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# A Case Control Study of Speed and Crash Risk

# Technical Report 2

# Bayesian Reconstruction of Traffic Accidents and the Causal Effect of Speed in Intersection and Pedestrian Accidents

**Final Report**

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Traffic accident reconstruction has been defined as the effort to determine, from whatever evidence is available, how an accident happened. Traffic accident reconstruction can be treated as a problem in uncertain reasoning about a particular event, and developments in modeling uncertain reasoning for artificial intelligence can be applied to this problem. Physical principles can usually be used to develop a structural model of the accident and this model, together with an expert assessment of prior uncertainty regarding the accident's initial conditions, can be represented as a Bayesian network. Posterior probabilities for the accident's initial conditions, given evidence collected at the accident scene, can then be computed by updating the Bayesian network. Using a possible worlds semantics, truth conditions for counterfactual claims about the accident can be defined and used to rigorously implement a "but for" test of whether or not a speed limit violation could be considered a cause of an accident. The logic of this approach is illustrated for a simplified version of a vehicle/pedestrian accident, and then the approach is applied to determine the causal effect of speeding in 10 actual accidents.

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**A Case Control Study of Speed and Crash Risk**

**Technical Report 2**

# Bayesian Reconstruction of Traffic Accidents and the Causal Effect of Speed in Intersection and Pedestrian Accidents

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In Technical Report 1 for this project we argued that traffic accidents should be treated as resulting from the workings of partially understood, deterministic processes, and that the causal effect of some variable could, at least in principle, be determined "bottom up," by considering that variable's effect on each of a set of individual accidents. Since this requires that we reason in a logical manner about the uncertainty attached to particular events, we are faced with a problem more like determining an individual's cause of death, or identifying the perpetrator of a crime, than like estimating a summary measure for some population of entities. At the First International Conference on Forensic Statistics Lindley (1991) argued that the probability calculus should be applied not only to statistical problems, but to forensic inference more generally. Lindley focused on a class of problems for which the hypotheses of interest were the guilt or innocence of a defendant, and the task was to weigh the plausibility of these alternatives in the light of evidence. His proposed solution was Bayesian, where one first determines a prior assignment of probability to the alternative hypotheses, along with the probability of the evidence given each alternative, and then uses Bayes theorem to compute posterior probabilities for the hypotheses. This approach has since been applied to increasingly more complicated problems in forensic identification (e.g. Balding 2000; Dawid and Mortera 1998), in part due to an intense interest in interpreting DNA evidence.

## Forensic Inference and Traffic Accidents

Lindley also referred briefly to a different class of problems, exemplified by "the case of a motorist charged with dangerous drivingYHere the possible criminal is known; what is in doubt is whether there was a crime" (1991, p. 86). Such problems are most prominent when a traffic accident has resulted in death or serious injury, and it must be determined if a driver should be subjected to criminal or civil penalties. The specific conditions which define this liability vary somewhat across jurisdictions, but it is generally possible to identify two basic issues, pertaining to the quality of driving, and to the causal relation between the driving and the accident's outcome. For example, in Great Britain "causing death by dangerous driving" occurs when "a person *causes* the death of another person by driving a mechanically propelled vehicle dangerously" while "dangerous driving" is in turn defined as driving that "falls far below what would be expected of a competent and careful driver" (Road Traffic Act 1991). In the United States, the Uniform Vehicle Code states that "homicide by vehicle" occurs when a driver "is engaged in the violation of any state law or municipal ordinance applying to the operation or use of a vehicle" and when "such violation is the *proximate cause* of said death" (NCUTLO 1992). (Italics have been added for emphasis.)

For some accidents, eyewitness testimony may be reliable enough, and the driving egregious enough, that questions concerning the quality of driving and its relation to the accident are readily resolved. In other cases however the evidence can be entirely circumstantial, collected by accident investigators after the event. The connections between the evidence and the basic legal issues can then be less clear, and over the past 70 years the practice of accident investigation and reconstruction has developed primarily to assist the legal system in resolving such questions (Traffic Institute 1946; Rudram and Lambourn 1981). Baker and Fricke define accident reconstruction as "the effort to determine, from whatever information is available, how the accident occurred" (1990, p. 50-3). Typical questions which a reconstructionist can be called on to address include ascertaining the initial speeds and locations of the involved parties, identifying the actions taken by these parties, and determining the degree to which avoidance actions might have been effective. Rarely however will direct observations alone be sufficient to answer these questions, and the reconstructionist may need to supplement the information collected at the accident's scene.

These issues can be illustrated by considering an instructional example used by Greatrix (2002). On a summer afternoon in Carlisle, England, a seven year-old boy attempted to cross a road ahead of an oncoming vehicle, and although the driver braked to a stop, the child was struck. Measurements made at the scene yielded a skidmark 22 meters long, with the point of collision being 12 meters from the start of the skid. It was also noted that the speed limit on the road was 30 mph (13.4 meters/sec). A standard practice in accident reconstruction is to use, as additional premises, variants of the kinematic formula

$$s=vt+(at^2)/2 \qquad (1)$$

which gives the distance $s$ traveled during time interval $t$ by an object with initial velocity $v$ while undergoing a constant acceleration of $a$. In this case, test skids conducted after the accident suggested that a braking deceleration $a$=-7.24 meters/sec$^2$ was plausible, and this value together with the measured skidmark of 22 meters gives the speed of the vehicle at the start of the skid as 17.8 meters/sec. Citing statistical evidence on the distribution of driver reaction times (denoted here by $t_p$), Greatrix used a representative value of 1.33 seconds to deduce that the vehicle was 35.7 meters from the collision point when the driver noticed the pedestrian. If the vehicle has been traveling instead at the posted speed of 30 mph (13.4 meters/sec), the driver would have needed only 30.2 meters to stop and so, other things equal, would not have hit the child. Thus the evidence could be taken as supporting claims that the driver was speeding, and that had the driver not been speeding, the accident would not have happened. At this point though a different expert could argue that no one knows for certain either the actual braking deceleration or the driver's actual reaction time. The alternative values of $a$=-5.5 meters/sec$^2$ and $t_p$=1.0 seconds imply that the magnitude of the speed limit violation was only about 5 mph, and that the vehicle would not have stopped before reaching to collision point, even if the initial speed had been 30 mph. It could then be claimed that both the degree to which the driver's speed violated the speed limit, and the causal connection between that violation and the result, are open to doubt.

One way to view this example is as an attempt to answer three types of questions. The first type, which can be called factual questions, concerns the actual conditions associated with this particular accident. These include the paths of the pedestrian and vehicle, the initial speed of the vehicle, and the relation between this speed and the posted speed limit. Answering this type of question is what Baker and Fricke consider the role of accident reconstruction proper. The second type, which can be called counterfactual questions, starts with a stipulation of the facts, but then goes beyond these and attempts to determine what would have happened had certain specific conditions been different. In the example, determining whether or not the vehicle would have stopped before reaching the pedestrian, had the initial speed been equal to the posted limit, is a question of this type. Counterfactual questions usually arise in accident reconstruction when attempting to identify what Baker called "causal factors," which are circumstances "contributing to a result without which the result could not have occurred" (Baker 1975, p.274). Posing and answering counterfactual questions has been referred to as "avoidance analysis" (Limpert 1989), or "sequence of events analysis" (Hicks 1989) in the accident reconstruction literature. Finally, the third type of question arises because of uncertainty regarding the conditions of the accident, which leads to uncertainty concerning the answers to the first two types of question.

How best to account for uncertainty in an accident reconstruction is currently an unresolved issue. A common recommendation has been to perform sensitivity analyses, in which the values of selected input variables are changed and estimates recomputed (Schochenhoff et al. 1985; Hicks 1989; Niederer 1991). When the conclusions of the reconstruction (e.g. that the driver was speeding) are insensitive to this variation they can be regarded as well-supported, but if different yet plausible combinations of input values produce differing results, this approach is inconclusive. This limitation has been recognized, and has motivated several authors to apply probabilistic methods. Brach (1994) has illustrated how the method of statistical differentials can be used to compute the variance of an estimate, given a differentiable expression for the desired estimate and a specification of the means and variances of distributions characterizing the uncertainty in the expression's arguments. Kost and Werner (1994) and Wood and O'Riordain (1994) have suggested that Monte Carlo simulation can be used for more complicated models where tractable solutions are not at hand. The Monte Carlo methods also produce approximate probability distributions characterizing the uncertainty of estimates, rather than just means and variances. Particularly interesting are the examples used by Wood and O'Riordain, where simulated outcomes inconsistent with measurements were rejected, in effect producing posterior conditional distributions for the quantities of interest. These posterior distributions were then used to compute the probability of a counterfactual claim, that the accident would not have occurred had a vehicle's initial speed been different. More recently, Rose et al. (2001) have employed a Monte Carlo approach similar to that of Wood and O'Riordain. A weakness of this approach, rooted in the appearance of the Borel paradox when making inferences using deterministic simulation models, has been identified by Hoogstrate and Spek (2002), who used Bayesian melding (Poole and Raftery 2000) to get around this problem. Roughly concomitant with the developing interest in probabilistic accident reconstruction has been an interest in using Bayesian networks to support more traditional forensic inference (Edwards 1991; Aitken et al. 1996; Dawid and Evett 1997; Curran et al., 1998). Bayesian networks can be used to represent an expert's knowledge concerning some class of systems, together with his or her uncertainty concerning the state of a particular system in that class. The system's state is characterized by the values taken on by a set of variables, and the dependencies among the system variables are described by specifying a set of deterministic and/or stochastic relationships (Jensen 1996). Davis (1999, 2001) has illustrated how Bayesian network methods and Markov Chain Monte Carlo (MCMC) computational techniques can be combined to accomplish a Bayesian reconstruction of vehicle/pedestrian accidents.

