more decentralized than the processes in $P_1(L_1, C_1)$.

Let $(D^i, Q^i, a_i)_{i \in I}$ be a decomposable environment satisfying Assumptions 1*, 2*, 3*, and 4*. Let

$$L_2 = \{U:\ U \text{ is a non-empty subset of } R^m\},$$
$$C_2 = R^{mn}.$$

The language L_2 can be regarded as the set of all non-empty sets of individual trades, and the outcome space C_2 as the set of all redistributions. L_2 is much simpler than L_1.

For each $i \in I$ and each $x_i \in R^m$, let

$$Z_i(x_i) = \{z_i \in R^m:\ (x_i + z_i + y_i,\ y_i) \in D^i \text{ for some } y_i \in Y_i\}.$$

The set $Z_i(x_i)$ stands for the individual trades of agent i that are possible given his predetermined resources x_i. In terms of the notation introduced for the "pure-exchange" economy, we may write

$$Z_i(x_i) = \{z_i \in R^m:\ z_i + x_i \in X_i^*\}.$$

Given $i \in I$ and a positive real number r, we define a function f_i^{*r}: $(L_2)^n \to R$, a function S_i^{*r}: $(L_2)^n \times R^m \to L_2$, and a function T_i^{*r}: $(L_2)^n \times R^m \to L_2 \cup \{\emptyset\}$ by

$$f_i^{*r}(M) = \begin{cases} r & \text{if } d(M_i, 0) = 0 \text{ or } F \cap \prod_{j \in I} M_j \neq \emptyset, \\ \dfrac{\Sigma_{j \in I-\{i\}} d(M_j, 0)}{\Sigma_{j \in I} d(M_j, 0)}\, d(M_i, 0) & \text{otherwise,} \end{cases}$$

$$S_i^{*r}(M, x_i) = \{z_i \in Z_i(x_i): \ d(z_i, 0) \leqq \max \{d(Z_i(x_i), 0), f_i^{*r}(M)\}\},$$

$$T_i^{*r}(M, x_i) = \{z_i^* \in Z_i(x_i): \text{ There exists } y_i^* \in Y_i \text{ such that } (x_i + z_i^* + y_i^*, y_i^*) \in D^i \text{ and } (x_i + z_i^* + y_i^*, y_i^*)Q^i(x_i + z_i + y_i, y_i) \text{ for every } (y_i, z_i) \in Y_i \times S_i^{*r}(M, x_i) \text{ such that } (x_i + z_i + y_i, y_i) \in D^i\},$$

respectively. In terms of the notation for the "pure-exchange" economy, we may write

$$T_i^{*r}(M, x_i) = \{z_i^* \in Z_i(x_i): \ z_i^* + x_i \succsim_i z_i + x_i \ \text{ for every } \ z_i \in S_i^{*r}(M, x_i)\}.$$

Given a positive real number r, we define a function T^{*r}: $(L_2)^n \times C_2 \to (L_2 \cup \{\emptyset\})^n$ by

$$T^{*r}(M, x) = (T_i^{*r}(M, x_i))_{i \in I}.$$

Actually, T^{*r} is a function from $(L_2)^n \times C_2$ to $(L_2)^n$ (cf. Lemma 3 in Section 4).

Given a positive real number r, we define a correspondence H^{*r}: $(L_2)^n \times C_2 \to C_2$ by

$$H^{*r}(M, x) = \begin{cases} \{x\} & \text{if } F \cap \prod_{i \in I} T_i^{*r}(M, x_i) = \emptyset, \\ (F \cap \prod_{i \in I} T_i^{*r}(M, x_i)) + \{x\} & \text{otherwise.} \end{cases}$$

Let

$P_2(L_2, C_2) = \{(T, h)$: There exists a positive real number r such that $T = T^{*r}$ and $h(M, x) \in H^{*r}(M, x)$ for every $(M, x) \in (L_2)^n \times C_2\}$.

Then any element (T, h) of $P_2(L_2, C_2)$ is a non-tatonnement resource allocation process. Interpretations of the response function T and the outcome function h are similar to those given to processes in $P_1(L_1, C_1)$, except that proposals here are made in terms of individual trades but not allocations. Since, for every $i \in I$, T_i is independent of (D^j, Q^j, a_j) for every $j \in I - \{i\}$, the response function T _is_ privacy-preserving. Also T is anonymous, so that the process (T, h) is more eligible as an informationally decentralized process than the processes in $P_1(L_1, C_1)$.

The _set of attainable outcomes_ of $(T, h) \in P_2(L_2, C_2)$ is always chosen to be equal to B, the set of attainable redistributions.

THEOREM 4: _Under Assumptions_ 1*, 2*, _and_ 3*, _for every_ $(T, h) \in P_2(L_2, C_2)$, _every weak equilibrium outcome of_ (T, h) _is a Pareto-optimal redistribution_.

THEOREM 5: _Under Assumptions_ 1*, 2*, _and_ 3*, _for every_ $(T, h) \in P_2(L_2, C_2)$, _every Pareto-optimal redistribution is a strong equilibrium outcome of_ (T, h).

THEOREM 6: _Suppose that Assumptions_ 1*, 2*, 3*, _and_ 4* _are satisfied_. _Let_ $(T, h) \in P_2(L_2, C_2)$ _and let_ $\{x^{t-1}\}_{t \in N}$ _be a sequence in_ R^{mn} _such that_ $x^0 = a$ _and there exists a sequence_ $\{M^{t-1}\}_{t \in N}$ _in_ $(L_2)^n$ _such that_ $M^t = T(M^{t-1}, x^{t-1})$ _and_ $x^t = h(M^{t-1}, x^{t-1})$ _for every_ $t \in N$. _Then every cluster point of_ $\{x^{t-1}\}_{t \in N}$ _is a Pareto-optimal redistribution_.

COROLLARY TO THEOREM 6: Suppose, in addition to Assumptions 1*, 2*, 3*, and 4*, that B is compact, and that, for each $i \in I$, $\succsim_i$ can be represented by a utility function u_i. Let $\{x^{t-1}\}_{t \in N}$ be as in Theorem 6. For each $t \in N$, let $u^t = (u_i(x_i^t))_{i \in I}$. Then the sequence $\{u^t\}_{t \in N}$ in R^n is convergent. If, furthermore, for each $i \in I$, u_i is lower semi-continuous, then $\lim_{t \to \infty} u^t = (u_i(x_i^*))_{i \in I}$ for some Pareto-optimal redistribution x^*.

