ON INVESTMENT IN HUMAN CAPITAL UNDER UNCERTAINTY

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Discussion Paper No. 19, July 1972

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I. Introduction

Rates of return to education even when current consumption components are excluded were observed by a number of economists to be higher than rates of return to investments in physical capital. This was partly explained by the fact that investment in human capital is more risky as compared with these physical capital investments.[1] Furthermore, insurance against acquisition of unwanted skills or lack of ability to complete education is almost absent and this is thought to lead to a deficiency in the demand for education from the society's standpoint.[2] This stresses the significance of the considerable risk involved in investment in education; yet, the theoretical analysis was solely confined to the deterministic case.[3] There does not exist a theoretical study of the effect of uncertainty in the process of education on investment in human capital.

The purpose of this paper is to partially close this gap in the literature by providing a theoretical analysis of investment in human capital under conditions of uncertainty with respect to the production of human capital. This study may also be regarded as an extension of the analysis of savings under uncertainty.[4]

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1. See Becker [1], chapter 5.

2. See Nerlove [4], pp. 185-188.

3. See Ben-Porath [2] and extension of his approach to economic growth in Razin [6].

4. See Phelps [5], Levhari and Srinivasan [3] and Samuelson [8]. We remark that in these papers labor income is either excluded from the analysis or assumed to be known with complete certainty.

In Section II we consider investment in human capital by a risk-neutral individual. Risk-neutrality together with perfect capital markets imply that the investment decision can be separated from the consumption decision, thus, the rate of investment will be determined by maximizing expected discounted sum of labor income stream. It is shown that the rate of investment in human capital will be affected by uncertainty in the educational production function in many possible ways depending upon which of the parameters of the production function is subject to risk. In particular, when riskiness of the rate of return to human capital is increased the rate of investment in human capital will not change.

Section III extends the analysis of investment in human capital under uncertainty to the case of risk-aversion (in which the consumption and the investment decisions are inseparable). In this case also, not all forms of risk in the production of human capital are deterrental to investment in human capital. In the special case where riskiness of the rate of return to human capital is increased, the demand for investment will decline and the expected rate of return to human capital will exceed the riskless rate of return to financial capital.

Section IV ends the paper with concluding remarks.

II. Risk-neutrality

Consider an individual at a period t in possession of earnings potential (human capital) L_t. Denote net wage (for consumption or savings purposes) by w_t and assume that by investing time and purchased inputs in the nominal amount $L_t - w_t$ $(= I_t)$ the next period human capital is increased according to the stochastic human-capital-production-function $g(I;\ \theta)$

$$L_{t+1} = g(I_t;\ \theta_t) \quad , \quad t = 0,\ 1,\ \ldots,\ N \tag{1}$$

L_o given, where θ_t is an independent random variable and N is the individual's horizon $(N \geq 1)$.[5] The production function $g(I;\ \theta)$ [6] exhibits positive and diminishing rates of return $g_I = \frac{\partial g()}{\partial I}$ (i.e., $g_I > 0$, $g_{II} < 0$) .

We first assume a risk-neutral individual operating in competitive capital markets facing a sequence of rates of interest r_t ($R_t = 1 + r_t$, $t = 0, 1, \ldots, N$) assumed to be <u>known</u> with complete certainty. Using the expected-utility-hypothesis, and since net wages are random variables, the individual will maximize the expected discounted sum of the stream of net wages with horizon N, V^N .[7]

$$V^N(L_o) = \max_{w_t} \mathop{E}_{\theta} \left[\sum_{t=0}^{N} \left(\frac{1}{R_t}\right)^t w_t \right] \tag{2}$$

subject to the stochastic constraint in (1) (where E is the expectation operator).

This is a problem in dynamic programming.[8] We can rewrite (2) to get

$$V^N(L_o) = \max_{w} \ \{w + \frac{1}{R_1} \mathop{E}_{\theta} \ [V^{N-1}(g(L_o - w;\ \theta))]\} \ . \tag{3}$$

A necessary condition for a regular interior maximum is obtained by setting the derivative of (3) with respect to w equal zero:

$$\frac{\partial V^N(L_o)}{\partial w} = 1 - \frac{1}{R_1} \mathop{E}_{\theta} \ [g_I(L_o - w_o;\ \theta) \ \frac{\partial V^{N-1}}{\partial L} \ (g(L_o - w_o;\ \theta))] = 0 \tag{4}$$

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5. The length of the horizon is at least two periods.

