THE USE OF OPERATIONAL TIME TO CORRECT FOR SAMPLING INTERVAL MISSPECIFICATION

by

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A. The Problem

Many discrete time series are generated by the observation of processes which are most naturally considered to be continuously changing with time, or (almost equivalently) which have a fundamental time interval of evolution which is very much smaller than the sampling interval. The case in which the sampling interval is constant has been studied at some length,[1] but for some important applications, the sampling intervals are not evenly spaced, and this factor adds considerable complication to analysis of the data.

Consider a continuous random process $X(t)$ which is covariance stationary, that is: $E(X(t)\cdot X(t+s)) = R(s)$ is a function of s only. Further, assume that $X(t)$ is "ergodic"; namely

$$\frac{1}{n}\sum_{s=0}^{n-1} E[X(s+t)\cdot X(s)\cdot X(t)\cdot X(0)] \quad \text{in mean} \quad R^2(t), \; n \to \infty \quad . \tag{1}$$

Condition (1) assures that time averages converge to expectations when calculating sample autocovariances.[2] Finally, to help simplify the analysis and notation, assume $E[X(t)] = 0$. Then the discrete process ...; X(0), X(1), X(2), ... has mean zero and is stationary and "ergodic" in the sense defined. It can be analyzed by standard statistical methods, although a "simple" $X(t)$ in continuous time may give rise to a more "complicated" process in discrete time.[3]

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* Thanks are in order (for helpful comments) to Christopher Sims and Gary Chamberlain. Remaining errors are my own.

The discrete random process $\ldots X(\tau_0),\ X(\tau_1),\ X(\tau_2),\ \ldots$ with $\ldots \le \tau_0 \le \tau_1 \le \tau_2 \le \ldots$, and $(\tau_i - \tau_{i-1})$ is not necessarily equal to $(\tau_{i+1} - \tau_i)$ is the topic of this paper. Section B discusses the errors in estimating the correlogram or parameters of such a process, in the case where it is falsely assumed that $\ldots = (\tau_i - \tau_{i-1}) = (\tau_{i+1} - \tau_i) = \ldots$. Section C discusses the incorporation of information about the time intervals of observation for more efficient estimation and prediction. Section D applies the methods proposed in Section C to a sample problem: time series of cotton futures prices.

B. The Results of Naive Estimation

The standard autocovariance estimator

$$\tilde{R}_n(i) = \frac{1}{n} \sum_{j=1}^{n-i} X(\tau_j)\ X(\tau_{j+i}) \qquad (2)$$

may be calculated for the discrete process $\ldots,\ X(\tau_1),\ X(\tau_2),\ X(\tau_3),\ \ldots$ but if the intervals $(\tau_j - \tau_{j-1})$ are unequal, this estimator does not estimate $R(i)$ at discrete points, as it does in the case when the intervals are evenly spaced. The sum

$$\sum_{j=1}^{n-i} X(\tau_j)\ X(\tau_{j+i})$$

contains some pairs of observations that are very close together, while other pairs are further apart. $\tilde{R}_n(i)$, then, must represent some average of various autocovariances for different time intervals.

Theorem 1: Let $F_i(\Delta\tau)$ be the distribution of the independent observation intervals $\Delta\tau_i = \tau_{k+i} - \tau_k$. Let the Lesbesque-Stieltjes integral $\int R(z)dF_i(z)$ exist, and assume that for any ε ,

there exists z_ε such that $1 - F(z_\varepsilon) < \varepsilon$. Then the estimator

$$\tilde{R}_n(i) = \frac{1}{n} \sum_{j=1}^{n-i} X(\tau_j)\, X(\tau_{j+i})$$

converges to:

$$\tilde{R}_\infty(i) = \int R(z)\, dF_i(z) \quad . \tag{3}$$

Proof: If $\tau_{j+i} - \tau_j$ were a constant z , then

$$\tilde{R}_n(z) = \frac{1}{n} \sum_{j=1}^{n-i} X(\tau_j)\, X(\tau_j + z)$$

would converge to $R(z)$ as $n \to \infty$, by the ergodicity assumption (1). For any i , the $\Delta\tau_i$ may be divided up into n classes:

