The Series Josephson Junction Array:

from collective modes to quantum phase transitions – with applications.

David B. Haviland

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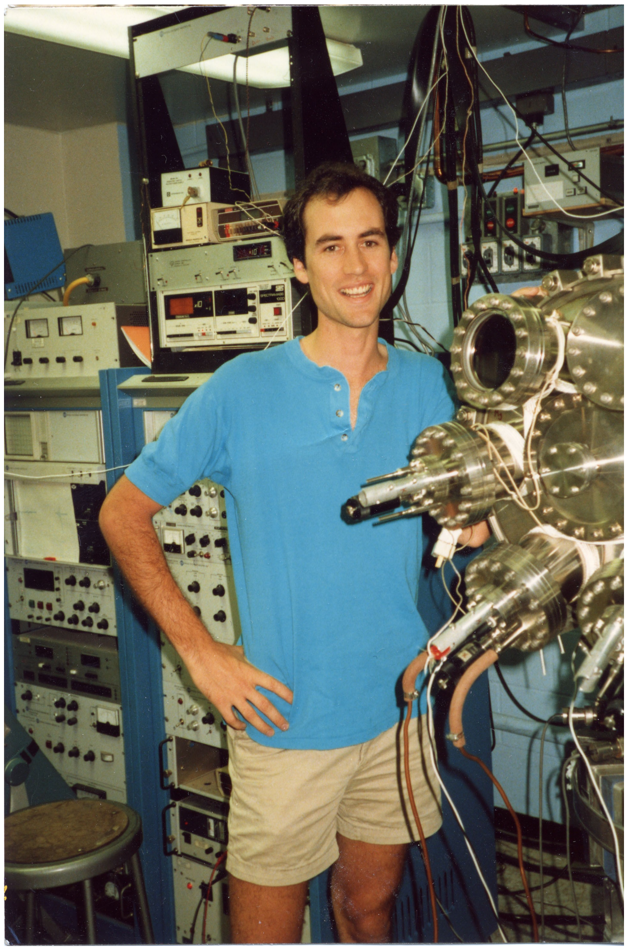
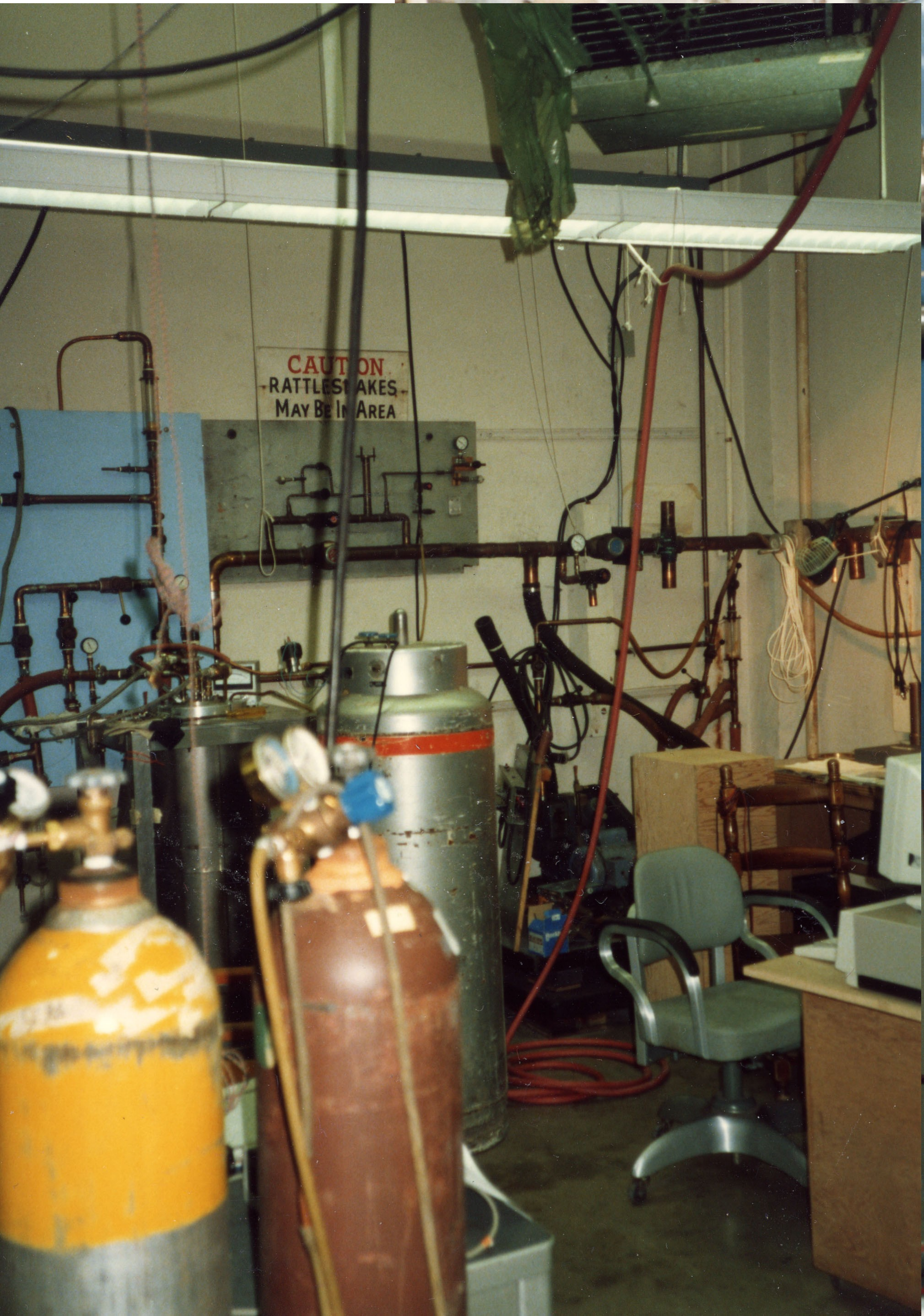
Royal Institute of Technology (KTH)
Stockholm, Sweden

Memories of working in Allen Goldman's Lab

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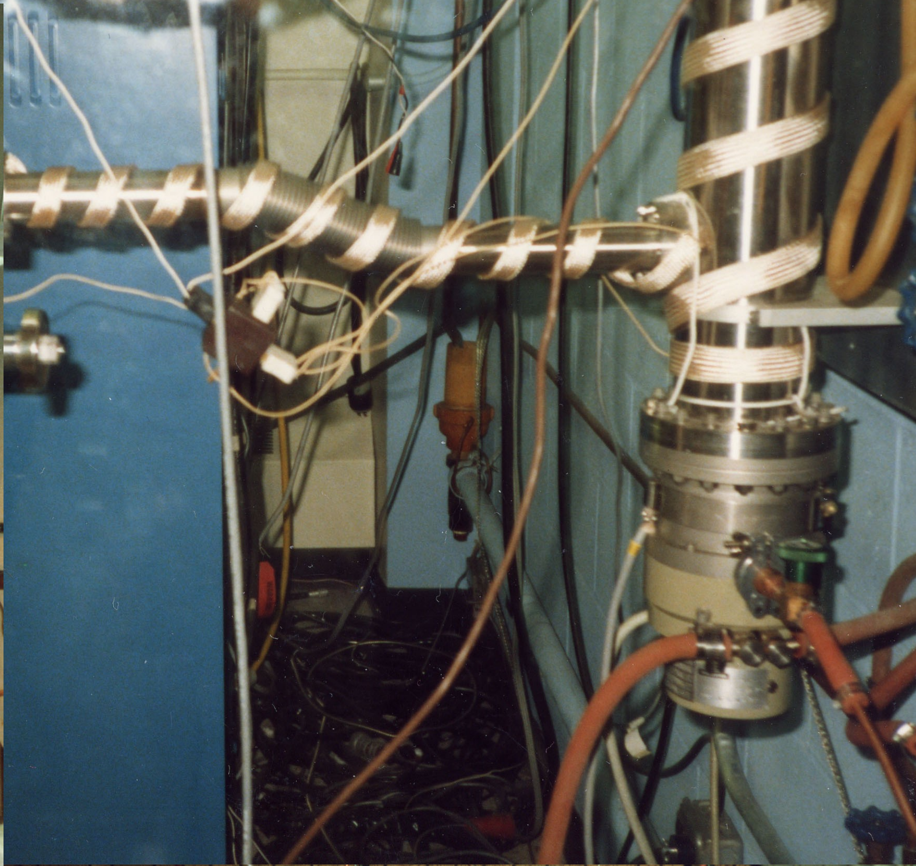
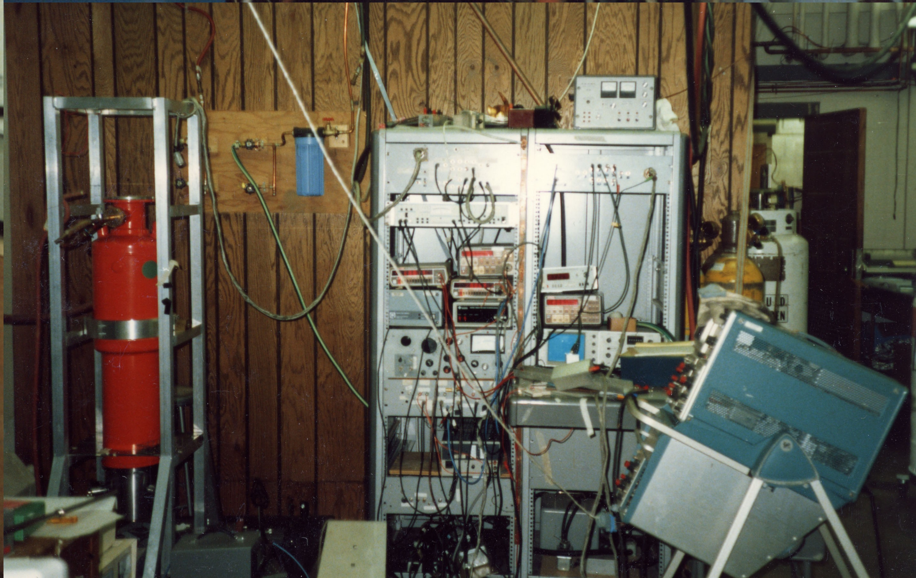
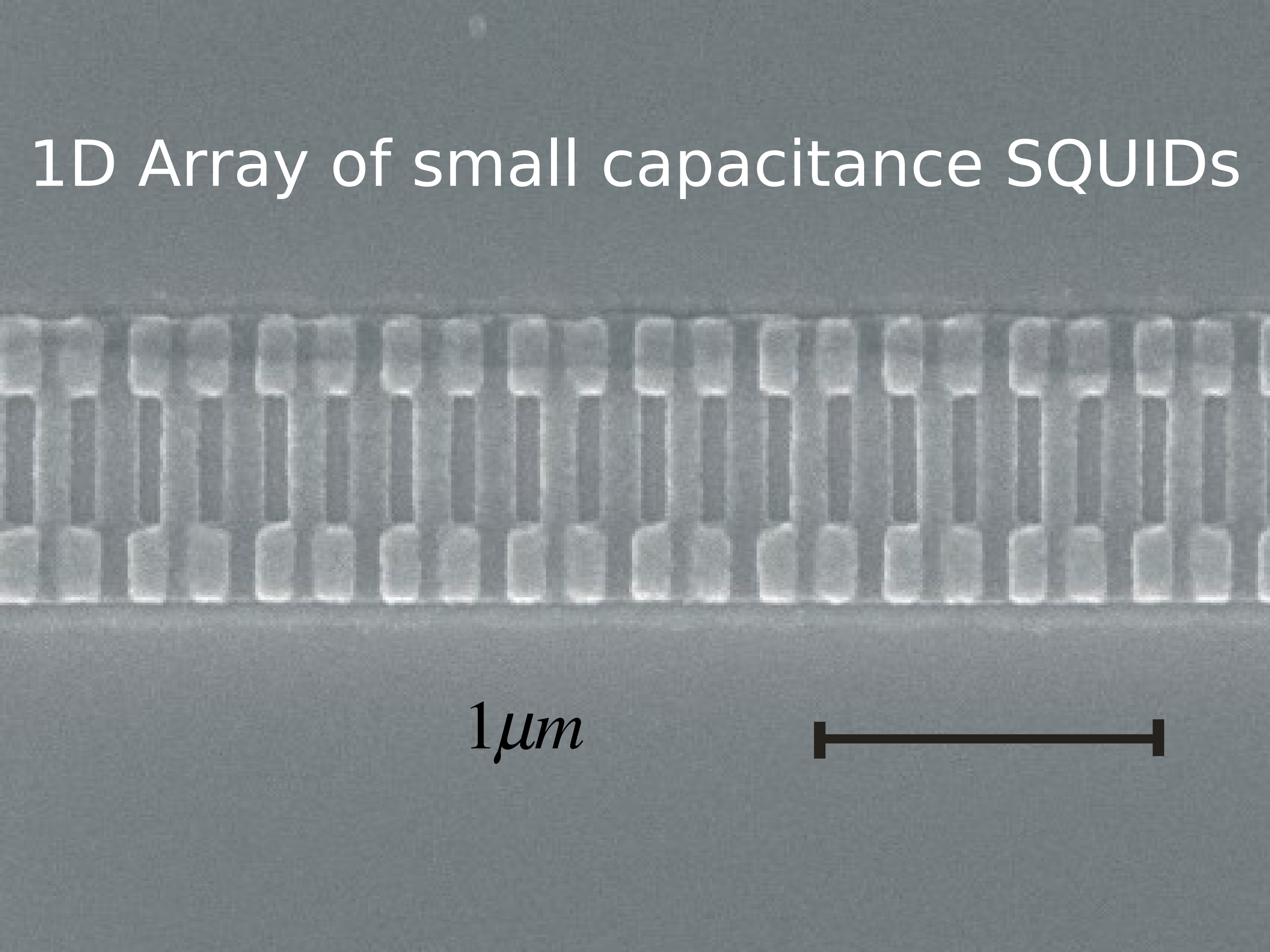


