# WAGE FUND, TECHNICAL PROGRESS

# AND ECONOMIC GROWTH

by

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Discussion Paper No. 12, December 1971

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## I. Rate of Profit and Income Distribution

### 1. Introduction

Of the manifold functions of capital, major attention is focused on its function as <u>circulating</u> capital in the Ricardian classical school, whereas principal emphasis is laid on its function as fixed capital in the neoclassical school. What plays the most important part in the realm of the neoclassical school such as theories of income distribution and economic growth is the very single $K$ which takes comfortably one of the two seats for both variables of a production function $F(K,L)$. Even apart from certain difficulties peculiar to macroscopic aggregative concepts as pointed out often, it seems to be doubtful that the neoclassical $K$ embodies exhaustively the functions of capital as a major driving force of the short-run as well as long-run working of an economy.

In this connection Marx's analysis of a capitalist economy seems to be full of insight as an extreme goal of the capital theory of the classical school. In his analysis, one of whose ultimate sources might be traced back to Ricardo on adverse influence on labor of the introduction of machinery in production, the function of capital as an entity consisting of constant capital and variable capital is thoroughly scrutinized.

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1. The study resulting in this publication was made at the University of Minnesota under a fellowship granted by the Ford Foundation. However, the conclusions, opinions and other statements in this publication are those of the author and are not necessarily those of the Ford Foundation. The study was initiated at Hitotsubashi University and benefited from the criticism of the author's colleagues in both institutions.

The neoclassical school has grown up on the basis of the rich nourishment fed by the classical school except for the concept of capital. The neoclassical school, which has by now reached maturity, could still learn much from the concept of capital of the classical school. There might be alternative ways in which the former could learn. One of them will be an attempt to enlarge the world of the neoclassical school so as to include that of the classical school in an enlarged world, as will be done in this paper.

In his pioneering paper [2] Lange has worked out a penetrating analysis of a capitalist economy in terms of the concept of money capital, which functions as purchasing power to employ fixed capital and labor. This paper explores how money capital governs dynamic performance of a capitalist economy, along with the line put forward by Lange, whose analysis is confined to the statistical aspects.

The circulating capital of the classical school is nothing but wage fund, that is composed of stocks of consumption goods for advanced wage payments, or its equivalent in money value. Wage fund makes no direct contribution to production, but only indirectly contributes to it by employing labor, which, in collaboration with fixed capital, does the job. How jointly fixed capital $K$ and labor $L$ contribute may be represented by a neoclassical production function $F(K,L)$ that indicates the real gross national product brought out under the given technology.

The capital which functions as a $K$ in the world of the neoclassical school need not be fixed capital in the narrow sense. $K$ can be a macroscopic aggregate of production facilities, machinery, raw material, semi-finished goods, inventories, and so on. However, if either or all of the biological necessities, mode of employment and social customs require an advanced payment of wage, the capital which a firm must have is $P = K + \theta w L$ rather than a mere $K$, where $w$ is the rate of real wage and $\theta$ is the institutionally given rate of the advance of wage to the total wage bill. Here we have in mind an economy which produces, consumes and invests a homogeneous good, which enables us to set the price of $K$ to unity.

As is well-known, the reward $\lambda$ per unit of $K$ equals the marginal productivity of $K$ and the reward $w$ per unit of $L$ equals the marginal productivity of $L$ in the competitive world of the neoclassical school. Here, a maximum $\lambda$, the rate of profit, brings about income distribution according to marginal productivities.

However, if a maximum rate of profit is pursued in terms of capital as a totality of material stocks for production and wage fund for the employment of labor in the very competitive world of the neoclassical school, a paradoxical situation arises where the rate of real wage falls short of the marginal productivity of labor. The purpose of this paper is to elucidate how far diverge income distribution in the short-run as well as in the long-run and economic growth under competitive profit rate maximization in terms of the capital $K + \theta w L$ from those in the original neoclassical world.

This paper intends to bring in light some failure of the marginal productivity theory of income distribution in a similar spirit as in a previous paper [1], though it is exclusively concerned with a competitive situation.

## 2. Income distribution in the short-run

We consider an economy where fixed capital $K$ (stocks of goods) and labor $L$ cooperate to produce gross national product (GNP). Let

$$(2.1) \qquad F(K,L)$$

be the given economy-wide production function of a neoclassical type. The products of the economy are composed of a single aggregative homogeneous good, which is used as both consumption goods and investment goods. The price of the good is set to unity regardless of whether it is used as consumption goods or investment goods. The production function (2.1) is assumed to be homogeneous of degree one and to have the well-behaved neoclassical properties. In what follows these nice properties will be utilized frequently without explicit reference thereto unless necessary.

As stated already, fixed capital $K$ as input for production is an aggregate composite of production facilities, raw materials, semifinished goods and inventories. It is assumed that $K$ depreciates radioactively at the rate of $100\mu$ percent at any moment of time when operated at full capacity, where $\mu$ is a given constant fulfilling

$$(2.2) \qquad 1 \geqq \mu \geqq 0 \, .$$

The depreciation of $K$ consists of the wear and tear of production facilities on one hand and consumption of raw materials and semifinished goods through the transformation thereof to products and decrease of inventories on the other hand.

This section analyzes the short-run working of the economy, with special regard to income distribution, under these basic premises. To this end it might be convenient to begin with a review of the neoclassical theory of short-run income distribution in perfect competition. As is well established, under a given rate of real wage $w$ an optimum capital-labor ratio $K/L$ is determined by maximizing the rate of profit of the neoclassical school

$$\frac{F(K,L) - wL - \mu K}{K} \quad . \tag{2.3}$$

The well-known condition for a maximum of (2.3) is

$$F_L(K/L,1) = w \quad , \tag{2.4}$$

that is, the equality of the marginal productivity of labor to the rate of real wage.

