A TWO-SECTOR MACROECONOMIC MODEL AND THE RELATIVE POTENCY OF MONETARY AND FISCAL POLICY

by

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INTRODUCTION

This paper attempts to highlight the importance of the distinction between stock and flow markets, and the consequences of such a distinction, with regard to relative potencies of monetary and fiscal policies, within the context of a two sector macroeconomic model. In particular we intend to focus our attention on the market for physical capital, characteristics of which, in our view, are the main determinants of the channel through which monetary policies affect the real sector.

Since Tobin's well known "Dynamic Aggregative Model" [5] the importance of a perfect market[1/] in physical capital in transmitting the monetary disturbances to the real sector has been recognized, in spite of Monetarist criticisms to Tobin's argument. However one good macroeconomic models are rather restrictive in the sense that either there exists a perfect market in existing stocks of physical capital in which firms can trade; or the capital market is characterized by transaction costs, bid-ask spreads, transportation costs, etc.[2/] Clearly the existance of a perfect capital market implies the lack of a flow demand for capital because firms hold the desired amount at every instant. However this lack of flow demand should not be interpreted as zero ex-post investment, for the firms will be indifferent between different levels of capital stock as long as capital-labour ratio remains the same.[3/] Under these circumstances ex-post investment will be determined as a residual; that part of output that is not consumed by the households and government. On the other hand if the stock market in physical capital is characterized by transaction costs or simply if it does not exist, then the firms will not hold the desired amount of physical capital at every instant. Consequently they try to adjust their stocks of capital at a finite rate per unit of time by investing. Such an approach to capital market not only implies a flow demand for investment but, in a sense, justifies it.

On the other hand the moment the economic theorist starts casting his models within the context of two sectors, he finds himself in a widened horizon. He can entertain a variety of assumptions regarding the labour and capital markets and his models will change accordingly. In such models one must specify not only intra-industry capital markets but also inter-industry transactions. Recently D. W. Henderson and T. J. Sargent have analyzed "the short-term influences exerted by monetary and fiscal policies in a two-sector macroeconomic model" [2]. As they have stated, their model contains Keynesian elements but can best be described as an extension of J. Tobin's "Dynamic Aggregative Model" [5]. The key assumption they make is that there exists a perfect market in existing capital stock. Firms, by operating in this market, are able to acquire all the capital they desire instantaneously. In addition to the assumption of perfect market they assume that it is the same physical capital which is used in producing the capital goods as well as producing the consumption goods. This assumption implies that existing stocks of physical capital are mobile not only among the firms producing the same commodity but also from one sector to the other.

In this paper we shall develop a model similar to that of Henderson and Sargent under a set of alternative assumptions:

(1) In the short run physical capital is "sector-specialized" implying that physical capital that is being used in producing, say, capital goods cannot be transformed to produce consumption goods, without transaction costs, and vice versa.

(2) In the sector that produces capital goods, physical capital is "firm specialized" in the short run. Consequently there is no market in existing physical capital that is being used in producing capital goods.

(3) In the sector that produces consumption goods, physical capital is not firm specialized and there exists a perfect market in existing stocks of physical capital in which firms can trade.

As a consequence of these assumptions, the model that emerges, contains more Keynesian elements in its structure, compared to that of Henderson's and Sargent's model, as well as retaining its similarity to Tobin's Dynamic Aggregative Model. Furthermore our method of analysis is essentially Keynesian in the sense that we shall confine ourselves to short-run, static questions even though the model presented will be capable of answering long-run, dynamic ones as well.

In Section I, we formalize the model by describing the behaviour of government, firms, and households, In Section II we present the solution of the model. In Section III the short-run effects of monetary and fiscal policy are studied by performing a number of static exercises. Conclusions are presented in Section IV.

I. DESCRIPTION OF THE BEHAVIOUR OF GOVERNMENT, FIRMS AND HOUSEHOLDS.

Government

Government is assumed to have money, M, and bonds, B, as outstanding liabilities. The nominal rate of return on money is fixed at zero, whereas the nominal rate of return on bonds, r, is market determined given the value of bonds. Government can also change its liability structure at a point in time by conducting open-market operations subject to the following constraint

$$dM + dB = 0$$

Government also collects taxes which total to

$$T_t = T_o + T_k + T_y$$

where T_o is total autonomous taxes, T_k is total profit taxes at a marginal rate of t_k , and T_y is total income taxes at a marginal rate of t_y .

Government makes net outlays which consist of G , a direct claim on consumption goods, and rB , nominal amount of transfer payments.

Firms

A) Firms Producing Capital Goods

Individual firms producing capital goods, all use capital and labor as inputs with a linearly homogeneous production function identical for all firms. There being no specific training or search costs they can hire any amount of labor they want at the fixed wage rate, w , at every instant. On the other hand due to very high transaction costs they can not alter their capital stock instantaneously. Instead, if if is profitable to do so, they change their capital stock by a finite amount per unit of time by investing. We shall assume that capital depreciates at the rate δ.

Furthermore all economic agents are assumed to expect the money wage rate and prices of capital and consumption goods to change at the rate π. It will also be assumed that government charges a corporate income tax at the proportion t_k where taxable corporate income is defined as cash receipts less the wage bill less the depreciation allowances.

Let the production function for capital goods be

$$I = I(K_I, N_I)$$

where K_I is the real capital stock in capital goods sector and N_I is the level of employment. We shall assume that the following hold regarding the first and second order partial derivatives of the production function

$$I_K > 0 \ , \ I_N > 0 \ , \ I_{KK} < 0 \ , \ I_{NN} < 0 \ , \ I_{KN} > 0 \ .$$

Net cash flow of a typical firm after taxes is equal to

$$(1 - t_k)[P_I I(K_I, N_I) - wN_I - P_I \delta K_I] - P_I \dot{K}_I \ . \tag{1}$$

Assuming that firms' objective is to maximize the present value of its net cash flow defined as

$$\int_o^\infty e^{-(r-\pi)t}\{(1 - t_k)[P_I I(K_I, N_I) - wN_I - P_I\delta K_I] - P_I\dot{K}_I\}\, dt \qquad (1a)$$

the Euler conditions for a maximum are:

$$I_N = \frac{w}{P_I} \qquad (2)$$

$$I_K = \delta + \frac{r - \pi}{1 - t_k} \qquad (3)$$

If we define a new variable, q, as follows[4/]

$$q = \frac{(I_k - \delta)(1 - t_k)}{r - \pi}$$

equation (3) implies $q = 1$.

The assumption that firms can alter the level of employment instantaneously implies that the first equality always holds. On the other hand the lack of a perfect market in physical capital prevents the attainment of second equality at every instant. I_K in equation (3) is evaluated at the given level of capital stock and the amount of employment implied by equation (2) with the given capital stock. Thus

$$I_K \gtreqless \delta + \frac{r - \pi}{1 - t_k} \qquad \text{or} \qquad q \gtreqless 1 \qquad (4)$$

What equations (3) and (4) say is that had there been a market in physical capital the firm would have been a buyer of capital stock if

$$I_K > \delta + \frac{r - \pi}{1 - t_k} \quad \text{or} \quad q > 1$$

and a seller if

$$I_K < \delta + \frac{r - \pi}{1 - t_k} \quad \text{or} \quad q < 1$$

Given the lack of a perfect market in physical capital, firm will do the second-best thing; namely, it will adjust its capital stock at a finite rate per unit of time by investing. We shall assume that the aggregate demand for investment by capital goods producing firms is of the following form

$$\dot{K}_I = g(q - 1) \tag{5}$$

with $g'(\cdot) > 0$.