4. PROOFS OF THE RESULTS

PROOF OF PROPOSITION 1: NECESSITY. There is $y \in \prod_{i\in I} Y_i$ such that (1) $(d_i + y_i, y_i) \in D^i$ for every $i \in I$ and (2) there is no $(d', y') \in B \times \prod_{i\in I} Y_i$ such that $(d_i' + y_i', y_i') \in D^i$ and $(d_i + y_i, y_i)\overline{Q}^i(d_i' + y_i', y_i')$ for every $i \in I$. Suppose $(d + y, y)$ were not a Pareto-optimal allocation. Then there would be $(x', y') \in A$ such that $(d_i + y_i, y_i)\overline{Q}^i(x_i', y_i')$ for every $i \in I$. Let $d' = x' - y'$. Then $d' - a \in F$ and $(d' + y', y') \in D = \prod_{i\in I} D^i$ so that $(d_i' + y_i', y_i') \in D^i$ for every $i \in I$. Hence $(d', y') \in B \times \prod_{i\in I} Y_i$ and $(d_i + y_i, y_i)\overline{Q}^i (d_i' + y_i', y_i')$ for every $i \in I$, a contradiction. SUFFICIENCY. $(d_i + y_i, y_i) \in D^i$ for every $i \in I$, $d \in B$, and there is no $(x', y') \in A$ such that $(d_i + y_i, y_i)\overline{Q}^i(x_i', y_i')$ for every $i \in I$. Suppose d were not a Pareto-optimal redistribution. Then there would be $(d', y') \in B \times \prod_{i\in I} Y_i$ such that $(d_i' + y_i', y_i') \in D^i$ and $(d_i + y_i, y_i)\overline{Q}^i(d_i' + y_i', y_i')$ for every $i \in I$. For each $i \in I$, let $x_i' = d_i' + y_i'$. Then $(x', y') \in A$ and $(d_i + y_i, y_i)\overline{Q}^i(x_i', y_i')$ for every $i \in I$, a contradiction.

PROOF OF PROPOSITION 2: Let $\{x_i^\nu\}_{\nu\in N}$ be any sequence in X_i^* converging to some $x_i \in R^m$. For each $\nu \in N$ there is $y_i^\nu \in R^m$ such that $(x_i^\nu + y_i^\nu, y_i^\nu) \in D^i$. Since, by Assumption 1(b), X_i is bounded from below, we may assume, without loss of genrality, that there is $b \in R^m$ such that $y_i^\nu \geqq b$, i.e., $y_i^\nu \in Y_i(b)$ for every $\nu \in N$. By Assumption 1(c), $Y_i(b)$ is bounded, so that we may assume, without loss of generality, that $\{y_i^\nu\}_{\nu\in N}$ converges to some $y_i \in R^m$. Since, by Assumption 1(a), D^i is closed in R^{2m}, it follows that $(x_i + y_i, y_i) \in D^i$ so that $x_i \in X_i^*$. Thus X_i^* is closed in R^m.

LEMMA 1: <u>Under Assumption</u> 1*, <u>for every</u> $i \in I$ <u>and every</u> $x_i \in X_i^*$, <u>the set</u> $Y_i'(x_i) = \{y_i \in R^m: (x_i + y_i, y_i) \in D^i\}$ <u>is non-empty and compact</u>.

PROOF: Non-emptiness is obvious. Since, by Assumption 1(b), X_i is bounded from below, $Y_i'(x_i)$ is also bounded from below, so that there is $b \in R^m$ such that $y_i \geqq b$ for every $y_i \in Y_i'(x_i)$. Hence $Y_i'(x_i) \subset Y_i(b)$ and hence $Y_i'(x_i)$ is bounded by Assumption 1(c). By Assumption 1(a), D^i is closed in R^{2m}, so that $Y_i'(x_i)$ is clearly closed in R^m.

PROOF OF PROPOSITION 3: To show that $\succsim_i$ is complete, suppose that x_i, $x_i^o \in X_i^*$ but not $x_i \succsim_i x_i^o$. Then

(1) there is no $y_i \in Y_i'(x_i)$ such that $(x_i + y_i, y_i)Q^i(x_i^o + y_i', y_i')$ for every $y_i' \in Y_i'(x_i^o)$.

Define a binary relation $\succsim_i^*$ on $Y_i'(x_i^o)$ by $\succsim_i^* = \{(y_i, y_i') \in (Y_i'(x_i^o))^2: (x_i^o + y_i, y_i)Q^i(x_i^o + y_i', y_i')\}$. It is easy to see that $\succsim_i^*$ is complete and transitive, and that the set $\{y_i' \in Y_i'(x_i^o): y_i' \succsim_i^* y_i\}$ is closed in $Y_i'(x_i^o)$ for every $y_i \in Y_i'(x_i^o)$. Since, by Lemma 1, $Y_i'(x_i^o)$ is non-empty and compact, it follows from a theorem due to Wallace [10] and Ward [11] that there is $y_i^o \in Y_i'(x_i^o)$ such that $y_i^o \succsim_i^* y_i'$, i.e., $(x_i^o + y_i^o, y_i^o)Q^i(x_i^o + y_i', y_i')$ for every $y_i' \in Y_i'(x_i^o)$. Let $y_i \in Y_i'(x_i)$. Then, by (1), $(x_i + y_i, y_i)\bar{Q}^i(x_i^o + y_i', y_i')$ for some $y_i' \in Y_i'(x_i^o)$. Since $(x_i^o + y_i^o, y_i^o)Q^i(x_i^o + y_i', y_i')$, it follows that $(x_i + y_i, y_i)\bar{Q}^i(x_i^o + y_i^o, y_i^o)$ so that $(x_i^o + y_i^o, y_i^o)Q^i(x_i + y_i, y_i)$. Since y_i is arbitrary in $Y_i'(x_i)$, this implies that $x_i^o \succsim_i x_i$. Thus $\succsim_i$ is complete.