Despite this growing interest in using probabilistic reasoning in accident reconstruction, there appears to be some confusion about the relation between these applications and traditional statistical reasoning. Brach faulted sensitivity analyses because "the statistical nature of the variations is not explicitly taken into account," (1994, p. 148), while Wood and O'Riordain referred to an expression giving the coefficient of variation for an individual speed estimate as "..a statistical approach," (1994, p. 130). Rose et al. recommended Monte Carlo simulation because it produces "statistically relevant conclusions regarding the probable $\Delta$V experienced by a vehicle" (2001, p. 2). These quotations suggest a tendency to view probabilistic accident reconstruction as an exercise in statistical inference. Lindley (1991), and more recently Schum (2000) have argued though that statistical inference problems are only a subset of the forensic problems to which probabilistic methods can be applied. Although reasonable people can disagree on how to define the discipline of statistics, statistical problems typically involve making inferences about how some characteristic is distributed over a population of entities, using measurements made on a sample from that population. Probabilities are used to express uncertainty about parameters characterizing the population distribution, and this uncertainty can in principle be reduced by increasing the sample's size. In contrast, the usual objective of an accident reconstruction is to make inferences about an individual event, and probabilities are assigned to statements about that event. When an individual accident can be regarded as exchangeable with the members of a reference population, pre-established characteristics of that population could be used to make these probability assignments, but once the characteristics of the reference distribution are known, information about additional accidents in the reference population reveals nothing more about the accident at hand.

The confusion concerning the appropriate role of probability in accident reconstruction can at least in part be attributed to two fundamentally different meanings attached to probability statements, on the one hand referring to expected relative frequencies in repeated trials of "chance setups," (Hacking 1965) and on the other referring to a degree of credibility assigned to propositions. This duality was apparently present in the initial development of probability theory in the 17th century (Hacking 1975), recurs in philosophical treatments of probability (e.g. Carnap 1945; Lewis 1980), and has reappeared in the study of logics for artificial reasoning (Halpern 1990). The view we shall adopt here is that an accident reconstruction produces expert opinion about a particular, past event. To quote Baker and Fricke: "Opinions or conclusions are the products of accident reconstruction. To the extent that reports of direct observations are available and can be depended on as facts, reconstruction is unnecessary." (1990, p. 50-4) Statements of opinion can be more or less certain however, depending on the certainty attached to their premises. If this uncertainty is graded using probabilities, then the probability calculus can be used as a logic to derive the uncertainty attached to conclusions. This approach is what Howson (1993) calls "personalistic Bayesianism" but before continuing it should be noted that it is by no means the only alternative available. Debate continues as to when or even if the probability calculus is the appropriate logic for uncertain reasoning, and the issue shows no signs of being resolved anytime in the near future. The development of logics to capture aspects of reasoning under uncertainty is an active area of research, and an overview of some of this work can be found in Dubois and Prade (1993). Summaries of the arguments for and against the Bayesian approach can be found in Howson and Urbach (1993) and Earman (1992).

**Accidents, Probability, and Possible Worlds**

This use of probability can be brought into sharper focus by considering a simpler model of a vehicle/pedestrian collision, having just three Boolean variables. Variable $v$ denotes the vehicle's initial speed, and takes on the value 0 if the vehicle was not speeding and the value 1 if it was. Variable $x$ denotes the vehicle's initial distance, and takes on the value 0 if this distance was "short" and 1 if it was "long." (The problem of determining exactly what is meant by "short" and "long" will, for the time being, be ignored.) Variable $y$ denotes whether or not a collision takes place, with 0 being no collision and 1 being collision. $y$ is assumed to be related to $v$ and $x$ via the structural equation

$$y = (1-x) + x*v \tag{2}$$

where + and * denote Boolean addition and multiplication, respectively. In words, a collision occurs if either the initial distance is short ($1-x=1$), or if the initial distance is long but the vehicle is speeding ($x*v=1$). Since all variables are Boolean, the relationship between $y$, $x$ and $v$ can be tabulated as in Table 1.

In Table 1 each assignment of values to $v$ and $x$ determines a possible way the vehicle/pedestrian encounter could have occurred. The rows of such tables have been variously referred to as "states of affairs," "scenarios," or "system states," but a long-running practice in philosophical logic (e.g. Lewis 1976), which is becoming increasingly common in research on artificial intelligence (e.g. Bacchus 1990; Halpern 1990) is to follow Leibniz, and call them "possible worlds." Uncertainty can then arise in an accident reconstruction when the available evidence is not sufficient to determine which possible world was the actual one. For example, suppose one is interested in whether or not the vehicle was speeding, but the only evidence is that the accident occurred ($y$=1). Table 1 shows that the condition $y$=1 eliminates world 3 as a possibility, but of the remaining three worlds at least one has $v$=0 and one has $v$=1, so the best that can be said is that it is possible, but not necessary, that the vehicle was speeding. On the other hand, suppose that a reliable witness reported that the initial distance was "long" ($x$=1) when the pedestrian entered the road. Only world 4 has $x$=1 and $y$=1, and in this world the $v$=1, so here the evidence implies that the vehicle was speeding.

In any given possible world a statement is either true or false, so uncertainty about a statement arises when a set of possible worlds contains some members where that statement is true, and other members where it is false. Uncertainty can be modeled by placing a probability distribution on the set of possible worlds, so that the probability attached to a statement is simply the probability assigned to the set of possible worlds where that statement is true. For example, suppose that each of the possible worlds in Table 1 is regarded as *a priori* equally probable, so that each has a prior probability of 1/4. One then observes that a collision has occurred. The conditional probability of speeding given that the collision occurred is then

$$P[v=1|y=1] = P[v=1 \ \& \ y=1]/P[y=1] = (1/4 + 1/4)/(1/4 + 1/4 + 1/4) = 2/3. \qquad (3)$$

This possible worlds approach can also be used to specify truth conditions for counterfactual statements, such as "if the vehicle had not been speeding, the collision would not have occurred," by considering what is the case in the "closest" possible world where the antecedent is true. That is, a counterfactual conditional is said to be true in this (the actual) world, if its consequent is true in the closest possible world where the antecedent is true. For instance, suppose the actual world is world 4 ($v$=1, $x$=1) and world 3 ($v$=0, $x$=1) is taken to be the world closest to world 4, but having $v$=0. Letting $y_{v=0}=0$ stand for the counterfactual claim that had $v$ been 0, $y$ would have been 0, Table 1 shows that since $y$=0 is true in the possible world 3, $y_{v=0}=0$ should be taken as true in the actual world 4. On the other hand, $y_{v=0}=0$ should not be taken as true in world 2 ($v$=1, $x$=0) if world 1 ($v$=0,$x$=0) is taken as the closest with $v$=0. As with indicative statements, probabilities of counterfactual statements can be determined by computing the probability assigned to the set of possible worlds where that statement is true. For example, again treating the possible worlds as *a priori* equally probable, the probability that speeding was a necessary cause of the collision can be evaluated as

$$P[y_{v=0}=0|y=1] = P[y_{v=0}=0 \text{ and } y=1]/P[y=1] = 1/3 \qquad (4)$$

This simple example illustrates both a deterministic and a probabilistic approach to accident reconstruction, and these two approaches have a similar structure. The deterministic reconstruction started with a statement describing an observation, ($y$=1) and then added a structural premise stating the assumed causal relation between this outcome and the variables describing the initial conditions. Boolean algebra, supplemented with a possible worlds semantics for counterfactual conditionals, was then used to derive statements about the initial conditions, and about the causal connection between the initial conditions and the outcome. Ideally, the premises of a reconstruction argument should be strong enough to logically imply definite conclusions about the accident, but as often as not the appearance of deductive certitude is achieved by adding premises whose certainty is questionable. The probabilistic reconstruction also began with the observation and the structural premises, but then supplemented these with a probabilistic premise, in this case a prior probability distribution over the possible worlds. It was then possible to use the probability calculus to derive probabilistic conclusions about the accident's provenance.

These ideas are not new. The distinction between probabilities as relative frequencies for actual world populations, and probabilities as measures on models of interpreted formal languages (what we are calling possible worlds) can be found in Carnap (1971) while Lewis (1973) and Stalnaker (1968) have developed the idea that the truth conditions for a counterfactual conditional are given by what is true in closest possible worlds. Lewis (1976) illustrates how these notions can be combined to define probabilities of counterfactual conditionals, and Balke and Pearl (1994) have shown how this approach can be applied to a wide class of inference problems using Bayesian network methods. Chapters 7-9 of Pearl (2000) describe a more detailed development of Balke and Pearl's approach, based on what Pearl calls "causal models." To specify a causal model, one first identifies a set of background variables and a set of endogenous variables, and then for each endogenous variable specifies a structural equation describing how that variable changes in response to changes in the background or other endogenous variables. A possible world is then determined by an assignment of values to the model's background variables. In Greatrix's example, the vehicle's initial distance ($x$), its speed ($v$), the braking deceleration ($a$) and the driver's reaction time ($t_p$) can be taken as background variables, while the length of the skidmark ($s$) and a collision indicator ($y$) are endogenous. The structural equation for the skidmark would then be

$$s(v,a) = -v^2/(2a), \tag{5}$$

while the structural equation of the collision indicator would be

$$y(x,t_p,v,a) = \begin{cases} 1, \text{ if } x < vt_p - v^2/2a \\ 0, \text{ otherwise}. \end{cases} \tag{6}$$

Structural models are especially useful in assessing the plausibility of causal claims because they allow one to give an unambiguous definition of truth conditions for a class of causal statements, along the lines of the closest possible world approach outlined above. For example, suppose that in the actual world $v$=18 meters/sec, $a$ = -7 meters/sec$^2$, $x$ = 40 meters, and $t_p$=1.5 seconds. Then in the actual world $x$=40 meters while $vt_p-v^2/2a$ = 50.1 meters, so by equation (6) $y$=1 and the collision occurs. Previously, the causal effect of speeding was informally defined as whether or not, other things equal, the collision would not have occurred if the vehicle had not been speeding. This "other things equal" condition can be made explicit by defining the closest possible world as the one where all background variables have the same value except for $v$, which is set to $v$=13.4 meters/sec. In this world $x$ = 40 meters, while $vt_p$ - $v^2/2a$ = 32.9 meters, implying $y$=0. So in the actual world, $y_{v=13.4}$=0 is true, and speeding could be considered a causal factor for the collision.

In the Greatrix example, a deterministic assessment of whether or not obeying the speed limit would have prevented a pedestrian accident consisted of three steps:

(a) estimating the vehicle's initial speed and location, using the measured skid marks and nominal values for $a$ and $t_p$,

(b) setting the vehicle's initial speed to the counterfactual value,

(c) using the same values of $a$ and $t_p$ along with the counterfactual speed to predict if the vehicle would then have stopped before hitting the pedestrian.