Firms are assumed to have equities as outstanding liabilities. Bonds and equities are assumed to be perfect substitutes when their yields are equal. To find yield on equities we must calculate the value of equities. Assuming that firms finance their investment by issuing new equities the value of outstanding equities is equal to the present value of dividends.

$$V_I = \int_o^\infty e^{-(r-\pi)^t}\{(1-t_k)[P_I I(K_I, N_I) - wN_I - P_I\delta K_I]\} \, dt \, . \tag{6}$$

Linear homogeneity of production function together with (2) implies[5/]

$$V_I = \frac{(1-t_k)(I_K-\delta)}{r-\pi} P_I K_I = P_I q K_I \tag{7}$$

Return on equities is defined as the ratio of dividends to the value of equities plus expected capital gains, and is equal to:

$$U = \frac{P_I I(K_I, N_I) - wN_I - P_I \delta K_I}{V_I} + \pi = r \quad .$$

B) Firms Producing Consumption Goods

We shall again assume that individual firms producing consumption goods use capital and labor as inputs with a linearly homogenous production function identical for all firms. Given our assumptions and a perfect market in existing physical capital used in the production of consumption goods, and a production function that has the following properties:

$$C = C(K_c, N_c)$$

$$C_K > 0 \ , \quad C_N > 0 \ , \quad C_{KK} < 0 \ , \quad C_{NN} < 0 \ , \quad C_{KN} > 0$$

the after tax economic profits of a typical firm can be written as

$$(1-t_k)[P_c C(K_c, N_c) - wN_c - P_I \delta K_c] - (r-\pi) P_I K_c \quad .$$

Profit maximizing conditions (which are the same conditions that maximize the present value of net cash flow) are:

$$C_N = \frac{w}{P_c} \tag{8}$$

$$(1-t_k)(P_c C_K - P_I \delta) = (r-\pi) P_I \quad . \tag{9}$$

Defining $P = \frac{P_I}{P_c}$ we can rewrite (9) as follows:

$$C_k = P \cdot \delta + \frac{r-\pi}{1-t_k} \quad . \tag{9a}$$

Under these conditions, the value of equities issued by consumption goods sector will be equal to[6/]

$$V_c = P_I K_c$$

Households

Real wealth of households in terms of consumption goods consists of money, bonds and equities with money and bonds being the liability of government and equities being the liability of firms.

$$Z = \frac{M+B+V_I+V_c}{P_c} = \frac{M+B}{P_c} + P(q K_I + K_c) \tag{10}$$

Real income of the public in terms of consumption goods consists of wages, dividends and interest income from government bonds. This implies that personal income before taxes is equal to

$$\frac{wN_I}{P_c} + \frac{wN_c}{P_c} + P(1-t_k)(I_K-\delta)K_I + (1-t_k)(C_K-P\delta)\ K_c + r\,\frac{B}{P_c} \quad .$$

Using the linear homogeneity of production functions and profit maximization conditions implies that personal income is

$$Y - P\delta K - T_k + r\frac{B}{P_c}$$

where $Y = PI + C$ and K total capital stock.

Out of this income public pays an income tax of a lump-sum amount T_o and proportional rate t_y . Consequently after-tax income of the public is equal to

$$= Y - P\delta K + r\frac{B}{P_c} - T_o - T_k - T_y \quad . \tag{11}$$

We shall assume, following Henderson and Sargent, that public base their consumption decisions or after-tax personal income as defined by equation (11).

$$C = \gamma(Y - P\delta K - T) \qquad 0 < \gamma' < 1$$

where $T = T_t - r\frac{B}{P_c}$. Notice that this national income accounts definition of disposable income does not imply that it is the amount that public expects to be able to spend while leaving their real wealth intact. To find the disposable income that is consistent with this latter definition take the time derivative of equation (10) and assume that $\dot{q} = 0$.

$$\dot{Z} = \frac{\dot{M}+\dot{B}}{P_c} - \pi\frac{M+B}{P_c} + Pq\dot{K}_I + P\dot{K}_c \tag{12}$$

Government budget constraint is

$$G - T = \frac{\dot{M}+\dot{B}}{P_c} \tag{13}$$

Equality of flow supply of capital goods to flow demand for them implies

$$I(K_I,\ N_I) = \dot{K}_I + \dot{K}_c + \delta K \tag{14}$$

Equality of supply and demand for consumption goods implies

$$C(K_c,\ N_c) = \tilde{C} + G \tag{15}$$

where $\tilde{C}$ is private consumption. Now if we substitue (13) - (15) into (12) we get

$$\dot{Z} = C(K_c,\ N_c) - \tilde{C} - T + P(q - 1)\dot{K}_I + PI(K_I, N_I)$$
$$- \pi \frac{M+B}{P_c} - P\delta K \ . \tag{16}$$

By using the definition of GNP we can rewrite this equation as

$$\dot{Z} = Y - T - P\delta K + P(q - 1)\dot{K}_I - \pi \frac{M+B}{P_c} - \tilde{C} \tag{17}$$

Finally to find the expression for disposable income set equation (17) equal to zero and solve for $\tilde{C}$ to get

$$\tilde{C} = Y - T - P\delta K + P(q - 1)\dot{K}_I - \pi \frac{M+B}{P_c} \tag{18}$$

In addition to flow consumption decision public makes stock portfolio decisions. The fact that bonds and equities are regarded as perfect substitutes when their yields are equal implies that public's portfolio decision consists of dividing their wealth between money on one hand and paper earning assets on the other. The portfolio balance of the public is satisfied when the demand for real balances is equal to supply,

$$\frac{M}{P_c} = m((1-t_y)r,\ Y) \qquad m_r < 0\ , \quad m_y > 0 \tag{19}$$

where after-tax return on government bonds is equal to $(1-t_y)r$.

II. SOLUTION OF THE MODEL

The complete model of the economy is described by the following equations:

$$I = I(K_I, N_I) \tag{20}$$

$$C = C(K_c, N_c) \tag{21}$$

$$I_N = \frac{w}{PP_c} \tag{22}$$

$$C_N = \frac{w}{P_c} \tag{23}$$

$$C_K = P(\delta + \frac{r-\pi}{1-t_k}) \tag{24}$$

$$\dot{K}_I = g(q-1) \tag{25}$$

$$C = \gamma(Y - T - P\delta K) + G \tag{26}$$

$$Y = PI + C \tag{27}$$

$$q = \frac{(I_k-\delta)(1-t_k)}{r - \pi} \tag{28}$$

$$I = \dot{K}_I + \dot{K}_c + \delta K \tag{29}$$

$$\frac{M}{P_c} = m((1-t_y)r, Y) \tag{30}$$

This is a system of eleven equations in eleven variables: I, C, P, P_c, Y, $\dot{K}_I$, $\dot{K}_c$, N_I, N_c, r, q. Parameters of the model are K_I, K_c, w, δ, π, G, t_k, t_y, T, M. It is assumed that variables of the model adjust instantaneously in response to a change in exogenous variables.

To solve the model we shall reduce the system into two equations in relative price and gross return on capital. One of these relations will give the pairs of relative price and gross return on capital that will ensure profit maximization and portfolio balance. The second relation will give pairs of relative price and gross return on capital, again satisfying profit maximization conditions, and ensuring the equality of demand for consumption goods to their supply. The first relationship we shall call LL schedule and the second SS schedule.