Suppose $x_i \succsim_i x_i'$ and $x_i' \succsim_i x_i''$. Then there is $y_i \in Y_i'(x_i)$ and $y_i' \in Y_i'(x_i')$ such that $(x_i + y_i, y_i)Q^i(x_i' + y_i'', y_i'')$ for every $y_i'' \in Y_i'(x_i')$ and $(x_i' + y_i', y_i')Q^i(x_i'' + y_i'', y_i'')$ for every $y_i'' \in Y_i'(x_i'')$. Hence $(x_i + y_i, y_i)Q^i$

$(x_i' + y_i', y_i')$ so that $(x_i + y_i, y_i)Q^i(x_i'' + y_i'', y_i'')$ for every $y_i'' \in Y_i'(x_i'')$; therefore $x_i \succsim_i x_i''$. Thus $\succsim_i$ is transitive.

Let $x_i \in X_i^*$ and let $\{x_i^\nu\}_{\nu \in N}$ be any sequence in the set $\{x_i' \in X_i^*: x_i' \succsim_i x_i\}$ converging to some $x_i^o \in X_i^*$. For each $\nu \in N$ there is $y_i^\nu \in Y_i'(x_i^\nu)$ such that $(x_i^\nu + y_i^\nu, y_i^\nu)Q^i(x_i + y_i, y_i)$ for every $y_i \in Y_i'(x_i)$. Since $\{x_i^\nu\}_{\nu \in N}$ converges to x_i^o and $(x_i^\nu + y_i^\nu, y_i^\nu) \in D^i$ for every $\nu \in N$, we may assume, as in the proof of Proposition 2, that the sequence $\{y_i^\nu\}_{\nu \in N}$ converges to some $y_i^o \in R^m$. By Assumption 1*, it follows that $y_i^o \in Y_i'(x_i^o)$. Let $y_i \in Y_i'(x_i)$. Then $(x_i^\nu + y_i^\nu, y_i^\nu)Q^i(x_i + y_i, y_i)$ for every $\nu \in N$, so that, by Assumption 2*, $(x_i^o + y_i^o, y_i^o)Q^i(x_i + y_i, y_i)$. Since y_i is arbitrary in $Y_i'(x_i)$, this implies that $x_i^o \succsim_i x_i$, i.e., $x_i^o \in \{x_i' \in X_i^*: x_i' \succsim_i x_i\}$. Thus (a) of Proposition 3 is established.

Let $x_i \in X_i^*$ and $\varepsilon > 0$. As in the proof of completeness of $\succsim_i$, there is $y_i \in Y_i'(x_i)$ such that

(2) $(x_i + y_i, y_i)Q^i(x_i + y_i', y_i')$ for every $y_i' \in Y_i'(x_i)$.

By Assumption 3*, for each $\nu \in N$ there is $(x_i^\nu, y_i^\nu) \in D^i$ such that $(x_i + y_i, y_i) \overline{Q^i}(x_i^\nu, y_i^\nu)$ and $d((x_i^\nu, y_i^\nu), (x_i + y_i, y_i)) < 1/\nu$. There is $\nu \in N$ such that $1/\nu < \varepsilon/2$. Let $x_i' = x_i^\nu - y_i^\nu$. Then $(x_i' + y_i^\nu, y_i^\nu) \in D^i$ so that $y_i^\nu \in Y_i'(x_i')$ and $x_i' \in X_i^*$. Furthermore, $d(x_i', x_i) \leqq d(x_i^\nu - y_i^\nu, x_i + y_i - y_i^\nu) + d(x_i + y_i - y_i^\nu, x_i) = d(x_i^\nu, x_i + y_i) + d(y_i^\nu, y_i) \leqq d((x_i^\nu, y_i^\nu), (x_i + y_i, y_i)) + d((x_i^\nu, y_i^\nu), (x_i + y_i, y_i)) < \varepsilon$. Suppose $x_i \succsim_i x_i'$. Then there would be $y_i' \in Y_i'(x_i)$ such that $(x_i + y_i', y_i')Q^i(x_i' + y_i'', y_i'')$ for every $y_i'' \in Y_i'(x_i')$, so that $(x_i + y_i', y_i') Q^i(x_i' + y_i^\nu, y_i^\nu)$. Since $(x_i + y_i, y_i)\overline{Q^i}(x_i^\nu, y_i^\nu)$ and $x_i^\nu = x_i' + y_i^\nu$, it follows that $(x_i + y_i, y_i)\overline{Q^i}(x_i + y_i', y_i')$, which contradicts (2). Thus $x_i' \succ_i x_i$ and (b) of Proposition 3 is established.

PROOF OF PROPOSITION 4: If (a) of Assumption 4* holds, then $a_i \in X_i^*$ for every $i \in I$, i.e., $a \in \prod_{i\in I} X_i^*$. Suppose (b) of Assumption 4* holds. Then there is $(z', y') \in A$, $(z'', y'') \in \prod_{i\in I} D^i$, and $\delta \in R^n$ such that, for every $i \in I$, $d(z_i'' - y_i'', a_i) < \delta_i$ and $(z_i', y_i')Q^i(x_i, y_i)$ for every $(x_i, y_i) \in D^i$ such that $d(x_i - y_i, a_i) < \delta_i$. Let $x' = z' - y'$ and $x'' = z'' - y''$. Then $x' \in B$, $x'' \in \prod_{i\in I} X_i^*$, and $d(x_i'', a_i) < \delta_i$ for every $i \in I$. Let $x_i \in X_i^*$ and $d(x_i, a_i) < \delta_i$. Then $(x_i + y_i, y_i) \in D^i$ for some $y_i \in R^m$. Since $d((x_i + y_i) - y_i, a_i) < \delta_i$, $(x_i' + y_i', y_i')Q^i(x_i + y_i, y_i)$; therefore $x_i' \succsim_i x_i$.

PROOF OF PROPOSITION 5: NECESSITY. There is $y^* \in \prod_{i\in I} Y_i'(x_i^*)$ such that

(1) there is no $(x, y) \in B \times \prod_{i\in I} Y_i$ such that $y_i \in Y_i'(x_i)$ and $(x_i^* + y_i^*, y_i^*)$ $\overline{Q}^i(x_i + y_i, y_i)$ for every $i \in I$.