Let $K_0$ and $L_0$ be the existing levels of capital $K$ and labor $L$, respectively, then, the rate of real wage is given by $w = F_L(K_0/L_0,1)$ in full employment, and the corresponding rate of profit is given by the marginal productivity of capital less the depreciation ratio

$$F_K(K_0/L_0,1) - \mu \tag{2.5}$$

in virtue of Euler's theorem on homogeneous functions.

The rate of profit in the spirit of the classical school is

$$\frac{F(K,L) - wL - \mu K}{K + \theta w L} \quad , \tag{2.6}$$

rather than (2.3). As was rightly pointed out and worked out by Lange, an optimum capital-labor ratio $K/L$ is determined in a perfectly competitive situation, where money capital is binding, by maximizing (2.6) under a given rate of real wage and an institutionally given rate $\theta$ of advanced wage payment to total wage bill $(1 \geqq \theta \geqq 0)$. The conditions for a maximum of (2.6), which is readily verified, are

$$(2.7) \qquad F_K(K/L,1) - \mu = \lambda$$

$$(2.8) \qquad F_L(K/L,1) - w = \lambda\, \theta\, w$$

for a value of $\lambda$ . $\lambda$ is a Lagrangian multiplier in the corresponding Lagrangian. By Euler's theorem on homogeneous functions, $\lambda$ equals the corresponding maximum of (2.6)

$$(2.9) \qquad \lambda = \frac{F(K,L) - wL - \mu K}{K + \theta\, w\, L} \quad .$$

Moreover, condition (2.8) can be implied by conditions (2.7) and (2.9), and conversely condition (2.7) can be implied by conditions (2.8) and (2.9).

It is noted that the case for $\theta = 0$ in (2.7) and (2.8) is the original neoclassical situation (2.4). On the other hand, conditions (2.7) and (2.8) for $\theta > 0$ can be rewritten as

$$(2.10) \qquad F_K(K/L,1) - \mu = \frac{F_L(K/L,1) - w}{\theta\, w} = \lambda \quad .$$

If the maximum rate of profit $\lambda$ is positive, which is most likely to be the case, the rate of real wage falls short of the marginal productivity of labor by (2.10). The reward per unit of capital is $\lambda = F_K(K/L,1) - \mu$ , which is the same as in the neoclassical situation. But what is worthwhile noting is the fact that the total reward to capital is

$$(2.11) \qquad \lambda\, (K + \theta\, w\, L)$$

and differs from the total reward to capital $\lambda K$ in the neoclassical situation. The reward per unit of labor is $w$ , the total wage bill being $wL$ . However, the rate of real wage $w$ falls short of the marginal productivity of labor, viz the rate of real wage in the neoclassical situation, and

$$(2.12) \qquad (F_L - w)L = \lambda\ \theta\ w\ L \quad ,$$

which is originally imputed to labor in the neoclassical world, is further rewarded to capital as wage fund at the same per unit rate of remuneration $\lambda$ .

In the above result, the case for $\theta = \mu = 1$ corresponds to situations with which Ricardo and Marx were concerned. It is also assumed in the above discussion that wage is so flexible that the existing capital and labor are fully employed. The corresponding rate of real wage in full employment is given by

$$(2.13) \qquad w = \frac{F_L(K/L,1)}{1 + \theta\{F_K(K/L,1) - \mu\}} \; ,$$

evaluated at the existing amounts of $K$ and $L$ . If $w$ is thought of, more faithfully to what Ricardo and Marx had in mind, as being given exogenously, then some factor of production is likely to suffer under-employment. However, only full employment situations will be discussed in this paper, since it intends to extend the neoclassical theory of income distribution so as to have a unified formulation of both theories.

## 3. Marginal productivity theory at fault

If a positive maximum rate of profit is achieved, the marginal productivity income distribution theory of the neoclassical school is no longer valid, because the denominator of (2.13) is greater than one so that the rate of real wage falls short of the marginal productivity of labor even in a competitive situation. This is due to the function of a certain part of capital as wage fund.

Nonetheless, the situation could be looked at in such an alternative way that it might appear as if the marginal productivity income distribution theory could still be at work. This could be done by reformulating the production function as a function of money capital $P = K + \theta w L$ .

In fact, if $K$ is eliminated from

$$F(K,L) - \mu K \tag{3.1}$$

by using

$$P = K + \theta w L \quad , \tag{3.2}$$

a function

$$H(P,L) = F(P - \theta wL, L) + \mu\theta wL - \mu P \tag{3.3}$$

of money capital $P$ and labor $L$ is obtained, and the rate of profit can be put in the following form

$$\frac{H(P,L) - wL}{P} = H(1,L/P) - w(L/P) \quad . \tag{3.4}$$

Then, the condition for a maximum rate of profit takes the form

$$H_L(1,L/P) = w \quad , \tag{3.5}$$

which states the equality of the rate of real wage to the "marginal productivity" of labor.

The above remark by no means revives the marginal productivity income distribution theory. For, first, the alleged production function (3.3) can hardly be thought of as a production function that is supposed to represent technological know-how, independent of the wage rate. Second, the function

$$F(P - \theta wL, L) \tag{3.6}$$

has a graph, as illustrated in Figure 1, that is drastically different from the favorite shape of a production function of the neoclassical school. The graph bends downward and has negative "marginal productivities" for larger amounts of labor input.