LL Schedule

Let ϵ be the gross return on capital defined as follows:

$$\epsilon = \delta + \frac{r-\pi}{1-t_k}$$

Then from profit maximizing conditions, equations (20) - (24), we can derive the following equations[7/]

$$I = I(P, \epsilon) \tag{31}$$

$$C = C(P, \epsilon) \tag{32}$$

$$P_c = P_c(P, \epsilon, w) \tag{33}$$

where all partial derivatives are positive. From the definition of gross return on capital we obtain

$$r = (1-t_k)(\epsilon-\delta) + \pi \tag{34}$$

Now if we substitute (27) and (31) - (34) into equation (30) that describes the portfolio balance we get

$$\frac{M}{P_c(P,\epsilon,w)} = m\{(1-t_y)[(1-t_k)(\epsilon-\delta) + \pi], PI(P,\epsilon) + C(P,\epsilon)\}$$

Defining LL schedule as a curve along which excess demand for real balances is zero, we can write it as follows:

$$\begin{aligned}&\Psi(P,\epsilon;M,w,t_y,t_k,\delta,\pi) = \\ &m\{(1-t_y)[(1-t_k)(\epsilon-\delta) + \pi], PI(P,\epsilon) + C(P,\epsilon)\} - \frac{M}{P_c(P,\epsilon,w)} = 0\end{aligned} \tag{35}$$

In order to determine the slope of LL schedule and the effects of a change in parameters we differentiate (35) to get

$$\Psi_\epsilon d\epsilon + \Psi_p dP + \Psi_m dM + \Psi_w dw + \Psi_{t_y} dt_y + \Psi_{t_k} dt_k + \Psi_\pi d\pi = 0 \tag{35a}$$

where

$$\Psi_\epsilon = M_r(1-t_y)(1-t_k) + M_y(PI_\epsilon + C_\epsilon) + \frac{M}{P_c^{\,2}} P_{c\epsilon}$$

$$\Psi_p = M_y(I + PI_p + C_p) + \frac{M}{P_c^{\,2}} P_{cp} > 0$$

$$\Psi_M = -\frac{1}{P_c} < 0$$

$$\Psi_w = \frac{M}{P_c^{\ 2}} P_{cw} > 0$$

$$\Psi_{t_y} = - m_r r > 0$$

$$\Psi_{t_k} = - m_r(1-t_y)(\epsilon-\delta) > 0$$

$$\Psi_\pi = m_r(1-t_y) < 0$$

$$\Psi_{k_I} = m_y PI_k > 0$$

$$\Psi_{k_c} = m_y C_k > 0 \quad .$$

Of these only the sign of Ψ_ϵ is ambiguous. Because Ψ represents excess demand for real balances Ψ_ϵ shows the effects of a rise in ϵ on this excess demand. A rise in ϵ tends to reduce excess demand for real balances by raising the nominal return on paper earning assets. We shall call this effect the interest elasticity effect (IEE), and it is given by:

$$m_r(1-t_y)(1-t_k) \quad .$$

On the other hand a rise in ϵ through profit maximizing conditions tends to increase the output of capital and consumption goods. Given relative price the increase in output in both sectors tends to increase aggregate output and hence the excess demand for real balances. This effect, which we shall call output effect (OE), is given by

$$m_y(PI_\epsilon + C_\epsilon) \quad .$$

On supply side of real balances a rise in ϵ tends to increase the excess demand by raising the price of consumption goods (P_c) and hence reducing the supply of real balances. This is equal to

$$\frac{M}{P_c^{\,2}} P_{c\epsilon}$$

and we shall call it the supply effect (SE).

The slope of LL curve is given by

$$\frac{\partial P}{\partial \epsilon}\Big|LL = -\frac{\Psi_\epsilon}{\Psi_p} \gtrless 0 \quad \text{as} \quad \Psi_\epsilon \lessgtr 0 \ .$$

SS Schedule

To derive SS schedule substitute (27), (31), (32) into (26) to get

$$C(P,\epsilon) = \gamma\{PI(P,\epsilon) + C(P,\epsilon) - T - P\delta K\} + G$$

which gives a curve along which aggregate demand for consumption goods is equal to their supply. Defining SS schedule as a relationship between P and ϵ such that excess demand for consumption goods is equal to zero, it can be written as:

$$\begin{aligned} &\phi(P,\epsilon;\delta,K,T,G) = \\ &\gamma\{PI(P,\epsilon) + C(P,\epsilon) - P\delta K - T\} + G - C(P,\epsilon) = 0 \end{aligned} \tag{36}$$

Again, in order to be able to determine the slope of SS schedule and the effects of a change in parameters, we differentiate (36) and get

$$\phi_\epsilon d\epsilon + \phi_P dP + \phi_T dT + \phi_G dG = 0 \tag{36a}$$

where

$$\phi_\epsilon = \gamma'(PI_\epsilon + C_\epsilon) - C_\epsilon$$

$$\phi_p - \gamma'(I + PI_p + C_p - \delta K) - C_p$$

$$\phi_T = - \gamma' < 0$$

$$\phi_G = 1 \quad > 0$$

Notice that signs of ϕ_ϵ and ϕ_p are ambiguous implying that the effects of a change in the gross return for capital and relative price, upon the excess demand for consumption goods can be positive or negative. If, for a moment, we assume $\delta = 0$, then we can write

$$\phi_p = \gamma'(I + PI_p) - (1 - \gamma')C_p$$

Using the results derived in the appendix we can express PI_p and C_p as follows:

$$PI_p = \sigma_I \frac{I\ c(k_c)}{i'Pk_I}$$

$$C_p = \sigma_c \frac{C}{P}$$

where σ_I and σ_c are the elasticities of substitution in the two sectors. Similarly for ϕ_ϵ we can write

$$\phi_\epsilon = \gamma' PI_\epsilon - (1-\gamma')C_\epsilon$$

and

$$PI_\epsilon = \sigma_I \frac{PI}{i'} \frac{k_c}{k_I}$$

$$C_\epsilon = \sigma_c \frac{PC}{c'}$$

As ϕ is the excess demand function for consumption goods a rise in P_c will be expected to reduce ϕ. Given that $P = (P_I/P_c)$, a rise in P_c will lower P. Therefore, for a reduction of ϕ we would expect $\phi_p > 0$. The necessary condition for this is given by

$$\sigma_c \frac{(1-\gamma')C}{\gamma' PI} - \sigma_I \frac{c(k_c)}{Pk_I i'} < 1 \tag{37}$$

which we shall assume to hold. On the other hand for $\phi_\epsilon > 0$ the necessary condition is

$$\sigma_c \frac{(1-\gamma')C}{\gamma' PI} - \sigma_I \frac{k_c c'}{Pk_I i'} < 0 \tag{38}$$

It is obvious that (38) implies (37) but (37) does not imply (38). Hence when $\phi_\epsilon > 0$ then $\phi_p > 0$ holds, whereas when $\phi_p > 0$ ϕ_ϵ can be of either sign.

The slope of SS schedule is given by

$$\left.\frac{\partial P}{\partial \epsilon}\right|_{SS} = - \frac{\phi_\epsilon}{\phi_p}$$

sign of which cannot be determined unless we make further assumptions.

Stability of the Model

Before turning to section III and performing the static exercises to study the relative potency of monetary and fiscal policies, it seems to be fruitful to establish the stability conditions of the model. Once these conditions are established we can easily confine our studies to those cases which are stable.

We shall assume that interest rate changes in response to an excess demand for money, and price of consumption goods change in response to an excess demand for consumption goods. The differential equations implied by these specifications are as follows:

$$\dot{r} = \alpha_1 \, [\Psi(P,\epsilon)] \tag{39}$$

$$\dot{P}_c = \alpha_2 \, [\phi(P,\epsilon)] \tag{40}$$

Using equations (33) and (34), and linearizing around equilibrium we get

$$\begin{bmatrix} (1-t_k) & 0 \\ P_{c\epsilon} & P_{cp} \end{bmatrix} \begin{bmatrix} \dot{\epsilon} \\ \dot{P} \end{bmatrix} = \begin{bmatrix} \alpha_1 \Psi_\epsilon & \alpha_1 \Psi_p \\ \alpha_2 \phi_\epsilon & \alpha_2 \phi_p \end{bmatrix} \begin{bmatrix} d\epsilon \\ dP \end{bmatrix} \tag{41}$$

or

$$\begin{bmatrix} \dot{\epsilon} \\ \\ \dot{P} \end{bmatrix} = \begin{bmatrix} \dfrac{\alpha_1 \Psi_\epsilon}{(1-t_k)} & \dfrac{\alpha_1 \Psi_p}{(1-t_k)} \\ \\ \dfrac{\alpha_2 \phi_\epsilon}{P_{cp}} - \dfrac{\alpha_1 P_{c\epsilon} \Psi_\epsilon}{P_{cp}(1-t_k)} & \dfrac{\alpha_2 \phi_p}{P_{cp}} - \dfrac{\alpha_1 P_{c\epsilon} \Psi_p}{P_{cp}(1-t_k)} \end{bmatrix} \begin{bmatrix} d\epsilon \\ \\ dP \end{bmatrix}$$