Each assignment of values to the background variables corresponds to a possible world, and placing a prior probability distribution over the possible worlds produces what Pearl calls a probabilistic causal model. As before, the probability assigned to a statement about the accident is determined by the probability assigned to the set of possible worlds in which that statement is true. This applies both to factual statements, such as "the vehicle was speeding," and to counterfactual statements, such as "if the vehicle had not been speeding, the collision would not have occurred." Steps (a)-(c) correspond, in probabilistic causal models, to what Pearl calls abduction, action, and prediction. For the problem of assessing the probability that $y_{v=v*}$=0, given a measured skidmark $s$, these would involve

(A) Abduction: compute P[$x$,$t_p$,$v$,$a$ | $s$];

(B) Action: set $v$=$v$*;

(C) Prediction: compute P[$y_{v=v*}$ =0 |$s$].

In our simple Boolean example, where the number of possible worlds was finite and small, the abduction step could be carried out by a simple application of the definition of conditional probability, while the prediction step simply required summing the posterior probabilities over possible worlds. For more complicated models however this direct approach is not computationally feasible, but Balke and Pearl (1994) have shown that for models which can be represented as Bayesian networks, steps (A)-(C) can be carried out by applying Bayesian updating to an appropriately constructed "twin network," where the original Bayesian network has been augmented with additional nodes representing the closest possible world. In accident reconstruction the structural equations will often be nonlinear and involve several arguments, while many of the underlying variables will be represented as continuous quantities. This means that the exact updating methods developed for discrete or normal/linear Bayesian networks (Jensen 1996) can applied only after constructing a discrete approximation of the reconstruction model. Alternatively, Monte Carlo computational methods could be used to compute approximate but asymptotically exact updates for the original reconstruction model, and this latter approach is used in what follows.

## Application to Four Illustrative Accidents

These ideas will be applied to four actual accidents, three involving a vehicle and a pedestrian, and one involving two vehicles at an intersection. The first of the vehicle/pedestrian accidents is Greatrix's example described above, while the other two are taken from a group of fatal accidents investigated by the University of Adelaide's Road Accident Research Unit (RARU) (McLean et al 1994). The scenario for the pedestrian accidents runs as follows. The driver of a vehicle traveling at a speed of $v$ notices an impending collision with a pedestrian, traveling at a speed $v2$, when the front of the vehicle is a distance $x$ from the potential point of impact. After a perception/reaction time of $t_p$ the driver locks the brakes, and the vehicle decelerates at a constant rate $-fg$, where $g$ denotes gravitational acceleration and $f$ is the braking "drag factor" which, following a standard practice in accident reconstruction, expresses the deceleration as a multiple of $g$. After a transient time $t_s$ the tires begin making skid marks, and the vehicle comes to a stop, leaving a skidmark of length $s1$. Before stopping, the vehicle strikes the pedestrian at a speed of $vi$, and the pedestrian is thrown into the air and comes to rest a distance $d$ from the point of impact. In addition, if the pedestrian was struck after the vehicle began skidding, it may be possible to measure a distance $s2$ running from the point of impact to the end of the skidmark. Figure 1 illustrates the collision scenario (with $x_s$ denoting the distance traveled during the braking transient.) The abduction step of Pearl's three-step method involves computing the posterior distributions of the model's unobserved variables, given some subset of the measurements $d$, $s1$, $s2$. Figure 2 represents the collision model as a directed acyclic graph summarizing the conditional dependence structure, and including nodes to represent the counterfactual speed $v^*$ and the counterfactual collision indicator $y^*$. To complete the model it is necessary to specify deterministic or stochastic relations for the arrows appearing in Figure 2, and prior distributions for the background variables $x$, $v$, $t_p$, $t_s$, $v2$, and $f$.

The structural equations for this model have been described elsewhere (Davis 2001; Davis et al. 2002), and so the details will not be repeated. Roughly, the relationship between the expected skidmark lengths and the background variables is governed by the kinematic equation (1), while the measured skidmark is the result of combining random measurement error with the expected length. The coefficient of variation for this measurement error was taken to be 10% (Garrott and Guenther 1982). An empirical relationship between impact speed and throw distance was determined by fitting a model to the results of 55 crash tests between cars and pedestrian dummies, and is described in Davis et al. (2002). Finally, the counterfactual collision variable $y^*$ was taken to be zero (i.e. the collision was avoided) if either the vehicle stopped before reaching the collision point, of if the pedestrian managed to travel an additional 3.0 meters before the vehicle arrived at the collision point.

Selection of the prior probability distributions for the background variables was less straightforward. As indicated earlier, these distributions should be interpreted as expert opinions concerning plausible ranges of values, although statistical information might, in some cases, be used to inform these opinions. The strategy used for these examples was to identify priors that appeared on their face to be consistent with current reconstruction practice. In deterministic sensitivity analyses it is often possible to identify defensible prior ranges for background variables (Niederer 1991), and Wood and O'Riordain argue that, in the absence of more specific information, uniform distributions restricted to these ranges offer a plausible extension of the deterministic sensitivity methods (1994, p. 137). Following these suggestions, the reconstructions described in this paper used uniform prior distributions. Specifically, the range for $f$ was [0.55,0.9], and was taken from Fricke (1990, p. 62-14), where 0.55 corresponds to the lower bound for a dry, traveled asphalt pavement and 0.9 is what Fricke considers a reasonable upper bound for most cases (1990, p. 62-13). The range for the perception/reaction time, $t_p$, was [0.5 seconds, 2.5 seconds], which brackets the values obtained by Fambro et al. (1998) in surprise braking tests, and the midpoint of which

(1.5 seconds) equals a popular default value (Stewart-Morris 1995). For the braking transient time, Neptune et al. (1995) reported values ranging between 0.1 and 0.35 seconds for a well-tuned braking system, while Reed and Keskin (1989) reported values in the range of 0.4-0.5 seconds, so the chosen range was [0.1 seconds, 0.5 seconds]. The bounds for the pedestrian speeds ($v2$) were different for the three cases, varying according to the age and sex of the pedestrian, and were selected to include the 15th and 85th percentile figures for children's running speeds tabulated in Eubanks and Hall (1998 pp. 82-86). The ranges for the initial distance and initial speed were chosen to be wide enough that no reasonable possibility would be excluded *a priori*. The range for $v$ was [5 meters/sec, 50 meters/sec], but initial attempts to apply Markov Chain Monte Carlo methods revealed convergence problems when the initial distance was selected as a background variable. To remedy this the model was re-parameterized with the distance from the collision point to the start of braking as a background variable, and the prior for this initial braking distance was taken to be uniform with range [0 meters, 200 meters]. Note that the initial distance is then simply the sum of this initial braking distance and the distance traveled during the perception/reaction time. Table 2 displays information for each of the three vehicle/pedestrian accidents, including the age and sex of the pedestrian, the lower and upper bounds for the pedestrian's running speed used in the reconstruction, and the skidmark and throw distance measurements. The computer program WinBUGS (Spiegelhalter et al 2000) was used to generate Monte Carlo samples of the quantities of interest. (An example of the WinBUGS code used for pedestrian collisions is listed in the Appendix.) In each case a 5000 iteration burn-in was followed by 150,000 iterations, with the outcome of every 10th iteration being saved for the MCMC sample. Inspection of traces and autocorrelations indicated no obvious problems with nonstationarity or failure to converge.

As noted earlier, determining liability involves addressing (at least) two basic issues, one concerning the actual driving, and one concerning the causal connection between that driving and the occurrence of the accident. With regard to the first issue, often an important concern is the speeds of the vehicles involved, and the relation of those speeds to any speed limits. With regard to the second issue, an important question is often whether or not an initial speed equal to the legal limit would, other things equal, have been sufficient to prevent the accident. Figures 3-5 display, for each of the three pedestrian accidents, a plot of the posterior probability density for the speed of the involved vehicle and a plot of the probability the accident would have been avoided, as a function of the counterfactual initial speed.

Figure 3, which shows results for Greatrix's example, indicates that the posterior distribution of the vehicle's initial speed is centered at about 45 mph (72 km/h), and that a probable range for the initial speed is between 35 mph (56 km/h) and 55 mph (88 km/h). The posterior probability that the vehicle was traveling at or below the posted speed limit of 30 mph (48 km/h) is essentially zero. Figure 3 also indicates that had the initial speed been at or below the posted speed limit it is very probable that either the driver would have been able to stop before hitting the pedestrian, or the pedestrian would have been able to clear the vehicle's path before collision. So even after allowing for reasonable uncertainty in the vehicle's braking deceleration, in the driver's reaction time and for a fairly substantial measurement error in measuring the skidmarks, it appears highly probable that the driver was speeding, and that speeding was a causal factor in this accident.

Figure 4 shows results for the RARU's case 89-H002, in which a 5 year-old boy ran into a road from behind a parked car, stopped briefly in the middle of the road, and was struck when he attempted to run across the far lane. The speed limit on this road was 60 km/h. In this case the posterior probability of the vehicle's initial speed is centered at about 73 km/h, with a probable range being between 60 km/h and 90 km/h. The posterior probability that the initial speed was greater than 60 km/h was about 0.985, and the probability the collision would have been avoided had the initial speed been equal to 60 km/h was equal to about 0.84. Although less obvious than in the Greatrix example, again it appears that the vehicle was probably speeding, and that speeding was probably a causal factor in this accident. Figure 5 shows similar results for the RARU's case 91-H025. For this case the posterior probability that the driver was exceeding the 60 km/h speed limit is only about 0.5, and the probability the accident would have been prevented had the initial speed been 60 km/h is only about 0.27. Unlike the first two cases, here it appears difficult to maintain that the driver should be held liable.

The fourth illustrative accident involved a collision between two vehicles at a two-way stop-controlled intersection in the United States, and is described in Fricke (1990, p. 68-27). In this accident vehicle #1 attempted to turn left onto a state highway from a stop-controlled approach, and was struck broadside by vehicle #2, which was westbound on the highway. Vehicle #2 left a skidmark of about 73 feet (22.3 meters) prior to impact, and after impact the two vehicles slid together in a northwesterly direction, across the concrete surface of the intersection and onto a grassy shoulder, before coming to a stop. Test skids indicated that drag factors of 0.75 and 0.45 were plausible for the concrete and grass surfaces, respectively. Because the two vehicles followed a common direction after the impact, a "forward" reconstruction, in which the conservation-of-momentum equations are used to predict the after-impact speeds and directions, was not feasible, so a "backward" approach, where one estimates speeds working back from the point of rest, was used instead. WinBUGS was used to compute Monte Carlo estimates of the posterior distributions for the background variables, including the initial speed of vehicle #2, as well as estimates of the probability the collision would have been avoided as a function of different counterfactual initial speeds. The WinBUGS code used to generate these estimates has been listed in the Appendix. The collision was treated as having been avoided if either vehicle #2 managed to stop before reaching the point of collision or if vehicle #1 managed to travel an additional 20 feet (6.1 meters) before vehicle #2 arrived at the collision point. Because the objective for this example was to see if Bayesian reconstruction could produce results similar to Fricke's deterministic approach, relatively narrow uncertainty ranges were used. Skidmark measurement error was assumed to be normal with a standard deviation of five feet (1.5 meters), the uncertainty for measured angles was taken to be "2.5$^{o}$ around Fricke's values, and the uncertainties in the drag factors were taken to be ".05.