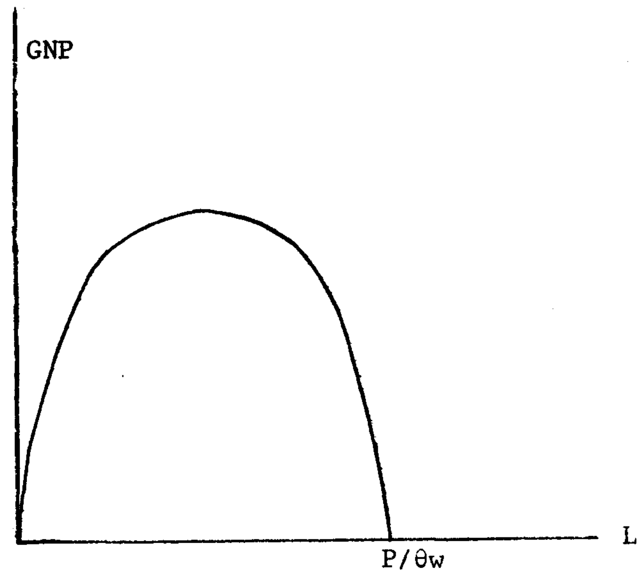


Figure 1

4. Harrod neutrality versus Hicks neutrality

A standard analytical formulation of the productivity increase caused by technical change is given in terms of an upward shift of the production function. From the income distribution theoretic viewpoint, it is important to single out certain types of technical change as neutral ones in the sense that they equally benefit capital and labor in some ways. Among various neutrality concepts those due to Harrod and Hicks are best known and play an important part in the neoclassical world. But what about their relevance in the world with which the classical school is concerned? It should be noted that the Harrod neutrality concept remains of much relevance to the income distribution aspect of the general situation, whereas the Hicks neutrality concept seems to lose its income distribution theoretic relevance there.

Mrs. Robinson [3] has put forward the Harrod neutrality of technical change as the invariance of the relative shares of capital and labor in the national product for a given rate of profit under the technical change in question.

Suppose that a shift of the production function has taken place in the general situation

$$F(K,L) - \mu K \Rightarrow G(K,L) - \nu K \tag{4.1}$$

in such a way that the relative shares of capital and labor remain unchanged for any given level $\lambda$ of the rate of profit under the technical change in question. Let $k_F$ and $k_G$ be the capital-labor ratios $K/L$ corresponding to $\lambda$, respectively, before and after the technical change. Then the following equations obtain

$$F_K(k_F, 1) - \mu = G_K(k_G, 1) - \nu = \lambda \quad , \tag{4.2}$$

and the corresponding rates of real wage are given by

$$w_F = \frac{F_L(k_F, 1)}{1 + \theta\lambda} \tag{4.3}$$

$$w_G = \frac{G_L(k_G, 1)}{1 + \theta\lambda} \tag{4.4}$$

by virtue of (2.13). From the assumed invariance of relative shares

$$\frac{\lambda(k_F + \theta w_F)}{w_F} = \frac{\lambda(k_G + \theta w_G)}{w_G}$$

follows immediately

$$\frac{\lambda k_F}{w_F} = \frac{\lambda k_G}{w_G} \quad , \tag{4.5}$$

which, in the light of (4.3) and (4.4), implies

$$(4.6) \qquad \frac{\lambda k_F}{F_L(k_F, 1)} = \frac{\lambda k_G}{G_L(k_G, 1)} \quad .$$

(4.6) tells that the neoclassical relative shares remain unchanged for any given level of rate of profit under the technical change. Conversely, the Harrod neutrality in the neoclassical situation implies that in the general situation. This is due to the fact that, given a rate of profit $\lambda$, the corresponding capital-labor ratio $K/L$ is uniquely determined by (4.2) up to the production function together with the depreciation ratio and is independent of the institutional parameter $\theta$. The only difference comes from the distortion in the rate of wage by the multiplier $1/(1 + \lambda\theta)$. The Harrod neutrality is therefore a meaningful concept in the world of the classical school as well as in that of the neoclassical school. The well-known characterization of the Harrod neutrality as labor-augmenting technical progress (Robinson [3]) is therefore valid in the general situation that includes the world of the classical school as well as that of the neoclassical school.

On the other hand, an output-augmenting technical change, which is nothing but a neutral technical change in Hicks' sense in the world of the neoclassical school, may cease to be neutral in the world of the classical school in the very sense of the Hicks neutrality. In fact, suppose that the technical change (4.1) is output-augmenting. Here, the Hicks neutrality means that the relative shares of capital and labor remain unchanged for any given capital-labor ratio under the technical change in question. Let $\lambda_F$ and $\lambda_G$ be the rates of profit for a given capital-labor ratio $k$, respectively, before and after the technical change.