As it is well known, for this system to be stable, the characterisic roots of the matrix must have negative real parts. The necessary and sufficient conditions for this to be so are

$$\frac{\alpha_1 \Psi_\epsilon}{(1-t_k)} + \frac{\alpha_2 \phi_p}{P_{cp}} - \frac{\alpha_1 P_{c\epsilon} \Psi_p}{P_{cp}(1-t_k)} < 0 \tag{42}$$

$$\frac{\alpha_1 \Psi_\epsilon}{(1-t_k)} \quad \frac{\alpha_2 \phi_p}{P_{cp}} - \frac{\alpha_1 P_{c\epsilon} \Psi_p}{P_{cp}(1-t_k)} \quad - \frac{\alpha_1 \Psi_p}{1-t_k} \quad \frac{\alpha_2 \phi_\epsilon}{P_{cp}} - \frac{\alpha_1 P_{c\epsilon} \Psi_\epsilon}{P_{cp}(1-t_k)} \quad > 0 \tag{43}$$

Equation (42) implies that

$$\frac{\alpha_1}{P_{cp}(1-t_k)} \quad (\Psi_\epsilon P_{cp} - \Psi_p P_{c\epsilon}) + \frac{\alpha_2 \phi_p}{P_{cp}} < 0 \tag{42a}$$

from which we get

$$\frac{\alpha_2}{\alpha_1} < \frac{\Psi_p P_{c\epsilon} - \Psi_\epsilon P_{cp}}{(1-t_k)\,|\phi_p|} \tag{44}$$

As α_1 and α_2 are both positive then it follows that $\Psi_p P_{c\epsilon} - \Psi_\epsilon P_{cp} > 0$.

On the other hand equation (43) implies that

$$\frac{\alpha_1\alpha_2}{P_{cp}(1-t_k)} \quad (\Psi_\epsilon\phi_p - \Psi_p\phi_\epsilon) > 0 \tag{45}$$

which means that $\Psi_\epsilon\phi_p - \Psi_p\phi_\epsilon > 0$.

Under our assumption that $\phi_p > 0$ we can rewrite the above inequality as

$$-\frac{\phi_\epsilon}{\phi_p} > -\frac{\Psi_\epsilon}{\Psi_p} \tag{46}$$

which means that in $P-\epsilon$ plane slope of the excess demand function for consumption goods must be greater than the slope of the excess demand function for real balances.

In addition to this from equation (44) we note that

$$\Psi_p P_{c\epsilon} - \Psi_\epsilon P_{cp} > 0$$

which implies

$$-\frac{P_{c\epsilon}}{P_{cp}} = -\frac{P}{\epsilon} < -\frac{\Psi_\epsilon}{\Psi_p} \tag{47}$$

Taking into account the fact that $\Psi_p > 0$, for equation (47) (and therefore the stability condition (42)) to hold excess demand function for real balances should either be positively sloped ($\Psi_\epsilon < 0$) or, if negatively sloped should not be "too steep." As a matter of fact if we combine equations (46) and (47) we see that slope of the excess demand function for real balances is bounded from below and from above

$$- \frac{P}{\epsilon} < - \frac{\Psi_\epsilon}{\Psi_p} < - \frac{\phi_\epsilon}{\phi_p} \tag{48}$$

III. RELATIVE POTENCY OF MONETARY AND FISCAL POLICIES UNDER ALTERNATIVE CIRCUMSTANCES

The relative potency of monetary and fiscal policies will be studied in terms of their effect upon the general equilibrium of the model as given by a pair of values of P and ϵ such that both the excess demand for real balances and excess demand for consumption goods are equal to zero. This point in P-ϵ plane is given by the intersection of LL and SS schedules.

To study the effects of a change in M, w, t_y, t_k, π, T, and G we shall use the system of equations given by (35a) and (36a) which can be written, in matrix notation, as follows:

$$\begin{bmatrix} \Psi_\epsilon & \Psi_p \\ \phi_\epsilon & \phi_p \end{bmatrix} \begin{bmatrix} d\epsilon \\ dP \end{bmatrix} = - \begin{bmatrix} \Psi_\theta \\ \phi_\theta \end{bmatrix} d\theta \tag{49}$$

where θ is a symbol that stands for any one of the exogenous parameters. Stability condition (45) implies that the determinant of the matrix is positive. Consequently we can invert it and solve for $d\epsilon$ and dP to get

$$\begin{bmatrix} d\epsilon \\ dP \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \Psi_p\phi_\theta - \phi_p\Psi_\theta \\ \phi_\epsilon\Psi_\theta - \Psi_\epsilon\phi_\theta \end{bmatrix} d\theta \tag{49a}$$

where $\Delta = \Psi_\epsilon\phi_p - \Psi_p\phi_\epsilon > 0$.

From equation (48) we know that if the SS schedule is positively sloped ($\phi_\epsilon < 0$), LL schedule could either be positively sloped ($\Psi_\epsilon < 0$) or negatively sloped ($\Psi_\epsilon > 0$). We shall call the former case interest elastic demand for money (IEDM) and the latter case interest inelastic demand for money (IIDM). If, on the other hand, SS schedule is negatively ($\phi_\epsilon > 0$) sloped then LL schedule must also be negatively sloped. This case we shall call interest elastic demand for consumption (IEDC).

We believe that the case of IEDC is relatively unimportant and therefore we shall study in detail only the cases of IEDM and IIDM. Nevertheless results of static exercises in the case of IEDC will be presented in Table 3.

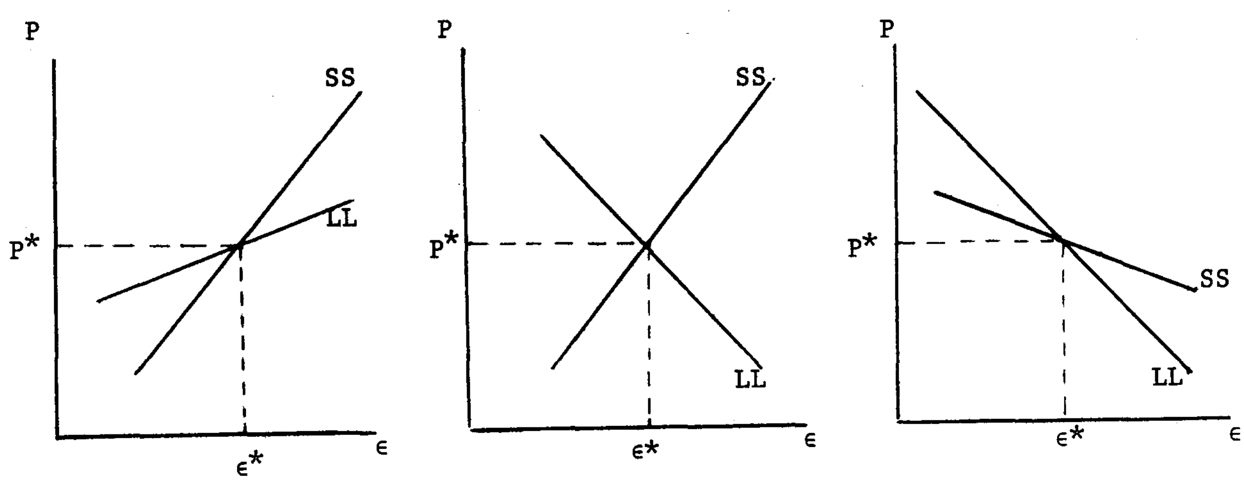


Figure 1

The working of the model could be summarized as follows: the intersection of SS and LL schedules determine the equilibrium values of P and ϵ. Given P and ϵ together with K_c equation (24) will determine N_c. Equation (23) then determines P_c for a given money wage w. Once P_c is determined then equation (22) will set N_I. With the levels of employment in each sector so determined the output levels will be given by (20) and (21), GNP by (27), and interest rate by (34).