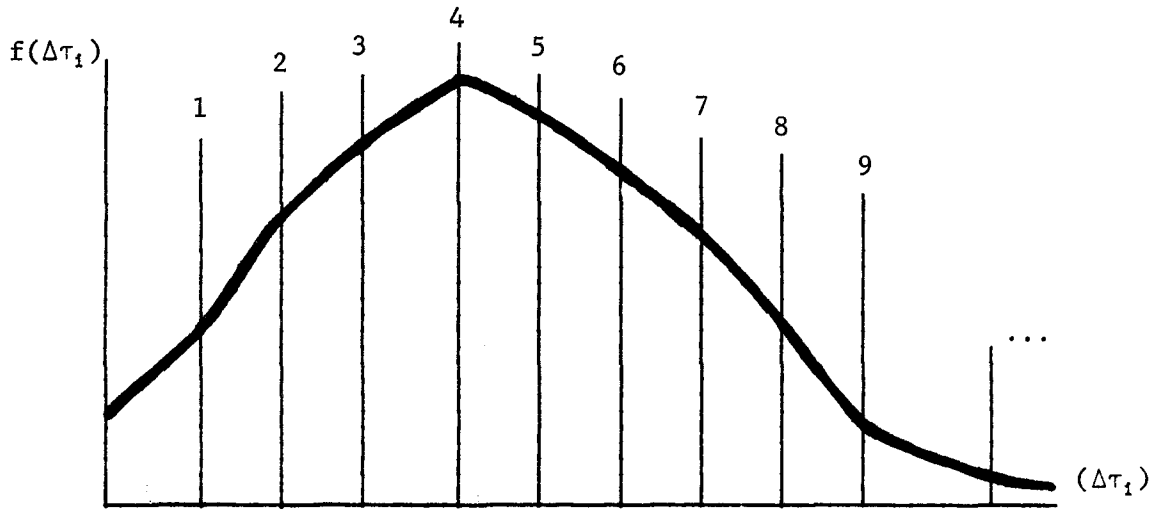

Figure 1

For any class k $(k = 1, 2, \ldots, r)$, let n_k be the number of observations in that class. Thus $n_1 + n_2 + \ldots + n_r = n$. Let

$$\frac{1}{n_k} \sum_{j_k=1}^{n_k^*} X(\tau_{j_k})\, X(\tau_{j_k+i}) = \tilde{R}_n^k(i)$$

where $n_k^* = n_k$ if none of the intervals in the class k are late enough so that no observation is i units later. Otherwise, reduce n_k^* appropriately. As n increases,

$$\min_{t\varepsilon\ (k^{th}\ \text{interval})} R(t) \leq \lim_{n\to\infty} \frac{1}{n_k} \sum_{j_k=1}^{n_k^*} X(\tau_{j_k})\, X(\tau_{j_k+i}) \leq \max_{t\varepsilon\ (k^{th}\ \text{interval})} R(t) .$$

For any fixed division of the time interval between observation i units apart,

$$\sum_{k=1}^{r} \frac{n_k}{n} \cdot \min_{t\varepsilon\ (k^{th}\ \text{interval})} R(t) \leq \lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n-i} X(\tau_j)\, X(\tau_{j+i}) \leq \sum_{k=1}^{r} \frac{n_k}{n} \max_{t\varepsilon\ (k^{th}\ \text{interval})} R(t) .$$