Suppose there were $x \in B$ such that $x_i \succ_i x_i^*$ for every $i \in I$. Then, for every $i \in I$, there is no $y_i' \in Y_i'(x_i^*)$ such that $(x_i^* + y_i', y_i')Q^i(x_i + y_i, y_i)$ for every $y_i \in Y_i'(x_i)$. Hence, for every $i \in I$, there is $y_i \in Y_i'(x_i)$ such that $(x_i^* + y_i^*, y_i^*)\overline{Q}^i(x_i + y_i, y_i)$, which contradicts (1). SUFFICIENCY. For every $i \in I$, since, by Lemma 1, $Y_i'(x_i^*)$ is non-empty and compact, it follows, as in the proof of Proposition 3, that there is $y_i^* \in Y_i'(x_i^*)$ such that $(x_i^* + y_i^*, y_i^*)Q^i$ $(x_i^* + y_i', y_i')$ for every $y_i' \in Y_i'(x_i^*)$. Suppose x^* were not a Pareto-optimal redistribution. Then there would be $(x, y) \in B \times \prod_{i\in I} Y_i$ such that $y_i \in Y_i'(x_i)$ and $(x_i^* + y_i^*, y_i^*)\overline{Q}^i(x_i + y_i, y_i)$ for every $i \in I$. Hence, for every $i \in I$, $(x_i^* + y_i', y_i')\overline{Q}^i(x_i + y_i, y_i)$ for every $y_i' \in Y_i'(x_i^*)$, which implies that $x_i \succ_i x_i^*$, a contradiction.

LEMMA 2: <u>Under Assumptions</u> 1, 2, <u>and</u> 3, <u>for every</u> $(T, h) \in P_1(L_1, C_1)$, <u>every</u> $i \in I$, <u>every</u> $M \in (L_1)^n$, <u>and every</u> $(x, y) \in R^{2mn}$, $T_i(M, x, y)$ <u>is</u>

<u>non-empty and closed in</u> R^{2mn} <u>and</u> $d(T_i(M, x, y), (x, y)) = \max \{d(D_i, (x, y)), f_i^r(M, x, y)\}$, <u>where</u> r <u>is determined by</u> $T = T^r$.

PROOF: Since, by Assumption 1, D_i is closed in R^{2mn}, $S_i^r(M, x, y)$ is compact and, as was noted earlier, non-empty. Hence, by Assumption 2, there is $(x^*, y^*) \in S_i^r(M, x, y)$ such that $(x^*, y^*)Q_i(x', y')$ for every $(x', y') \in S_i^r(M, x, y)$; therefore $(x^*, y^*) \in T_i(M, x, y)$ so that $T_i(M, x, y)$ is non-empty. Since $T_i(M, x, y) = \{(x', y') \in D_i\colon (x', y')Q_i(x^*, y^*)\}$ and D_i is closed in R^{2mn}, it follows from Assumption 2 that $T_i(M, x, y)$ is closed in R^{2mn}. Since $(x^*, y^*) \in S_i^r(M, x, y)$, $d((x^*, y^*), (x, y)) \leqq \max \{d(D_i, (x, y)), f_i^r(M, x, y)\}$ and hence $d(T_i(M, x, y), (x, y)) \leqq \max \{d(D_i, (x, y)), f_i^r(M, x, y)\}$. Suppose $d(T_i(M, x, y), (x, y)) < \max \{d(D_i, (x, y)), f_i^r(M, x, y)\}$. Then there would be $(x', y') \in T_i(M, x, y)$ such that $d((x', y'), (x, y)) < \max \{d(D_i, (x, y)), f_i^r(M, x, y)\}$. Let $0 < \varepsilon < \max \{d(D_i, (x, y)), f_i^r(M, x, y)\} - d((x', y'), (x, y))$. By Assumption 3, there is $(x'', y'') \in D_i$ such that $(x', y')\overline{Q_i}(x'', y'')$ and $d((x', y'), (x'', y'')) < \varepsilon$. Then $d((x'', y''), (x, y)) \leqq d((x'', y''), (x', y')) + d((x', y'), (x, y)) < \varepsilon + d((x', y'), (x, y)) < \max \{d(D_i, (x, y)), f_i^r(M, x, y)\}$ and hence $(x'', y'') \in S_i^r(M, x, y)$, implying that $(x', y')Q_i(x'', y'')$, a contradiction.

LEMMA 3: <u>Under Assumptions</u> 1*, 2*, <u>and</u> 3*, <u>for every</u> $(T, h) \in P_2(L_2, C_2)$, <u>every</u> $i \in I$, <u>every</u> $M \in (L_2)^n$, <u>and every</u> $x_i \in R^m$, $T_i(M, x_i)$ <u>is non-empty and closed in</u> R^m <u>and</u> $d(T_i(M, x_i), 0) = \max \{d(Z_i(x_i), 0), f_i^{*r}(M)\}$, <u>where</u> r <u>is determined by</u> $T = T^{*r}$.

PROOF: Similar to that of Lemma 2, in view of Propositions 2 and 3.

LEMMA 4: Under Assumptions 1, 2, and 3, for every $(T, h) \in P_1(L_1, C_1)$, every sequence $\{M^t\}_{t\in N}$ in $(L_1)^n$, every sequence $\{(x^t, y^t)\}_{t\in N}$ in D, and every $t^* \in N$, if $M^{t+1} = T(M^t, x^t, y^t)$ and $(x^{t+1}, y^{t+1}) = h(M^t, x^t, y^t)$ for every $t \in N$ and $A \cap (\cap_{i\in I} M_i^{t^*+t}) = \emptyset$ for every $t \in N$, then $\lim_{t\to\infty} d(M_i^t, (x^{t^*}, y^{t^*})) = 0$ for every $i \in I$.