The Case of IEDM

a) An increase in money supply

An increase in money supply causes LL schedule to shift upwards -- see Figure 2 -- thereby creating an excess supply of money at the initial levels of P and ϵ. Public, in an attempt to achieve portfolio equilibrium will dump money and demand bonds, thus pushing the interest rate down. As ϵ falls firms in the consumption goods sector will bid for capital and therefore push P up. Both the increase in P and fall in ϵ will create an excess demand for consumption goods. Consequently the fall in ϵ and rise in P should be such that P_c goes up. These forces will eventually create an excess demand for money and ϵ starts increasing. At this point with both ϵ and P rising so will P_c. However our stability condition (44) tells us that the increase in P_c is relatively "slow" implying that costs are rising faster than revenues. Hence, firms will start dumping capital and P starts falling with ϵ

and P_c rising. These forces will bring the system back to equilibrium with a higher P and ϵ than originally.[8/]

As the new equilibrium if characterized by a higher P and ϵ, from (24) we see that marginal product of capital in consumption goods sector must rise.

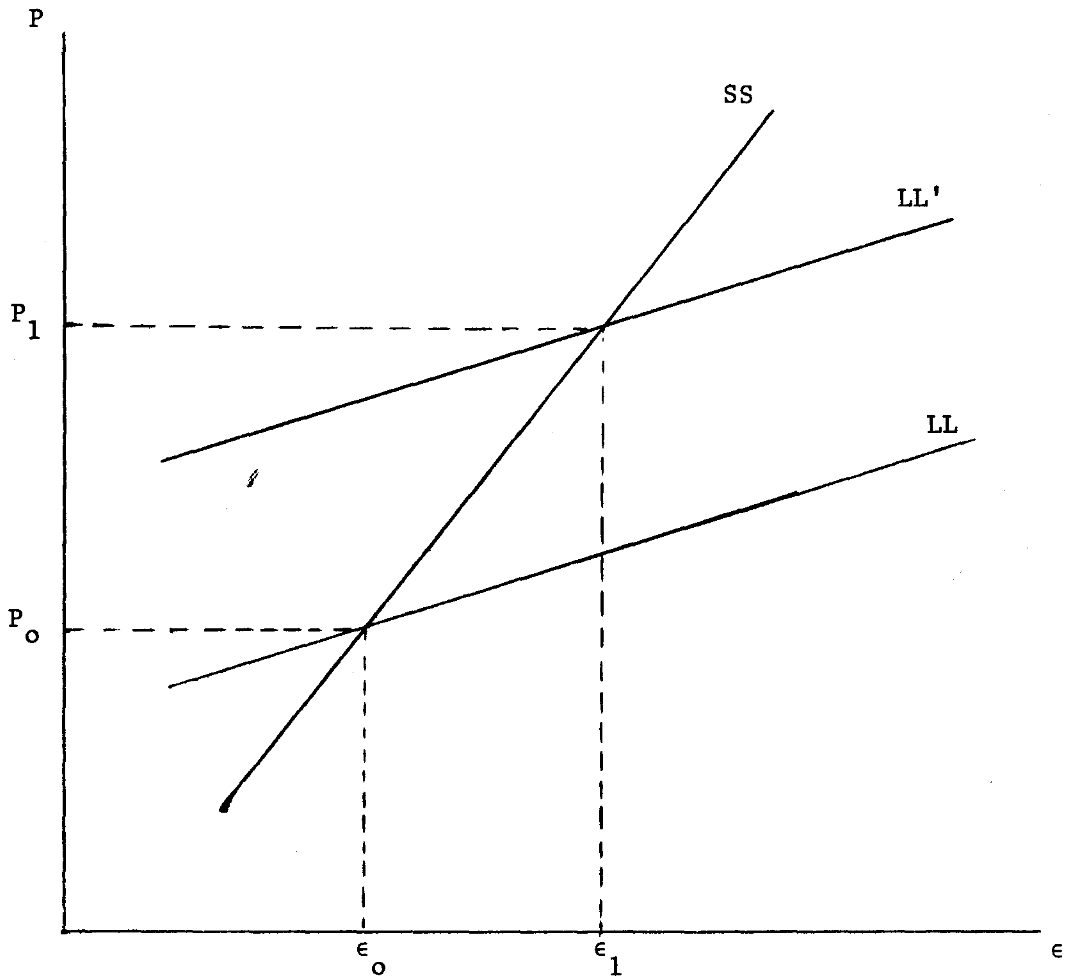


Figure 2

Given that K_c is fixed marginal product of capital will rise only if the level of employment in the consumption goods sector increases. With higher N_c and lower marginal product of labour profit maximizing conditions require that P_c must be higher. Similarly an increase in both P and P_c will lower the real wages in capital goods industry, pushing up the level of employment (N_I). As the level of employment in both sectors go up so will the GNP.

b) An increase in government expenditures

An increase in government expenditures will shift SS schedule to the right (see Figure 3), thus creating an excess demand for consumption goods at original values of P and ϵ . This will push P_c up thereby lowering real wages. Firms noticing that real wages are falling start increasing their employment and consequently the marginal product of capital. As marginal product of capital increases firms will bid for capital raising P . These effects will, in turn, create an excess demand for money and ϵ starts increasing. At this point we again have P, ϵ, and P_c rising, but because P_c is rising "slowly" firms will eventually start dumping capital and lowering P . The system will achieve equilibrium at a higher P , and ϵ than originally.

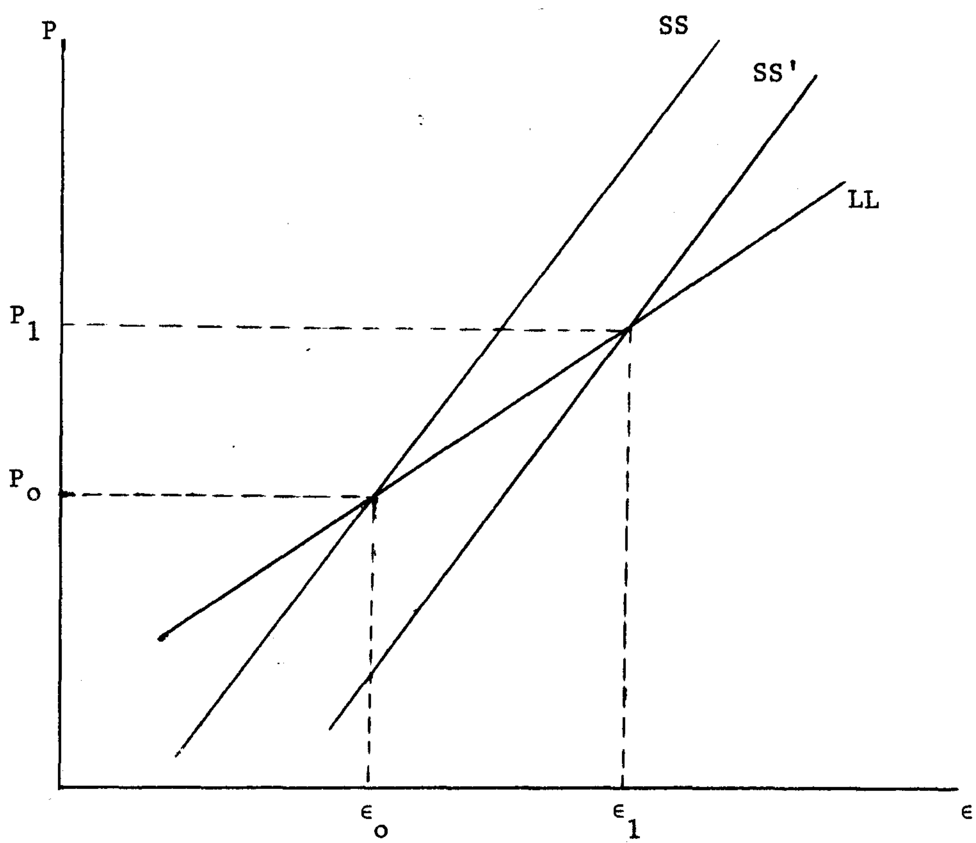


Figure 3

As P and ϵ are higher so will be P_c, N_c, and N_I.

c) Effects of changes in other parameters.

