LIFETIME UNCERTAINTY, HUMAN CAPITAL
AND PHYSICAL CAPITAL

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I. Introduction

The effect of lifetime uncertainty on investment in human capital has not been thoroughly investigated in the literature, which traditionally dealt with other forms of risk, and especially that associated with either market conditions or technology of producing human capital. The significance of risk involved in the production of human capital, in the context of education, was stressed by Becker, in [2, Ch. 5]. He suggests that the observed (positive) divergence between mean values of rates of return to investment in human capital and rates of return to investments in physical capital is due to excessive risk involved in producing the former. Nerlove, in [5], suggests that due to the absence of insurance against lack of ability to complete education the demand for education, from the society's standpoint, is deficient. In [7] I analyzed a model of investment in human capital under conditions of uncertainty regarding production of human capital. The analysis shows that investment in human capital will not necessarily decrease with increases in risk associated with the production of human cpaital. It seems, therefore, that there might be other major risk characteristics which distinguish investment in human capital from investments in physical capital. Of major importance is the risk associated with the individual's lifetime. Underlying this paper is the idea that since in general human capital, as opposed to physical capital, is not inheritable, the effect of lifetime uncertainty on investment in human capital is unique.[1/]

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[1/] There are, however, some kinds of investment in human capital, such as migration, which are inheritable.

Using a simple two-period model, the paper considers the desired rate of investment in human capital for a risk-averse individual whose lifetime is uncertain. Owing to the bequest motive, and since only physical capital is inheritable, there is certain "advantage" to investing in physical over human capital. As a result, the marginal rate of return to investment in human capital exceeds the marginal rate of return to physical capital (Section II).[2/] If there is available actuarially fair insurance against the loss of human capital in the event of premature death, the ratio of the marginal rates of return of physical and human capital is equal to the probability of surviving full lifetime. However, it is found that the introduction of insurance of this kind does not necessarily increase the desired rate of investment in human capital (Section III).

In the context of a commonly used and slightly more simplified model, it is shown that a necessary and sufficient condition for accepting fair insurance against the loss of human capital is that the individual plans to bequeath all his physical wealth and a positive portion of his labor earnings in the last period of his life (Section IV). Physical capital holdings are shown to have a positive effect on the desired rate of investment in human capital (Section V). Finally, Section VI discusses briefly some implications of the model for the real world.

II. A Two-Period Model

Consider an individual whose lifetime is uncertain. The first period of life is an investment period. In the second period, benefits to investment are obtained. A probability p is associated with two-period lifetime and a probability $1 - p$ is associated with lifetime of one period.[3/] In order not to detract attention from the main issue, the model assumes away non-pecuniary returns to human capital; also leisure is assumed exogenous. The individual is endowed with K and L units of physical and human capital

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[2/] Obviously, the existence of substantial consumption components in marginal benefits to investment in human capital, which is another unique feature of this kind of investment, will tend to make the marginal rate of return in physical capital higher than that of human capital.

[3/] The model can be straightforwardly generalized to a multiperiod model. The probability, $1 - p$, is then a conditional probability of dying at each age given survival to that age. Studies of the effect of lifetime uncertainty on consumption, using a multiperiod model, are found in Yaari [8] and Dar, Razin and Yahav [3].

respectively. The amount of human capital is measured by potential wage earnings. The individual may forego a part of L in order to invest his time in human-capital generating activities. Actual wage earnings are denoted by W ; the level of investment in human capital is denoted by $I(= L - W)$. Using superscripts 1 and 2 to denote first and second period respectively, the second-period amount of human capital, generated by a human capital production function $g(\ )$, is given by:

$$(1) \qquad L^2 = L^1 + g(I)$$

where the marginal rate of return to investment in human capital, g' , is assumed to be positive and diminishing (i.e., $g' > 0$, $g'' < 0$).

Denote the amounts of consumption in the first and second period by C^1 and C^2 respectively. Savings in the first period is $K^1 + W^1 - C^1$, the interest factor is R ; the second period amount of physical capital is given by (2).

$$(2) \qquad K^2 = R(K^1 + W^1 - C^1)$$

The individual can borrow and lend freely, at R , a given constant. It will be regarded as the marginal rate of return to investment in physical capital.

The individual's concern for the well-being of his children is indicated by the amount of bequest, B . In the event of full lifetime, the second period constraint is:

$$(3) \qquad B + C^2 = K^2 + L^2$$

In the event of inter-period mortality, since human capital is not inheritable, the constraint is:

$$(4) \qquad B^* = K^2$$

where B^* is the amount of bequest in the event of premature death.