PROOF: Note that $(x^{t^*+t}, y^{t^*+t}) = (x^{t^*}, y^{t^*})$ for every $t \in N$. By Lemma 2,

$$d(M_j^{t^*+t+1}, (x^{t^*}, y^{t^*})) = \left(1 - \frac{d(M_j^{t^*+t}, (x^{t^*}, y^{t^*}))}{\Sigma_{k\in I} d(M_k^{t^*+t}, (x^{t^*}, y^{t^*}))}\right) d(M_j^{t^*+t}, (x^{t^*}, y^{t^*}))$$

for every $j \in I$ and every $t \in N$, so that the sequence $\{d(M_j^{t^*+t}, (x^{t^*}, y^{t^*}))\}_{t\in N}$ is decreasing for every $j \in I$. Let $s = \Sigma_{j\in I} d(M_j^{t^*+1}, (x^{t^*}, y^{t^*}))$. Then $\Sigma_{j\in I} d(M_j^{t^*+t}, (x^{t^*}, y^{t^*})) \leqq s$ for every $t \in N$. Suppose $\lim_{t\to\infty} d(M_i^t, (x^{t^*}, y^{t^*})) \neq 0$ for some $i \in I$. Then there would be $\varepsilon > 0$ such that $d(M_i^{t^*+t}, (x^{t^*}, y^{t^*})) > \varepsilon$ for every $t \in N$, since the sequence $\{d(M_i^{t^*+t}, (x^{t^*}, y^{t^*}))\}_{t\in N}$ is decreasing. Hence

$$\begin{aligned} \varepsilon &< d(M_i^{t^*+t+1}, (x^{t^*}, y^{t^*})) \\ &= \left(1 - \frac{d(M_i^{t^*+t}, (x^{t^*}, y^{t^*}))}{\Sigma_{j\in I} d(M_j^{t^*+t}, (x^{t^*}, y^{t^*}))}\right) d(M_i^{t^*+t}, (x^{t^*}, y^{t^*})) \\ &\leqq (1 - \varepsilon/s) d(M_i^{t^*+t}, (x^{t^*}, y^{t^*})) \leqq \cdots \\ &\leqq (1 - \varepsilon/s)^t d(M_i^{t^*+1}, (x^{t^*}, y^{t^*})) \end{aligned}$$

for every $t \in N$, so that $\varepsilon \leqq \lim_{t\to\infty} (1 - \varepsilon/s)^t d(M_i^{t^*+1}, (x^{t^*}, y^{t^*})) = 0$, a contradiction.

LEMMA 5: Under Assumptions 1*, 2*, and 3*, for every $(T, h) \in P_2(L_2, C_2)$ and every sequence $\{M^t\}_{t\in N}$ in $(L_2)^m$, if there exists a sequence $\{x^t\}_{t\in N}$ in $\Pi_{i\in I} X_i^*$ such that $M^{t+1} = T(M^t, x^t)$ and $x^{t+1} = h(M^t, x^t)$ for every $t \in N$, and

<u>if there exists</u> $t^* \in N$ <u>such that</u> $F \cap \prod_{i\in I} M_i^{t^*+t} = \emptyset$ <u>for every</u> $t \in N$, <u>then</u> $\lim_{t\to\infty} d(M_i^t, 0) = 0$ <u>for every</u> $i \in I$.

PROOF: Similar to that of Lemma 4, in view of Lemma 3.

LEMMA 6: <u>Under Assumptions</u> 1, 2, <u>and</u> 3, <u>for every</u> $(T, h) \in P_1(L_1, C_1)$ <u>and every sequence</u> $\{M^t\}_{t\in N}$ <u>in</u> $(L_1)^n$, <u>if there exists a sequence</u> $\{(x^t, y^t)\}_{t\in N}$ <u>in</u> A <u>such that</u> $M^{t+1} = T(M^t, x^t, y^t)$ <u>and</u> $(x^{t+1}, y^{t+1}) = h(M^t, x^t, y^t)$ <u>for every</u> $t \in N$, <u>then, for every</u> $t^* \in N$ <u>such that</u> (x^{t^*}, y^{t^*}) <u>is not a Pareto-optimal allocation, there exists</u> $t' \in N$ <u>such that</u> $t' \geqq t^*$ <u>and</u> $A \cap (\cap_{i\in I} M_i^{t'}) \neq \emptyset$.

PROOF: There is $(x, y) \in A$ such that $(x^{t^*}, y^{t^*})\overline{Q_i}(x, y)$ for every $i \in I$. By Assumption 2, there is $\varepsilon > 0$ such that, for every $i \in I$, $(x', y')\overline{Q_i}(x, y)$ for every $(x', y') \in D_i$ such that $d((x', y'), (x^{t^*}, y^{t^*})) < \varepsilon$. Suppose $A \cap (\cap_{i\in I} M_i^{t'}) = \emptyset$ for every $t' \geqq t^*$. By Lemma 4, there would be $t > t^*$ such that $d(M_i^{t-1}, (x^{t^*}, y^{t^*})) < \varepsilon$ for every $i \in I$. Note that $(x^{t-1}, y^{t-1}) = (x^{t^*}, y^{t^*})$. If $(x, y) \notin M_i^t$ for some $i \in I$, then $(x, y)\overline{Q_i}(x', y')$ for some $(x', y') \in S_i^r(M^{t-1}, x^{t^*}, y^{t^*})$ and hence $d((x', y'), (x^{t^*}, y^{t^*})) \leqq$
$$\frac{\Sigma_{j\in I-\{i\}} d(M_j^{t-1}, (x^{t^*}, y^{t^*}))}{\Sigma_{j\in I} d(M_j^{t-1}, (x^{t^*}, y^{t^*}))} d(M_i^{t-1}, (x^{t^*}, y^{t^*})) < d(M_i^{t-1}, (x^{t^*}, y^{t^*})) < \varepsilon,$$ so that $(x', y')\overline{Q_i}(x, y)$, a contradiction. Therefore $(x, y) \in M_i^t$ for every $i \in I$, implying that $A \cap (\cap_{i\in I} M_i^t) \neq \emptyset$, a contradiction.

LEMMA 7: <u>Under Assumptions</u> 1*, 2*, <u>and</u> 3*, <u>for every</u> $(T, h) \in P_2(L_2, C_2)$ <u>and every sequence</u> $\{M^t\}_{t\in N}$ <u>in</u> $(L_2)^n$, <u>if there exists a sequence</u> $\{x^t\}_{t\in N}$ <u>in</u>

B <u>such that</u> $M^{t+1} = T(M^t, x^t)$ <u>and</u> $x^{t+1} = h(M^t, x^t)$ <u>for every</u> $t \in N$, <u>then, for every</u> $t^* \in N$ <u>such that</u> x^{t^*} <u>is not a Pareto-optimal redistribution, there exists</u> $t' \in N$ <u>such that</u> $t' \geqq t^*$ <u>and</u> $F \cap \prod_{i \in I} M_i^{t'} \neq \emptyset$.