Due to the fact that changes in w, t_y, t_k, π, and T are similar to either a change in money supply or government expenditures we feel that it suffices to summarize their results in Table 1, without going into detail.

TABLE 1 (IEDM)

	dM	dG	dT	dt_k	dt_y	dw	$d\pi^{(a)}$
dp	+	+	-	-	-	-	+
$d\epsilon$	+	+	-	-	-	-	+
dP_c	+	+	-	-	-	-	+
dN_I	+	+	-	-	-	-	+
dN_c	+	+	-	-	-	-	+
dY	+	+	-	-	-	-	+
dr	+	+	-	-	-	-	+

(a) $$d\epsilon = - \frac{\phi_p m_r (1-t_y)}{\Delta} d\pi > 0$$

$$dr = (1-t_k)d\epsilon + d\pi \quad > 0$$

The Case of IIDM

As we have seen before, in the case of IIDM, LL schedule is negatively sloped but less steep than $-P/\epsilon$.

a) An increase in money supply.

As Ψ_p is still positive an increase in money supply will result in a rightward shift of the LL curve (see Figure 4) and create an excess supply of money at initial values of P and ϵ . Public will react to this by bidding for paper earning assets thus forcing ϵ down.

As ϵ goes down firms in the consumption goods sector, realizing that public is willing to hold capital at a lower return will bid for capital raising P. An increase in P and a reduction in ϵ both tend to create an excess demand for consumption goods thereby forcing P_c up. The reduction in ϵ and an increase in P will bring the money market into equilibrium. However the excess demand for consumption goods will continue to push P and P_c upwards. As P_c rises, and reduces the supply of real balances an excess demand will result forcing ϵ up. Again by appealing to our stability condition (44) we can conclude that eventually costs will start increasing faster than revenues forcing firsm to dump capital, pushing P down. Equilibrium will be reached at a higher P and ϵ than originally, with a higher P_c, N_I, N_c, Y, and r.

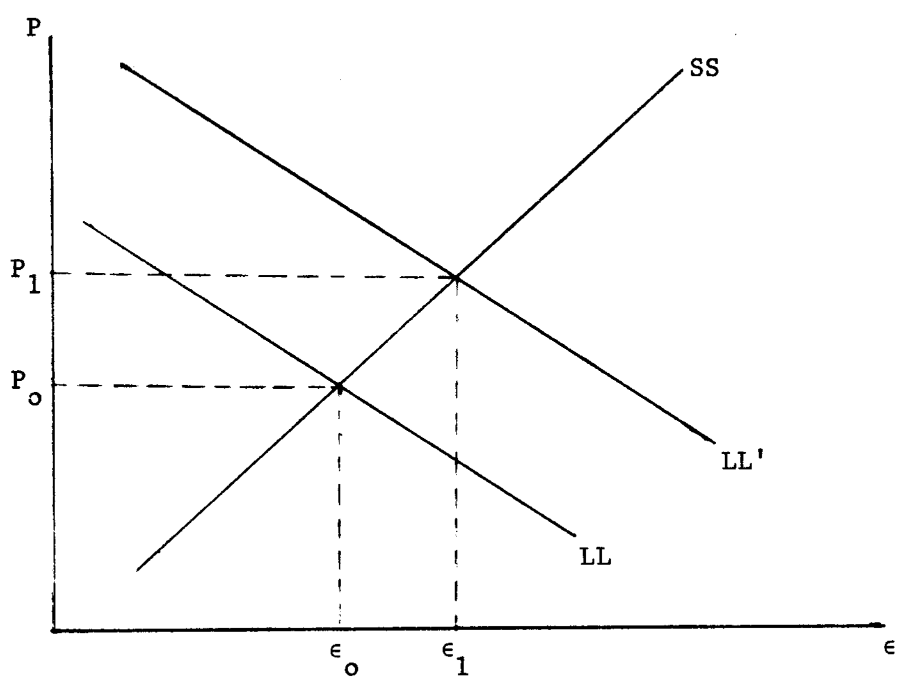


Figure 4

b) An increase in government expenditures.

An increase in government expenditures will result in an excess demand for consumption goods at initial levels of ϵ and P thus forcing P_c up. As P_c increases supply of real balances will fall creating an excess demand for money (see Figure 5) and, therefore, pushing ϵ up. At this time when P and ϵ are both rising P_c will rise as well, but at a "slower" rate. Due to this slowness eventually costs will start increasing more rapidly than revenues forcing firms to dump capital and lowering P. The new equilibrium will be reached at a higher ϵ and lower P. In general when P and ϵ move in different directions between two equilibriums we cannot, off hand, say what happens to the levels of employment in the two sectors. The important thing is what happens to $P.\epsilon$; in other words we are interested in the sign of $Pd\epsilon + \epsilon dP$, which, if negative, implies that N_c falls and, if positive, implies that N_c rises.

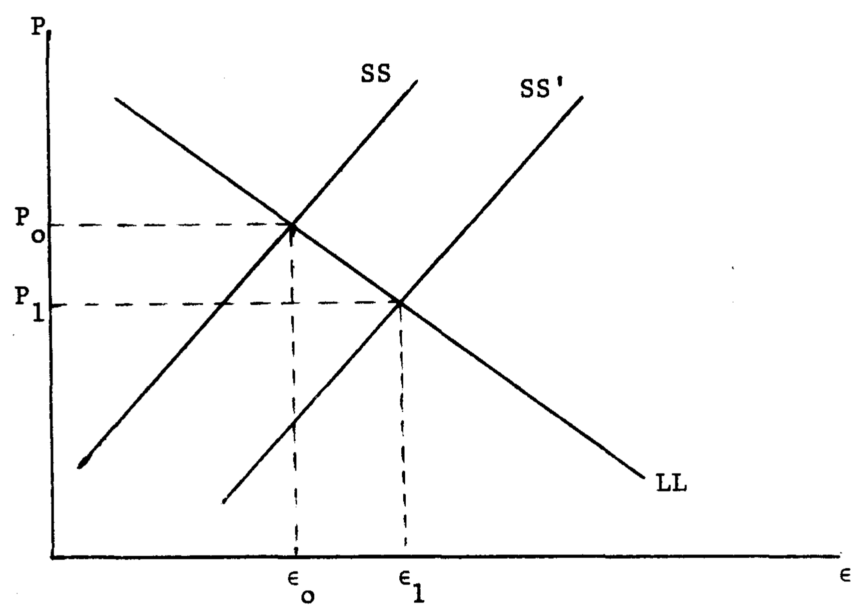


Figure 5

To find this out let us consider the explicit solution for $d\epsilon$ and dP . From equations (49a) we see that

$$\begin{aligned} d\epsilon &= \frac{1}{\Delta}\Psi_p\phi_G\, dG \\ dP &= -\frac{1}{\Delta}\Psi_\epsilon\phi_G\, dG \end{aligned} \tag{50}$$

If we multiply the first equation by P and second equation by ϵ and add the two equations up, we get

$$Pd\epsilon + \epsilon dP = \frac{1}{\Delta}(P\Psi_p - \epsilon\Psi_\epsilon)dG$$

From equation (47) we can show that $P\Psi_p - \epsilon\Psi_\epsilon > 0$, which makes the right hand side of the above equation positive. This, in turn, implies that as a result of an increase in government expenditures $P.\epsilon$ will go up. As profit maximizing conditions must continue to hold an increase in marginal product of capital must occur. With K_c fixed this will happen only if N_c goes up.