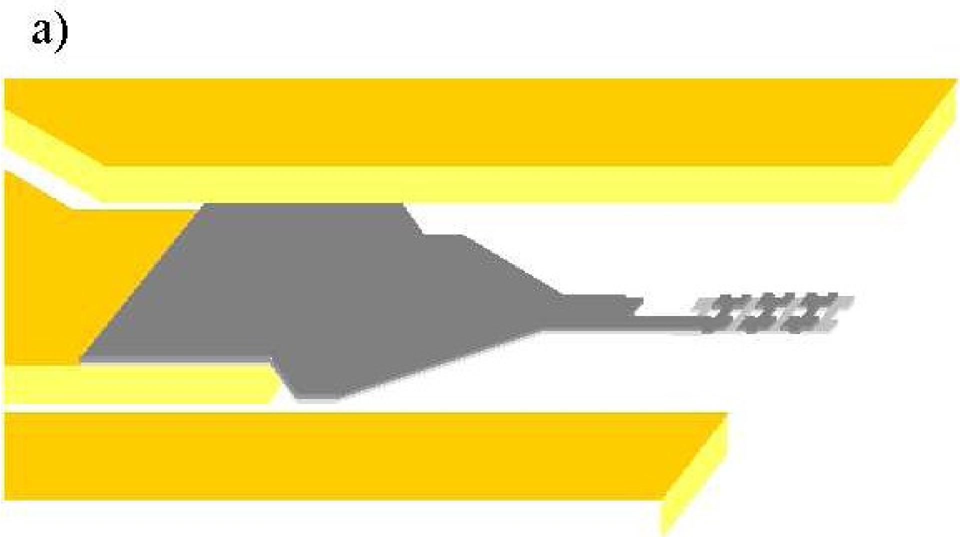


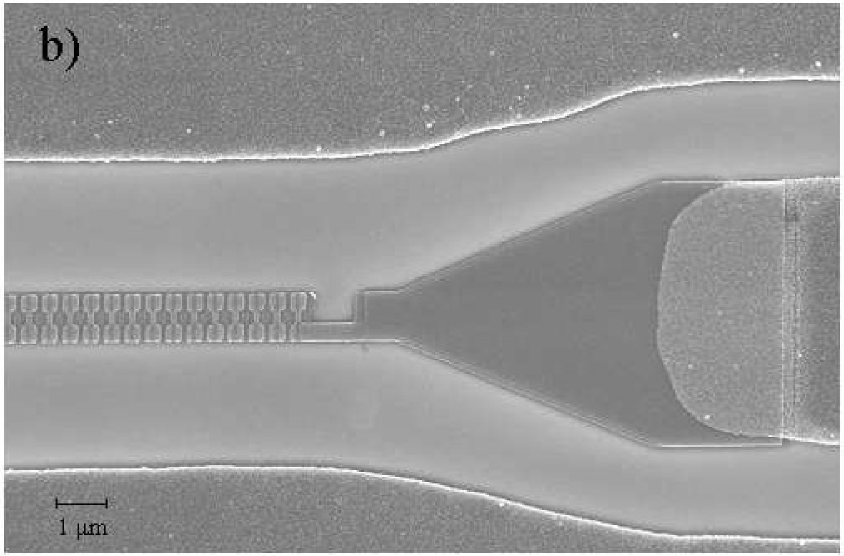

MALLORY

Conference on Josephson Junction Arrays
Jackson Hole, 1994

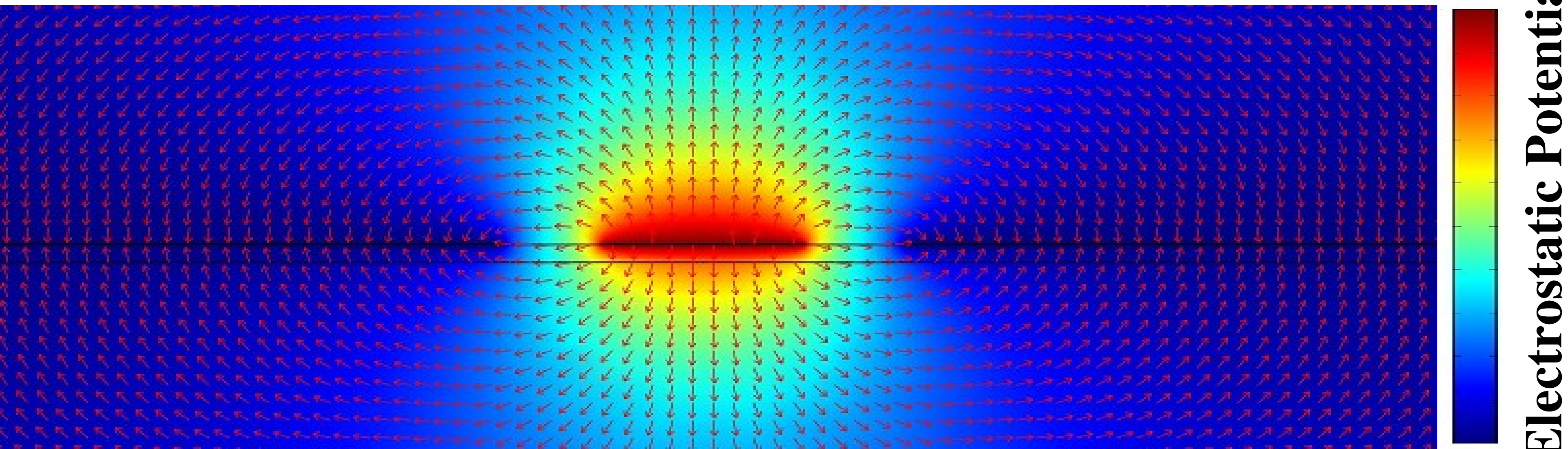

1D Array of small capacitance SQUIDs
1μm

Array as a Transmission Line





Coplanar Wave Guide (CPW) - really a transmission line



Three Limits Examined

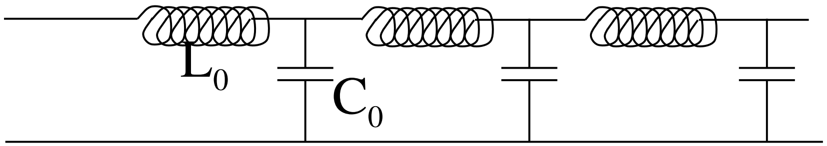
1. Classical, Linear Electrodynamics

2. Classical, Nonlinear Electrodynamics

3. Quantum Electrodynamics

Circuit model of a Transmission Line

Discrete, lumped element model of TEM transmission line



$$Z_L = \sqrt{\frac{L_0}{C_0}} \approx \frac{Z_0}{2\pi} = 60\ \Omega$$

Another Natural Impedance for Superconductors:

Quantum Resistance $R_Q = \Phi_0/2e = h/4e^2 = 6.45\ \mathrm{k}\Omega$

Fact of Nature: $Z_0 \ll R_Q$

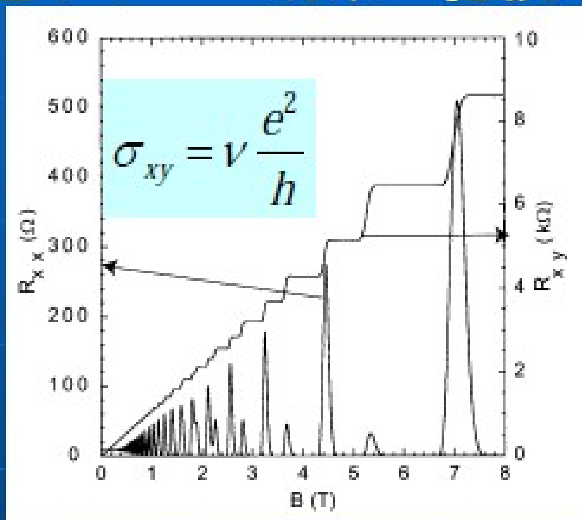
$$\frac{Z_L}{R_Q} = \frac{2e^2}{\pi h c \varepsilon_0} = \frac{4}{\pi}\alpha$$

Fine structure constant $\alpha = \dfrac{1}{137.035999679(94)}$ $\qquad \alpha_{CGS} = \dfrac{e^2}{\hbar c}$

量子コンダクタンス

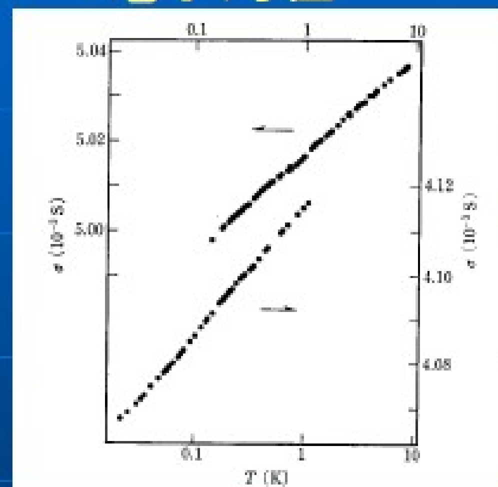
$$\frac{e^2}{h} = \frac{1}{25.813\,\mathrm{k\Omega}}$$

量子ホール効果⇒抵抗の国際標準



$$\sigma_{xy} = \nu \frac{e^2}{h}$$

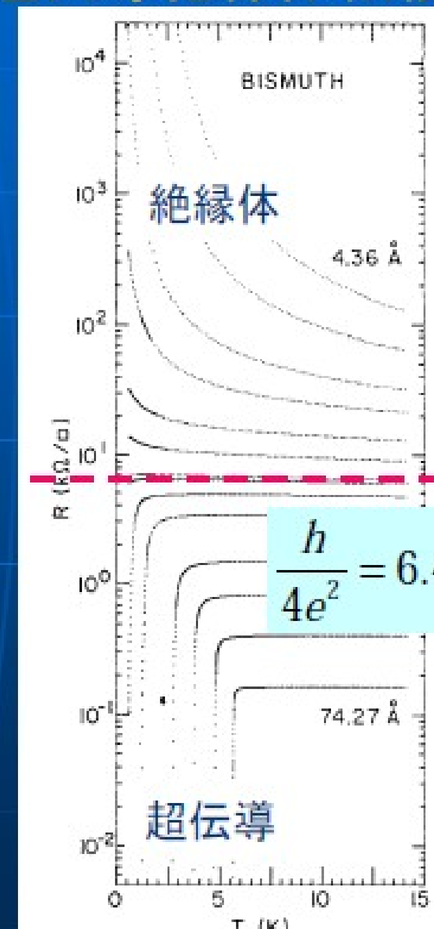
電子局在



$$\sigma_{2D} = \frac{ne^2\tau}{m} - \frac{P}{\pi}\frac{e^2}{h}\ln\frac{T}{T_0}$$

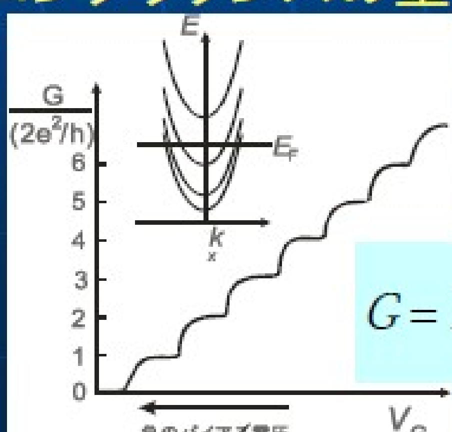
古典伝導度＋量子補正

超伝導絶縁体転移



$$\frac{h}{4e^2} = 6.4\,\mathrm{k\Omega}$$

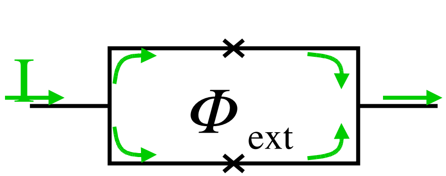
コンダクタンスの量子化

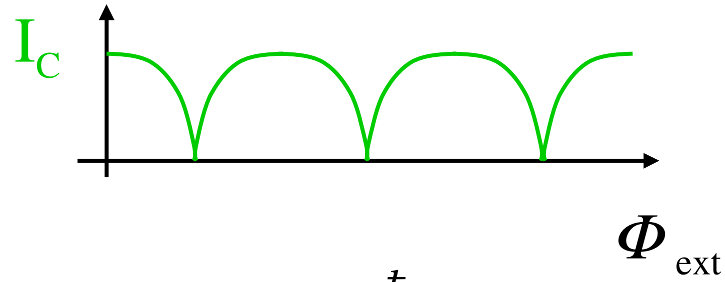


$$G = N\frac{2e^2}{h}$$

物質によらない普遍的物理

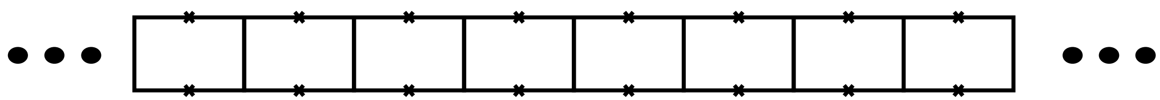
1D SQUID array as *tunable* Josephson transmission line

