RECONTRACTING IN SPECULATIVE MARKETS

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Although the demand for risky assets has been studied from a number of points of view, the speculative (or capital gains) aspects of futures markets and stock markets have been largely avoided.[1] The purchase of a stock in the expectation that it may be resold later for a profit is an act which is at odds with most notions of equilibrium in the market for the stock. Nevertheless, such behavior is very common and modeling such behavior may give insights into the determination of speculative prices.

In the following discussion, a risk-averse decision maker is faced with prices for a particular "future." By studying his behavior, we will try to discover something about the equilibrium price, without explicitly presenting an entire system for equilibrium.

The simplest model for the study of speculative behavior has three periods or dates. The first is the date at which trading in a particular "futures" contract may first be traded. The last is the "spot" or delivery date when all uncertainty is resolved and the contractual obligations in the "futures" contracts are resolved. The third is at some point between these two where buyers and sellers of futures contracts meet and re-contract, realizing capital gains (or capital losses). The notation for this three-period model is presented in Table 1.

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[1] See Keynes [6], Chapter 12 and Samuelson [7] for some discussion of the speculation problem.

Table 1

NOTATION FOR THE THREE-PERIOD RECONTRACTING MODEL

Time	A	B	C
Dollar Price (of one "futures" contract)	p	q	r
Wealth (before trading)	W_A	W_B	W_C
Demand	x	y	
States of the World		s	t

Thus at time A, the investor has certain wealth W_A and is faced with market price \$p for a futures contract that will be worth r if held to C, the maturity or delivery date. Viewed from A, q and r are random variables, and the investor must use his subjective probability distribution for r and q to find x, his demand for futures. There are many possibilities for s and t, and knowledge of s, the state of the world that has occurred at B, does not imply full knowledge about t. Between A and B, traders learn something about what t (and thus r) will eventually be, but not everything. In the case of wheat futures, s would be the Agriculture Department crop forecasts that have become available between A and B, while r would be the spot price at the delivery date in the future.

This model has four realistic features. First, expectations are heterogeneous. There is no presumption that the investor has the same expectations about future events and prices as any other investor in the market. The investor being studied is presumed to be small relative to the market, so that prices faced by him are constant at all levels of demand. Second, this is an "incomplete markets" model. There is only one futures contract, but many states of the world. This is in direct contrast with the mathematically convenient but unrealistic Arrow-Debreu framework. The "state of the world" jargon is retained since states of expectation might not be uniquely related to prices.

Third, the model allows for price uncertainty. Thus we are assuming that the only way to know a market price is to convene the market and allow the equilibrium price to be determined. The market at B has not yet occurred (when viewed from A) ; investors are uncertain about the price q.[2] Fourth, recontracting (and hence speculation) are possible. An investor who expected $(r - p)$ to be negative still might buy futures at A if he expected $(q - p)$ to be positive.

The investor's decision problem at A may be solved recursively:

$$\operatorname*{Max}_{x} V_A(x) = E_q V_B(q, x)$$

where E_q is the expectation operator with respect to his beliefs about q, and

$$V_B(x) = \operatorname*{Max}_{y} \left\{ E_{r|q} U \left[W_B(q, x) + (r - q) y \right] \right\}$$

$U(\cdot)$ is the investor's risk-averse utility function for certain wealth. This is, of course, the standard backwards iteration for any dynamic decision process.

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[2] Price uncertainty poses a fundamental problem for the Arrow-Debreu model, and particularly Arrow's securities-market result [1], Chapter 4.

Two points can be made about this decision process. First, any decision to buy at A (or sell, since x may be negative) depends on the investor's beliefs about the joint probability distribution of q, r and various states of the world. The distribution of q cannot be ignored when buying futures for "long term" gains to be realized at C. Furthermore, the distribution of r cannot be ignored except by investors who would not be in the market between B and C under any circumstances. This is because opportunities at B depend on decisions at A, a standard case of interdependent ventures as studied by Hildreth [5]. This is at odds with (at least one interpretation of) Meiselman's expectations hypothesis.

Second, if the investor feels that other investors will have the same opinion that he does at time B, then his position in the market at A will agree with his long run expectations (about r).

Proposition 1. If at time A an investor feels that all other investors will have expectations $E_{r|q}$ at time B which are identical with his own, then $E(r) \gtreqless p$ at time A if and only if $x \gtreqless 0$ respectively for this investor.

Proof: Arrow's proposition that a small part of a favorable bet will always be taken[3] implies that at time B, $E_{r|q} = q$ will be the market price. Consequently, y will be zero, and the decision problem at A becomes a one-period problem with certain wealth. In this situation, Arrow's "favorable bet" proposition indicates that the investor will have $x > 0$ if and only if $E(r) > p$, with the analogous results for $=$ and $<$.

[3] See Arrow [1], page 100.