Let utility be a concave function (reflecting risk aversion) of first- and second-period consumption and of bequest $U(C^1, C^2, B)$. Using the expected utility hypothesis, the maximand for the individual is [4/]:

$$(5) \qquad p\, U(C^1, C^2, B) + (1 - p)\, U(C^1, 0, B^*)$$

subject to (1) - (4).

III. The Desired Rate of Investment in Human Capital

Carrying out the optimization problem (5) we get necessary conditions for an (interior) equilibrium [5/]:

$$(6) \qquad p\, U_1(C^1, C^2, B) + (1 - p)\, U_1(C^1, 0, K^2) = R[p\, U_2(C^1, C^2, B) + (1 - p)\, U_B(C^1, 0, B^*)]$$

$$(7) \qquad R[p\, U_2(C^1, C^2, B) + (1 - p)\, U_B(C^1, 0, B^*)] = p\, g'(L^1 - W)\, U_2(C^1, C^2, B)$$

$$(8) \qquad U_B(C^1, C^2, B) = U_2(C^1, C^2, B)$$

Interpreting equation (6), on its left hand side is the expected value of the marginal utility of the first-period consumption; on its right hand side is the expected value of the additional utility obtained through alternatively using physical capital to increase next period consumption if the individual survives this period or to increase bequest in the event of premature death. Equation (7) is an optimum rule for investment in human capital.

[4/] In the analysis of Sections III and V, the utility function U( ) may be allowed to be piecewise continuous in the argument C^2 . Under this more general formulation the utility function can be made state dependent. Namely, $U(C^1, 0, B^*)$ need not equal to $\lim U(C^1, C^2, B^*)$ as $C^2 \to 0$. Therefore, the effect of death is not equivalent to the effect of zero consumption.

[5/] Subscripts denote partial derivatives; primes denote derivatives.

On the right hand side of (6) is the expected value of additional utility obtained from one additional unit of investment in human capital. (Note that if death occurs prematurely all benefits of the investment in human capital are lost for the individual and his children.) On the left hand side of equation (7) is the expected value of additional utility obtained when first-period wage earnings are increased by one unit. This increment to wages is augmented by the interest factor and the resulting sum can be used either for consumption or bequest depending on the state of the world. Finally, equation (8) implies that if the individual survives the second-period of his lifetime, he will allocate his resources between consumption and bequest as to equalize marginal utilities of these variables. Obviously, there is no investment in human capital at that period since it is the last period of life; thus, wage earnings equal the amount of human capital.

Let us now give an alternative economic interpretation of the optimum rule for investment in human capital. Rewriting equation (7), using (8), we get

$$(9) \qquad g' - R = \frac{(1 - p)\ R\ U_B(C^1,\ 0,\ B^*)}{p\ U_B(C^1,\ C^2,\ B)}$$

The difference between the rates of return to investment in human capital g' and physical capital R is <u>positive</u> as long as the probability of premature death, $1 - p$, is not zero. As an intuitive explanation, we may say that there exists certain "advantage" to holding assets in the form of physical capital rather than in the form of human capital as long as there is some risk of premature death since only physical capital can be left over to children. Therefore, in equilibrium, the rate of return to investment in human capital has to equalize the sum of the pecuniary rate of return to physical capital and the non-pecuniary rate of return, stemming from the physical capital inheritability.

Another implication of equation (9) is that the introduction of lifetime uncertainty will lower the rate of investment in human capital.[6/]

[6/] Note that this statement holds for fairly general class of utility functions. In particular, it is not necessary for this result to hold that the (intertemporal) utility be separable.

This is a result of the assumption that the marginal rate of return to investment decreases with the rate of investment.

IV. Life Insurance

If insurance is available against the loss of human capital, in the event of premature death, benefits of (insured) investments in human capital can be left over to children; thus, human capital becomes inheritable. In this section we consider the effect of insurance on the desired rate of investment in human capital[7/].

Denote the costs of insuring one dollar of human capital π . Insurance premium is paid in the first period. The net amount of human capital that the individual plans to bequeath is the difference between labor earnings and consumption in the second period, $L^2 - C^2$. Consider a full coverage of losses of this quantity. Expected utility, under insurance, is given by

$$(10) \qquad p\, U(C^1, C^2, K^2 + (L^2 - C^2)\,(1 - R\pi)) + (1 - p)\, U(C^1, 0, K^2 + (L^2 - C^2)(1 - R\pi))$$

differentiating (10) with respect to W and setting the derivative equal to zero we get:

$$(11) \qquad R = (1 - R\pi)\, g'$$

Equation (11) implies that the rate of investment in human capital has to be such so that the rate of return to physical capital is a fraction of the marginal rate of return on human capital; the fraction being equal to one minus the insurance premium.