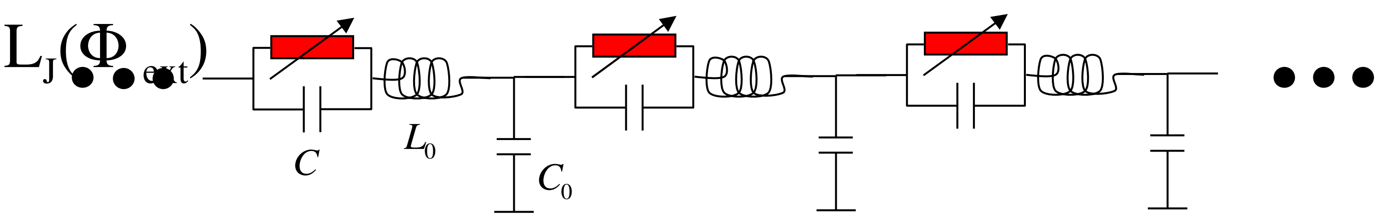




Tunable Josephson Inductance

$$L_J = \frac{\hbar}{2 e I_C(\Phi_{\text{ext}})}$$

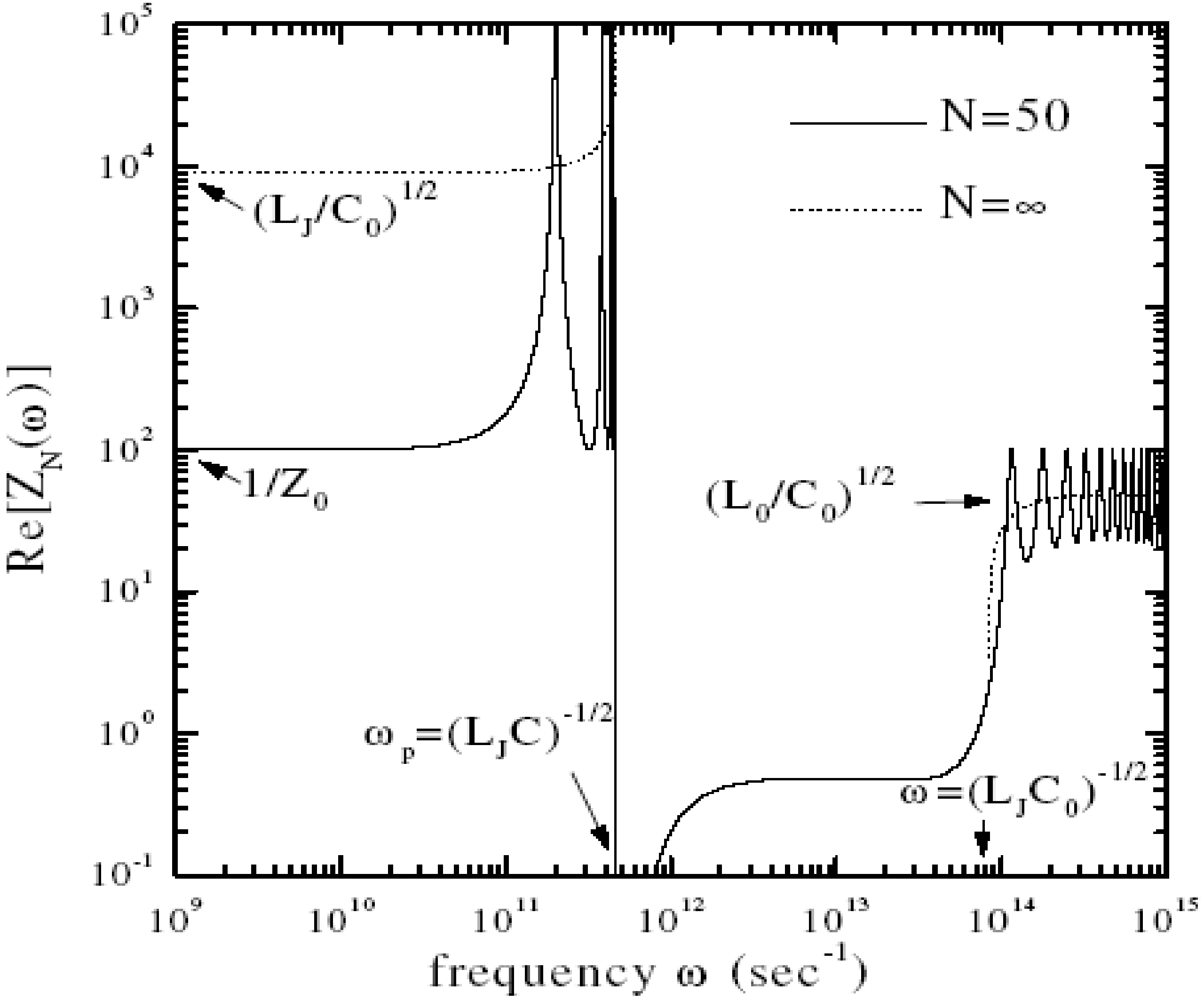




Transmission Line Impedance:

$$N \to \infty, \quad \omega << \omega_p = \frac{1}{L_J C} \quad \Rightarrow \quad Z_A = \sqrt{\frac{L_J + L_0}{C_0}} \simeq R_Q \sqrt{\frac{4E_C}{E_J}} \sqrt{\frac{C}{C_0}}$$

Linear model - impedance



$$C = 3.5\,\mathrm{fF}$$

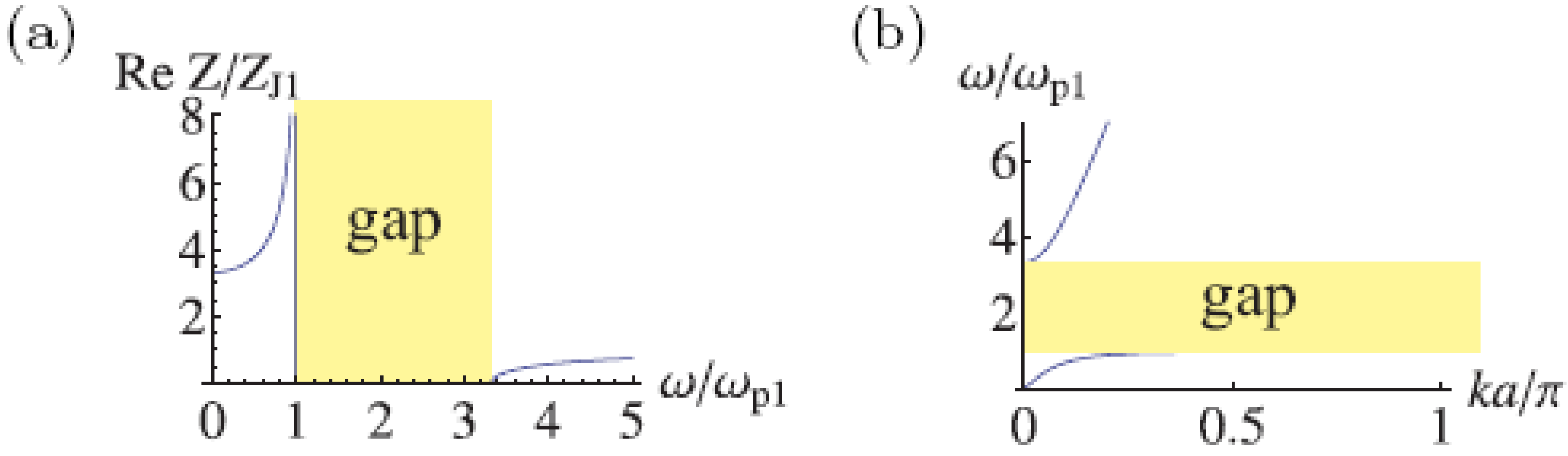
$$C_0 = 0.016\,\mathrm{fF}$$

$$L_J = 1.1\,\mathrm{nH}$$

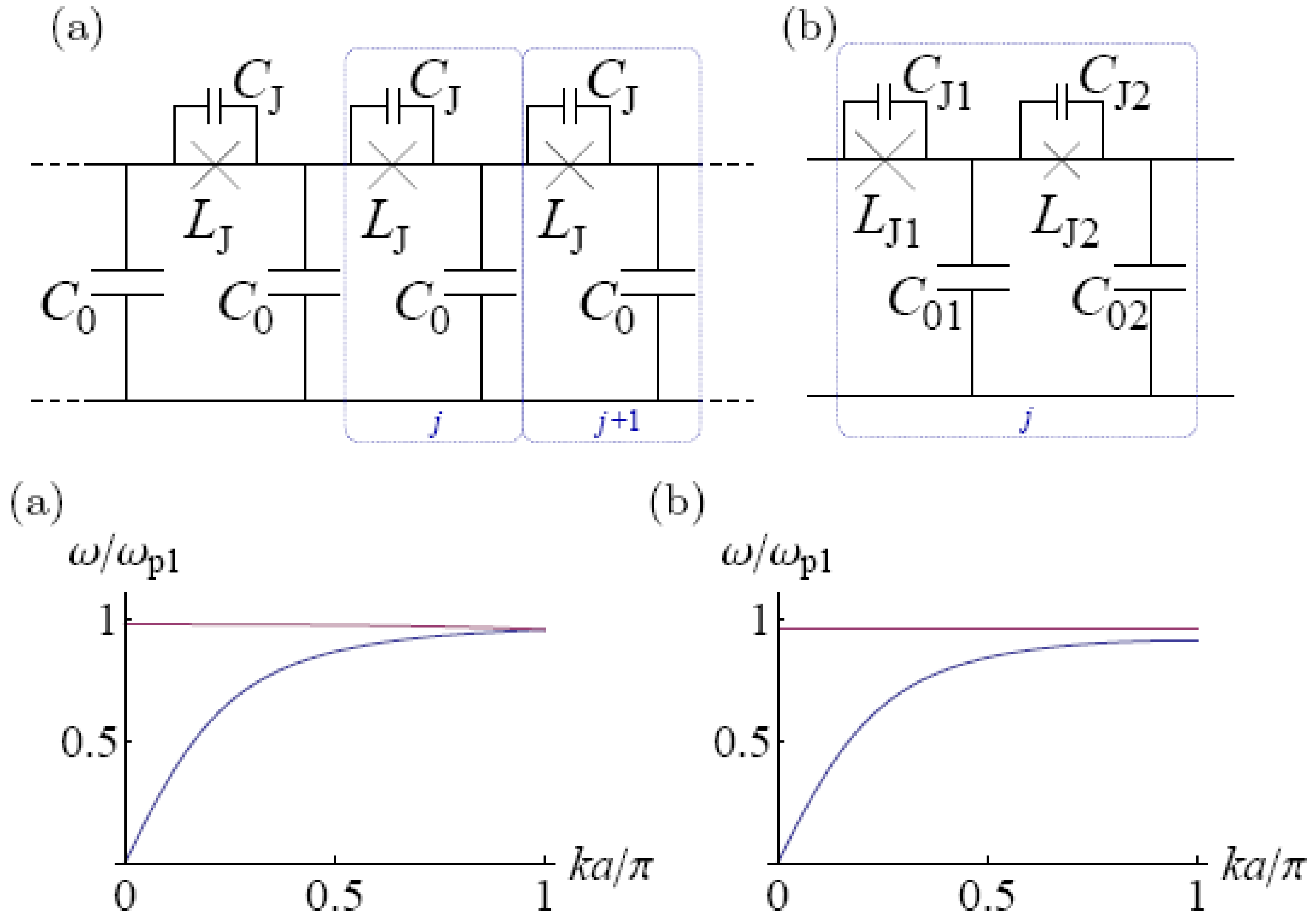
$$Z_0 = \sqrt{\frac{L_0}{C_0}} = 50\,\Omega$$

Circuit model of a “photonic band gap”