Proposition 1 says that if the investor feels there will be no chance for advantageous futures trading at B, he will never "speculate" at A. That is, he will never take a position in futures at A which contradicts his long-run expectations E_r about the "spot price" or "final outcome" r. It is reasonable in this case to presume that the equilibrium price p will be an average of investors' expectations about r.[4] This is a small generalization of the case in which investors' expectations are completely homogeneous; in that case (a common one studied in the literature) $p = E(q) = E(r)$ for all investors.

If a sizable proportion of investors do not expect the intermediate price q to be the mean of their expectations about the spot price r, then it is possible that the equilibrium p is not an average of expectations about r, as would be the case if no intermediate market at B existed. Example 1 illustrates this sort of behavior for an individual facing a very simple "futures" market.

Example 1

Let r be determined by drawing a ball from an urn containing five balls which are either red or white. If a red ball is drawn, $r = 1$, but if a white ball is drawn, $r = 0$. Assume that the general consensus at time A is that there are three red and two white balls in the urn. Thus $p = .6$. Between time A and time B, one ball will be drawn from the urn, yielding new information which will be used in determining q.

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[4]See Green [4] or Clark [3] for a discussion of such an equilibrium.

Consider a trader who thinks that there are three white and two red balls in the urn, and that his "inside information" will not be discovered by the market. Let his utility for wealth (at C) be $U = \log(\cdot)$ and his initial wealth (at A) be W_A. It is easy to show that $x = \frac{10\ W_A}{3}$ in this case.

Thus our investor with inside information is buying futures even though his long-run expectations are bearish $(E(r) = .4)$. His expectations are that the information that will be received between A and B will be better than what other traders expect. On the other hand, in the long run he expects the outcome to be worse than the market's expectations, and plans to change his position from long to short at B.

If there were many traders with these "speculative" expectations, then p might differ from a weighted average of expectations, and also differ from the price that would clear the market if trading were eliminated at time B. In certain circumstances then, a "recontracting market" such as the one at B, while making trading at A more attractive (since "futures" will be more liquid) might also encourage speculation in the sense defined above, and lead market price to diverge from the standard weighted average of opinion about r.

The simple example and the discussion above are no more than a reiteration (in terms of a specific model) of Keynes' Chapter 12: "The State of Long-Run Expectation."[5] It is a departure from the standard

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[5]Keynes [6].

sorts of models that assume identical expectations on the part of all investors and complete unanimity about the distribution of future returns. In some circumstances it may be a closer approximation to the workings of the real world.

An extension of Keynes' ideas may be obtained by reformulating the three-period model above, viewing time as a continuous, rather than discrete, parameter.

Gaussian Three-Period Continuous Time Parameter Model

Let the price of a "futures" contract at time t be $P(t)$, where

$P(t) = v(t) + e(t)$.

$v(t)$ = value at time t ; $E(P(T)) = v(t)$, where the expectation is taken at time t . T is the "delivery date" on the "futures" contract.

$e(t)$ = error caused by misinterpretation of information, divergence of opinion, speculation, etc. $e(t)$ is thus the difference between the present price and the investor's long-run expectation $E(P(T))$.

Assume that $v(t)$ is a Wiener process: $(v(t) - v(\tau))$ is distributed normally with mean zero and variance $\sigma^2(\tau - t)$.

$$E(v(t_1) - v(t_2))(v(t_3) - v(t_4)) = 0, t_1 > t_2 \geq t_3 > t_4 \quad .$$

Assume also that $e(t)$ is a Wiener process, independent of the first, with variance δ^2 (instead of σ^2) . On the delivery date, we know that $e(T) = 0$. This assumption essentially makes explicit how the investor feels the market price will converge to his expectations over time.

Without loss of generality, assume that

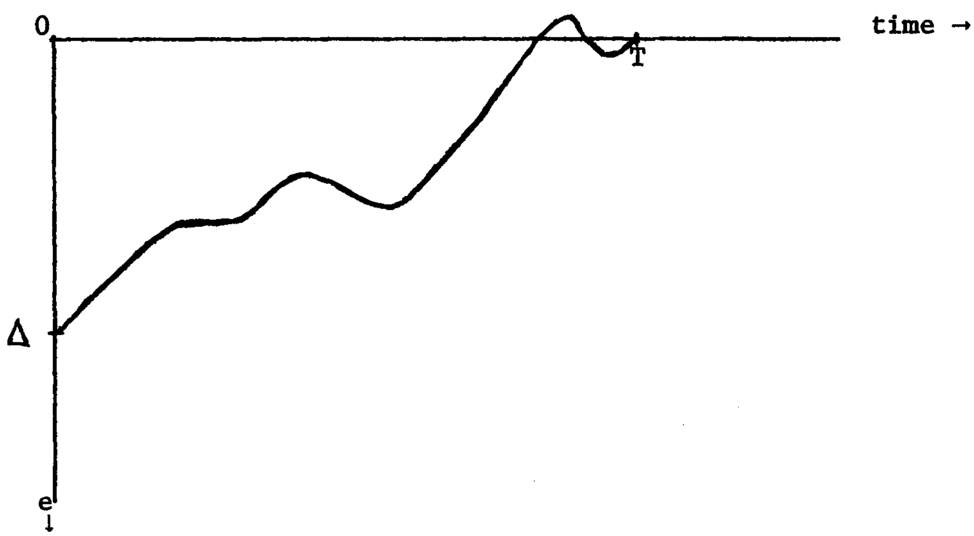
$$v(t) \equiv E(P(T)) = p(0) + \Delta , \quad \Delta > 0 \quad .$$