If the insurance is actuarially fair then the expected value of cost (per unit of value insured) to the insurer, $1 - p$, is equal to the expected

[7/] For analyses of insurance in different economic contexts, see Arrow [1] and Ehrlich and Becker [4].

(future) value of receipts (per unit of value insured), i.e., the premium $R\pi$.

(12) $$R\pi = 1 - p$$

Substituting (12) into (11) we get the ratio between the rates of return under fair insurance

(13) $$R/g' = p$$

The ratio of the rates of return of physical and human capital is equal to the probability of surviving full lifetime. Putting it differently, under full-coverage and fair insurance the expected values of the marginal rates of returns in the two forms of capital are equalized.

When the individual is risk-neutral in bequest, the marginal utility U_B is constant. In this case (9) reduces to (13). Therefore, the desired rate of uninsured investment for an individual who is risk-neutral is identical with the rate of investment under fully covering and fair insurance.

One obvious implication of this analysis is that if probability of death is positively correlated with age, the older the individual the lower is the rate of his investment in human capital.

If the individual is averse to risk, the introduction of fair insurance against human capital losses does not necessarily increase the desired rate of investment in human capital. To see this, compare (13) with (9). It is easily verified that the rate of insured investment in human capital is greater, equal or smaller than the rate of uninsured investment according to whether the sign of (14) is positive, zero or negative, respectively.

(14) $$U_B(C^1, 0, K^2) - U_B(C^1, C^2, K^2 + L^2 - C^2)$$

However, the sign of the difference in (14) cannot a priori be determined.

V. Some Conditions for Insurance

In order to discuss conditions for insurance we need to put more stringent conditions on the utility function. We assume that the utility function can be expressed by the sum of two subutility functions, one pertaining to the individual consumption and the other pertaining to bequest, i.e., utility = $U(C^1, C^2) + v(B)$. Let α be the fraction of the loss in human capital covered by the insurance policy. Denote the maximized value of expected utility under insurance by J ; the quantity J is a function of α .

$$(15) \qquad J(\alpha) = \max_{C^1,\, C^2,\, B} E\{U(C^1, C^2) + v(B)\}$$

where E is the expectation operator.

Using (3) - (4), the last term on the right hand side of (15) is made explicit in (16).

$$(16) \qquad E\{v(B)\} = p\ v(K^2 + L^2\ (1 - \alpha R\pi) - C^2) + (1 - p)\ v(K^2 + \alpha L^2(1 - R\pi))$$

To get the desired fraction of insurance, α , we differentiate (15) with respect to α , using the maximizing expected utility properties: $\partial E/\partial i = 0$ $(i = C^1, C^2, B)$, and (16). This yields (17).

$$(17) \qquad \frac{dJ(\alpha)}{d\ \alpha} = -pL^2\ R\pi\ v'(K^2 + L^2(1 - \alpha R\pi) - C^2) \\ + (1 - p)\ L^2\ (1 - R\pi)\ v'\ (K^2 + \alpha L^2\ (1 - R\pi))$$

Under actuarially fair insurance (see (12)), for positive L^2 , we get

$$(18) \qquad \text{sign}\ [\frac{dJ(\alpha)}{d\ \alpha}] = \text{sign}\ [v'(K^2 + \alpha L^2(1 - R\pi)\ ) - v'(K^2 + L^2(1 - \alpha R\pi) - C^2]$$

If the individual is risk averse in bequest, then $v'' < 0$; we simplify (18) to get:

(19) $$\text{sign}\left[\frac{dJ(\alpha)}{d\alpha}\right] = \text{sign}\,[L^2(1-\alpha) - C^2], \qquad 0 \leq \alpha \leq 1 .$$

Equation (19) implies that the individual will reject, be indifferent to, or accept fair insurance according to whether labor earnings (in the second period) fall short, are equal, or exceed consumption (in the second period). Furthermore, if the individual accepts insurance, the optimal value of α is equal to

(20) $$\alpha = \frac{L^2 - C^2}{L^2}$$

Putting the result in (20) somewhat differently, if the individual accepts insurance, he will have a <u>full</u> coverage for the losses in $L^2 - C^2$.