Impedance, dispersion relation, Density of modes

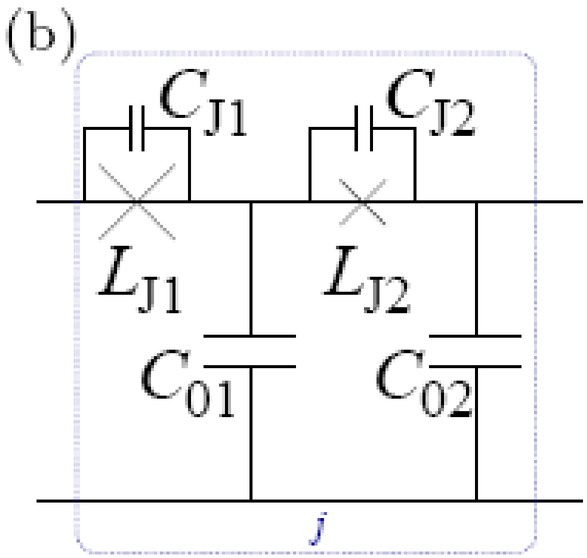


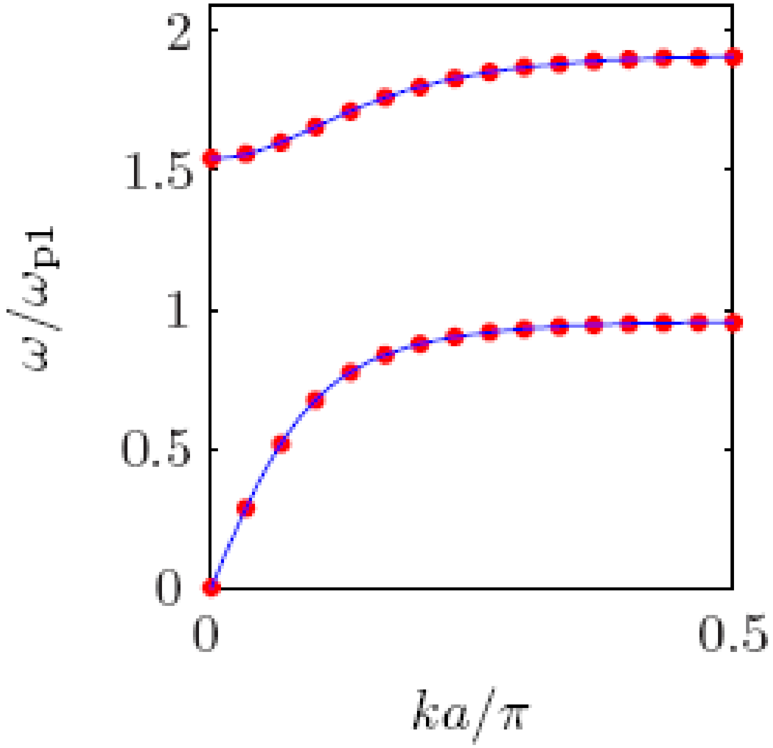
Lattice with a basis



Gap when different plasma frequencies

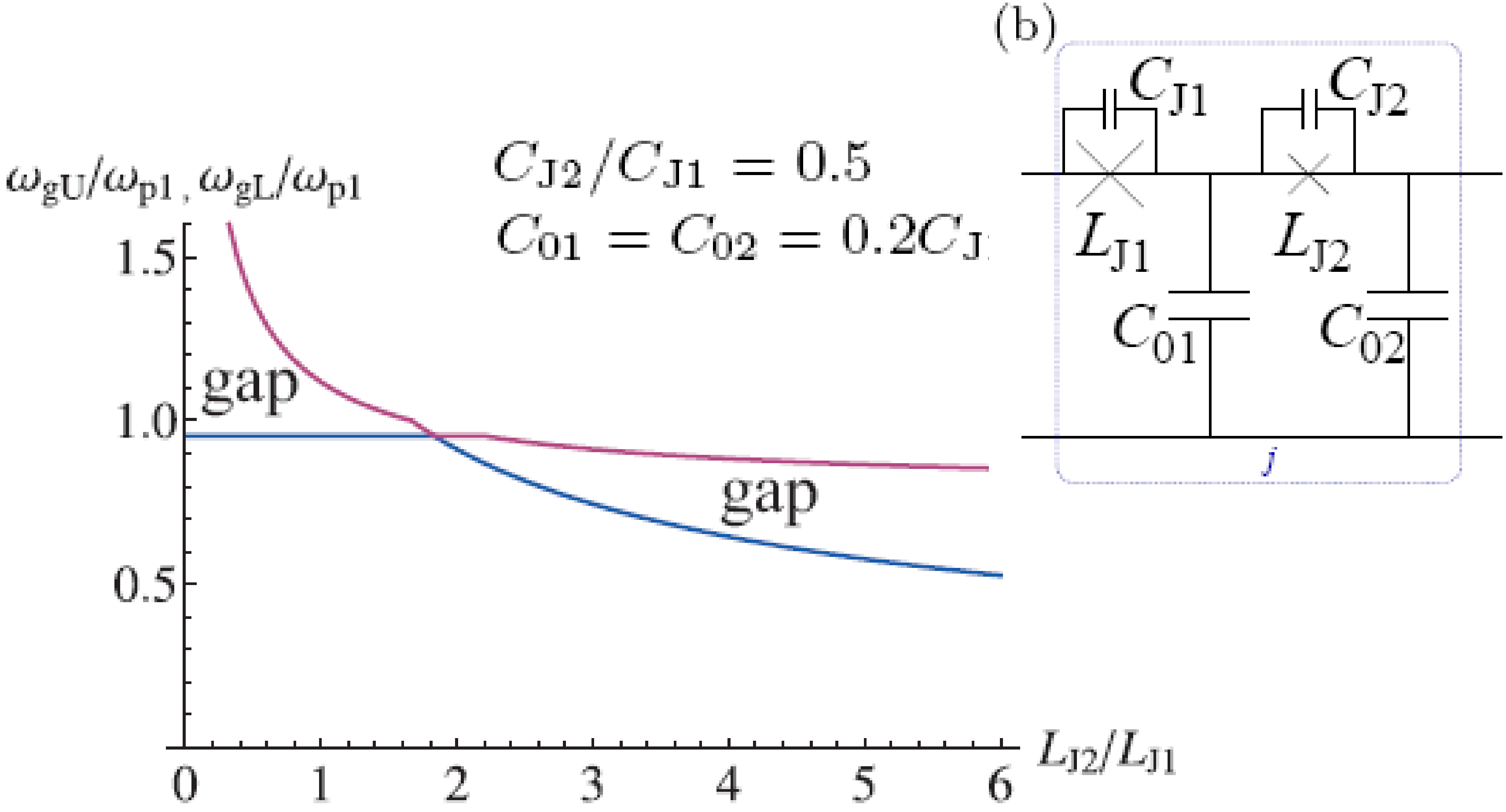
When Junction Capacitance > Capacitance to ground, $C_J > C_0$ we need $\omega_{p1} \neq \omega_{p2}$ in order to get gap.





$$L_{J2} = 0.25 L_{J1} \text{ and } C_{01} = C_{02} = 0.2 C_1 = 0.2 C_2$$

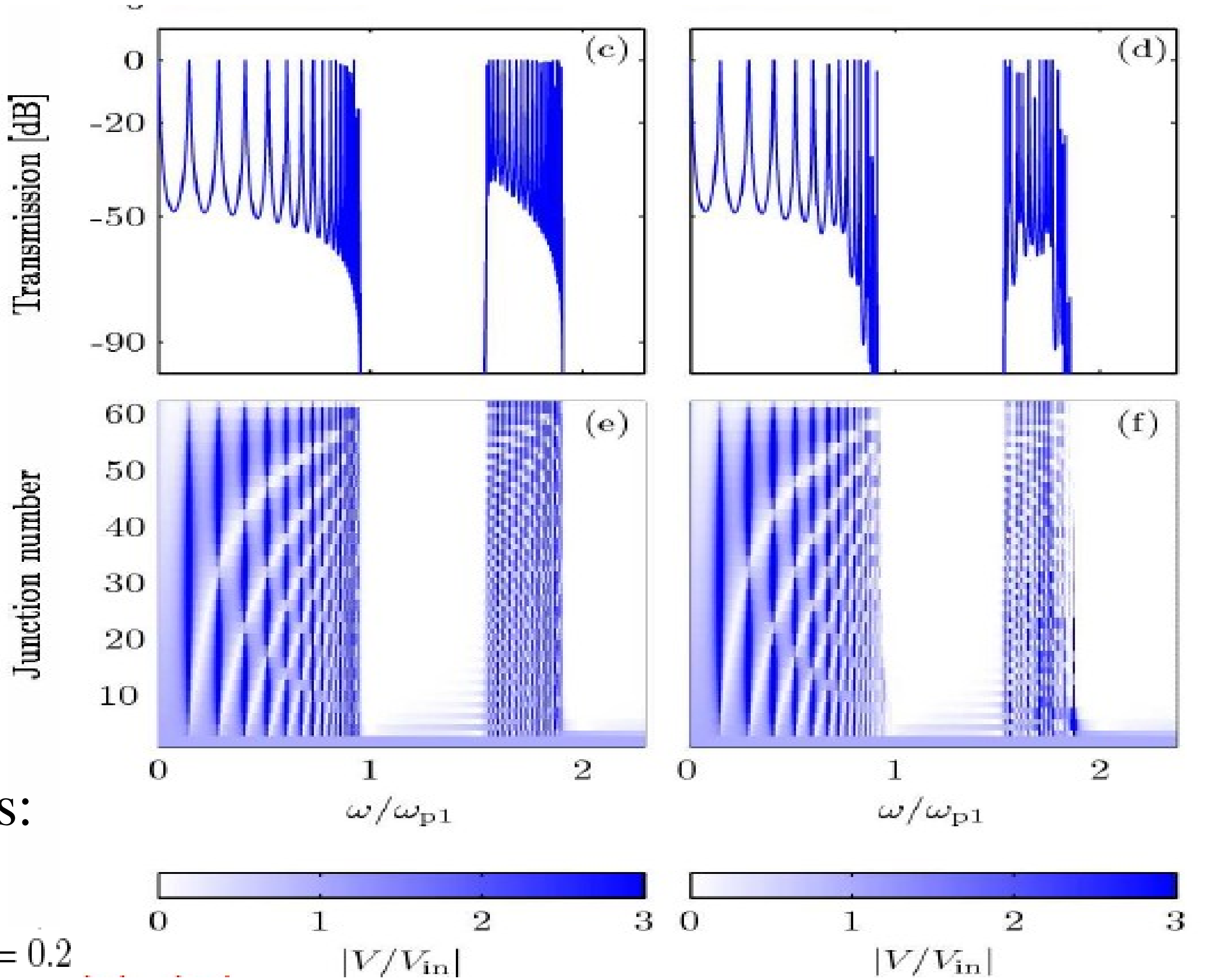
Gap engineering by changing Plasma frequency



Transmission simulation, with disorder

Transmission line impedance: $Z_A(0) = \frac{\sqrt{C_{01}C_{02}}}{C_{01}+C_{02}} \cdot \sqrt{\frac{L_{J1}}{C_{01}} + \frac{L_{J2}}{C_{01}} + \frac{L_{J1}}{C_{02}} + \frac{L_{J2}}{C_{02}}}$.

- **Impedance mismatch gives standing waves**
- **resonator without capacitor at ends**
- **Note: voltage note (current anti-node) in middle ($Z_A > Z_0$).**
- **Gap is a robust feature, nearly unaffected parameter spread (5% SD)**



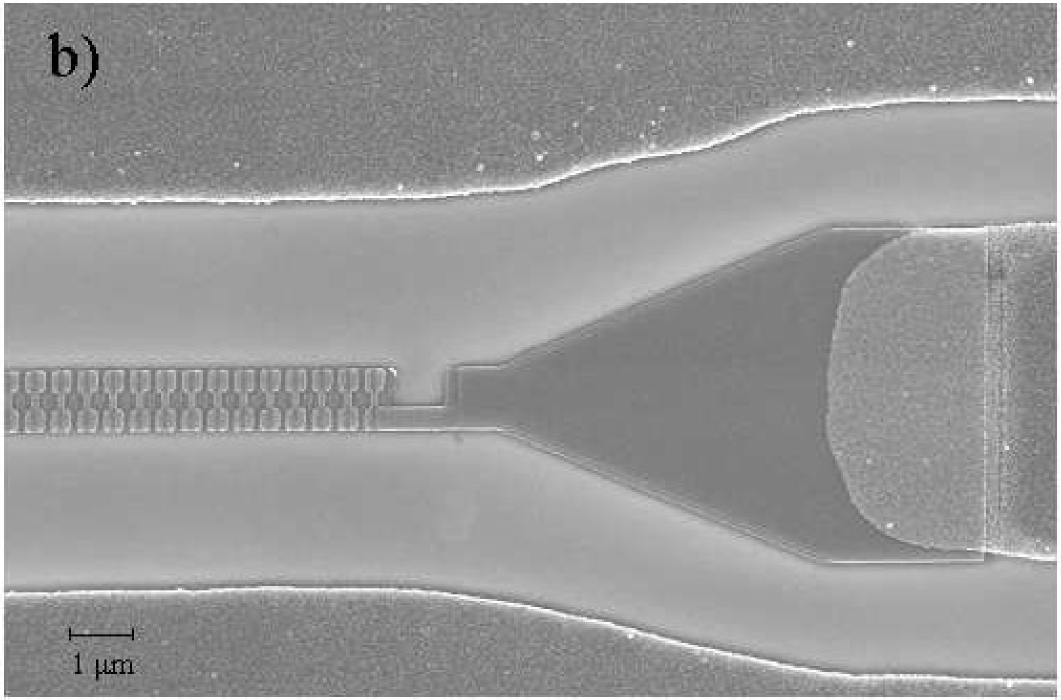
Simulation parameters:

$L_{J2} = 0.25 L_{J1}$

$C_{01} = C_{02} = 0.2C_1 = 0.2C_2$

$Z_{in} = Z_{out}$, and $Z_{in}/Z_A(0) = 0.2$

$C_0 < C_J$ or $C_0 > C_J$



CPW geometry:

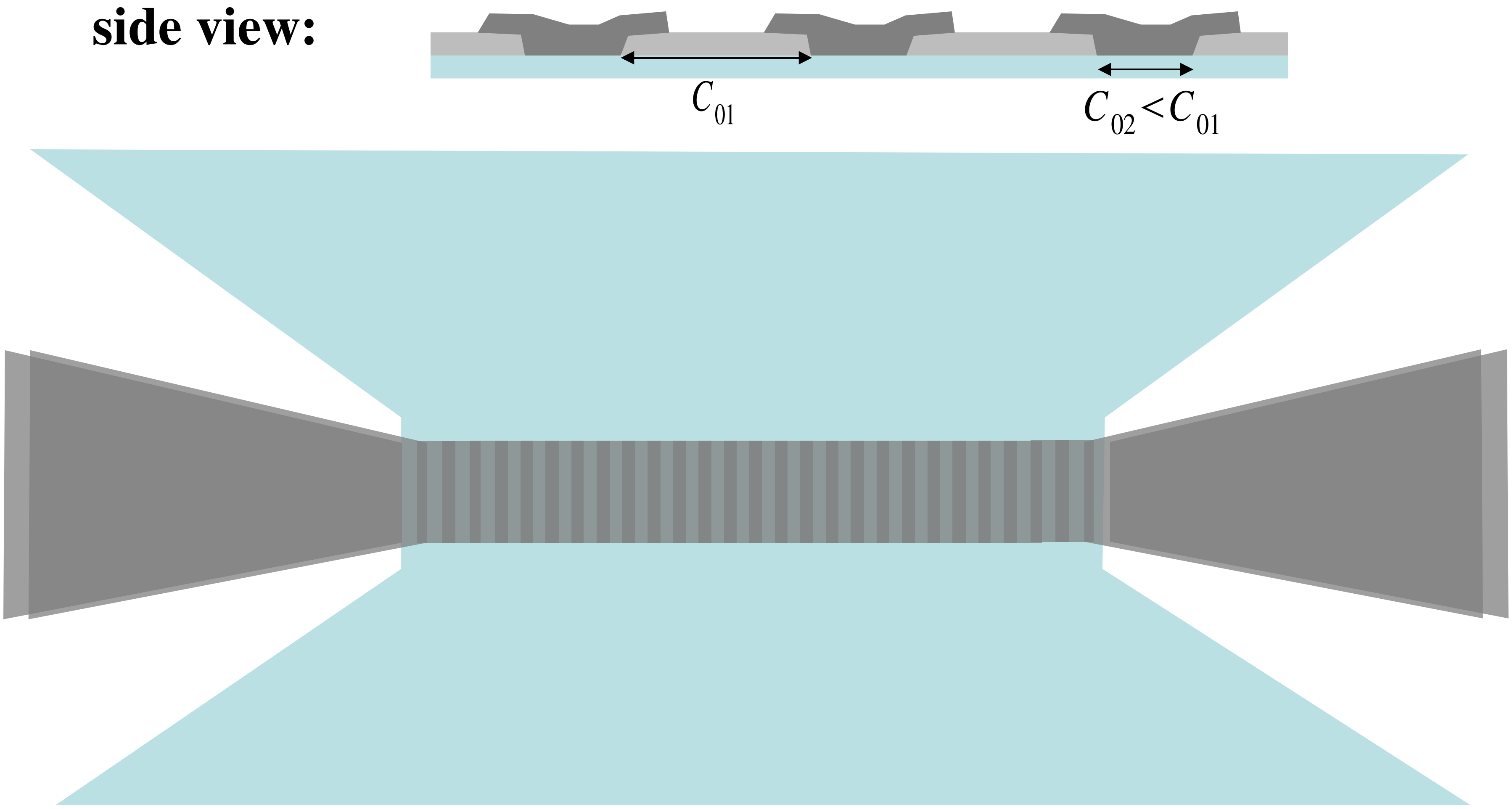
naturally $C_0 << C_J$

$\omega_{p1} \neq \omega_{p2}$ necessary for gap

Not easy to realize for Al shadow evaporation junctions.

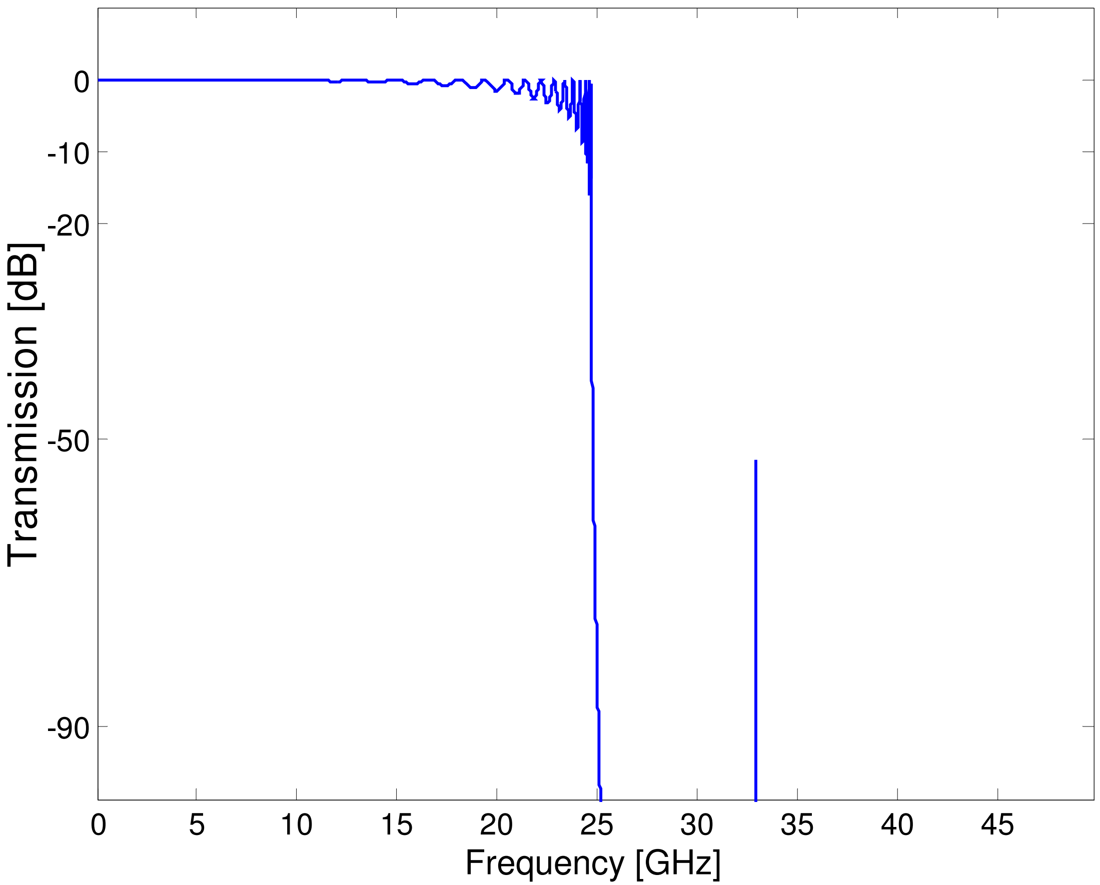
For $C_J < C_0$ we can have gap with $\omega_{p1} = \omega_{p2}$, if $C_{01} \neq C_{02}$.

Stripline geometry to get large C_0.



Layer 1 ground plane w/ heavy oxidation
Layer 2,3 Array, shadow evaporation

Simulation for Stripline geometry



Aluminum junctions on Heavily oxidized Al groud plane (insulating to gnd plane)

Heavy junction oxidation -
ω_p=33GHz
Z_{in} = Z_{out} = 60 Ω
N=20 unit cells
Shadow Mask dimensions:

L=8μ m
w=2μ m
D=0.2μ m
s=0.8μ m

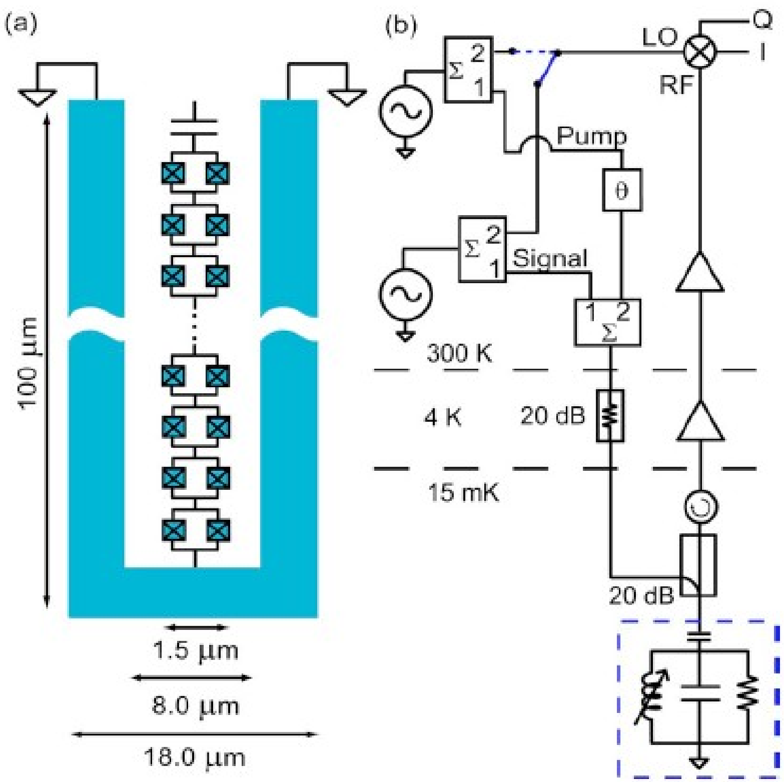
→ **Z_A=61.4 Ω**

Classical Nonlinear Electrodynamics

APPLIED PHYSICS LETTERS 91, 083509 (2007)

Widely tunable parametric amplifier based on a superconducting quantum interference device array resonator

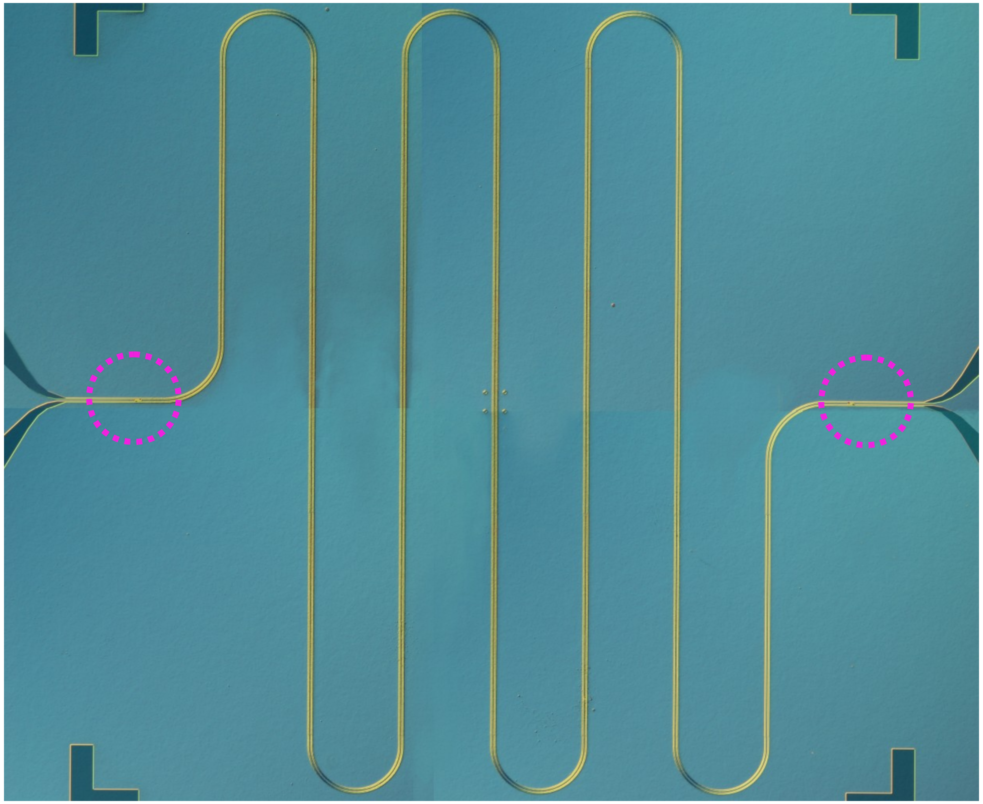
M. A. Castellanos-Beltran[a)] and K. W. Lehnert



- JJ gives current dependent inductance – nonlinear inductance
- SQUID makes for tunable nonlinearity
- Array resonator ⇒nonlinear oscillator, narrow band parametric amplifier, band is tunable with SQUID.
- Matched array ⇒wide band parametric amplifier. Attempted by Yurke et al.