Now suppose that as $n \to \infty$, r is increased but moves slowly, say at rate $\sqrt{n}$, and in such a way that each interval gets shorter. (Do this by dividing only a large but fixed amount of the positive real $\Delta\tau_i$ line $\leq V$.) If the Lesbesque-Stieltjes integral $\int_0^V R(t)dF_i(t)$ exists, then both

$$\sum_{k=1}^{r} \frac{n_k}{n} \min_{t\varepsilon\ (k^{th}\ \text{interval})} R(t) \quad \text{and} \quad \sum_{k=1}^{r} \frac{n_k}{n} \max_{t\varepsilon\ (k^{th}\ \text{interval})} R(t)$$

converge to it as $n \to \infty$ and $r \sim \sqrt{n}$. Thus we have proved the theorem if $\Delta\tau_i$ always belongs to the interval $[0, V]$. If not, assume that $F(V_\varepsilon) < \varepsilon$ and reapply the above result to a limit when $\varepsilon \to 0$. Clearly, $\int R(t)dF_i(t)$ is this limit. Q.E.D.

The above theorem states how the true autocovariance function is averaged when the unequal interval spacing is ignored. Notice that if the parameters of $F_i(\Delta\tau)$ are known (or have been estimated), then (3) gives a formula for estimating $R(t)$. However, as Quenouille [12] points out, this estimator will be inefficient if the time intervals between observations in the sample are known.

The sample periodogram obtained from naive estimation must be in some

sense an "averaged" version of the spectral density of the underlying continuous-time process, but here the "averaging" is not straightforward as it is for the autocovariance function. The interested reader is referred to Beutler and Leneman [1] for an analysis of the case when the spacing of the observations forms a stationary point process. The spectrum for a process sampled at Poisson intervals is a weighted average of the spectrum of the true process and a constant. Thus in this case the relative size of peaks in the spectral density is reduced.

C. Improved Estimates Incorporating Operational Time

Unless $R(t)$ can be presented using a small number of parameters, even the large-sample properties of autocovariance estimators may be hard to obtain. Suppose for instance, that $F_1(\Delta\tau)$ is absolutely continuous on the positive part of the real line. Then almost surely, even for a large sample, there will be as many points at which to estimate $R(t)$ as there are observations. Unless bounds are placed on $R'(t)$, or the space of admissible $R(t)$ is somehow restricted, it will be impossible to obtain good estimates.

In many applications, estimates may be best judged on the basis of prediction error. Predictions which are marginal on operational time must of necessity use averaged autocovariances, and the standard theory for equally spaced intervals is applied. For a prediction conditional on the time interval between the sample and the predicted observation, the true autocovariance function is better than the averaged one.

Example: Let $X(t+1) = \rho X(t) + u(t)$ where $|\rho| \leq 1$ and $E(u(t)) = 0$

for all t, and $E(u(t)u(s)) = \begin{cases} \sigma^2 & t = s \\ 0 & t \neq s \end{cases}$. The number of steps in this discrete process between observations follows a Poisson process with parameter λ; that is, the probability of k steps between observations is $\frac{e^{-\lambda}\lambda^k}{k!}$. Thus the observed series is $X(\tau_1)$, $X(\tau_2)$, $X(\tau_3)$ where $\Delta\tau_j = \tau_j - \tau_j$ is an integer with Poisson distribution. Theorem 1 tells us that if this process is assumed to have equal intervals, that is, $X(\tau_1)$, $X(\tau_2)$, ... = y(1), y(2) ... will have also been a first-order Markov process with parameter $r = \sum_{j=0}^{\infty} \frac{e^{-\lambda}\lambda^j}{j!} \rho^j = e^{-\lambda(1-\rho)}$. If observations on $\Delta\tau$ are unobtainable, predictions using this parameter r (or the usual estimate of it) are the best to be obtained. If λ is known, but the spacing between observations is not observed, a maximum likelihood estimate of ρ may be easily obtained. If the intervals in the particular sample are known, then the maximum likelihood estimator for ρ may be obtained by finding $\hat{\rho}$, $\hat{\sigma}^2$ which simultaneously minimize:

$$n \log \hat{\sigma}^2 + \sum_{j=1}^{T-1} \log \left(\sum_{k=1}^{\Delta\tau_j} \hat{\rho}^{2(k-1)} \right) + \sum_{j=1}^{T-1} \frac{(\hat{\rho}^{\Delta\tau_j} y(j) - y(j+1))^2}{\hat{\sigma}^2 \sum_{k=1}^{\Delta\tau_j} \hat{\rho}^{2(k-1)}} .$$