To find out what happens to N_I we must consider whether $P.P_c$ increases or decreases. We can write

$$dP_c = P_{cp}dP + P_{c\epsilon}d\epsilon$$

From this it follows that

$$PdP_c + P_c dP = (PP_{cp}+P_c)dP + PP_{c\epsilon}d\epsilon$$

If we substitute equations (50) on the right hand side we get

$$PdP_c + P_c dP = (PP_{cp}+P_c)(-\frac{\Psi_\epsilon}{\Delta}dG) + PP_{c\epsilon}(\frac{\Psi_p}{\Delta}dG)$$

$$= [PP_{c\epsilon}\Psi_p - \Psi_\epsilon(PP_{cp}+P_c)]\frac{dG}{\Delta}$$

$$= \frac{P_{c\epsilon}}{P_{cp}} - \frac{\Psi_\epsilon}{\Psi_p} \quad 1 + \frac{P_c}{PP_{cp}} \quad \frac{PP_{cp}\Psi_p}{\Delta} dG \tag{51}$$

In equation (51) the expression $\frac{P_c}{PP_{cp}}$ is nothing but inverse of the elasticity of the price of consumption with respect to relative price. In view of this, we can show that

$$dN_I \gtreqless 0 \qquad \text{as} \qquad \varepsilon_{P_cP} \gtreqless \frac{1}{\frac{P\Psi_p}{\epsilon\Psi_\epsilon} - 1}$$

Notice that by virtue of our stability condition (47) $\frac{P\Psi_p}{\epsilon\Psi_\epsilon} > 1$ and therefore $\varepsilon_{PcP} > 0$.

Due to the same reasons given under the case of IEDM, effects of changes in other parameters will be summarized in Table 2.

TABLE 2 (IIDM)

	dM	dG	dT	dt_k	dt_y	dw	$d\pi$
dP	+	-	+	-	-	-	+
$d\epsilon$	+	+	-	-	-	-	+
dP_c	+	+	-	-	-	-	+
dN_I	+	?	?	-	-	-	+
dN_c	+	+	-	-	-	-	+
dY	+	?	?	-	-	-	+
dr	+	+	-	-	-	-	+

TABLE 3 (IEDC)

	dM	dG	dT	dt_k	dt_y	dw	$d\pi$
dP	-	-	+	+	+	+	-
$d\epsilon$	+	+	-	-	-	-	+
dP_c	+	+	-	-	-	-	+
dN_I	?	?	?	?	?	?	?
dN_c	+	+	-	-	-	-	+
dY	?	?	?	?	?	?	?
dr	+	+	-	-	-	-	+

IV. CONCLUSIONS

The model developed in this paper has a stronger Keynesian outlook than the Henderson-Sargent model in the sense that government expenditures and tax collections are more potent stabilization instruments, in manipulating the level of employment, under our model than they are under Henderson-Sargent model. In particular when the demand function for real balances is interest elastic we see that an increase in government expenditures tends to increase the level of employment in our model whereas Henderson-Sargent find that the direction of change of the level of employment is ambiguous. With our assumptions an increase in government expenditures always increases the level of employment in consumption goods sector. Consequently, even when there is an adverse aggregate employment effect, government expenditures and taxes can be used to re-allocate resources.

Noting the similarity of our model to that of Henderson-Sargent one may inquire into the source of the differences cited above. Henderson-Sargent conclude their paper by stating that "...the Keynesian model posits investment demand curves for firms...That feature of the Keynesian model is an extremely important one in making variations in government expenditures a potent instrument for stabilization with predictable effects on output and employment in the short run." Notice that a necessary condition for the existance of investment demand curves is the immobility of capital between the two sectors. In particular we feel that assumptions regarding the mobility of capital between the sectors and assumptions regarding the existance of investment functions, should be handled very carefully. First of all we should observe that in a two sector model where marginal conditions for capital are satisfied it is necessary to assume that capital is costlessly mobile between the sectors. If one assumes that marginal conditions are satisfied but that capital is not mobile between the sectors, then there will be two prices of capital stock (one for that capital which is used in consumption goods sector and one for that which is used in capital goods sector). However, because there exists a flow of newly produced capital, which must possess a unique price, a short-run equilibrium of the system may not -- and in general will not -- exist.

Secondly, with capital mobile between the sectors of the economy, interaction of two stock markets, namely, the money market and the market in existing stocks of capital good, determines the equilibrium. Flow markets effect this equilibrium to the extent that they can alter the equilibrium in the market for the stocks of capital goods.

However, in the model we have presented in this paper it is the interaction of a stock market (money) and a flow market (consumption goods) that determines the equilibrium. The stock market in capital goods provides the main link through which the monetary disturbances effect the flow market. Although our model posits investment demand functions for firms, they do not play any role in the determination of instantaneous equilibrium. Hence "Keynesian" results that make fiscal policy a potent instrument for stabilization do not necessarily depend upon the form and characteristics of the investment demand schedules but depend upon the immobility of physical capital between the sectors of the economy. With mobile capital the transmission mechanism involves a movement of capital stock between the sectors and consequently plays a dampening role upon changes in the level of employment.

FOOTNOTES

1/ Perfect in the sense that firms can adjust their stocks of physical capital instantaneously in response to disturbances, without incurring any transaction costs.

2/ For a paper that highlights the importance of perfect capital markets and transaction costs within the context of a one good macroeconomic model see Sargent and Wallace [4].

3/ Assuming, of course, that the production function is linearly homogenous.

4/ The variable q is the ratio of the internal value of a unit of capital to its market price. It is the source variable q as defined by Tobin [6].

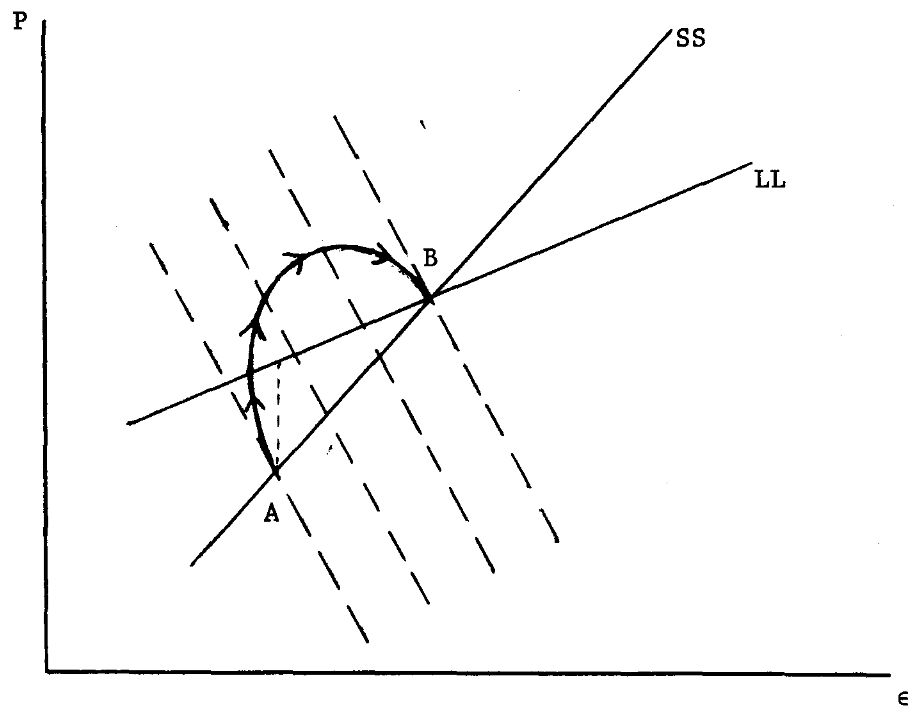
5/ Equation (7) helps to establish the interpretation of q rather clearly. If the marginal product of a unit of capital exceeds the real rate of interest, implying $q > 1$, the internal value of a unit of capital, $P_I q$, will exceed the market price of the same unit of capital, P_I . Consequently firms find it profitable to expand their capital stock whenever $q > 1$.