PROOF: Similar to that of Lemma 6, in view of Lemma 5.

PROOF OF THEOREM 1: Let (x, y) be a weak equilibrium outcome. Then there is $M \in (L_1)^n$ such that $(x, y) = h(G^{t-1}_{(x,y)}(M), (x, y))$ for every $t \in N$. For each $t \in N$, let $M^t = T(M^{t-1}, x, y)$ with $M^0 = M$. Then $(x, y) = h(M^{t-1}, x, y)$ for every $t \in N$, so that $A \cap (\cap_{i \in I} M_i^t) = \emptyset$ for every $t \in N$ and hence, by Lemma 6, (x, y) is a Pareto-optimal allocation.

PROOF OF THEOREM 4: Similar to that of Theorem 1, in view of Lemma 7.

PROOF OF THEOREM 2: Let (x, y) be a Pareto-optimal allocation. Let $M \in (L_1)^n$. For each $t \in N$, let $M^t = T(M^{t-1}, x^{t-1}, y^{t-1})$ and $(x^t, y^t) = h(M^{t-1}, x^{t-1}, y^{t-1})$, with $M^0 = M$ and $(x^0, y^0) = (x, y)$. It suffices to show that $A \cap (\cap_{i \in I} M_i^t) = \emptyset$ for every $t \in N$. Suppose $A \cap (\cap_{i \in I} M_i^t) \neq \emptyset$ for some $t \in N$. Without loss of generality, we may assume that $A \cap (\cap_{i \in I} M_i^{t'}) = \emptyset$ for every $t' < t$. Then $(x^{t-1}, y^{t-1}) = (x, y)$. Take any $(x', y') \in A \cap (\cap_{i \in I} M_i^t)$. Let $i \in I$. Then $(x', y') Q_i (x'', y'')$ for every $(x'', y'') \in S_i^r(M^{t-1}, x, y)$. Let

$$\varepsilon = \frac{\Sigma_{j \in I - \{i\}} d(M_j^{t-1}, (x, y))}{\Sigma_{j \in I} d(M_j^{t-1}, (x, y))} d(M_i^{t-1}, (x, y)).$$

Then $\varepsilon > 0$. By Assumption 3, there is $(x'', y'') \in D_i$ such that $(x, y) \overline{Q_i} (x'', y'')$ and $d((x, y), (x'', y'')) < \varepsilon$. It follows that $(x'', y'') \in S_i^r(M^{t-1}, x, y)$ and hence that $(x', y') Q_i (x'', y'')$. Thus $(x, y) \overline{Q_i} (x', y')$. Since i is arbitrary in I, this contradicts the assumption that (x, y) is a Pareto-optimal allocation.

PROOF OF THEOREM 5: Similar to that of Theorem 2, in view of Proposition 3.

PROOF OF THEOREM 3: Let (x, y) be any cluster point of $\{(x^{t-1}, y^{t-1})\}_{t\in N}$. Then there is a subsequence $\{(x^{t(\nu)}, y^{t(\nu)})\}_{\nu\in N}$ converging to (x, y). Suppose (x, y) were not a Pareto-optimal allocation. Then there would be $(x', y') \in A$ such that $(x, y)\overline{Q_i}(x', y')$ for every $i \in I$. By Assumption 2, there is $\varepsilon > 0$ such that, for every $i \in I$, $(x'', y'')\overline{Q_i}(x', y')$ for every $(x'', y'') \in D_i$ such that $d((x'', y''), (x, y)) < \varepsilon$. Hence there is $t' \in N$ such that $2r(\frac{n-1}{n})^{t'} < \varepsilon$. Without loss of generality, we may assume that $t(\nu) > t(\nu - 1) + t' + 1$ for every $\nu \in N$. Since $\{(x^{t(\nu)}, y^{t(\nu)})\}_{\nu\in N}$ is convergent and hence Cauchy-convergent, there is $\mu \in N$ such that

(1) $d((x^{t(\nu)}, y^{t(\nu)}), (x, y)) < r(\frac{n-1}{n})^{t'}$,

(2) $d((x^{t(\nu)}, y^{t(\nu)}), (x^{t(\nu+1)}, y^{t(\nu+1)})) < r(\frac{n-1}{n})^{t'}$

for every $\nu \geqq \mu$. Without loss of generality, we may assume that $(x^{t(\mu)}, y^{t(\mu)}) \neq (x^{t(\mu)-1}, y^{t(\mu)-1})$ and hence that $A \cap (\cap_{i\in I} M_i^{t(\mu)}) \neq \emptyset$, which, by Lemma 2, implies that $d(M_i^{t(\mu)+1}, (x^{t(\mu)}, y^{t(\mu)})) = r$ for every $i \in I$. Since for every $t \in N$ if $A \cap (\cap_{i\in I} M_i^{t(\mu)+s}) = \emptyset$ for every $s \in \{1, \ldots, t\}$ then $(x^{t(\mu)}, y^{t(\mu)}) = (x^{t(\mu)+s}, y^{t(\mu)+s})$ for every $s \in \{1, \ldots, t\}$, it follows from Lemma 2 that

(3) for every $t \in N$, if $A \cap (\cap_{i\in I} M_i^{t(\mu)+s}) = \emptyset$ for every $s \in \{1, \ldots, t\}$ then $d(M_i^{t(\mu)+s}, (x^{t(\mu)}, y^{t(\mu)})) = r(\frac{n-1}{n})^{s-1}$ for every $s \in \{1, \ldots, t\}$ and every $i \in I$.