Although the normal equation for $\hat{\rho}$ can be obtained, estimation of $\hat{\rho}$ is best carried out by direct numerical minimization of the above sum of squares. Further, if normality of the error terms is assumed, an α percent confidence interval around $\hat{\rho}$ may be constructed. $\hat{\rho}$, of course, is the necessary estimate for conditional predictions of $y(T+1)$ given $y(T)$ and $\Delta\tau_T$.

One would like to show that the mean square error of prediction from the sample $E(z-y(T+1))^2$ is lower when z is the conditional prediction

$\hat{\rho}^{\Delta\tau_T}\ y(T)$ than when it is the marginal predictor $\hat{r}\ y(T)$. Notice that for any predictor z which is independent of the error terms between T and $T+1$:

$$E(z-y(T+1))^2 = E\{z-\rho^{\Delta\tau_T}\ y(T)\}^2 + E\{v(T)\}^2$$

where $v(T) = \sum_{t=\tau_T}^{\tau_{T+1}-1} \rho^{\tau_{T+1}-t-1}\ u(t)$.

Since the variance of the "innovations" of the process [the second term on the right] is the same for all predictors, only the first term on the right need be compared.

$$E(\hat{\rho}^{\Delta\tau_T}\ y(T) - \rho^{\Delta\tau_T}\ y(T))^2 = y(T)^2\ E(\hat{\rho}^{\Delta\tau_T} - \rho^{\Delta\tau_T})^2$$

and

$$E(\hat{r}y(T) - \rho^{\Delta\tau_T}\ y(T))^2 = y(T)^2\ E(\hat{r} - r)^2 + y(T)^2\ E(r - \rho^{\Delta\tau_T})^2 .$$

Evidently, one must compare $E(\hat{\rho}^{\Delta\tau_T} - \rho^{\Delta\tau_T})^2$ with $E(\hat{r} - r)^2 + E(r - \rho^{\Delta\tau_T})^2$. A large enough sample will insure that the mean square error of the conditional prediction is lower than that of the marginal prediction, if $0 < \rho < 1$. In small samples, however, no analytical comparison has been achieved, due to the nonlinearity of the $\hat{\rho}$ estimator. Intuition suggests that even for small samples of size 50, $E(\hat{\rho}^{\Delta\tau_T} - \rho^{\Delta\tau_T})^2$ and $E(\hat{r} - r)^2$ are of very similar size, thus giving the conditional predictor lower mean square error.[4]

In order to compare the small-sample properties of predictors based on $\hat{\rho}$ and $\hat{r}$, a sampling experiment was carried out. Time series of length 50 were generated by the process $X(t+1) = \rho X(t) + u(t)$, where $u(t)$ were independent normal errors of mean 0 and variance 1. The observed series $y(0)$, $y(1)$, ..., $y(50)$ were generated by observing the $X(t)$ process at Poisson intervals; $y(j) = X(\tau_j)$. $\Delta\tau_j \sim$ Poisson (λ). The Poisson process

and the error process were sampled independently, and $X(0) = y(0)$ was always taken as zero.

Table 1 presents the results of these sampling experiments; for each (ρ, λ) parameter point, 20 series of size 50 were generated, and then estimated with and without the use of operational time.