Figure 6 shows the posterior probability density for vehicle #2's initial speed and the probability of avoidance as a function of initial speed. Inspection of this figure reveals that the initial speed of vehicle #2 was most probably around 92 mph (148 km/h), and the bounds of a 95% credible interval for this speed were 86 mph (141 km/h) and 99 mph (162 km/h). Also, for initial speeds below about 60 mph (97 km/h) it is almost certain, other things equal, that this accident would have been avoided. Taking the posted speed limit on the state highway as 55 mph (88.5 km/h), it can be concluded that vehicle #2 was quite probably speeding, and that speeding was quite probably a causal factor for this accident.

**Application to Fatal Accidents in Minnesota**

As part of this study, Minnesota Dept. of Transportation (MNDOT) personnel identified all fatal accidents listed as occurring on state highways between January 1 1997 and June 30 2000 that were near on of MNDOT's automatic speed recording stations. Visits to district offices of the Minnesota State Patrol then produced detailed investigation information for 46 fatal accidents, and of these seven involved collisions between motor vehicles and pedestrians while nine were multiple vehicle collisions at intersections. For four of the pedestrian collisions and two of the intersection collisions it was possible to apply the Bayesian network methods described above to estimate initial vehicles speeds and compute probabilities of avoidance. The results of these computations are displayed in Figures 7-12.

Figure 7 shows the results for a collision between a passenger car and a pedestrian running across on an on-ramp which entered into a four-lane divided highway. The posted speed limit for the highway was 55 mph, and it is unclear whether or not the driver was exceeding this limit. However, the conditions of this collision were such that only at speeds below about 45 mph would the probability of avoidance be greater than 0.5, and so it would be difficult to conclude that speeding was a cause of this accident. Figure 8 depicts results from a collision between a passenger car and a pedestrian running across an Interstate highway, where the posted speed limit was 70 mph. Here it is very clear that the driver was probably not exceeding the posted limit, and that only for initial speeds below about 48 mph is the probability of avoidance greater than 0.5. So for this accident we would be inclined to conclude that with high probability the driver was not speeding, and that speeding was not a causal factor for this accident. Interestingly, if the driver had been travelling at the old National Maximum Speed Limit of 55 mph, the probability the collision would have been avoided is only about 0.18. Figure 9 shows the results for a collision between a passenger car and a pedestrian running across part of a four-lane divided highway, in a construction zone. The posted speed limit was 50 mph, and it is unclear whether or not the driver was exceeding this limit. But exceeding the posted limit was probably not a causal factor for this accident. Finally, Figure 10 shows results from a collision between a passenger car and a pedestrian running from a median across two lanes of a four-lane highway. The collision occurred at a signalized intersection where the pedestrian did not have the right of way. The posted speed limit was 50 mph, and here the probability the driver was exceeding the limit equals about 0.82. However, since the probability of avoidance at 50 mph is only about 0.20 it does not appear the speeding should be considered a causal factor for this accident.

Figure 11 shows results from a collision between a left-turning vehicle and an eastbound vehicle on a four-lane county highway. The intersection was not signalized, the eastbound driver had the right-of-way, and the posted speed limit was 55 mph. The results shown in Figure 11 are for the eastbound vehicle. Here it is clear that the eastbound driver was not exceeding the posted limit, and in fact was probably travelling well below the limit. Clearly, exceeding the posted limit was probably not a causal factor in this collision. Finally, Figure 12 depicts results from a collision between two pickup trucks at a two-way stop controlled intersection. The major street was a four-lane divided highway with a posted speed limit of 55 mph, and the results in Figure 12 are for the major road vehicle. Here it is quite probable that the major road driver was exceeding the posted speed limit, and the probability of avoidance at the posted limit is about 0.56. Here it appears quite probable that the major road driver was speeding, and it is more probable than not that speeding was a causal factor for this accident.

**Conclusion**

Statistical methods are now commonly used to study relationships between traffic engineering actions and the incidence of accidents, but in Technical Report 1 we pointed out how failure to attend to the underlying processes which generate accidents can lead to misinterpretation of statistical results. In Technical Report 1 we alluded to Snow's work on identifying the mechanism of transmission for cholera, which utilized both statistical and clinical methods to test hypotheses about transmission. The main objective of the work described in this report was to develop and apply a method for using clinical investigations of road accidents to support and inform traffic engineering decisions. The basic idea is straightforward: if the oft-reported statistical associations between average speed, or changes in speed limits, and the occurrence of fatal and serious road accidents reflect a causal connection, then it ought to be possible to identify speed as a causal factor in at least a substantial fraction of actually occurring accidents. Implementing this idea is bit more difficult however, and our general approach has been adapt ideas from formal logic and artificial intelligence in order to specify truth conditions for causal claims, and then use probability theory to assess the degree to which evidence supports or contradicts those claims. This leads to what is essentially a

Bayesian network approach to doing traffic accident reconstruction, and in this report we developed some of the connections between accident reconstruction, forensic inference, and probabilistic reasoning.

Our Bayesian network approach was applied to ten actual accidents involving either collisions between vehicles and pedestrians or collisions between vehicles at intersections. Three of the vehicle/pedestrian collision occurred on lower speed roads, and in two of these it appeared both that the vehicle was probably speeding prior to the collision and that speeding could be considered a causal factor for these collisions. In the third low-speed road collision it was unclear whether or not the driver was speeding but speeding probably was not a causal factor. The other four pedestrian collisions occurred on higher speed roads in Minnesota, and in each the conditions were such that the driver had the right of way. In three of the four it was not obvious that the driver was speeding, and speeding could not be considered a causal factor, while in the fourth it did appear that the driver was speeding but speeding still could not be considered a causal factor. When considering the two-vehicle intersection collisions, all three occurred on higher speed roads, and in two it was clear that the vehicle with the right-of-way speeding, and that speeding was probably a causal factor for these collisions. In the third intersection collision it was clear the right-of-way vehicle was not speeding and that speeding was not a causal factor.

With the exception of Fricke's (1990) example collision, it does not appear that the vehicles involved in these collisions were travelling at exceptionally high speeds, and it seems reasonable to suspect that other drivers traversed the accident scene at about the same time as fast or even faster without being involved in collisions. This in turn suggests that speeding by itself is not sufficient to cause a crash, nor does it appear to be necessary. Rather, for these crashes it appears that first an accident avoidance situation is triggered, by a pedestrian or other vehicle attempting to cross an oncoming vehicle's path, and then speed interacts with the other characteristics of that particular situation to determine whether or not the crash occurs. The focus of this report has been on determining whether or not speeding should be considered a causal factor in road accidents, and causal factors are analogous to what Pearl (2000) calls necessary causes, since their absence is sufficient to prevent the accident. Road accidents usually result though from a particular combination of several causal factors, none of which was sufficient in and of itself to produce the accident. An intriguing extension of this Bayesian approach would be toward identifying what Baker calls the cause of an accident, that is, the complete set of causal factors which, if reproduced, would result in an identical accident (1975, p. 284).

Finally, the tentative conclusions advanced in this report apply only to the types of accidents considered here. Single vehicle run-off road accidents will be considered in Technical Report 3.

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**Table 1. Possible Worlds and a Probability Distribution for the Simple Boolean Collision Model**

| World | *v* | *x* | *y* | $y_{v=0}$ | P[.] |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 1/4 |
| 2 | 1 | 0 | 1 | 1 | 1/4 |
| 3 | 0 | 1 | 0 | 0 | 1/4 |
| 4 | 1 | 1 | 1 | 0 | 1/4 |

**Table 2. Features of Three Reconstructed Pedestrian Accidents. Distances are in Meters, Speeds in Meters/Second.**

| | | | | Pedestrian Characteristics | | | |
|---|---|---|---|---|---|---|---|
| | Scene Data | | | | | Running Speed | |
| Case | *s1* | *s2* | *d* | Sex | Age | Lower | Upper |
| Greatrix | 22 | 10 | - | M | 7 | 1.8 | 5.1 |
| RARU 89-H002 | 23.5 | 8.1 | 14.8 | M | 5 | 2.5 | 4.5 |
| RARU 91-H025 | 14.9 | 4.5 | - | F | 9 | 3.3 | 5.6 |