We shall now show that $A \cap (\cap_{i\in I} M_i^{t(\mu)+t}) = \emptyset$ for every $t \in \{1, \ldots, t' + 1\}$. Suppose that $A \cap (\cap_{i\in I} M_i^{t(\mu)+t}) \neq \emptyset$ for some $t \in \{1, \ldots, t' + 1\}$. Without loss of generality, we may assume that $A \cap (\cap_{i\in I} M_i^{t(\mu)+s}) = \emptyset$ for every $s \in \{1, \ldots, t-1\}$. Hence, by (3), $d(M_i^{t(\mu)+t}, (x^{t(\mu)}, y^{t(\mu)})) = r(\frac{n-1}{n})^{t-1} \geqq r(\frac{n-1}{n})^{t'}$ for every $i \in I$. It follows from (2) that $(x^{t(\mu+1)}, y^{t(\mu+1)}) \notin M_i^{t(\mu)+t}$ for every $i \in I$. Since $(x^{t(\mu)+t}, y^{t(\mu)+t}) \in M_i^{t(\mu)+t}$ for every $i \in I$, if $(x^{t(\mu+1)}, y^{t(\mu+1)}) Q_i (x^{t(\mu)+t}, y^{t(\mu)+t})$ for some $i \in I$ then $(x^{t(\mu+1)}, y^{t(\mu+1)}) \in M_i^{t(\mu)+t}$, a contradiction. Hence $(x^{t(\mu+1)}, y^{t(\mu+1)}) \overline{Q_i} (x^{t(\mu)+t}, y^{t(\mu)+t})$ for every $i \in I$. But $t(\mu) + t \leqq t(\mu) + t' + 1 \leqq t(\mu + 1)$ and the sequence $\{(x^{t-1}, y^{t-1})\}_{t\in N}$ is non-decreasing with respect to Q_i for every $i \in I$, so that $(x^{t(\mu+1)}, y^{t(\mu+1)}) Q_i (x^{t(\mu)+t}, y^{t(\mu)+t})$ for every $i \in I$. This is a contradiction. Thus $A \cap (\cap_{i\in I} M_i^{t(\mu)+t}) = \emptyset$ for every $t \in \{1, \ldots, t'+1\}$.

Therefore, by (3), $d(M_i^{t(\mu)+t'+1}, (x^{t(\mu)}, y^{t(\mu)})) = r(\frac{n-1}{n})^{t'}$ for every $i \in I$. Let $i \in I$. Since, by Lemma 2, $M_i^{t(\mu)+t'+1}$ is closed in R^{2mn}, there is $(x'', y'') \in M_i^{t(\mu)+t'+1}$ such that $d((x'', y''), (x^{t(\mu)}, y^{t(\mu)})) = r(\frac{n-1}{n})^{t'}$. By (1), $d((x'', y''), (x, y)) \leqq d((x'', y''), (x^{t(\mu)}, y^{t(\mu)})) + d((x^{t(\mu)}, y^{t(\mu)}), (x, y)) < 2r(\frac{n-1}{n})^{t'} < \varepsilon$, so that $(x'', y'') \overline{Q_i} (x', y')$. Since $(x'', y'') \in M_i^{t(\mu)+t'+1}$, this implies that $(x', y') \in M_i^{t(\mu)+t'+1}$. Since i is arbitrary in I, $(x', y') \in A \cap (\cap_{i\in I} M_i^{t(\mu)+t'+1})$, which contradicts the fact that $A \cap (\cap_{i\in I} M_i^{t(\mu)+t}) = \emptyset$ for every $t \in \{1, \ldots, t'+1\}$.

PROOF OF COROLLARY TO THEOREM 3: Since A is compact, the sequence $\{(x^{t-1}, y^{t-1})\}_{t\in N}$ has a cluster point (x^*, y^*), which is, by Theorem 3, a Pareto-optimal allocation. Hence there is a subsequence $\{(x^{t(\nu)}, y^{t(\nu)})\}_{\nu\in N}$ converging to (x^*, y^*). Since the sequence $\{(x^{t-1}, y^{t-1})\}_{t\in N}$ is non-decreasing with respect to Q_i for every $i \in I$, it follows that, for every $i \in I$, $(x^{t(\nu')}, y^{t(\nu')}) Q_i (x^{t(\nu)}, y^{t(\nu)})$ whenever $\nu' \geqq \nu$. By Assumption 2, $(x^*, y^*) Q_i (x^{t(\nu)}, y^{t(\nu)})$ for every $\nu \in N$ and every $i \in I$. If $u_i(x^t, y^t) > u_i(x^*, y^*)$

for some $t \in N$ and some $i \in I$, then $u_i(x^{t(\nu)}, y^{t(\nu)}) \geqq u_i(x^t, y^t) > u_i(x^*, y^*)$, i.e., $(x^*, y^*)\overline{Q_i}(x^{t(\nu)}, y^{t(\nu)})$ for some $\nu \in N$ such that $t(\nu) > t$, a contradiction. So $(u_i(x^*, y^*))_{i\in I} \geqq u^t$ for every $t \in N$. Thus, for every $i \in I$, the sequence $\{u_i^t\}_{t\in N}$ is non-decreasing and bounded from above, so that $\lim_{t\to\infty} u_i^t$ exists. Hence the sequence $\{u^t\}_{t\in N}$ is convergent. Now suppose that u_i is lower semi-continuous for every $i \in I$. Clearly $u_i(x^*, y^*) \geqq \lim_{t\to\infty} u_i^t$ for every $i \in I$. Suppose $u_i(x^*, y^*) > \lim_{t\to\infty} u_i^t$ for some $i \in I$. Since u_i is lower semi-continuous and $\{(x^{t(\nu)}, y^{t(\nu)})\}_{\nu\in N}$ converges to (x^*, y^*), $u_i(x^{t(\nu)}, y^{t(\nu)}) > \lim_{t\to\infty} u_i^t$ for some $\nu \in N$ so that $u_i(x^t, y^t) \geqq u_i(x^{t(\nu)}, y^{t(\nu)}) > \lim_{t\to\infty} u_i^t$ for some $t \in N$ such that $t \geqq t(\nu)$, a contradiction. Thus $u_i(x^*, y^*) = \lim_{t\to\infty} u_i^t$ for every $i \in I$, i.e., $\lim_{t\to\infty} u^t = (u_i(x^*, y^*))_{i\in I}$.