Table 1

Sampling Results for the Poisson-Normal Process[5]

Parameter Set	$\hat{E}(\hat{\rho} - \rho)^2$	$\hat{E}(\hat{\rho} - \rho)^{2\Delta\tau_T}$	$\hat{E}(\hat{r} - r)^2$	$\hat{E}(\hat{r} - \rho^{\Delta\tau_T})^2$
λ=1, ρ=0.0	.087	.033	.024	.256
λ=1, ρ=0.5	.014	.005	.010	.114
λ=1, ρ=0.9	.006	.002	.015	.023
λ=1, ρ=1.0	.002	.001	.002	.002
λ=5, ρ=0.0	.541	.094	.038	.044
λ=5, ρ=0.2	.469	.064	.019	.027
λ=10, ρ=0.0	.715	.058	.016	.016
λ=10, ρ=0.5	.128	.000	.030	.030
λ=10, ρ=0.9	.002	.000	.020	.034
λ=10, ρ=1.0	.000	.000	.002	.002

There are a number of conclusions to be drawn from this experiment:

1. Comparison of the relevant results for prediction error (the columns under $\hat{E}(\hat{\rho} - \rho)^{2\Delta\tau_T}$ and $\hat{E}(\hat{r} - \rho^{\Delta\tau_T})^2$) indicates that $E(\hat{\rho} - \rho)^{2\Delta\tau_T}$ and $E(\hat{r} - r)^2$ are of similar size, and the unavoidable error in

using a predictor which is marginal on operational time makes the nonlinear estimation worthwhile when ρ is larger than zero.

2. For $\rho = 1.0$, the prediction error using operational time is smaller even though the unavoidable error $E(r - \rho^{\Delta\tau_T})^2$ is zero. This result is not intuitive, and might be amenable to analytic proof.

3. For small λ, where the probability of $\Delta\tau = 0$ (which is $e^{-\lambda}$) is not small, the prediction conditional on operational time has an obvious advantage: if $\Delta\tau_j = 0$, then $z = y(j)$ with prediction error zero, regardless of ρ.

4. As ρ gets close to zero, a larger and larger sample is necessary to make the conditional prediction better. For $0 \leq \rho \leq 0.2$, say, 50 is not a large sample, and the conventional methods are better for prediction, as long as λ is large enough so that considerations in (3) above do not take precedence.

Although it might be true in the case where $\Delta\tau$ is continuous and Prob $(\Delta\tau = 0) = 0$, and $\rho = 0$, that the prediction error for ordinary methods might be better for any size sample, this case is not very useful. If you know $\rho = 0$, you predict $y(j) = 0$ always. In other cases of known ρ (or more generally, known $R(t)$), very precise results for interpolation and extrapolation of time series samples at irregular intervals are available See [1], [10], and [11].

D. Application to Cotton Futures Prices

In an earlier article [3] I showed that cotton futures price series may be characterized as realizations of a subordinated stochastic process, or equivalently, that the observed price changes, while equally spaced in standard time measure were in fact randomly and unequally spaced in the relevant time dimension for price evolution. Cotton prices were taken as observations from a continuous stochastic process, with trading volume as an imperfect measure of the time intervals between observations.[6]

It is hypothesized that observed prices $X(\tau_0)$, $X(\tau_1)$, ..., $X(\tau_T)$ follow a process described by the stochastic differential equation:

$$\frac{dX(t)}{dt} + aX(t) = u(t) \tag{4}$$

where $u(t)$ is a stationary white-noise input with mean zero and variance

$$\int u(t)^2 \, dF_u(t) = \sigma^2 \quad .$$

A test of the "random-walk hypothesis" is a test of $a = 0$. Solving (4) for $X(t)$, we find

$$X(t) = e^{-at} \int_0^t u(s) \, e^{as} ds$$

or

$$X(\tau_{j+1}) = e^{-a\Delta\tau_j} \, X(\tau_j) + e^{-a\tau_{j+1}} \int_{\tau_j}^{\tau_{j+1}} u(s) \, e^{as} ds \quad .$$

If we now define $\rho = e^{-a}$ and assume a is close to zero or $\Delta\tau$ large, so that

$$v(j) = e^{-a\tau_{j+1}} \int_{\tau_j}^{\tau_{j+1}} u(s) \, e^{as} \, ds \approx \int_{\tau_j}^{\tau_{j+1}} u(s) \, ds \quad ,$$

and therefore independent of any other errors, we obtain:

$$X(\tau_{j+1}) = \rho^{\Delta\tau_j} X(\tau_j) + v(j) \qquad (5)$$

where ρ in (5) can be estimated using the observations on operational time as well as price data. The results of the nonlinear estimation of ρ are given below in Table 2.