**Figure 1. Major Variables Appearing in the Vehicle/Pedestrian Collision Model.**

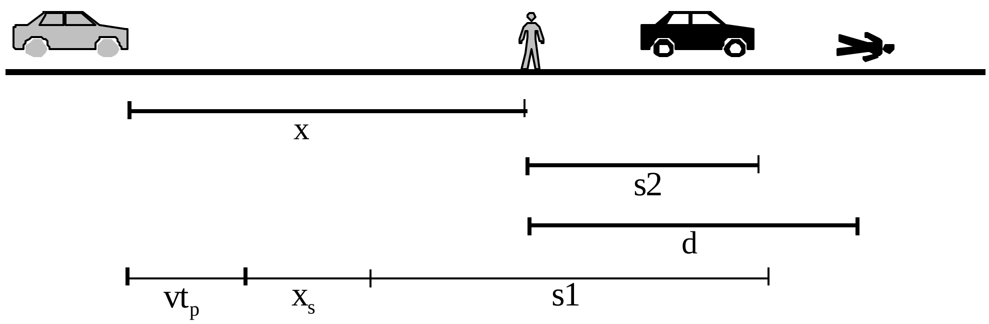


**Figure 2. Directed Acyclic Graph Representation of Vehicle/Pedestrian Collision Model.**

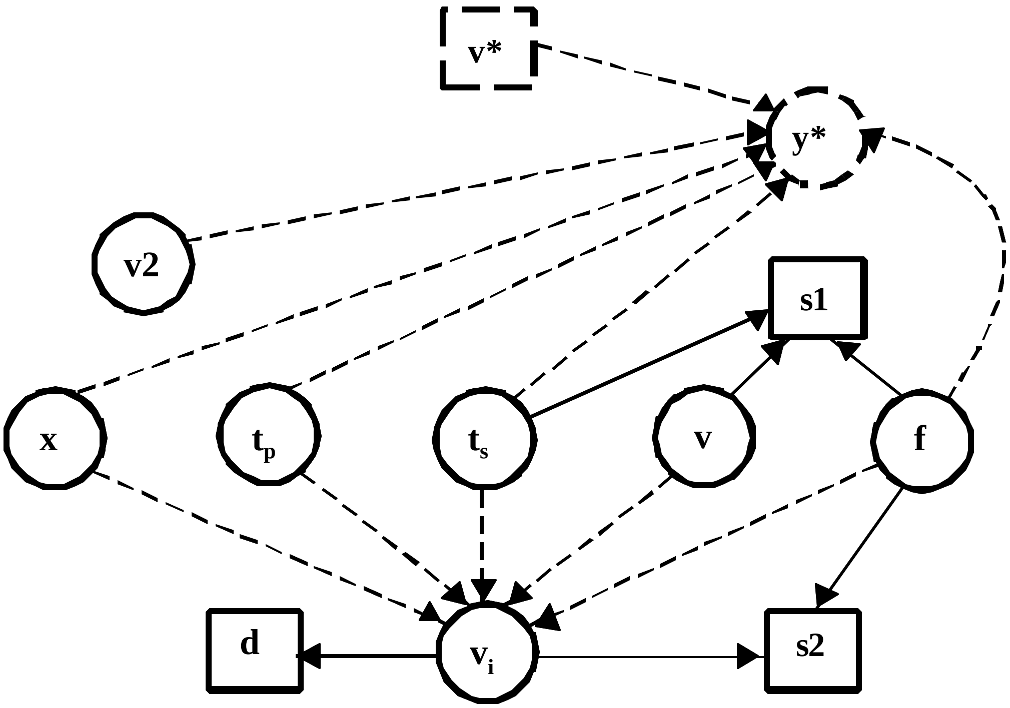


**Figure 3. Posterior Density for Vehicle's Initial Speed, and Probability of Avoidance as a Function of Initial Speed: Greatrix's Example.**

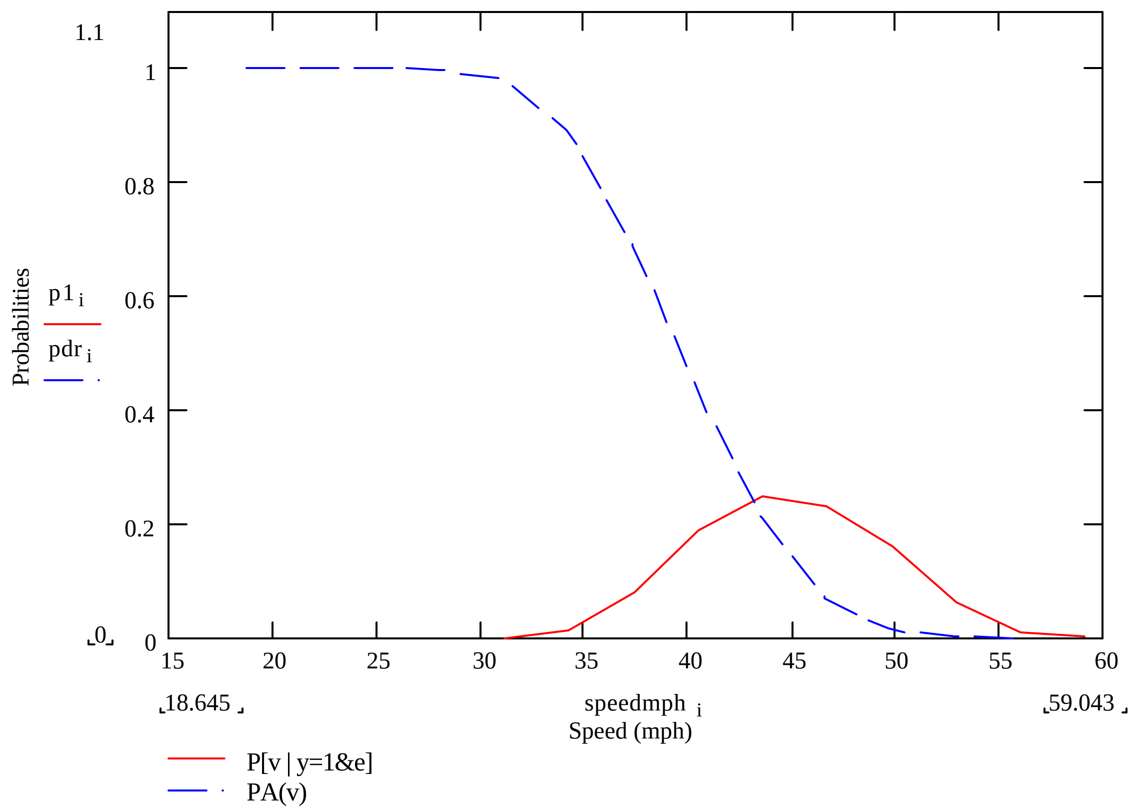


**Figure 4. Posterior Density for Vehicle's Initial Speed, and Probability of Avoidance as a Function of Initial Speed: RARU Case 89-H002.**

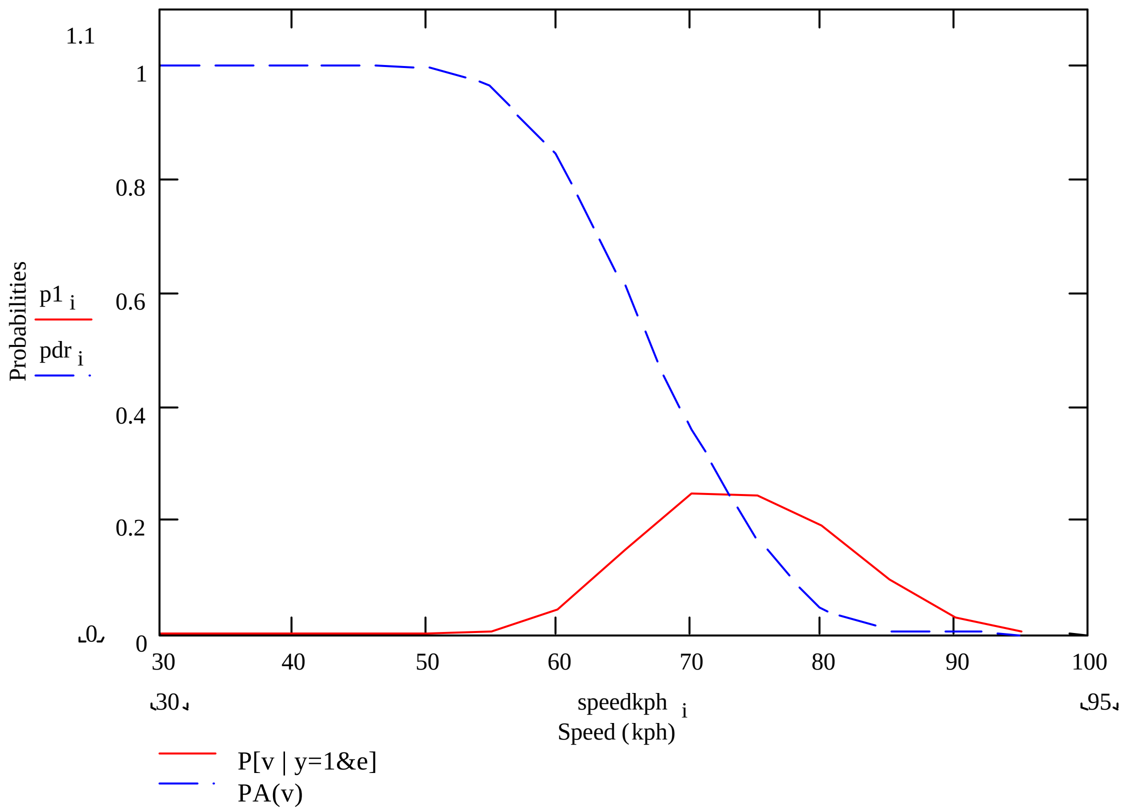


**Figure 5. Posterior Density for Vehicle's Initial Speed, and Probability of Avoidance as a Function of Initial Speed: RARU Case 91-H025.**

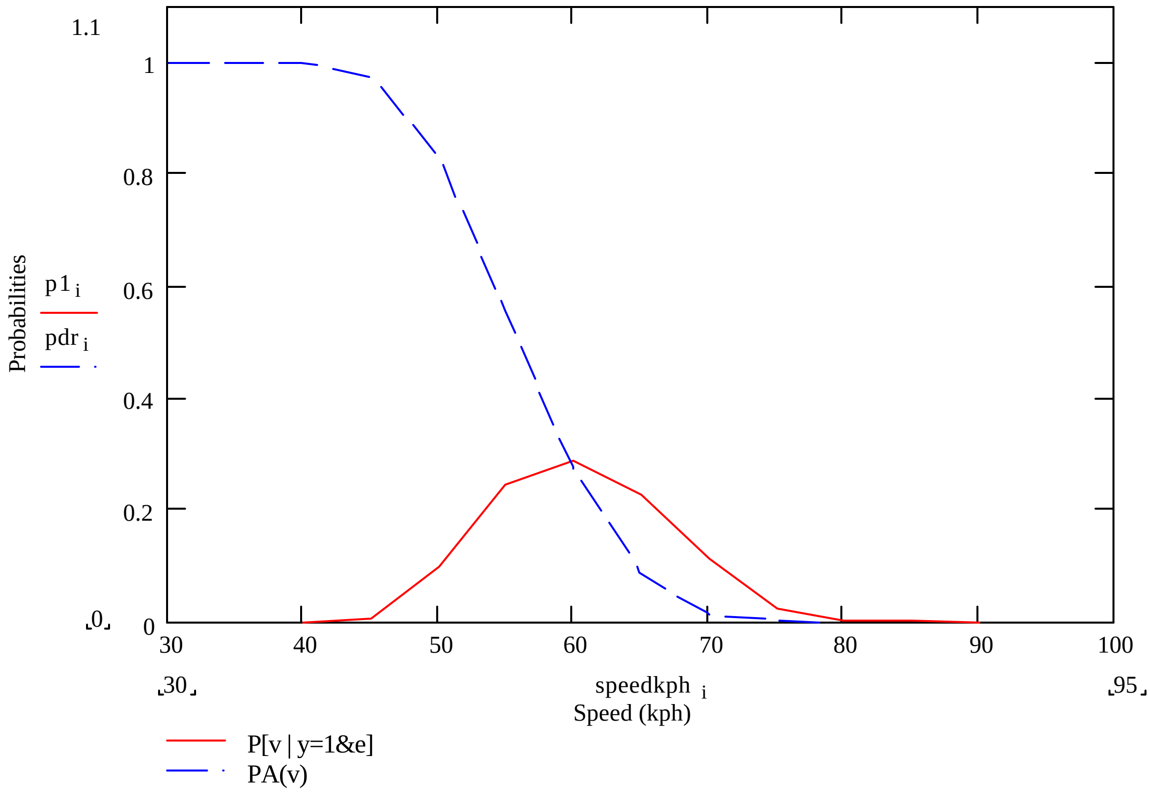


**Figure 6. Posterior Density for Vehicle's Initial Speed, and Probability of Avoidance as a Function of Initial Speed: Vehicle 2 in Fricke 1990 Two-Vehicle Example.**