LEMMA 8: <u>Under Assumptions</u> 1*, 2*, 3*, <u>and</u> 4*, <u>for every</u> $(T, h) \in P_2(L_2, C_2)$ <u>and every sequence</u> $\{x^{t-1}\}_{t\in N}$ <u>in</u> R^{mn}, <u>if</u> $x^0 = a$ <u>and there exists a sequence</u> $\{M^{t-1}\}_{t\in N}$ <u>in</u> $(L_2)^n$ <u>such that</u> $M^t = T(M^{t-1}, x^{t-1})$ <u>and</u> $x^t = h(M^{t-1}, x^{t-1})$ <u>for every</u> $t \in N$, <u>then there exists</u> $t \in N$ <u>such that</u> $x^t \in B$.

PROOF: If $a \in \prod_{i\in I}X_i^*$ then the lemma is trivial. So we assume that $a \notin \prod_{i\in I}X_i^*$. Suppose $x^t = x^0$ for every $t \in N$. Then $F \cap \prod_{i\in I}M_i^t = \emptyset$ for every $t \in N$. It follows from Lemma 3 that, for every $i \in I$ and every $t \in N$, if $d(Z_i(a_i), 0) \geqq \dfrac{\Sigma_{j\in I-\{i\}}d(M_j^t, 0)}{\Sigma_{j\in I}d(M_j^t, 0)} d(M_i^t, 0)$ then $d(M_i^{t+1}, 0) = d(Z_i(a_i), 0) \leqq d(M_i^t, 0)$; while if $d(Z_i(a_i), 0) < \dfrac{\Sigma_{j\in I-\{i\}}d(M_j^t, 0)}{\Sigma_{j\in I}d(M_j^t, 0)} d(M_i^t, 0)$ then $d(M_i^{t+1}, 0) = \dfrac{\Sigma_{j\in I-\{i\}}d(M_j^t, 0)}{\Sigma_{j\in I}d(M_j^t, 0)} d(M_i^t, 0) < d(M_i^t, 0)$. Hence $d(M_i^{t+1}, 0) \leqq d(M_i^t, 0)$ for every

$i \in I$ and every $t \in N$. Let $s = \Sigma_{i\in I} d(M_i^1, 0)$. If $\lim_{t\to\infty} d(M_i^t, 0) > d(Z_i(a_i), 0)$ for some $i \in I$, then $d(M_i^t, 0) > d(Z_i(a_i), 0)$ for every $t \in N$, so that

$$d(M_i^{t+1}, 0) = \left(1 - \frac{d(M_i^t, 0)}{\Sigma_{j\in I} d(M_j^t, 0)}\right) d(M_i^t, 0) \leqq \left(1 - \frac{d(Z_i(a_i), 0)}{s}\right) d(M_i^t, 0) \leqq \cdots$$

$\leqq (1 - d(Z_i(a_i), 0)/s)^t d(M_i^1, 0)$ for every $t \in N$ and therefore $\lim_{t\to\infty} d(M_i^t, 0) \leqq 0$, a contradiction. Hence $\lim_{t\to\infty} d(M_i^t, 0) \leqq d(Z_i(a_i), 0)$ for every $i \in I$. By Proposition 4, there is $x' \in B$ and $\delta \in R^n$ such that, for every $i \in I$, $d(Z_i(a_i), 0) < \delta_i$ and $x_i' \succsim_i x_i$ for every $x_i \in X_i^*$ such that $d(x_i, a_i) < \delta_i$. So there is $t \in N$ such that $d(M_i^t, 0) < \delta_i$ for every $i \in I$. For every $i \in I$, since, by Lemma 3, M_i^t is closed in R^m, there is $z_i \in M_i^t$ such that $d(z_i, 0) = d(M_i^t, 0)$. Hence, for every $i \in I$, $z_i + a_i \in X_i^*$ and $d(z_i + a_i, a_i) < \delta_i$, which implies that $x_i' \succsim_i z_i + a_i$; therefore $x_i' - a_i \in M_i^t$. Thus $x' - a \in F \cap \prod_{i\in I} M_i^t$, a contradiction. Hence $x^t \neq x^0$ for some $t \in N$. Without loss of generality, we may assume that $x^{t'} = x^0$ for every $t' < t$. Then $x^t \neq x^{t-1}$, so that $x^t \in (F \cap \prod_{i\in I} M_i^t) + \{x^{t-1}\}$ and hence $x^t = z + x^{t-1}$ for some $z \in F \cap \prod_{i\in I} M_i^t$. For every $i \in I$, $z_i \in M_i^t \subset Z_i(x_i^{t-1})$, so that $x_i^t = z_i + x_i^{t-1} \in X_i^*$. Therefore $x^t \in B$.

PROOF OF THEOREM 6: Similar to that of Theorem 3, in view of Lemma 8.

PROOF OF COROLLARY TO THEOREM 6: Similar to that of Corollary to Theorem 3.

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FOOTNOTES

1 The greed process has another performance characteristic, called <u>essential single-valuedness</u>. In this paper, we shall be concerned only with non-wastefulness, unbiasedness, and stability.

2 For a study in this problem, see Mount and Reiter [9].

3 H. Kanemitsu has, independently, developed a similar non-tatonnement resource allocation process in a paper presented at the Third World Congress of the Econometric Society, Toronto, Canada, 1975.

4 $(x, y)Q_i(x', y')$ is interpreted to mean that (x, y) is at least as desired as (x', y') for agent i. Completeness of Q_i means that, for every $(x, y), (x', y') \in D_i$, either $(x, y)Q_i(x', y')$ or $(x', y')Q_i(x, y)$. Hence, completeness implies reflexivity.

5 Throughout this paper, d stands for the distance function in some Euclidean space R^k, defined by $d(p, q) = \max \{|p_1 - q_1|, \ldots, |p_k - q_k|\}$. The number k may be either $2mn$, mn, or m, in each case of which the same symbol d will be used. The only place in which the reader should be careful is in the proof of part (b) of Proposition 3, where the distance in R^m and that in R^{2m} appear in a series of inequalities.

6 Given two points p and q of R^m, we write $p \geqq q$ if and only if $p_h \geqq q_h$ for every $h \in H$.

7 $x_i' \succ_i x_i$ means that $x_i' \succsim_i x_i$ but not $x_i \succsim_i x_i'$.

8 Given a subset U of R^{2mn} and a point v of R^{2mn}, the distance of U from v is defined by $d(U, v) = \inf_{u \in U} d(u, v)$. The same definition will be used for the distance between a subset and a point of R^m.