Then, they are determined by

$$\lambda_F = F_K(k,1) - \mu$$

$$\lambda_G = G_K(k,1) - \nu$$

independently of the institutional parameter $\theta$ , a fortiori in the neo-classical situation where the rates of real wage corresponding to $k$ , respectively, before and after the technical change equal the marginal productivities of labor $F_L(k,1)$ and $G_L(k,1)$ . Then for a positive $\sigma$

$$\lambda_G = \sigma\lambda_F \tag{4.7}$$

$$G_L(k,1) = \sigma F_L(k,1) \tag{4.8}$$

by virtue of the assumption that the technical change is output-augmenting. The rates of real wage, respectively, before and after the technical change in the general situation are given by

$$w_F = \frac{F_L(k,1)}{1 + \theta\lambda_F} \tag{4.9}$$

$$w_G = \frac{G_L(k,1)}{1 + \theta\lambda_G} \quad . \tag{4.10}$$

Then, the relative shares of capital and labor before and after the technical change, respectively,

$$\frac{\lambda_F(k + \theta w_F)}{w_F} \tag{4.11}$$

$$\frac{\lambda_G(k + \theta w_G)}{w_G} \tag{4.12}$$

coincide with each other if and only if $\theta = 0$ (i.e. in the neoclassical case), provided the rates of profit are positive. For the difference of (4.11) and (4.12) can be expressed as

$$(4.12) - (4.11) = \frac{\theta\lambda_F(\sigma - 1)\{F(k,1) - \mu k\}}{F_L(k,1)} \tag{4.13}$$

which vanishes if and only if $\theta = 0$ , since by assumption $\lambda_F > 0$ , $\sigma \neq 1$ . Put slightly another way, an output-augmenting technical progress ($\sigma > 1$) is always biased in favor of capital in the general situation ($\theta > 0$) except in the neoclassical situation, as is obvious from (4.13).

## II. Dynamic Process of Growth and Capital Accumulation

### 5. Dynamic process of growth

Now that certain of the static features of the system is brought in light, the system will be set in motion. This will be done by weaving short-run temporary equilibria into a dynamic process. The way to set the system in motion is based on the principal views of neoclassical economists, notably, on that of Solow [4] and extends it so as to include the world of the classical economics.

Given the existing levels of (fixed) capital and labor at time $t$ , income distribution is thoroughly determined by (2.10). Moreover, it is assumed that there is wage fund $\theta wL$ at time $t$ enough to employ the existing volume of labor $L$ at the going rate of real wage determined by (2.13). In this short-run temporary equilibrium the net national product is $F(K,L) - \mu K$ , of which the share of labor is $wL$ and the share of capital is $F(K,L) - wL - \mu K$ .

The consumption-saving hypothesis to be assumed here is the familiar one that (a) workers only consume but never save, while capitalists only save but never consume.

Moreover, the special investment decision, peculiar to the neoclassical school, is also assumed; that is, (b) the volume of investment is determined in such a way that it is set equal to the given volume of saving.

The system is now geared by a dynamic mechanism by means of assumptions (a) and (b), so that a process of capital accumulation is generated. It is of vital importance to realize that the capital whose accumulation is to be chased is not the fixed capital $K$ but the money capital $P = K + \theta wL$ . Therefore the net investment is allocated to two parts, namely, the net increase of $K$ and the increase of wage fund $\theta wL$ . Thus if time is measured continuously, the resulting differential equation is

$$\frac{d}{dt}(K + \theta wL) = F(K,L) - wL - \mu K \quad . \tag{5.1}$$

To make the dynamic mechanism completely determined, (c) the growth rate of labor force is assumed to be a function $\phi(w)$ of the rate of real wage $w$ , which is continuous and nondecreasing, and takes on a non-negative value for some $w$ . Accordingly, the differential equation

$$\frac{dL}{dt} = \phi(w)L \tag{5.2}$$

is the dynamic law which governs the growth of labor force.

Mathematically, the money capital $P = K + \theta wL$ , which is more important from the basic economic viewpoint in this paper than $K$ , is, however, not an independent variable, but a simple function of $K$,$L$ and $w$ . The rate of real wage $w$ is determined by (2.13) at each moment of time, so that it is also a function of $K$ and $L$ . Therefore, the system of differential equations (5.1) and (5.2) in the unknown functions of time $K$ and $L$ governs the complete dynamic behaviors of $K$ and $L$ , and hence $P$ and $w$ .

The system (5.1) and (5.2) can be transformed to a differential equation in the capital-labor ratio

$$k = \frac{K}{L} \tag{5.3}$$

as the unknown function of time. In what follows a dot over a variable is used to mean differentiation with respect to time. If differentiation is performed and the terms are rearranged in (5.1), it becomes

$$\dot{K} = F(K,L) - (w + \theta\dot{w})L - \theta w\dot{L} - \mu K \quad , \tag{5.4}$$

which can further be converted by using (5.2) to

$$\dot{K} = F(K,L) - \{w + \theta\dot{w} + \theta w\phi(w)\}L - \mu K \quad . \tag{5.5}$$

From (5.3) it follows that

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} \quad . \tag{5.6}$$

Hence, if the right-hand side of (5.6) is evaluated by (5.2) and (5.5), then (5.6) can be reduced to

$$\dot{k} = F(k,1) - \{w + \theta\dot{w} + \theta w\phi(w)\} - \{\phi(w) + \mu\}k \quad . \tag{5.7}$$

Finally, in view of the fact that $w$ is a function of $k$ by (2.13), it can be differentiated with respect to time, obtaining

$$\dot{w} = \frac{dw}{dk}\dot{k} \quad . \tag{5.8}$$

If (5.8) is substituted for $\dot{w}$ in (5.7), a final form of the basic differential equation governing the dynamic behavior of $k$ will be

$$\left(1 + \theta\frac{dw}{dk}\right)\dot{k} = F(k,1) - w\{1 + \theta\phi(w)\} - \{\phi(w) + \mu\}k \quad . \tag{5.9}$$

Certain remarks on (5.9) seem to be now in order. First, it is noted that the coefficient of $\dot{k}$ on the left-hand side of (5.9) can be regarded as

$$1 + \theta\frac{dw}{dk} = \frac{d}{dk}(k + \theta w) \quad , \tag{5.10}$$

and the coefficient has proved to be the derivative with respect to $k$ of the per capita money capital, because