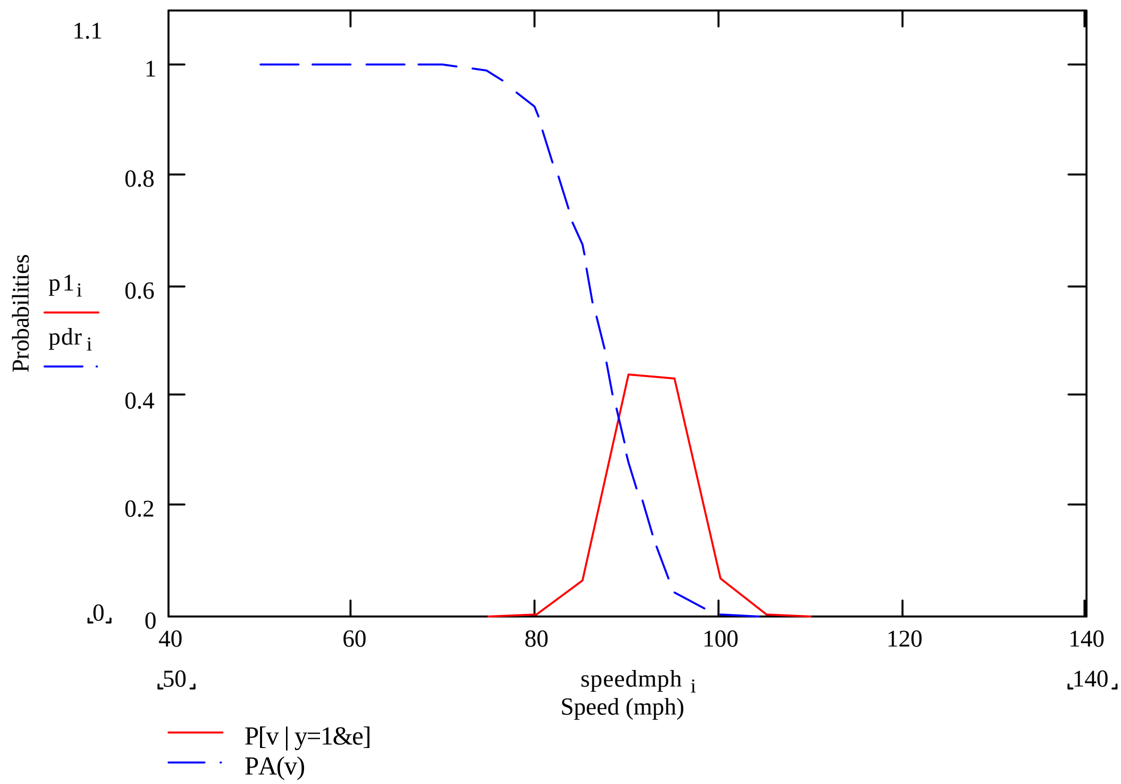


**Figure 7. Posterior Distribution for Vehicle's Initial Speed, and Probability of Avoidance, for a Vehicle/Pedestrian Collision at On-Ramp onto USTH 52 in Rochester. Posted Speed Limit on Highway was 55 mph.**

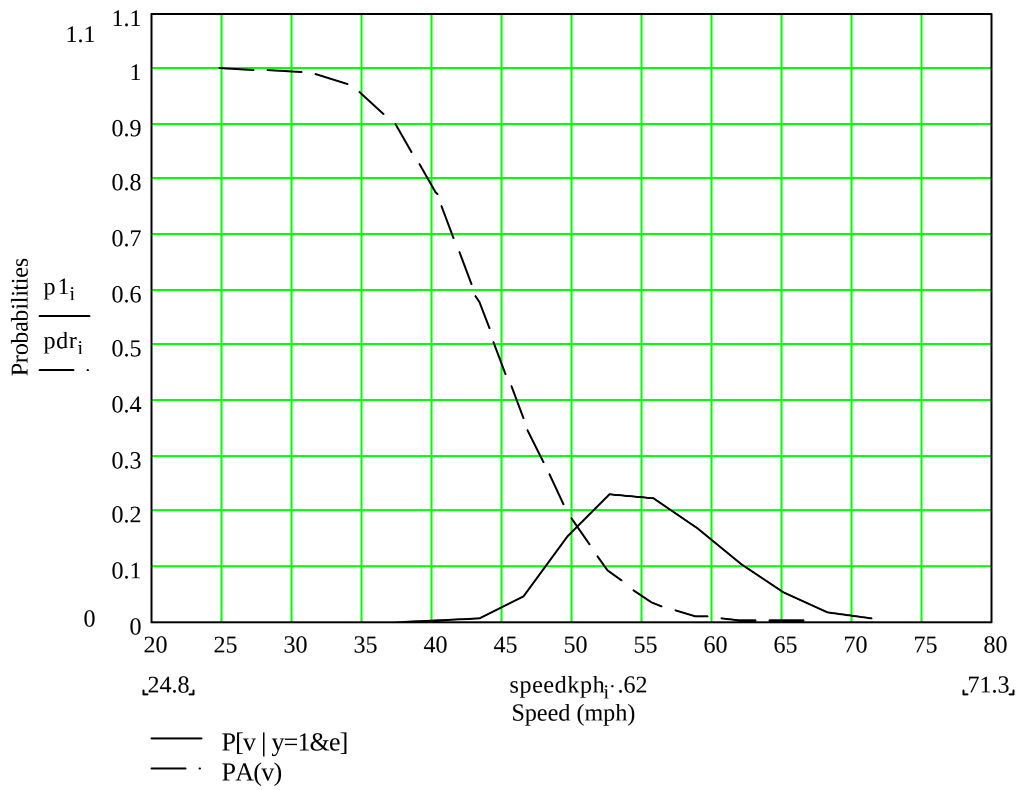


**Figure 8. Posterior Distribution for Vehicle's Initial Speed, and Probability of Avoidance, for a Vehicle/Pedestrian Collision on Northbound ISTH 35W. Posted Speed Limit on Highway was 70 mph.**

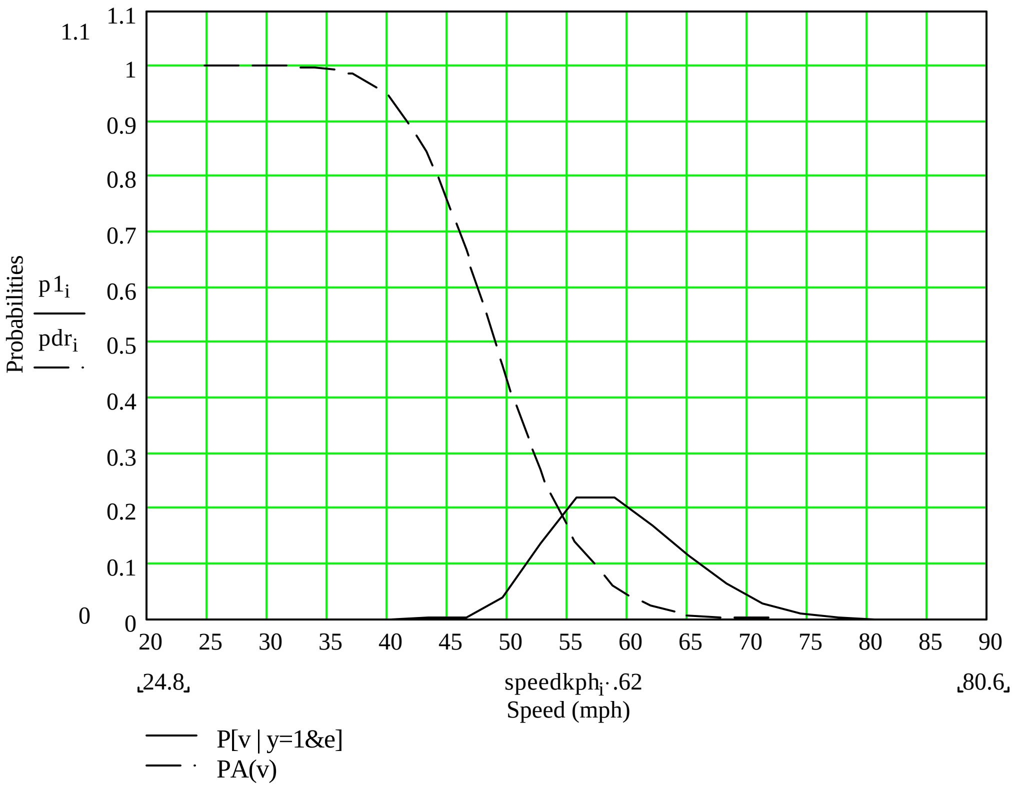


**Figure 9. Posterior Distribution for Vehicle's Initial Speed, and Probability of Avoidance, for a Vehicle/Pedestrian Collision at Northbound USTH 61, in Newport. Posted Speed Limit on Highway was 50 mph.**