6. The first model with a production function of human capital is due to Ben-Porath [2]. To facilitate the exposition, subscript t will be omitted from the random variable θ .

7. Under the conditions stated above, namely, risk-neutrality and perfectly competitive capital markets, the maximization of expected discounted sum of consumptions will necessarily lead to (2). See Equation (17).

8. See Levhari and Srinivasan [3] and Samuelson [8] for a treatment of problems using the dynamic programming approach.

On the other hand, differentiating (2) with respect to L yields

$$\frac{\partial V^N(L_o)}{\partial L} = (1-\frac{1}{R_1}\underset{\theta}{E}[g_I(L_o-w_o;\theta)\frac{\partial V^{N-1}}{\partial L}(g(L_o-w_o;\theta))])\frac{\partial w}{\partial L}+\frac{1}{R_1}\underset{\theta}{E}[g_I(L_o-w_o;\theta)\frac{\partial V^{N-1}}{\partial L}(g(L_o-w_o;\theta))]$$

$$= 1 \tag{5}$$

by using (4). Substituting (5) into (4) and rearranging we get

$$\underset{\theta}{E}\,[g_I(L_o - w_o;\ \theta)] = R_1 \ .\text{[9]} \tag{6}$$

Equation (6) is interpreted as the equality of expected rates of return. When w is chosen optimally the expected marginal rate of return from investment in human capital $Eg_I()$, should be equal to unity plus the known interest rate.

Consider now the effect of increasing riskiness of the production of human capital on the investment in human capital. "Increasing the riskiness" of a random variable is done by increasing the probability weight in the tails of its probability distribution while preserving its mean.[10] We can straightforwardly state that, as riskiness of θ is increased, investment in human capital I will be higher, equal or lower according to whether the rate of return to human capital $g_I(I;\ \theta)$ is a convex, linear or concave function of the random variable θ.[11]

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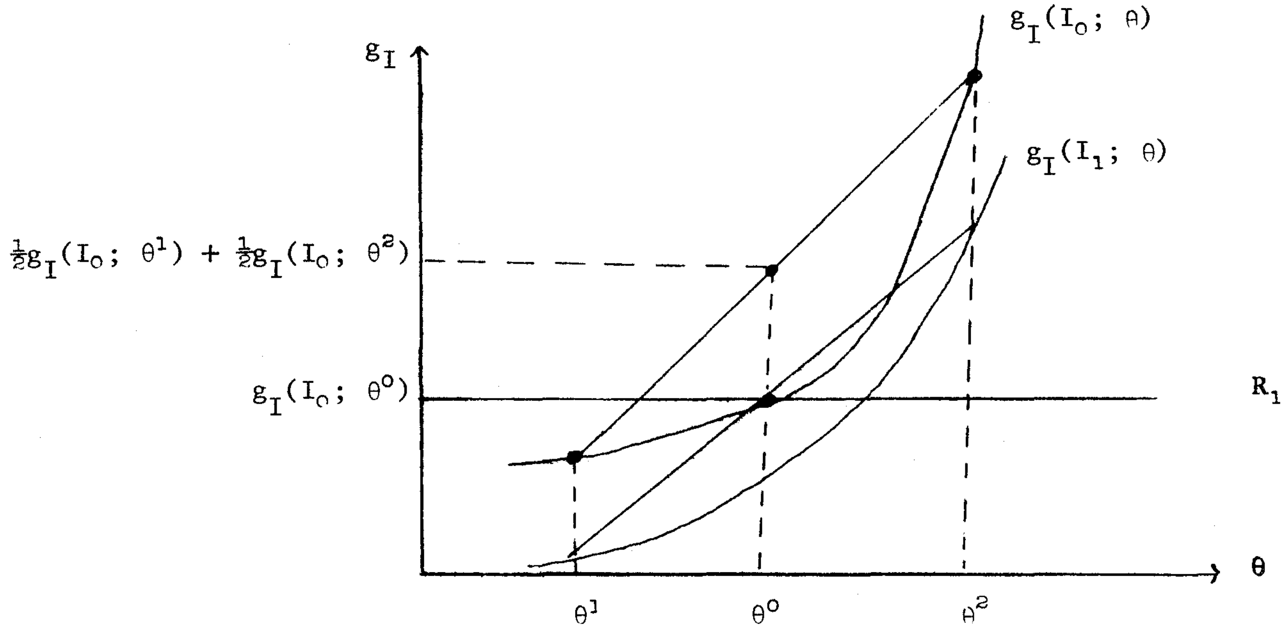
9. Sufficient conditions for a maximum are given by (1), (6) and by $\underset{\theta}{E}\ g_{II}(L_o-w_o;\ \theta) < 0$ (which is fulfilled since $g_{II} < 0$). Therefore, from (6), a higher rate of interest is associated with a smaller amount of investment in human capital.