(5.11) $$k + \theta w = \frac{P}{L} \quad .$$

Second, from (2.13) follows

(5.12) $$\frac{dw}{dk} = \frac{F_{LK}(k,1)[1 + \theta\{F_K(k,1) - \mu\}] - \theta F_L(k,1)F_{KK}(k,1)}{[1 + \theta\{F_K(k,1) - \mu\}]^2} > 0 \quad ,$$

whose positivity is obvious by

$$F_K, F_L, F_{LK}, 1 - \theta\mu > 0, \theta \geqq 0$$

$$F_{KK} < 0 \quad .$$

The above remarks have clarified that the coefficient of $\dot{k}$ on the left-hand side of (5.9) is always positive, so that (5.9) is a well posed differential equation of a normal type. It is obvious that (5.9) reduces to the familiar neoclassical case for $\theta = 0$ . (5.9) depicts capital accumulation in the world of the classical school for $\theta > 0$ $(1 \geqq \theta > 0)$.

## 6. Steady growth

Steady growth is a moving equilibrium state of the system, in which the capital-labor ratio $k$ is kept a constant over time. From the mathematical viewpoint a steady growth path is nothing but a stationary solution of equation (5.9). In this section the existence and stability of steady growth will be examined. To this end it is convenient to rearrange the right-hand side in the following way. In fact, first, from (2.13) follows

(6.1) $$F_L(k,1) = w[1 + \theta\{F_K(k,1) - \mu\}]$$

Moreover, Euler's theorem on homogeneous functions gives

(6.2) $$F(k,1) = F_K(k,1)k + F_L(k,1) \quad .$$

In the light of (6.1) and (6.2), the right-hand side of (5.9) can be expressed as follows

(6.3) $$\text{r.h.s of (5.9)} = [F_K(k,1) - \{\phi(w) + \mu\}](k + \theta w) \quad .$$

Therefore, in view of (5.10), equation (5.9) can be put in the form

(6.4) $$\left\{\frac{d}{dk}(k + \theta w)\right\}\dot{k} = [F_K(k,1) - \{\phi(w) + \mu\}](k + \theta w) \quad ,$$

or

(6.5) $$\dot{k} = [F_K(k,1) - \{\phi(w) + \mu\}] \frac{k + \theta w}{\frac{d}{dk}(k + \theta w)} \quad .$$

Here, both the numerator and denominator of

(6.6) $$\frac{k + \theta w}{\frac{d}{dk}(k + \theta w)}$$

are positive by (5.12), and hence (6.6) is also positive.

From what has been elucidated above it is clear that a steady growth solution $(k,w)$ is nothing but a solution of the system of equations

(6.7) $$\begin{cases} F_K(k,1) = \phi(w) + \mu & (6.7.1) \\ w = \dfrac{F_L(k,1)}{1 + \theta\{F_K(k,1) - \mu\}} & (6.7.2) \end{cases}$$

It is readily seen that (6.7) has a unique solution. In fact the left-hand side of (6.7.1) is a strictly decreasing continuous function whose value ranges from $+\infty$ to $0$ as $k$ increases. On the other hand, the right-hand side of (6.7.2) is a strictly increasing continuous function whose value ranges from $0$ to $+\infty$ as $k$ increases. If the right-hand side of (6.7.2) is substituted for $w$, the right-hand side of (6.7.1) is a nondecreasing continuous function of $k$, which takes on a positive value for some $k$. Hence, both sides of (6.7.1) can be equated once and only once at some value of $k$, say $k^*_\theta$, as is illustrated in Figure 2.

(6.8) $$k(t) = k^*_\theta$$

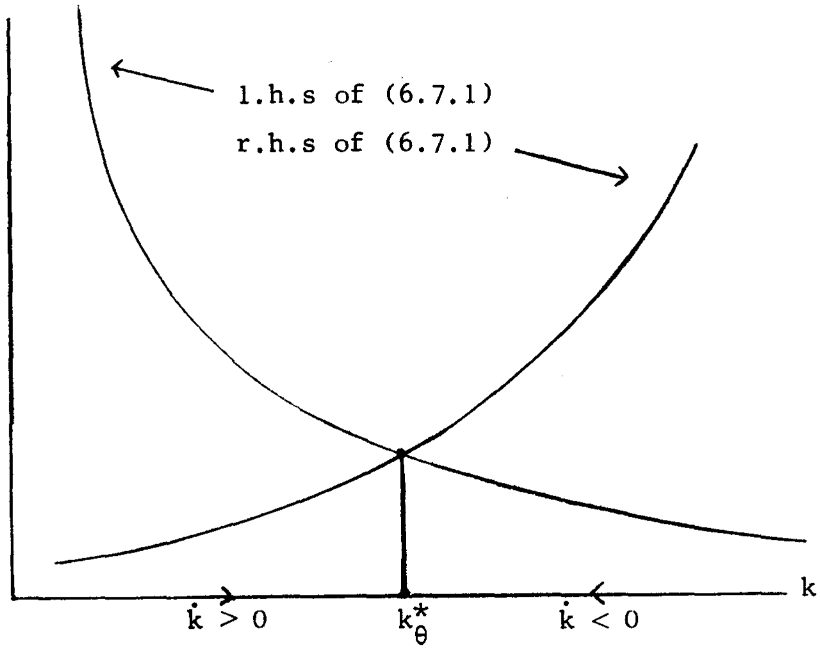


Figure 2

is therefore a unique steady growth solution of the system. The above argument has also clarified that in (6.7.1) with $w$ substituted by (6.7.2) the left-hand side is greater than the right-hand side for $k < k^*_\theta$ and the former is less than the latter for $k > k^*_\theta$ . Substantially, this ensures the stability of the steady growth solution, because the second factor (6.6) on the right-hand side of equation (6.5) is always positive.