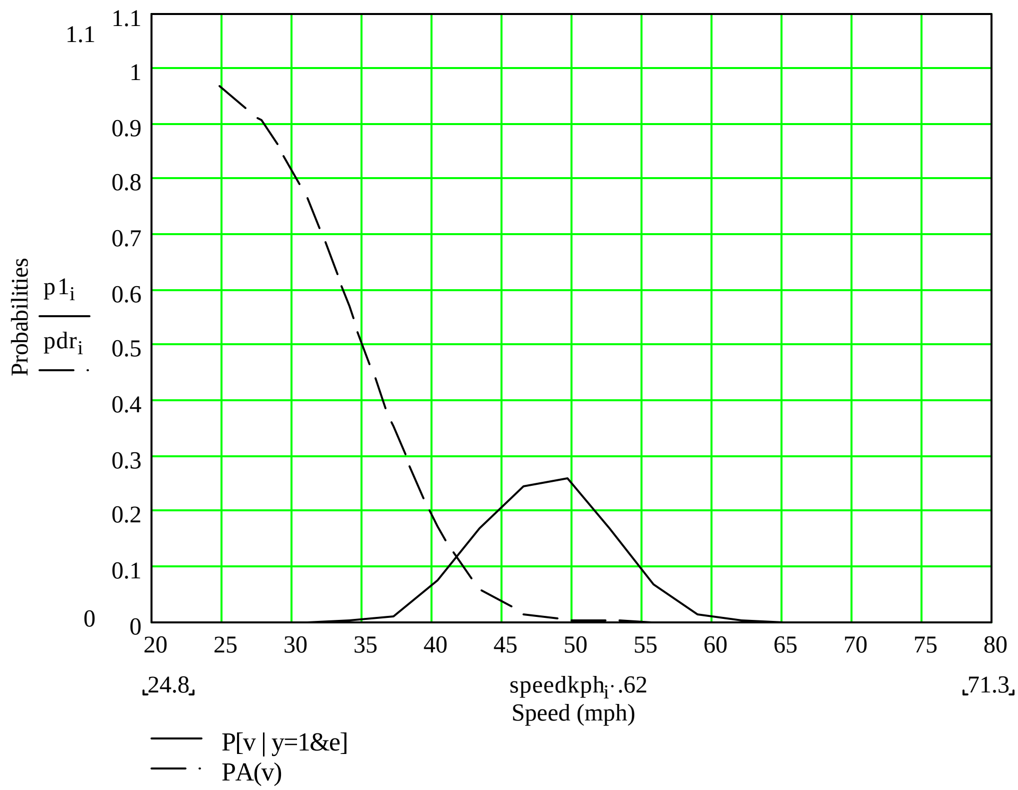


**Figure 10. Posterior Distribution for Vehicle's Initial Speed, and Probability of Avoidance, for a Vehicle/Pedestrian Collision on Westbound MNTH 36, in Stillwater. Posted Speed Limit on Highway was 50 mph.**

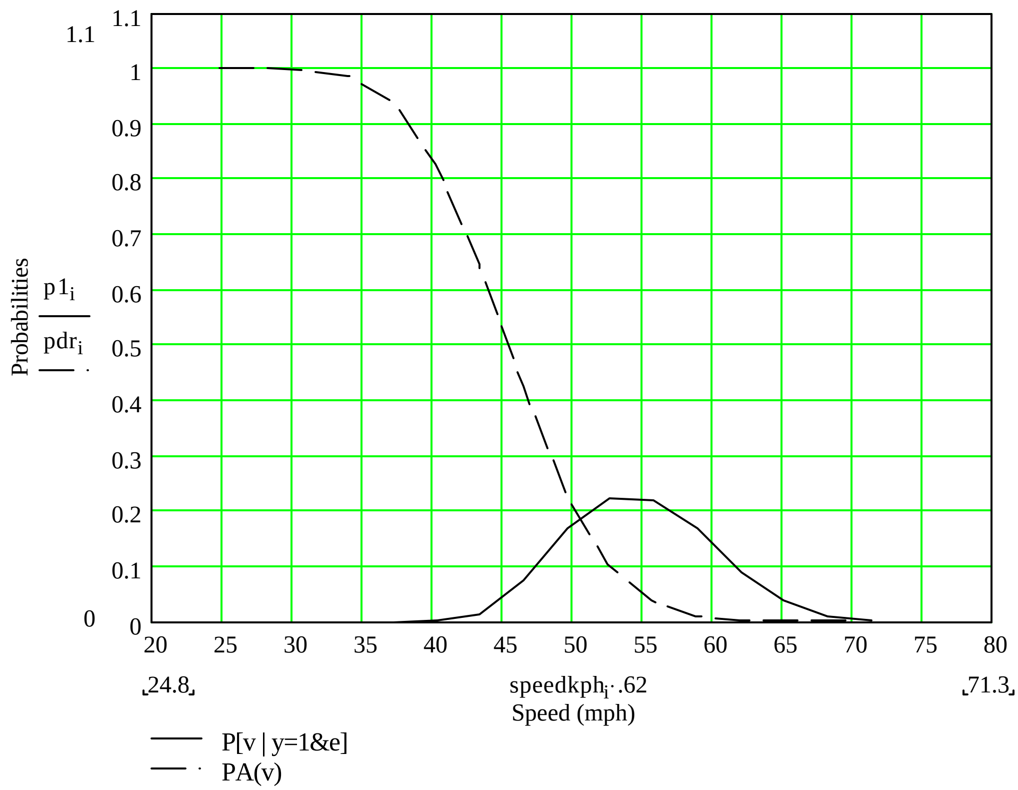


**Figure 11. Posterior Distribution for Vehicle's Initial Speed, and Probability of Avoidance, for Major-Road Vehicle in Two-Vehicle Collision on Anoka CSAH 23. Posted Speed Limit on Highway was 55 mph.**

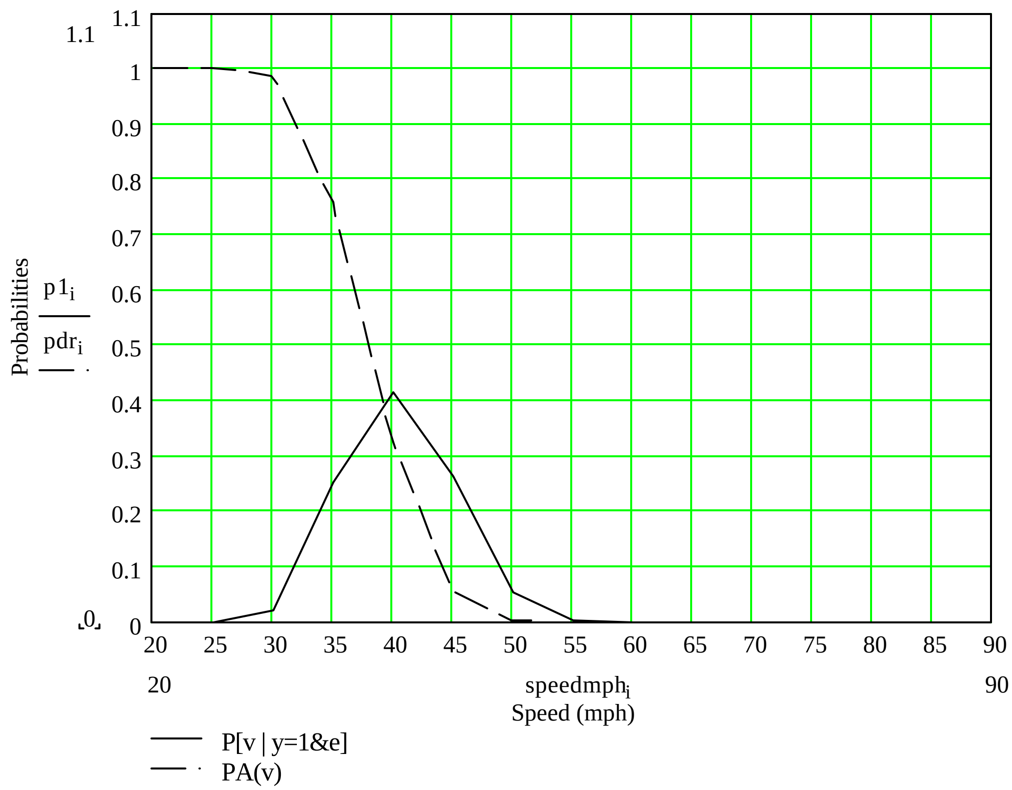
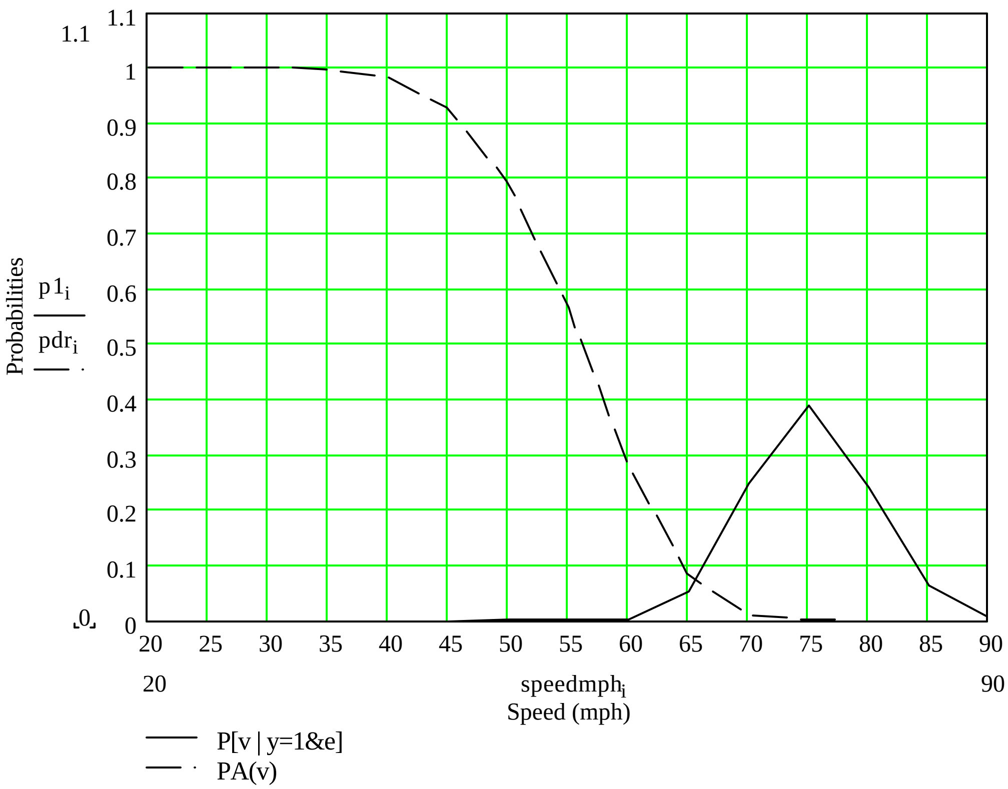