10. See Rothschild and Stiglitz [7]. They show that if F is a function of a random variable θ then increasing the risk of this random variable will increase, leave constant or decrease $\underset{\theta}{E}(F)$ according to whether F is convex, linear or concave in the random variable θ, respectively.

11. Consider the human-capital-production-function (assumed in Ben-Porath [2]) $g(I;\ \theta^1,\ \theta^2) = \theta^1\ I^{\theta^2}$ (where $P_r(0 < \theta^2 < 1) = 1$). Increasing the risk of θ^1 will leave investment unchanged whereas increasing the risk of θ^2 will decrease investment.

This result follows from the equality (6). By increasing the risk of the random variable θ the left-hand side of (6) will increase, remain constant or decrease as $g_I(I;\theta)$ is convex, linear or concave in θ ; to restore equality I must increase, remain constant or decrease, respectively.

To demonstrate this result, we draw in Figure 1 the rate of return to human capital g_I as a function of the random variable θ under the assumption that g_I is a convex function of θ . Consider alternatively a known value θ^0 with the corresponding optimal investment I_0 and a two-values probability distribution with values θ^1 and θ^2 each with probability one-half such that $\frac{1}{2}\theta^1 + \frac{1}{2}\theta^2 = \theta^0$. It is clear that for I_0 the expected value of the rate of return is greater in the risky case and therefore, in order to restore optimality investment must increase to, say, I_1 .



g_I
$g_I(I_0;\ \theta)$
$g_I(I_1;\ \theta)$
$\frac{1}{2}g_I(I_0;\ \theta^1) + \frac{1}{2}g_I(I_0;\ \theta^2)$
$g_I(I_0;\ \theta^0)$
R_1
θ
θ^1
θ^0
θ^2

Figure 1

III. Risk-aversion

In the case of risk-aversion consumption and investment decisions must be made simultaneously. We therefore reformulate the model of Section II. Using the expected utility hypothesis, the consumer will seek to maximize expected discounted sum of utilities:

$$\max_{\{w_t, c_t\}} \; \underset{\theta}{E}\left[\sum_{t=0}^{N} \beta^t \, u(c_t)\right] \quad \text{subject to the stochastic constraints} \tag{7}$$

$$K_{t+1} = R_t(K_t + w_t - c_t) \quad , \; K_o \text{ given} \tag{8}$$

$$L_{t+1} = g\,(L_t - w_t;\, \theta) \quad , \; L_o \text{ given} \tag{9}$$

where at time t we have,

β : discount factor

c_t: consumption

R_t: unity plus the (known) rate of interest

K_t: financial asset

and $u(\;)$ is a utility function with $u_c = \frac{\partial u}{\partial c}(\;) > 0$, $u_{cc} = \frac{\partial^2 u}{\partial c^2} < 0$.

In (8) we assume that wage payments w_t are received at the beginning of the period t .[12]

Let W^N stand for the maximized value of the problem (7) - (9) with horizon N , then

$$W^N(K_o,\, L_o) = \max_{c,w} \{u(c) + \beta \underset{\theta}{E}\, [W^{N-1}(R_1(K_o + w - c),\; g(L_o - w;\, \theta))]\} \tag{10}$$

Following the steps of the analysis in Section II we derive necessary conditions for an interior regular maximum as follows.