7. Long-run equilibrium rates of real wage

Given an initial level of capital-labor ratio $k(0)$ , the corresponding initial rate of real wage $w(0)$ is determined by (2.13), and hence the corresponding initial level of money capital per unit of labor is also determined. Starting at this initial position the system tends to the steady state $k^*_\theta$ , that is, an equilibrium in the long-run, as was established in the preceding section.

The long-run equilibrium, to which the system converges, depends upon the magnitude of the institutional parameter $\theta$ . $\theta$ indicates how much wage fund firms need to meet the advance of wage bill. It is the purpose of this section to examine to what extent $\theta$ influences the long-run features of income distribution. In particular, the relationship between $\theta$ and the corresponding rate of real wage $w_\theta^*$ in the long-run equilibrium will be analyzed.

Here, it is recalled that in the short-run temporary equilibrium in which the existing K and L are fully employed the greater $\theta$ the lower the rate of real wage, provided the rate of profit is positive, i.e.

$$(7.1) \qquad F_K(K/L,1) > \mu \quad .$$

This is clear from equation (2.13) which determines the rate of real wage in the short-run temporary equilibrium.

Now, as was remarked, the long-run equilibrium levels of capital-labor ratio $k_\theta^*$ and of rate of real wage $w_\theta^*$ are determined as a solution of the system of equations (6.7). In what follows the correspondence between the magnitude of $\theta$ and those of $k_\theta^*$ and $w_\theta^*$ will be brought in light.

First, it is useful to start with a special situation where the growth rate of labor force is a simplest function, i.e.

$$\phi(w) = n \text{ (a positive constant).}$$

In this special case equation (6.7.1) is

$$(7.2) \qquad F_K(k,1) = n + \mu \quad ,$$

which is independent of $\theta$ . The long-run equilibrium level of capital-labor ratio $k^*$ determined by (7.2) gives the common level of $k_\theta^*$ for all $\theta$ . Therefore, the relationship between the magnitude of $\theta$ and that of $w^*_\theta$ can be elucidated by exactly the same reasoning for the analysis of the short-run temporary equilibrium features of income distribution. In fact, $k^*_\theta = k^*$ , which is determined by (7.2), determines in turn $w^*_\theta$ by (6.7.2), so that

(7.3)
$$w^*_\theta = \frac{F_L(k^*,1)}{1 + \theta n} \quad .$$

Accordingly, the greater $\theta$ the lower $w^*_\theta$ .

Next, the case of $\phi(w)$ of a more general type will be examined. If (6.7.2) is substituted for the $w$ in (6.7.1), the equation

(7.4)
$$F_K(k,1) = \phi\left(\frac{F_L(k,1)}{1 + \theta\{F_K(k,1) - \mu\}}\right) + \mu$$

will be obtained.

Now, Figure 2 will be reproduced for two different values of $\theta$ , say, $\theta_1$ and $\theta_2$ $(1 \geqq \theta_2 > \theta_1 \geqq 0)$, to obtain Figures 3, 4 and 5, depending on the magnitude of a certain level of capital-labor ratio $\bar{k}$ determined by the equation

(7.5)
$$F_K(k,1) = \mu \quad .$$

Figures 3, 4 and 5 correspond to the cases

(7.6)
$$\phi(F_L(\bar{k},1)) > 0 \quad ,$$

(7.7)
$$\phi(F_L(\bar{k},1)) < 0 \quad ,$$

and

(7.8)
$$\phi(F_L(\bar{k},1)) = 0 \quad ,$$

respectively.

Clearly,

$$\frac{F_L(k,1)}{1+\theta_1\{F_K(k,1)-\mu\}} \gtreqless \frac{F_L(k,1)}{1+\theta_2\{F_K(k,1)-\mu\}}$$

if and only if $k \lesseqgtr \bar{k}$ . Therefore, by virtue of the nondecreasingness of $\phi(w)$ ,

$$\phi\left(\frac{F_L(k,1)}{1+\theta_1\{F_K(k,1)-\mu\}}\right) \geqq \phi\left(\frac{F_L(k,1)}{1+\theta_2\{F_K(k,1)-\mu\}}\right)$$

if $k \leqq \bar{k}$ , and

$$\phi\left(\frac{F_L(k,1)}{1+\theta_1\{F_K(k,1)-\mu\}}\right) \leqq \phi\left(\frac{F_L(k,1)}{1+\theta_2\{F_K(k,1)-\mu\}}\right)$$

if $k \geqq \bar{k}$ .

Consequently, the case for (7.6) is illustrated as in Figure 3, so that

(7.6*)
$$\begin{cases} k^*_{\theta_1} \leqq k^*_{\theta_2} \\ F_K(k^*_{\theta_1},1) \geqq F_K(k^*_{\theta_2},1) \end{cases} .$$

The corresponding rates of real wage $w^*_{\theta_1}$ and $w^*_{\theta_2}$ satisfy