**Figure 12. Posterior Distribution for Vehicle's Initial Speed, and Probability of Avoidance, for Major-Road Vehicle in Two-Vehicle Collision on MNTH 13, near Manchester. Posted Speed Limit on Highway was 55 mph.**



## Appendix

## Example WinBUGS Code for Bayesian Reconstruction of Intersection and Pedestrian Collisions

### Two-Vehicle Collision in Fricke 1990.

```
model momentum
# Fricke 1990  Momentum Example; English units
{
# counterfactual world(s)

        for (I in 1:M) {
        u2.star[i] <- u2.star.mph[i]*1.47
        xbrake.star[i] <- pow(u2.star[i],2)/(2*ap2)
        xprt.star[i] <- u2.star[i]*tp
        xstop.star[i] <- xbrake.star[i]+xprt.star[i]
        stop.star[i] <- step(xinit2-xstop.star[i])
        fullhit.star[i] <- step(xprt.star[i]-xinit2)
        tc1.star[i] <- xinit2/u2.star[i]
        xcrit.star[i] <- (1-stop.star[i])*max(xinit2-xprt.star[i],0)
        tc2.star[i] <-  tp+(u2.star[i]-sqrt(pow(u2.star[i],2)-2*ap2*xcrit.star[i]))/ap2
        tc.star[i] <- fullhit.star[i]*tc1.star[i] + (1-fullhit.star[i])*tc2.star[i]
        pass.star[i] <- step(tc.star[i]-(t.crit+20/v1))
        nohit.star[i] <- 1-(1-stop.star[i])*(1-pass.star[i])
        }

# estimate post-impact speeds from skidmarks

        ag <- mug*g*(wheels/4)
        vp <- sqrt(2*ag*skidg.bar)
        ap <- mup*g*(wheels/4)
        vfin <- sqrt(vp*vp+2*ap*skidp.bar)
        skidg ~ dnorm(skidg.bar,.04)
        skidp~dnorm(skidp.bar,.04)
        mug ~ dunif(.4,.5)
        mup ~ dunif(0.7,.8)
        skidg.bar ~ dunif(20,60)
        skidp.bar ~ dunif(60,100)
```

```
# estimate pre-impact speeds using momentum conservation

        m1 <- wcar1
        m2 <- wcar2

        alpha1 ~ dunif(a1low,a1up)
        alpha2 ~ dunif(a2low,a2up)
        beta ~ dunif(blow,bup)

        c <- 3.141592/180

        v2 <- (vfin*m1*sin(beta*c)+vfin*m2*sin(beta*c))/(m2*sin(alpha2*c))
        v1<-(vfin*m1*cos(beta*c)+vfin*m2*cos(beta*c)-v2*m2*cos(alpha2*c))/(m1*cos(alpha1*c))


# estimate vehicle 2 initial speed and distance

        ap2 <- mup*g
        xx <- max(0,x)
        u2 <- sqrt((v2*v2+2*ap2*xx))
        x.tran2 <- u2*ts-(ap2*pow(ts,2))/2
        x.prt <- tp*u2

        skid21.bar <- x-x.tran2
        skid21 ~ dnorm(skid21.bar,.04)

        xinit2 <-x+x.prt
        limit.fps <- limit*1.47
        speeding <- step(u2-limit.fps)

# estimate vehicle 1's initial speed and critical time

        notime <- step(-x.prt-x)
        fullhit <- step(-x)
        littletime <- step(-x)*(1-notime)
         t.crit <- 0*notime+littletime*(tp+(x/u2))+(1-fullhit)*(tp+((u2-v2)/ap2))
        temp <- step(v1*v1-2*a1*xinit1)
        u1 <- sqrt(temp*(v1*v1-2*a1*xinit1))

        a1 ~ dunif(0,8)
```

```
    tp ~ dunif(0.5, 2.5)
    ts ~ dunif(.1,.5)
    x ~ dunif(-10,150)

    u1.mph <- u1/1.47
    u2.mph <- u2/1.47
    v1.mph <- v1/1.47
    v2.mph <- v2/1.47

 }
```

Data list(g=32.2, skidg=40, skidp=80,wcar1=3600,wcar2=3700,skid21=73,xinit1=53, a1low=-2.5,a1up=2.5, a2low=272.5, a2up=277.5, blow=287.5, bup=292.5,wheels=3, limit=55,M=15, u2.star.mph=c(40,45,50,55,60,65,70,75,80,85,90,95,100,105,110))

Inits list(mup=0.75, mug=.45,alpha1=0, alpha2=275, beta=290 )

**Vehicle Pedestrian Collision Depicted in Figure 10.**

```
model pedmodel
# Case #98409793
# includes counterfactual  injury model
# damage model parameters taken from MNDOT report: Davis, Sanderson and Davuluri (2002)
# metric units
{

# counterfactual world
for (i in 1:M) {
    v.star[i] <- v.star.kph[i]/3.6;
    xbrake.star[i] <- pow(v.star[i],2)/(2*a);
    xprt.star[i] <- v.star[i]*tp;
    xstop.star[i] <- xbrake.star[i] + xprt.star[i]
    stop.star[i] <- step(x.init-xstop.star[i]);
    fullhit.star[i] <- step(xprt.star[i]-x.init);
    tc1.star[i] <- x.init/v.star[i];
    xcrit.star[i] <- (1-stop.star[i])*max(x.init-xprt.star[i],0)
    tc2.star[i] <-tp+(v.star[i]-sqrt(pow(v.star[i],2)-2*a*xcrit.star[i]))/a;
    tc.star[i] <- fullhit.star[i]*tc1.star[i] + (1-fullhit.star[i])*tc2.star[i];
    pass.star[i] <- step(tc.star[i]-(t.ped+t.buffer));
    nohit.star[i] <- 1-(1-stop.star[i])*(1-pass.star[i])
```

```
        v2.imp.star[i] <- fullhit.star[i]*pow(v.star[i],2) +
          (1-fullhit.star[i])*(pow(v.star[i],2)-2*a*(max(0,x.init-xprt.star[i])))
    vi.kph.star[i] <-3.6* sqrt(max(0,v2.imp.star[i]));
    damage.star[i] <- b*vi.kph.star[i] + randam
    slight.star[i] <- step(a1-damage.star[i])*(1-nohit.star[i])
    serious.star[i] <- step(damage.star[i]-a1)*step(a2-damage.star[i])*(1-nohit.star[i])
    fatal.star[i] <- step(damage.star[i]-a2)*(1-nohit.star[i])  }

# data models

      dam.bar <- b*vi.kph
      randam ~ dlogis(0,1)
      damage <- dam.bar+randam
      slight <- step(a1-damage)
      serious <- step(damage-a1)*step(a2-damage)
      fatal <- step(damage-a2)
      psev[1] <- .999*slight+.001/3
      psev[2] <- .999*serious+.001/3
      psev[3] <- .999*fatal+.001/3
      injury ~ dcat(psev[])

   throw.bar <- ahat+2*log(vi);
   throw.tau <- 1/(throw.sig2);
   throw ~ dnorm(throw.bar,throw.tau);

   skid1.bar <- max(0.1,x.brake-x.tran);
   lvi <- max(0.1,log(vi));
   logskid1.bar <- log(skid1.bar);
   logskid2.bar <- 2*lvi-log(2*a);
#     skid2.bar <- pow(vi,2)/(2*a);
   skid1 ~ dnorm(logskid1.bar,skid.tau);
   skid2 ~ dnorm(logskid2.bar,skid.tau);

# braking model
   a <- f*(9.807);
   x.brake <- pow(v,2)/(2*a);
   x.tran <- v*ts-(a*pow(ts,2))/2;
   x.prt <- v*tp;
   xx <- max(x,-x.prt);
   nohit <- step(x-x.brake);
```

```
      fullhit <- step(-x);
      skidmark <- step(x.brake-x.tran);
      v2.imp <- (1-nohit)*((fullhit*pow(v,2)) +
              (1-fullhit)*(pow(v,2)-2*a*(max(0,x))));
      vi <- sqrt(v2.imp);
      x.init <- xx + x.prt;
      notime <- step(-x.prt-x);
      littletime <- step(-x)*(1-notime);
      t.ped <- 0*notime+littletime*(tp+x/v)+(1-fullhit)*(tp+(v-vi)/a);
      vi.kph <- (3.6)*vi;
      v.kph <- (3.6)*v;
      speeding <- step(v- v.limit);
      t.buffer <- clear/ped.speed;

# prior distributions
      tp ~ dunif(tp.lower,tp.upper);
      ts ~ dunif(ts.lower,ts.upper);
      x ~ dunif(x.lower, x.upper);
      v ~ dunif(v.lower, v.upper);
      f ~ dunif(f.lower, f.upper);
          a1.tau <- 1/(a1.sig*a1.sig)
          a2.tau <- 1/(a2.sig*a2.sig)
          b.tau <- 1/(b.sig*b.sig)
      a1~dnorm(a1.bar,a1.tau)I(,a2)
           a2~dnorm(a2.bar,a2.tau)I(a1,)
           b~dnorm(b.bar,b.tau)
      # speed.tau <- 1/(speed.sig*speed.sig);
      # speed.mph ~dnorm(speed.bar.mph,speed.tau);
      # speed.kph <- speed.mph*1.609;
      ped.speed ~ dunif(ped.lower,ped.upper);
}


Data   list(M=17,
skid1=3.58,
skid2=3.07,
throw=3.19,
injury=3,
a1.bar=4.97,
a1.sig=.531,
a2.bar=8.87,
```

```
a2.sig=.822
b.bar=0.127,
b.sig=.018,
v.lower=10,
v.upper=45;
f.lower=0.55,
f.upper=0.9,
x.lower=0,
x.upper=60,
ts.lower=0.1,
ts.upper=0.5,
tp.lower=0.5,
tp.upper=2.5,
ahat=-2.24,
throw.sig2 = 0.084,
skid.tau= 100,
v.limit=22.35,
ped.lower=1.0,
ped.upper=4.5,
clear=3.0,
v.star.kph=c(40,45,50,55,60,65,70,75,80,85,90,95,100,105,110,115,120))
```

```
Inits list(tp=1.5,
x=12,
v=15,
f=.73,
ts=.3,
randam=0,
a1=4.9,
a2=8.7,
b=.12)
```