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12. Clearly, changing this assumption will not qualitatively alter any results of this section.

$$\frac{\partial W^N(K_o,\ L_o)}{\partial c} = u_c(c_o) - \beta\ R_1 \underset{\theta}{E}\left[\frac{\partial W^{N-1}}{\partial K}(K_1,\ L_1)\right] = 0 \tag{11}$$

$$\frac{\partial W^N(K_o,\ L_o)}{\partial w} = \beta \underset{\theta}{E}\left[R_1\frac{\partial W^{N-1}}{\partial K}(K_1,\ L_1) - \frac{\partial W^{N-1}}{\partial L}(K_1,\ L_1)\ g_I\ (L_o - w_o;\ \theta)\right] = 0 \tag{12}$$

where $K_1 = R_1(K_o + w_o - c_o)$, $L_1 = g\ (L_o - w_o;\ \theta)$. (Under the assumptions made regarding the concavity of $g\ (\ )$ and $u\ (\ )$ (8) - (9) and (11) - (12) constitute a set of sufficient conditions for a maximum.) On the other hand, differentiating (10) with respect to K and L, using (11) - (12), we get:

$$\begin{aligned}\frac{\partial W^N}{\partial K}(K_o,\ L_o) &= \beta\ R_1 \underset{\theta}{E}\left[\frac{\partial W^{N-1}}{\partial K}(K_1,\ L_1)\right] \\ &= u_c\ (c_o)\end{aligned} \tag{13}$$

$$\frac{\partial W^N}{\partial L}(K_o,\ L_o) = \beta \underset{\theta}{E}\left[\frac{\partial W^{N-1}}{\partial L}(K_1,\ L_1)\ g_I(L_o - w_o;\ \theta)\right] \tag{14}$$

Denoting the consumption with horizon N by $c^N = c^N(K,\ L)$ and substituting (13) into (11) we get

$$u_c(c^N(K_o,\ L_o)) = \beta\ R_1 \underset{\theta}{E}\ [u_c(c^{N-1}(K_1,\ L_1))] \tag{15}$$

Equation (15) restates the well known equality between the discounted marginal expected utility when an extra unit of financial capital is used to increase next period consumption and the marginal utility resulting from an extra unit of current consumption.

Similarly, substituting (12) and (13) into (14) we get

$$\frac{\partial W^N}{\partial L}(K_o,\ L_o) = u_c(c^N(K_o,\ L_o)) \tag{16}$$

Substituting (16) back into (12) yields

$$R_1 \underset{\theta}{E} [u_c \ (c^{N-1}(K_1,\ L_1))] = \underset{\theta}{E} [u_c(c^{N-1}(K_1,\ L_1)\ g_I\ (L_o - w_o;\ \theta)] \tag{17}$$

Equation (17) is interpreted as an equality of the expected additional utility resulting from using an extra unit of human capital for investment in human capital, and using its returns to increase next period consumption (the right-hand side of (17)) with the expected additional utility resulting from lending out an extra unit of capital and using the returns to increase next period consumption (the left-hand side of (17)). Note that when the consumer is risk-neutral (i.e., $u_c\ (c) = 1$) then (17) reduces to (6).

It can be shown[13] that the maximized value of problem (7) - (9) $W(K,L)$ is a monotone increasing and concave function in its arguments K and L. This implies[14] that

$$\frac{\partial W^N(k,L)}{\partial K} > 0 \ ; \ \frac{\partial W^N(K,L)}{\partial L} > 0 \ ; \ \frac{\partial^2 W^N(K,L)}{\partial K^2} < 0 \ ; \ \frac{\partial^2 W^N(K,L)}{\partial L^2} < 0 \ , \tag{18}$$

for all $N > 1$.

Define <u>wealth</u> by $Y = K + L$. We can now state that <u>consumption is an increasing function of wealth (i.e., $c = c\ (Y),\ \frac{\partial c}{\partial Y} > 0$).</u>

To prove the assertion we differentiate (13) with respect to K to get

$$\frac{\partial^2 W^N(K,L)}{\partial K^2} = u_{cc}\ (c^N)\frac{\partial c^N}{\partial K} \tag{19}$$

which implies that $\frac{\partial c^N}{\partial K} > 0$.

From (13) and (16) we have

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13. The proof is similar to the one given in Levhari and Srinivasan [3], page 158.

14. We assume that $W\ (K,\ L)$ is twice continuously differentiable.

$$\frac{\partial^2 W^N(K,L)}{\partial K \partial L} = u_{cc}(c^N)\frac{\partial c^N}{\partial L} \tag{20}$$

$$\frac{\partial^2 W^N(K,L)}{\partial L \partial K} = u_{cc}(c^N)\frac{\partial c^N}{\partial K} \tag{21}$$

Equating the right-hand sides of (20) and (21) we get

$$\frac{\partial c^N}{\partial K} = \frac{\partial c^N}{\partial L} \tag{22}$$

for all pairs (K, L) and for all $N > 1$.