Appl. Phys. Lett. **69** (20), 11 November 1996

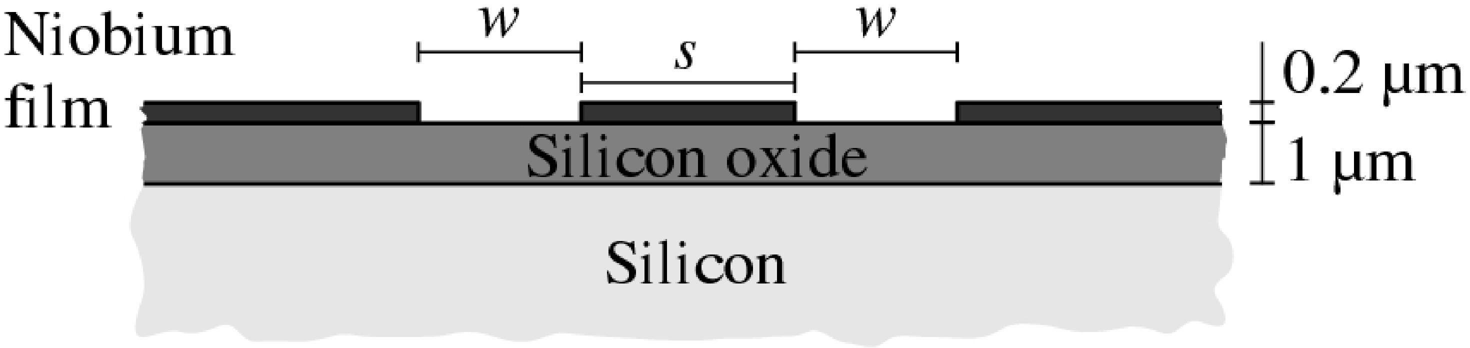
Array not really necessary: Thin-film, narrow strip, low critical current

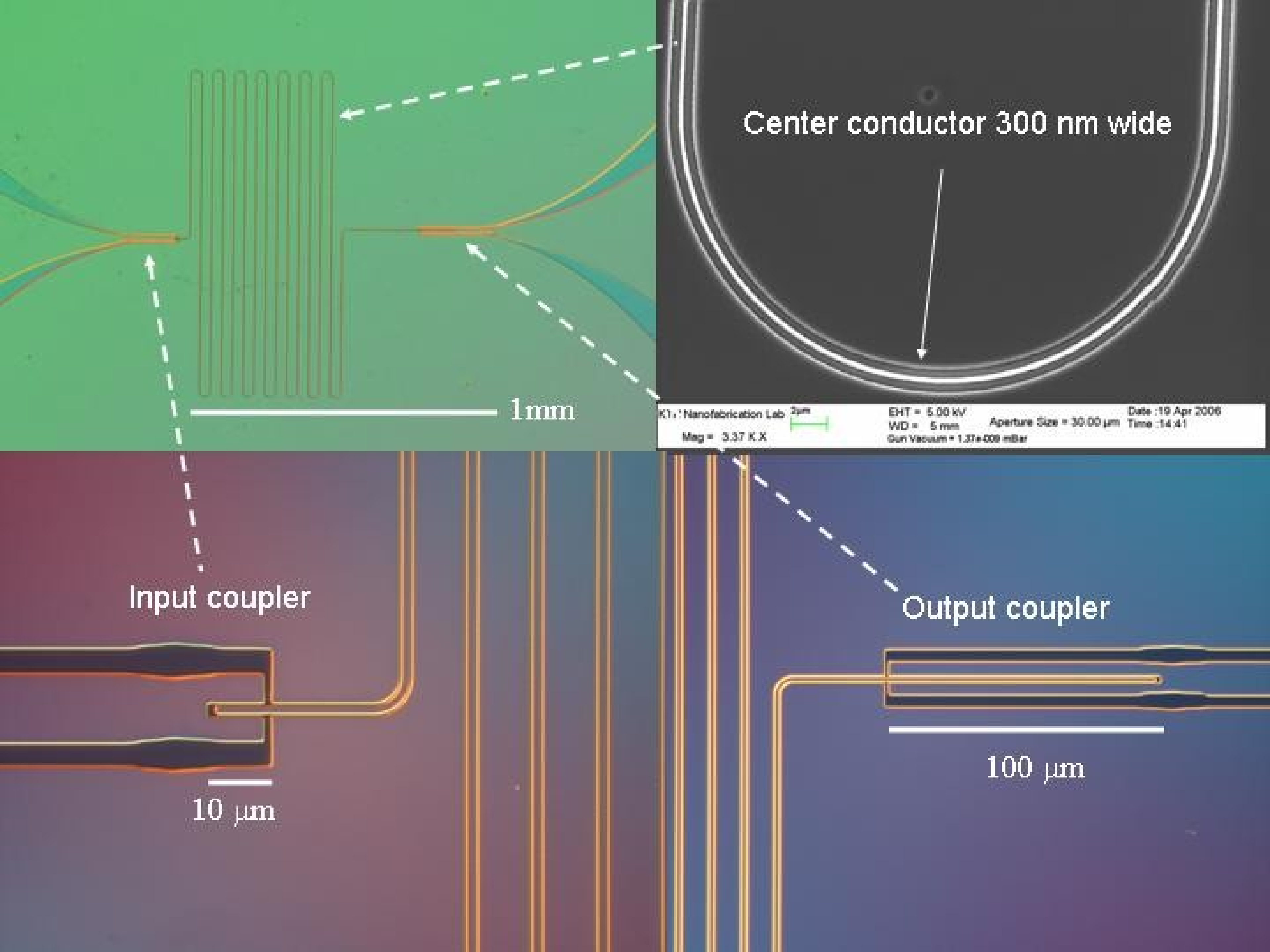


CPW resonators at KTH

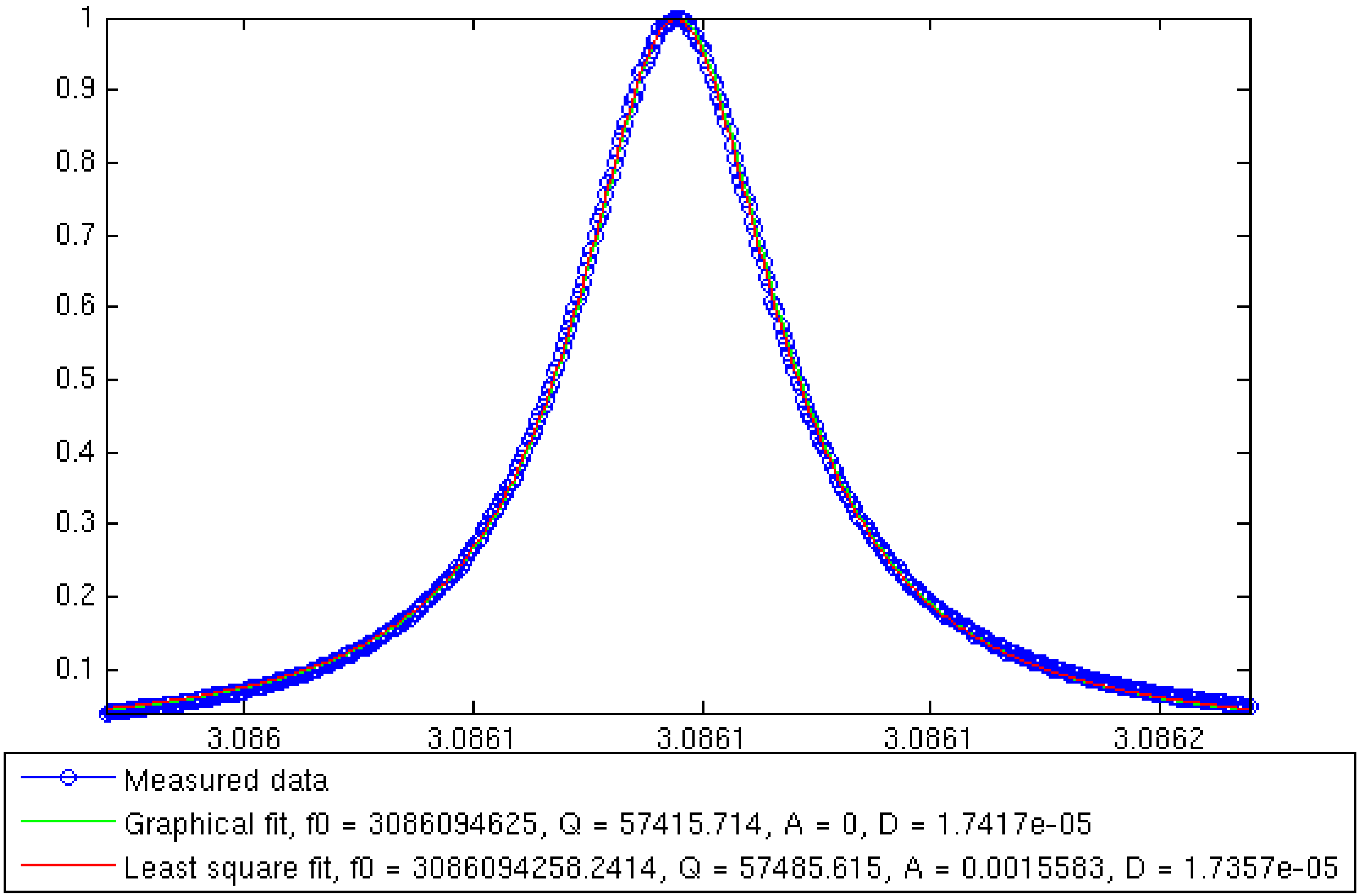
Niobium process:

- Nb thin film
- E-Beam Lithography
- Etch gapsBCl$_3$,




Center conductor 300 nm wide
1mm
Nanofabrication Lab
Mag = 3.37 K X
EHT = 5.00 kV
WD = 5 mm
Aperture Size = 30.00 µm
Gun Vacuum = 1.37e-009 mBar
Date :19 Apr 2006
Time :14:41
Input coupler
10 µm
Output coupler
100 µm

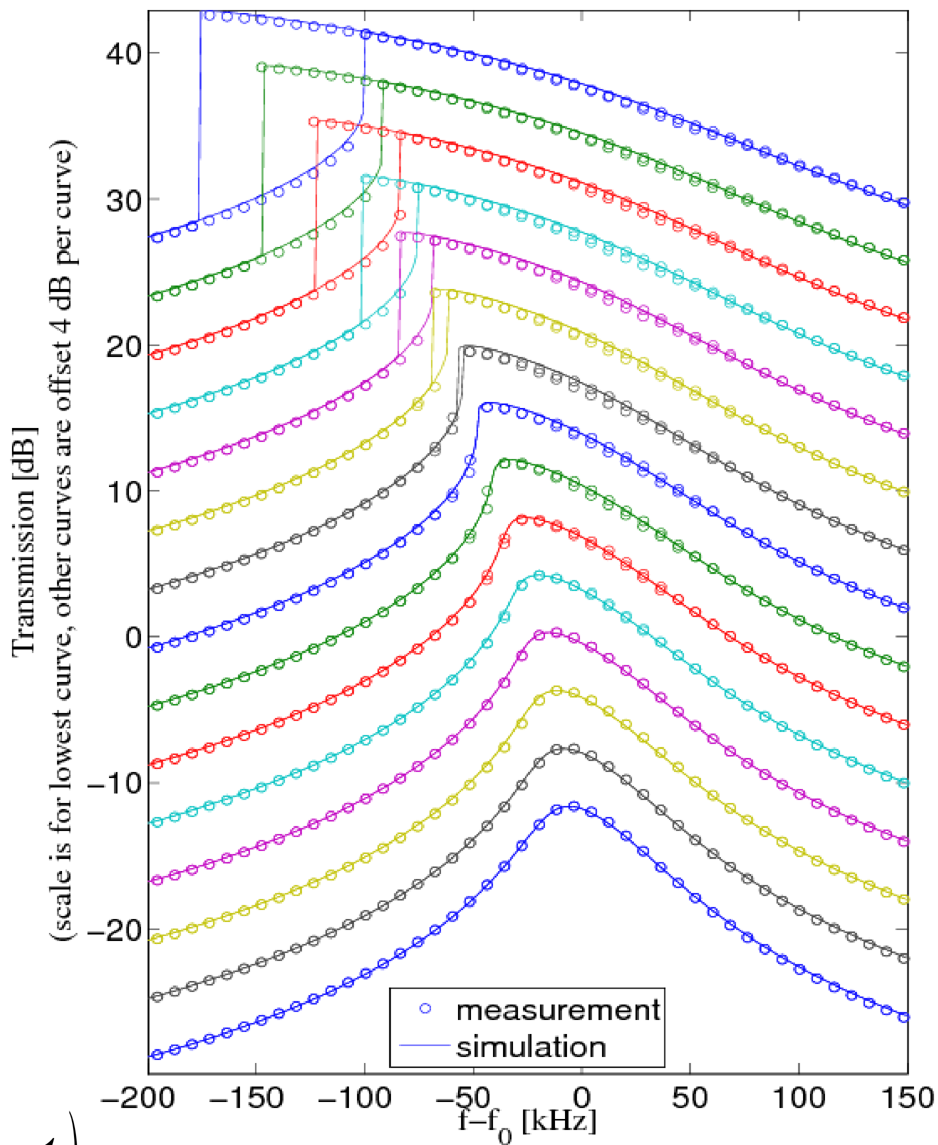
Low drive power, High Q oscillator



Increasing drive Power

- Bending of resonance curve to lower frequency
- Bifrication at critical power
- Well fit by nonlinear transmission line model
- Current dependent kinetic inductance:

$$L(I) = L_0 + \Delta L \left(\frac{I}{I_b} \right)^2$$



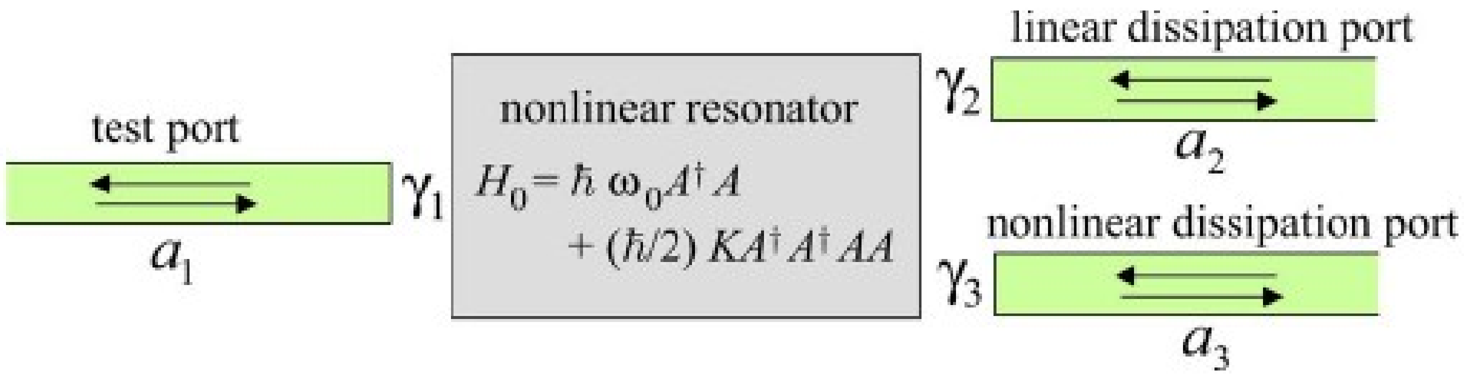
Duffing Oscillator

$$Q^2 \ddot{I} - \dot{I} - I - \beta I^3 = I_D \cos(\omega_D t)$$

$$\beta = \frac{\Delta L}{L_0 I_b{}^2}$$

Kerr Nonlinearity for Parametric Amplification

Yurke and Buks, Jour. Lightwave Tech. **24**, 5054 (2006)



Condition for having a bistable regime $|K| > \sqrt{3}\,\gamma_3$

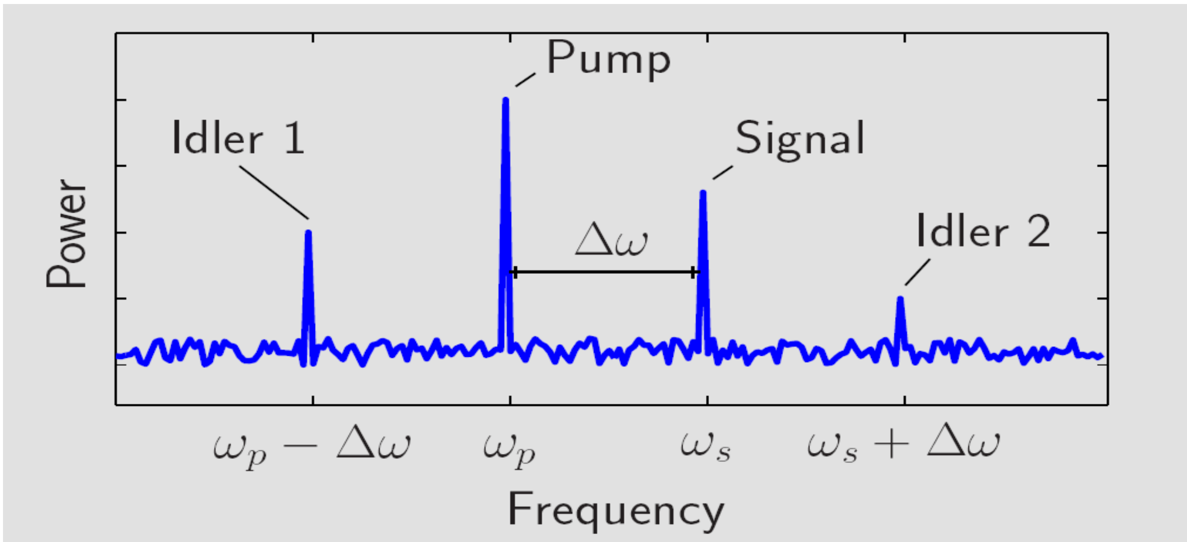
Critical input power at onset of bistability $\dfrac{4}{3\sqrt{3}} \dfrac{\left(\gamma_1 + \gamma_2\right)^3 \left(K + \gamma_3\right)}{\left(|K| - \sqrt{3}\,\gamma_3\right)^3}$

Also calculate parametric gain and intermodulation gain….. formulas--

Intermodulation and Parametric Amplification

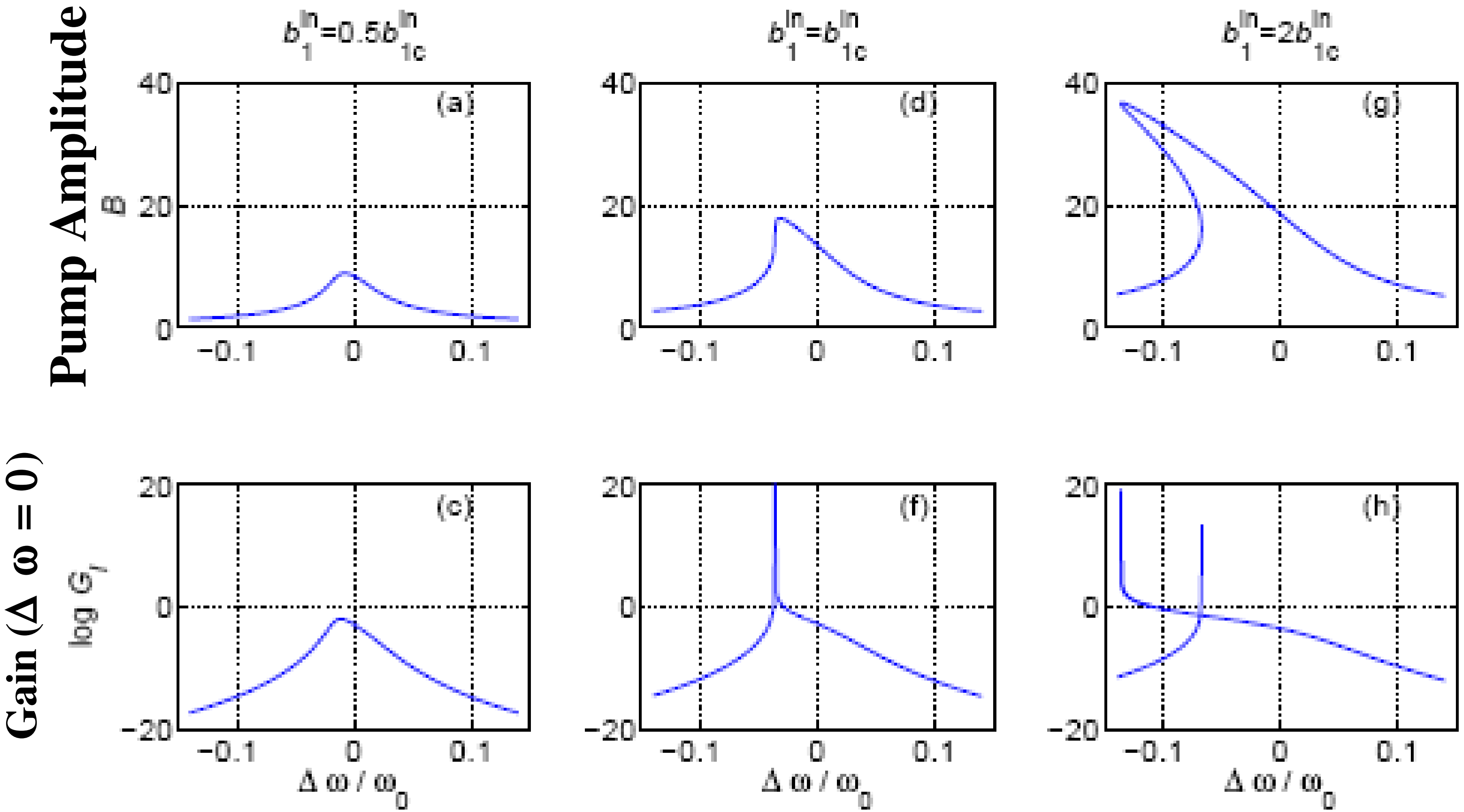
$$I^3 = \left[I_p \cos(\omega_p t) + I_s \cos(\omega_s t)\right]^3 =$$

$$6I_p^2 I_s \cos\left(\omega_s t\right) + 3I_p^2 I_s \cos\left((\omega_p - \Delta\omega)t\right) + 3I_p I_s^2 \cos\left((\omega_s + \Delta\omega)t\right)$$



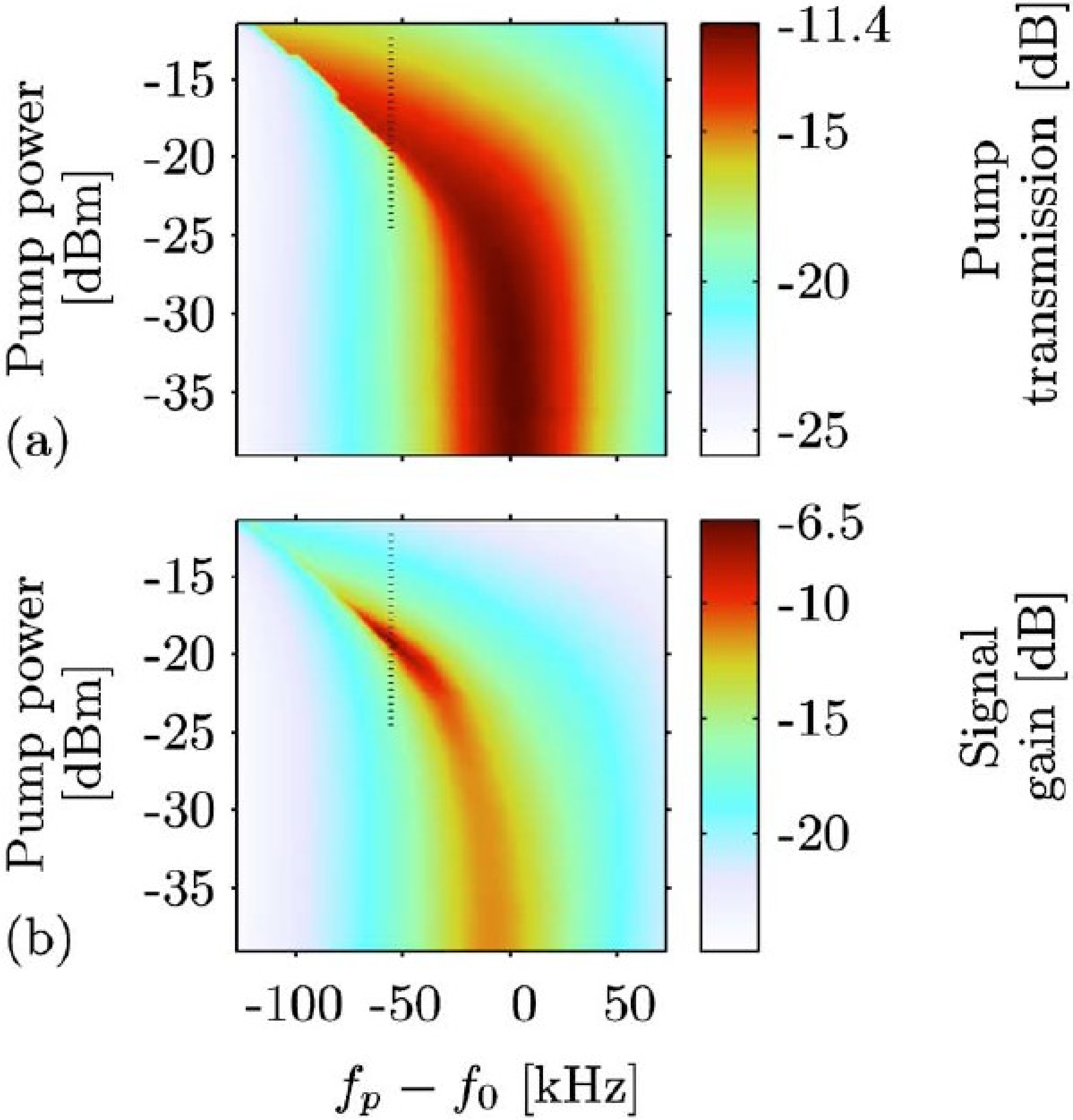
- Idlers - intermodulation products (IMPs), generated by nonlinearity
- Hierarchy of IMPs separated by $\Delta\omega$
- Degenerate mode $\omega_p = 2\omega_s$ ($\Delta\omega = \omega_s$)
- Doubly Degenerate mode $\omega_p = \omega_s$ ($\Delta\omega = 0$)
- Non-degenerate mode $\Delta\omega \neq 0 \ll \omega_s$