To interpret these results we observe that life insurance is relevant (for the individual) only as far as the amount of human capital bequested $(L^2 - C^2)$ is concerned. If this quantity is positive (in the event of full lifetime), a risk averse individual will benefit from fully insuring it; this will make the quantity of this "good" equal in the two states of the world. If $L^2 - C^2$ is negative, insurance is not useful since it will increase the difference between the quantities of this "good" in the two states of the world.

Under more general conditions regarding the utility function, it is expected that in addition to the quantity $L^2 - C^2$ there will be some other parameters that will determine whether or not life insurance is accepted. One such factor may be the extent to which premature death affects the immediate needs of children.

VI. Physical Capital and Investment in Human Capital

The model can now be used to analyze the effect that the amount of physical capital in the individual's possession has on the rate of his investment in human capital.

Under complete certainty, and some additional conditions [8/], the rate of investment in human capital is entirely independent of the amount of physical capital. When risks are associated with labor earnings, or with the production of human capital, physical capital does affect investment in human capital. However, the direction this effect takes is not unique. Under some simplifying assumptions (when utility is intertemporally additive) it is determined by what kind of a function (decreasing or increasing) the Arrow-Pratt measure of absolute risk aversion is of wealth [9/]. In this section we show that lifetime uncertainty has different implications than risks involved in the production of human capital for the effect of physical capital on investment in human capital.

For simplicity assume that utility can be expresses as the sum of three subutilities, each pertaining to a different "good". Formally,

$$U_{ij} = 0 \quad \text{for } i, j = C^1, C^2, B, \ (i \neq j) \quad . \tag{21}$$

Differentiating totally (1) - (8) we get

$$\frac{dW}{dK^1} = \frac{U_{11}[R^2(1 - p)\ U_{BB}(B^*)(U_{22}(C^2) + U_{BB}(B)) + RpU_{BB}(B)(R - g')]}{\Delta} \tag{22}$$

where Δ is the determinant of the matrix of the second-order derivatives of equations (1) - (8) with respect to the endogeneous variables [10/]. By the second-order conditions for the maximization problem, Δ must be positive. Since the utility function is concave the second-order derivatives U_{ii} ($i = C^2$, B) must be negative. From (9), the difference $R - g'$ is negative. This implies that as long as p is not one, $dW/dK^1 < 0$. Putting this

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8/ Additional conditions are: perfect capital markets and exogeneity of variables such as leisure and age of retirement in the decision-making model.

9/ See [7].

10/ See Appendix.

differently, as long as there exists a positive probability of premature death, the larger is the amount of physical capital the higher is the rate of investment in human capital.

VII. Concluding Remarks

Lifetime uncertainty, considered in this paper, brings out sharply the distinction between investment in human and physical capital. Empirical analysis is needed, however, in order to establish the significance of this theory for the real world. Some of the implications of the model are now briefly stated.

One obvious implication of the analysis is that if the conditional probability of dying at each age (given survival to that age) is an increasing function of age [11/] then, the older is the individual, other things being equal, the lower is the rate of his investment in human capital. Another implication of the analysis is that the more able or the more educated is the individual (other things being equal), the greater is his demand for life insurance.

The model also implies that the wealthier is the individual, other things being equal, the higher is the rate of his investment in human capital.

[11/] See O'Hara [6] for a discussion of mortality in a different context.

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Appendix

In order to evaluate the sign of dW/dK^1, differentiate (1) - (8) totally to get:

$$A \cdot \begin{bmatrix} dC^1 \\ dC^2 \\ dW \\ dB \end{bmatrix} = D \cdot [dK^1] \quad , \text{ where,}$$

$$A = \begin{bmatrix} U_{11}(C^1) + R^2(1-p)U_{BB}(B^*) & 0 & -R^2(1-p)U_{BB}(B^*) & -RpU_{BB}(B) \\ R^2(1-p)U_{BB}(B^*) & 0 & \begin{array}{c} R^2(1-p)U_{BB}(B^*) \\ + pU_B(B)g'' \end{array} & pU_{BB}(B)(R-g') \\ 0 & -U_{22}(C^2) & 0 & U_{BB}(B) \\ -R & -1 & R - g' & -1 \end{bmatrix}$$

and

$$D = \begin{bmatrix} R^2(1-p)U_{BB}(B^*) \\ -R^2(1-p)U_{BB}(B^*) \\ 0 \\ -R \end{bmatrix}$$

Solving the above equations for dW/dK^1 we get equation (22) where,

$$\Delta = \det [A] \quad .$$