6/ In this sector dividends are equal to $(r-\pi)P_I K_c$. Value of equities is equal to the present value of the flow of dividends.

$$V_c = \int_o^\infty e^{-(r-\pi)^t} (r-\pi)P_I K_c dt = P_I K_c$$

7/ See appendix.

8/ A possible path of adjustment is given below. The dashed lines represent constant P_c at different levels. Whenever we are to the left of SS curve P_c must rise due to excess demand.



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APPENDIX

We can write the profit maximizing subsystem, equations (20) through (24) in intensive form as follows

$$I = K_I/\beta_I \tag{A1}$$

$$C = K_c/\beta_c \tag{A2}$$

$$\beta_I = k_I/i(k_I) \tag{A3}$$

$$\beta_c = k_c/c(k_c) \tag{A4}$$

$$\frac{w}{PP_c} = i(k_I) - k_I\ i'(k_I) \tag{A5}$$

$$\frac{w}{P_c} = c(k_c) - k_c\ c'(k_c) \tag{A6}$$

$$P\epsilon = c'(k_c) \tag{A7}$$

$$\epsilon = \delta + \frac{r - \pi}{1 - t_k} \tag{A8}$$

where β_I and β_c are capital-output ratios in two sectors, k_I and k_c are capital labor ratios in two sectors, ϵ is the gross return on capital.

If $c''(k_c) \neq 0$ then we can invert (A7) to get

$$k_c = k_c(P,\epsilon) \tag{A9}$$

with

$$k_{cP} = \frac{\partial k_c}{\partial P} = \frac{\epsilon}{c''} < 0$$

$$k_{c\epsilon} = \frac{\partial k_c}{\partial \epsilon} = \frac{P}{c''} < 0 \ .$$

By equating the ratio of marginal products of labor in the two sectors to the ratio of real wages, from (A5) and (A6), and using (A9) we get

$$\frac{c[k_c(P, \epsilon)] - k_c(P, \epsilon)\ c'[k_c(P, \epsilon)]}{P} = i(k_I) - k_I i'(k_I) \ . \qquad \text{(A10)}$$

Equation (A10) defines k_I as a function of P and ϵ provided that $i''(k_I) \neq 0$

$$k_I = k_I(P, \epsilon) \qquad \text{(A11)}$$

with

$$k_{IP} = \frac{\partial k_I}{\partial P} = \frac{\frac{k_c\ c'' k_{cP}}{P} + \frac{\frac{w}{P_c}}{P^2}}{k_I i''} < 0$$

$$k_{I\epsilon} = \frac{\partial k_I}{\partial \epsilon} = \frac{\frac{k_c\ c'' k_{c\epsilon}}{P}}{k_I i''} < 0 \ .$$

From equation (A3) we have

$$\beta_I = \frac{k_I(P, \epsilon)}{i[k_I(P, \epsilon)]} = \beta_I(P, \epsilon)$$

with

$$\beta_{IP} = \frac{\partial \beta_I}{\partial P} = \frac{w}{PP_c} \ \frac{k_{IP}}{i(k_I)^2} < 0$$

$$\beta_{I\epsilon} = \frac{\partial \beta_I}{\partial \epsilon} = \frac{w}{PP_c} \ \frac{k_{I\epsilon}}{i(k_I)^2} < 0$$

and similarly from equation (A4)

$$\beta_c = \frac{k_c(P, \epsilon)}{c[k_c(P, \epsilon)]} = \beta_c(P, \epsilon)$$

with

$$\beta_{cP} = \frac{\partial \beta_c}{\partial P} = \frac{w}{P_c} \cdot \frac{k_{cP}}{c(k_c)^2} < 0$$

$$\beta_{c\epsilon} = \frac{\partial \beta_c}{\partial \epsilon} = \frac{w}{P_c} \cdot \frac{k_{c\epsilon}}{c(k_c)^2} < 0 .$$

Equation (A6) implies

$$\frac{w}{P_c} = c[k_c(P, \epsilon)] - k_c(P, \epsilon)\ c'[k_c(P, \epsilon)] \tag{A12}$$

$$P_c = P_c(P, \epsilon, w) \tag{A13}$$

where

$$P_{cP} = \frac{\partial P_c}{\partial P} = \frac{P_c^2}{w} k_c\ c''(k_c)k_{cP} > 0$$

$$P_{c\epsilon} = \frac{\partial P_c}{\partial \epsilon} = \frac{P_c^2}{w} k_c\ c''(k_c)k_{c\epsilon} > 0$$

$$P_{cw} = \frac{\partial P_c}{\partial w} = \frac{P_c}{w} > 0 .$$

Finally from equations (A1) and (A2) we can derive

$$I = K_I/\beta_I(P, \epsilon) = I(P, \epsilon, K_I) \tag{A14}$$

$$C = K_c/\beta_c(P, \epsilon) = C(P, \epsilon, K_c) \tag{A15}$$

with

$$I_P = \frac{\partial I}{\partial P} = - I \frac{\beta_{IP}}{\beta_I} > 0$$

$$I_\epsilon = \frac{\partial I}{\partial \epsilon} = - I \frac{\beta_{I\epsilon}}{\beta_I} > 0$$

$$C_P = \frac{\partial C}{\partial P} = - C \frac{\beta_{cP}}{\beta_c} > 0$$

$$C_\epsilon = \frac{\partial C}{\partial \epsilon} = - C \frac{\beta_{c\epsilon}}{\beta_c} > 0 \ .$$

Now consider the expression

$$PI_P = - PI \frac{\beta_{IP}}{\beta} = - PI \frac{w}{PP_c} \cdot \frac{k_{IP}}{i(k_I)^2} \cdot \frac{i(k_I)}{k_I}$$

$$= - PI \frac{w}{PP_c} \frac{1}{k_I i(k_I)} \cdot \left[\frac{\epsilon k_c}{Pk_I i''} + \frac{w/P_c}{P^2 \ k_I i''} \right]$$

$$= - I \frac{w}{PP_c} \frac{1}{k_I^2 P \ i'' i(k_I)} (P\epsilon k_c + w/P_c)$$

$$= \frac{I}{i'Pk_I} \quad - \frac{\frac{w}{PP_c} i'}{k_I i'' i(k_I)} \quad (P\epsilon k_c + w/P_c)$$

$$= \frac{I}{i'Pk_I} \ \sigma_I \ C(k_c) \qquad \text{(A16)}$$

where σ_I is the elasticity of substitution in capital goods sector.

In a similar way we can consider

$$C_p = - C \frac{\beta_{c\epsilon}}{\beta} = - C \frac{w}{P_c} \ \frac{k_{cP}}{C(k_c)^2} \frac{C(k_c)}{k_c}$$

$$= - C \frac{w}{P_c} \frac{1}{k_c C(k_c)} \cdot \frac{\epsilon}{c''} = \frac{c}{P} \quad - \frac{\frac{w}{P_c} \quad P_\epsilon}{k_c C(k_c) c''} \quad = \frac{C}{P} \sigma_c \tag{A17}$$

By similar algebraic manipulations we can also derive

$$PI_\epsilon = \frac{PI}{i'} \frac{k_c}{k_I} \sigma_I \tag{A18}$$

$$C_\epsilon = \frac{PC}{c'} \sigma_c \tag{A19}$$