Thus the price is too low, according to the particular investor we are now studying.[6]

$e(t)$ becomes a Wiener process on which two observations are known. $e(0) = -\Delta$ and $e(T) = 0$. This situation is illustrated in Figure 1.

Figure 1

PATH OF $e(t)$ OVER TIME



It is straightforward to show that $e(t)$ for $0 \leq t \leq T$ is distributed normally with:

$$E\,(e(t)) = \frac{\Delta t}{T} - \Delta$$

$$Var\,(e(t)) = \frac{\delta^2 t(T - t)}{T} \quad .$$

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[6] As a link with the earlier model, time A is now time 0 , time B is now time t , and time C is now time T .

The assumption that $e(t)$ is a Wiener process makes the investor's inside information (or opinion divergence) drift toward zero as other investors more correctly interpret information.

Now assume that at fixed t, the investor in question must convert his portfolio to cash, and that he must maintain the same position (of x futures) all the time between zero and t. (These assumptions eliminate a host of stochastic programming problems.) We propose to look at dx/dT (with T held constant) to discover what happens to the investor's demand for "futures" as the "day of account" is moved into the future.

If we let $V = E(U(W_t))$ be the investor's utility function, then the normality of his portfolio allows us to write:

$$V = V\ (\text{mean, variance})^7 \quad .$$

If W_o is his initial (certain) wealth and W_t his wealth at t,

$$E\ (W_t) = W_A + \frac{x\Delta t}{T}$$

$$\mathrm{Var}\ (W_t) = x^2 \left[\sigma^2 t + \frac{\delta^2 t(T - t)}{T} \right] \quad .$$

If we assume that $U(W_t)$ is strictly concave, then the set

$$\{\mathrm{Var}\ (W_t) \mid V(E(W_t)\ ,\ \mathrm{Var}\ (W_t)) > C\}$$

is strictly convex. Since the constraint along which the investor may choose is the upper boundary of another strictly convex set, the existence of a unique equilibrium x is assured.

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[7]See Chipman [2].

Necessary conditions for the negativity of dx/dT are complicated and not particularly interesting. The mean of the investor's portfolio decreases with T, while the variance of his portfolio increases with T. Although it is possible for x to increase with T for some risk-averse investors, dx/dT will be negative if reasonable behavior (such as decreasing absolute risk aversion) is assumed. We may state the result above as follows.

Proposition 2. If an investor's demand for a normally-distributed asset decreases when the asset's mean decreases and variance increases, then in the Continuous Three-Period Model an increase in the "day of reckoning" T will decrease the investor's demand for futures contracts.

Note that the above phenomenon results from the assumptions about $v(t)$, the expected long-run value of a futures contract, and $e(t)$, the path over which the investor's expectations differences are dissipated. The interest rate and/or discounting behavior play no role whatsoever. If the reader is bothered by the simplifying assumption that the interest rate is zero, he may easily construct a model with discount factors. If present values are used, the results will be identical.

The conclusion that must be drawn from this result is clear: the farther away that the contract date is, the smaller will be the influence of expectations about "fundamental" or long-run considerations. If the market is made up of many investors similar to the one just studied (except for different Δ and W), then it is reasonable to say that if T is farther away, short-run "speculative" considerations will have more weight. In the case of commodity futures, there should be more speculation early in the life of a particular contract.

The inference for stock prices is also interesting. Usually corporations have no fixed maturity or "contract" date at which the assets will be auctioned off to the highest bidder. If no dividends are paid, the above model with $T \rightarrow \infty$ is the obvious analogy. When $T \rightarrow \infty$, the model becomes a pure Gaussian random walk; no speculator will buy the stock on the basis of an expected present value which is different from the current market price. Short-run speculation becomes the only determinant of price.

Usually corporations have dividends and earnings reports; these two things should determine share prices as well as pure speculation. If one makes the assumption that dividends are perceived as more concrete evidence of "value" than reported (but undistributed) earnings, then speculation should be more important for stocks whose dividend/earnings ratio is low. This observation coincides with casual empiricism, but could be checked against the data if one were willing to identify some measure of volatility with speculation.

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