Table 2

Estimation of the First-Order Autocorrelation Coefficient of Cotton Futures Prices Incorporating Operational Time[7, 8]

	Sample 1 (1/17/47 - 8/31/50)	Sample 2 (3/26/51 - 2/10/55)
$\hat{\rho}$	1.000	1.000
99 percent Confidence Interval	.999 - 1.001	.999 - 1.001

Table 3 presents the results of estimating

$$y(j + 1) = ry(j) + w(j) \quad ,$$

that is, when the effect of operational time is ignored, and the spaces between observations are assumed equal.

Table 3

Least Squares Estimates of the First-Order Autocorrelation Coefficient of Cotton Futures Prices

	Sample 1	Sample 2
$\hat{r}$	1.002	1.000
99 percent Confidence Interval	.996 - 1.008	.998 - 1.002

These estimates are less precise than those in Table 1. Furthermore, since the regression does not adjust for the heteroscedasticity in the error terms, the calculated confidence regions are underestimates of the true confidence regions.

Another indication of the results of adjusting for operational time may be seen by testing the series of price differences against white noise. Table 4 presents the results of testing the periodogram of unadjusted and operational-time adjusted price differences against white noise.

Table 4

Kolmogorov-Smirnov Tests of a White Noise Null Hypothesis for Cotton Futures Price Differences[9]

	Unadjusted	Adjusted
Sample 1	K - S = .047	K - S = .016
Sample 2	K - S = .028	K - S = .021

5 percent Critical Region: KS ≥ .061

Note that the larger K - S statistic for the unadjusted data biases an investigator towards rejecting the hypothesis of uncorrelated errors. Although the null hypothesis is accepted in all cases, the K - S statistics from the unadjusted data are higher, reflecting the fact that these data are not normally distributed, and thus, the periodogram ordinates are not independent.

FOOTNOTES

1. See Durbin [4], [5], [6], and Telser [13].

2. See Hannan [9] for a proof of this statement.

3. For instance, the process $\left\{\frac{d^2X(t)}{dt^2} + a\,\frac{dX(t)}{dt} + b\,X(t) = u(t) \quad \text{where } u(t) \text{ is continuous white noise}\right\}$ has a very complicated expression in a and b to estimate when observations are spaced unevenly.

4. The above discussion should serve to convince the reader about the earlier statement that prediction error of conditional estimators must be approached on a case-by-case basis since the general problem is too complicated.

5. The $\hat{E}$ notation means an estimated expectation using the distribution obtained from sampling results.

6. As explained in [3], trading volume itself is an imperfect measure of operational time. Thus $\Delta\tau_j = v_j^{2.18}$ where v_j is the trading volume between price observations, $y(j)$ and $y(j+1)$.

7. Each cotton future price series is an index of daily cotton futures prices of length 1000. The index is constructed to avoid the problem arising because each contract is in existence for only a year and a half. The index is constructed so that the price is the price of cotton some given length of time in the future, thus avoiding appreciation over time. See Clark [2], [3] for further details.

8. These estimates are not invariant with changes in time units. $\Delta\tau_j$ is $v_j^{2.18}/10^7$, where v_j is trading volume on day j. Observed prices are closing prices. Mean v_j is approximately 3000, so that $\overline{\Delta\tau_j}$ varies around 1.

9. See Durbin [7], [8] for a discussion of the use of Kolmogorov-Smirnov-like statistics in testing the periodogram ordinates.

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