(7.6$^*$w)
$$w^*_{\theta_1} > w^*_{\theta_2} .$$

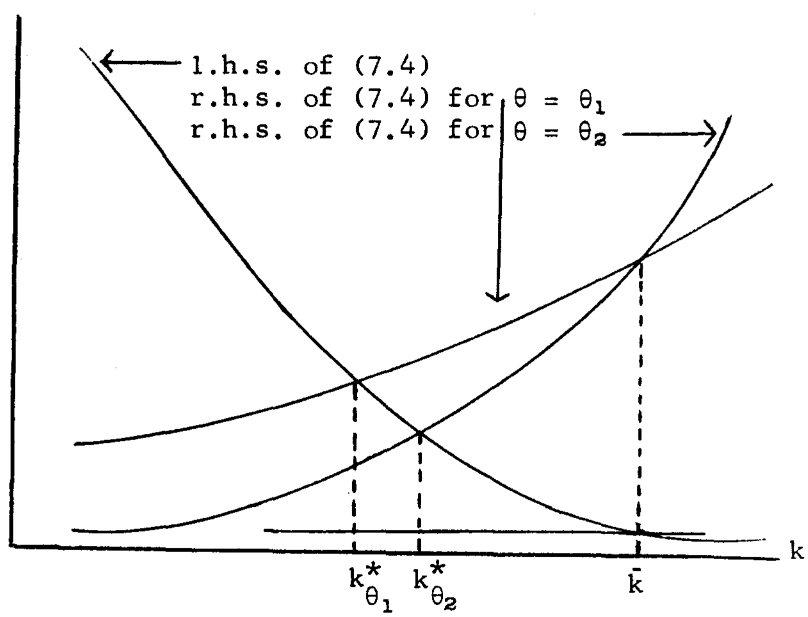


Figure 3

In fact, if equality sign obtains in (7.6*), the (7.6*w) can be ensured by exactly the same reasoning for the special case of a constant growth rate of labor force. On the other hand, if strict inequality sign obtains in (7.6*), the $k^*_{\theta_1}$ and $k^*_{\theta_2}$ fulfill (7.4) for $\theta = \theta_1$ and $\theta_2$, respectively. Hence

$$\phi(w^*_{\theta_1}) > \phi(w^*_{\theta_2})$$

so that (7.6*w) must hold by virtue of the nondecreasingness of $\phi(w)$.

The case for (7.7) can be likewise discussed and illustrated as in Figure 4, except for the sense of inequality signs reversed, and the corresponding results are

$$(7.7^*)\qquad \begin{cases} k^*_{\theta_1} \geqq k^*_{\theta_2} \\ F_K(k^*_{\theta_1},1) \leqq F_K(k^*_{\theta_2},1) \end{cases}$$

$$(7.7^*w)\qquad w^*_{\theta_1} < w^*_{\theta_2} \quad .$$

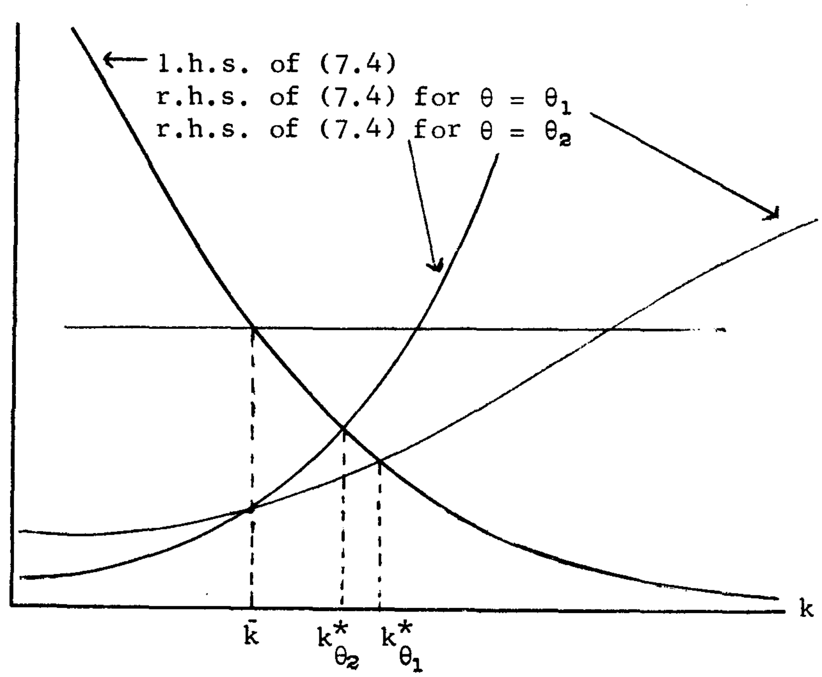


Figure 4

In the case for (7.8), which is illustrated in Figure 5, the rate of real wage is equated to the marginal productivity of labor at the capital-labor ratio $\bar{k}$ , independently of the magnitude of $\theta$ ,

$$(7.8^*w) \qquad w^*_{\theta_1} = w^*_{\theta_2} = F_L(\bar{k},1) \quad .$$

The case for (7.7) is a balanced decay situation in which all of the net rate of profit, net investment and growth rate of labor force are negative, which could not be thought of as a workable state. Therefore, only the cases for (7.6) and (7.8) are workable, and especially, the former is most likely to obtain in an economy endowed with sufficiently high productivity. Thus it may be concluded that the greater the institutional parameter $\theta$ the lower the long-run equilibrium rate of real wage $w^*_\theta$ in the normal workable situation.

The above analysis is for the situation without technical change. However, it can be as readily adapted to the situation with a Harrod neutral technical change as in the well-known treatment of the neoclassical case.