Equation (22) implies that $c(K, L) = c(K + L) = c(Y)$ which together with (19) proves the assertion.

We are now in a position to investigate the effect of introducing uncertainty in the rate of return to human capital on the investment in human capital.

Suppose we start with a complete certainty with respect to the production function g , then (17) reduces to

$$g_I(I) = R_1 \tag{23}$$

Uncertainty is now introduced by adding a random variable with a zero mean to the rate of return[15]

$$g_I(I;\ \theta) = G_I(I) + \theta\ ,\ \theta \text{ is an independent random variable with}$$

$$E\ \theta = 0 \tag{24}$$

where G_I is the <u>expected rate of return</u> to human capital.

We will show that, <u>for a risk-averse individual under uncertainty in the</u>

15. The implied production function in human capital is $g(I;\ \theta) = G(I) + \theta I$ Note that if $g(I;\ \theta) = G(I) + \theta$ then equation (17) reduces to (23) and increasin the risk of θ will not affect investment in human capital at all. (However, it will affect consumption.)

<u>rate of return to human capital the amount of investment in human capital is smaller than the certainty amount of investment in human capital; and the expected rate of return to human capital exceeds the (known) rate of return to financial capital</u>.

To prove this assertion we first note that the next period consumption is a random variable depending on θ (i.e., $c = c(\theta)$) . Rewriting (17) we have

$$R_1 - G_I(I) = \frac{\underset{\theta}{E}\,[U_c(c^{N-1}(\theta))\ \theta]}{\underset{\theta}{E}[U_c(c^{N-1}(\theta))]} \tag{25}$$

Now, for any given optimum values w_o, c_o , since consumption is increasing function of human capital and since $L_1 = G_I(I) + \theta$, the next period consumption $c(\theta)$ is an increasing function of θ . As a result of the strict concavity of the utility function, we therefore have

$$\begin{aligned} U_c\ (c^{N-1}(\theta)) &< U_c\ (c^{N-1}(0)) \quad , \quad \theta > 0 \\ U_c\ (c^{N-1}(\theta)) &> U_c\ (c^{N-1}(0)) \quad , \quad \theta < 0 \quad . \end{aligned} \tag{26}$$

Therefore,

$$U_c\ (c^{N-1}(\theta))\ \theta < U_c\ (c^{N-1}(0))\ \theta \qquad \text{for all} \quad \theta \neq 0 \quad . \tag{27}$$

Taking expectation on both sides of (27) we get[16]

$$\underset{\theta}{E}\ U_c\ (c^{N-1}(\theta))\ \theta < \underset{\theta}{E}\ U_c\ (c^{N-1}(0))\ \theta = U_c\ (c^{N-1}(0))\ \underset{\theta}{E}\ \theta = 0 \quad . \tag{28}$$

Equation (28) implies that the numerator in the right-hand side of (25) is negative whereas the denominator is clearly positive. We thus get

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16. We assume that the probability weight of $\theta \neq 0$ is positive.

$$R_1 - G_I(I) < 0 \tag{29}$$

The assertion is proved after comparing (29) with (23).

Since for a risk-averse individual increasing the risk will decrease the amount of investment in human capital when the rate of return to human capital relates linearly to the random variable θ, a fortiori it will do so when the rate of return is a strictly concave function of θ. Note, however, that when the rate of return to human capital $g_I(I; \theta)$ is a strictly convex function of θ then increasing the spread of θ while preserving its mean will increase the expected rate of return $\underset{\theta}{E}\, g_I(I; \theta)$ for any I so that the resulting (optimal) excess of $\underset{\theta}{E}\, g_I(I; \theta)$ over R needs not obtain through a decrease in I and, in fact, as a result, I may increase.

IV. Conclusion

The paper investigates the effect of uncertainty in the production of human capital on the investment in human capital. The forms of risk in the production function that lead to a decline in the demand for investment in human capital, and those forms of risk which do not, are separately analyzed in the cases of risk-neutrality and risk-aversion. As a by-product, the consumption function is derived.

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