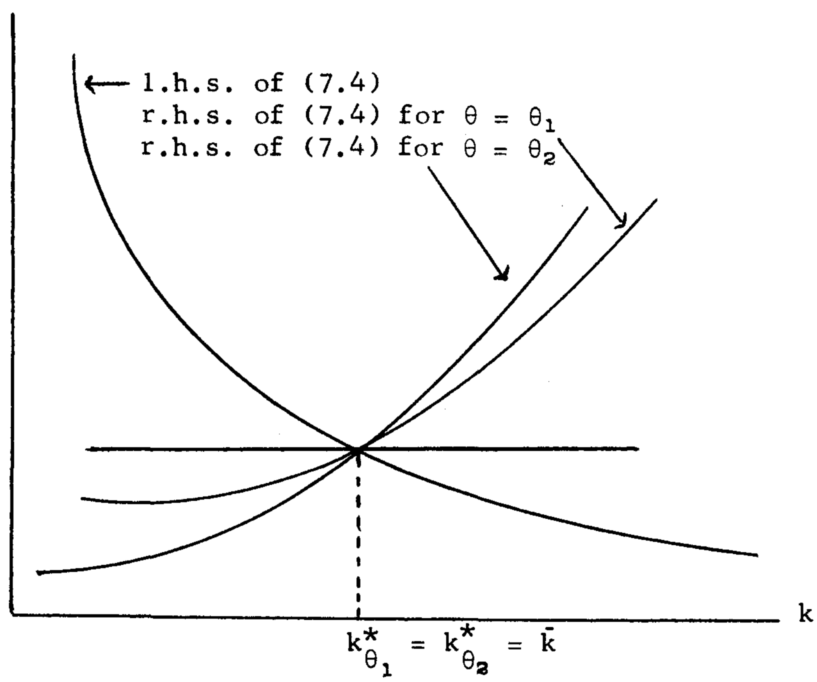


Figure 5

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## APPENDIX

### Geometry of the Classical Profit Rate Maximization

A maximum rate of profit will be located for the typical case $\theta = 1$ in Figure 6, in which conditions (2.8) and (2.9) take the form

$$\frac{F_L(1,\ell) - w}{w} = \frac{F(1,\ell) - (w\ell + \mu)}{1 + w\ell} \tag{A.1}$$

if $\lambda$ is eliminated and the labor-capital ratio (reciprocal of capital-labor ratio) is denoted by $\ell$

$$\ell = L/K \quad . \tag{A.2}$$

To locate a value of $\ell$ diagrammatically that satisfies condition (A.1) for a given level of $w$ one can proceed as follows. In Figure 6 $\ell$ is measured along the horizontal axis and output is measured along the vertical axis.

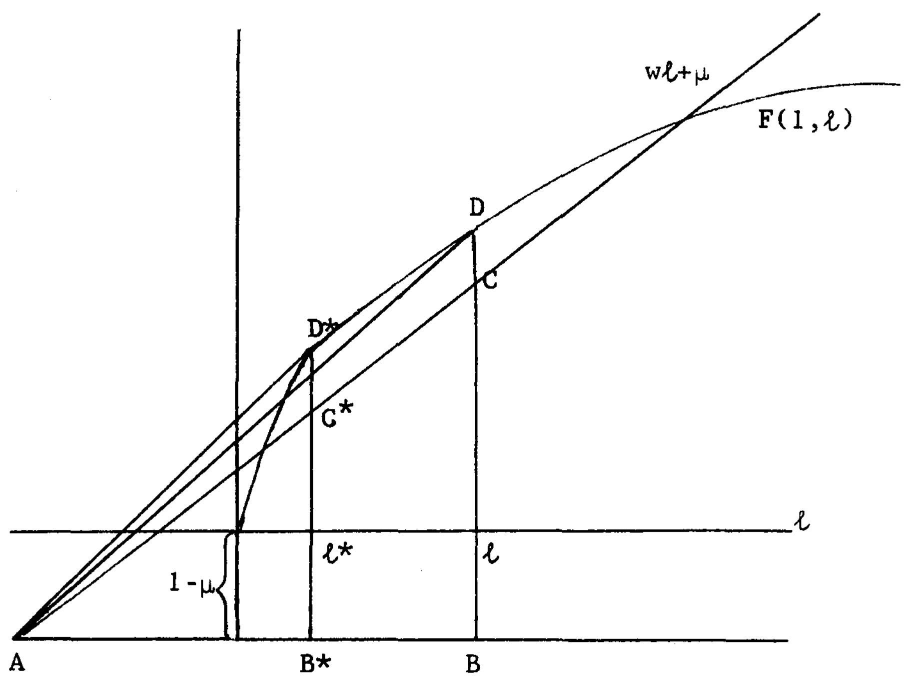


Figure 6

Let A be the intersecting point of the straight line $w\ell + \mu$ and the horizontal line obtained by translating the horizontal axis downward by $1 - \mu$. Take an $\ell$ on the horizontal axis and let its perpendicular at $\ell$ intersect at B, C, and D with the above two straight lines and the graph of the production function. Then

$$\text{(A.3)} \qquad \text{rate of profit at } \ell = \frac{CD}{BC} \ .$$

Now, (A.3) is a maximum, if and only if $\ell$ is chosen in such a way that

$$\text{(A.4)} \qquad 1 + \text{rate of profit at } \ell = \frac{BD}{BC} = \frac{BD}{AB} \Big/ \frac{BC}{AB} = \frac{BD}{AB} \Big/ w$$

is a maximum. (A.4) is in turn a maximum if and only if the angle B A D is a maximum, that is, A D is tangent to the graph of the production function. Therefore the solution can be located by choosing $D^*$, the point of tangency of the tangent to the graph of the production function passing through A and finding $\ell^*$ at the intersecting point of the vertical line through $D^*$ with the horizontal axis. At $D^*$ the ratio of the slope of the tangent to w equals 1 plus the maximum rate of profit.