Kerr Non-Linearity for parametric Amplification

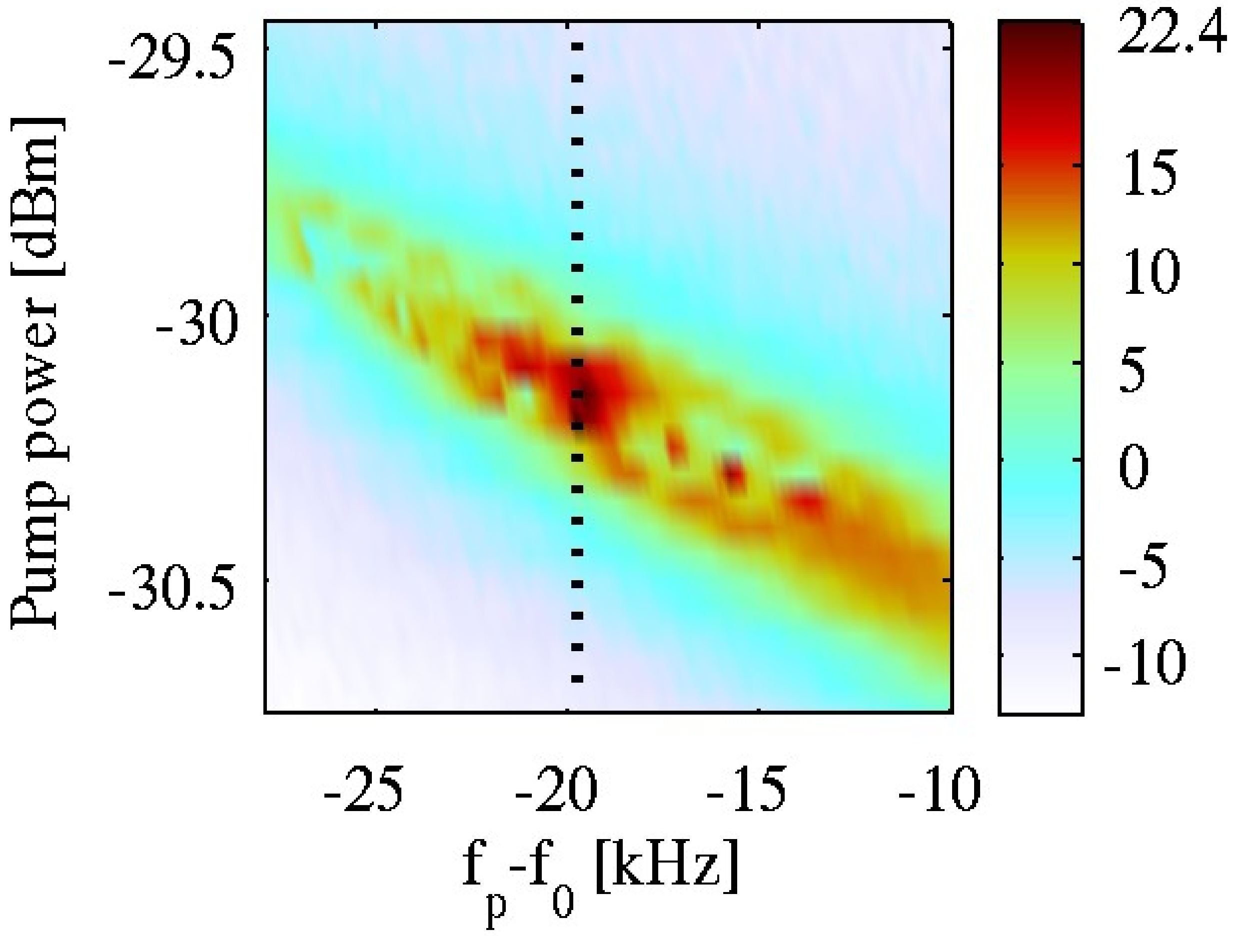


Yurke and Buks, Jour. Lightwave Tech. **24**, 5054 (2006)

Pump power dependence : Sample 1



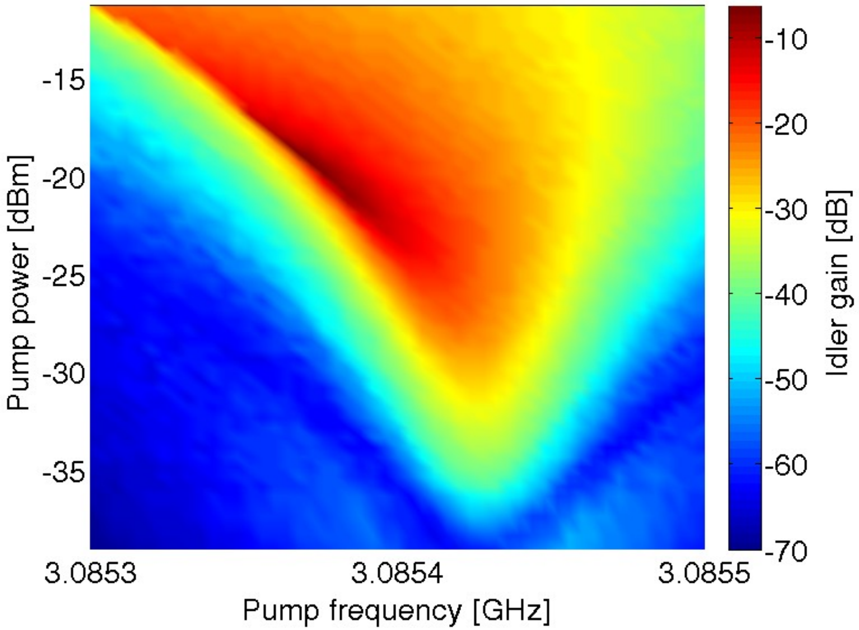
Zoom around maximum gain: sample 2



Nonlinearities and parametric amplification in superconducting stripline resonators. E. Tholén, A. Ergül, E. Doherty, F. M. Weber, F. Gregris and D. B. Haviland. Appl. Phys. Lett.

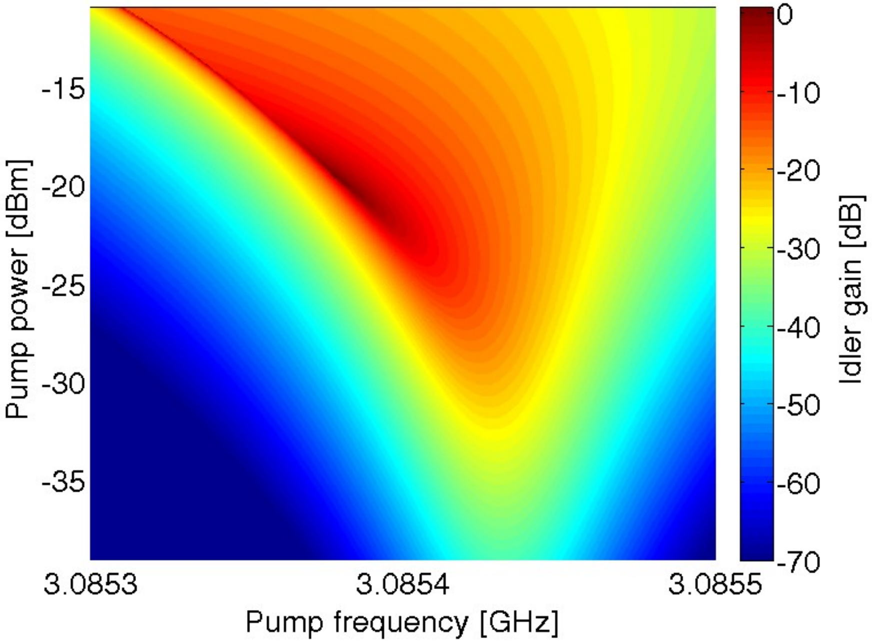
Intermodulation (Idler) Gain

Experiment (Sample 1)



E. Tholén *et al.* unpublished

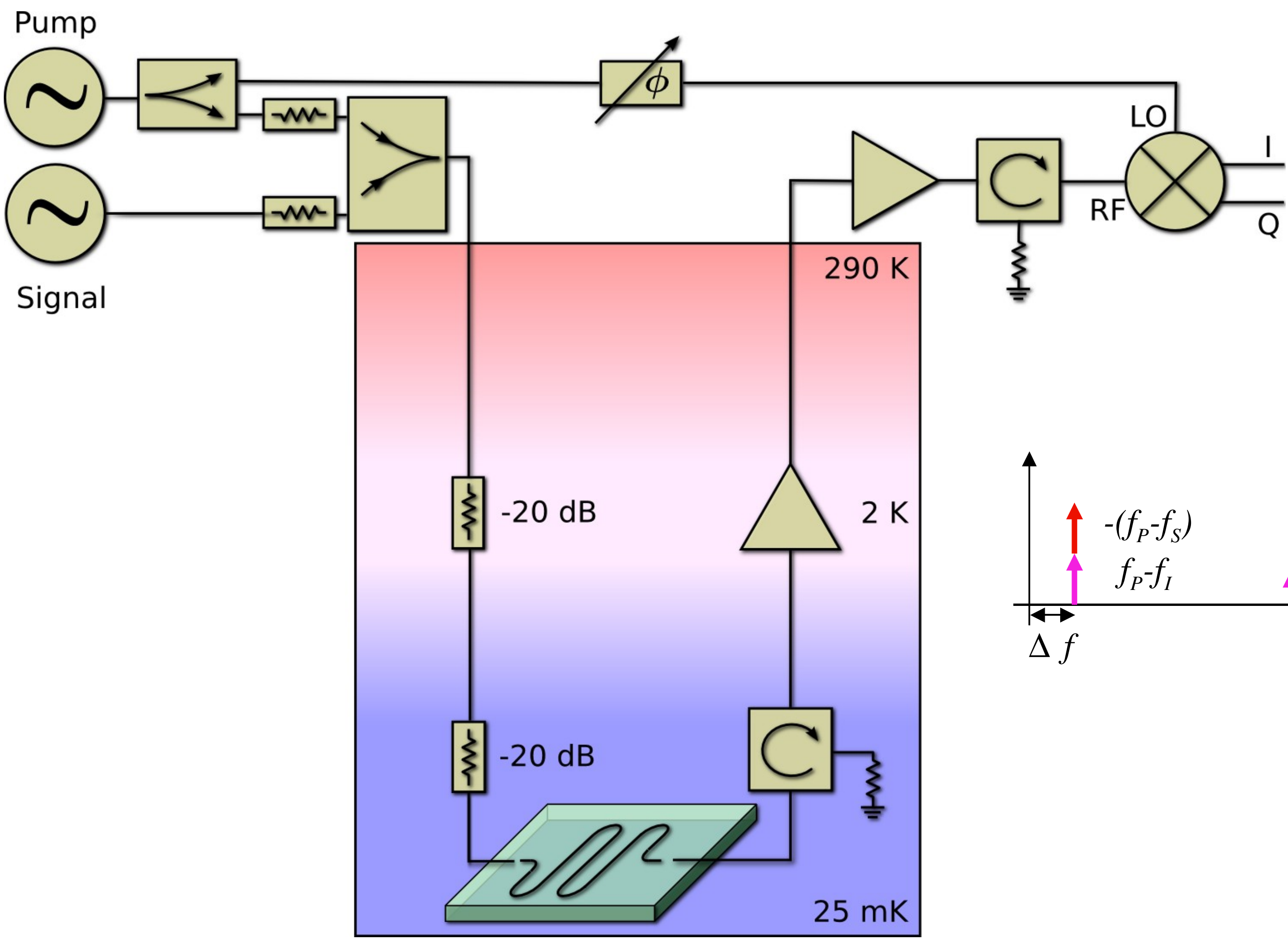
Theory

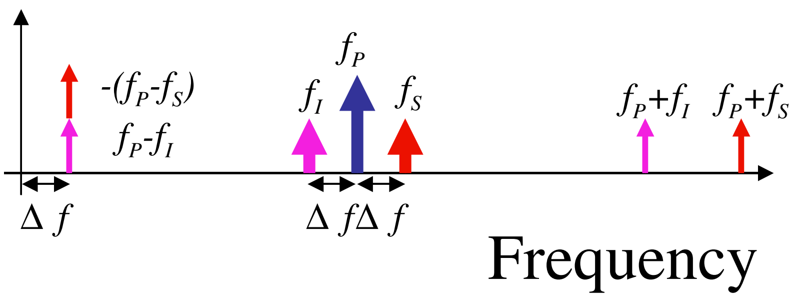


Yurke and Buks, Jour. Lightwave Tech. **24**, 5054 (2006)

Plots: E. Tholén

Phase sensitive amplification

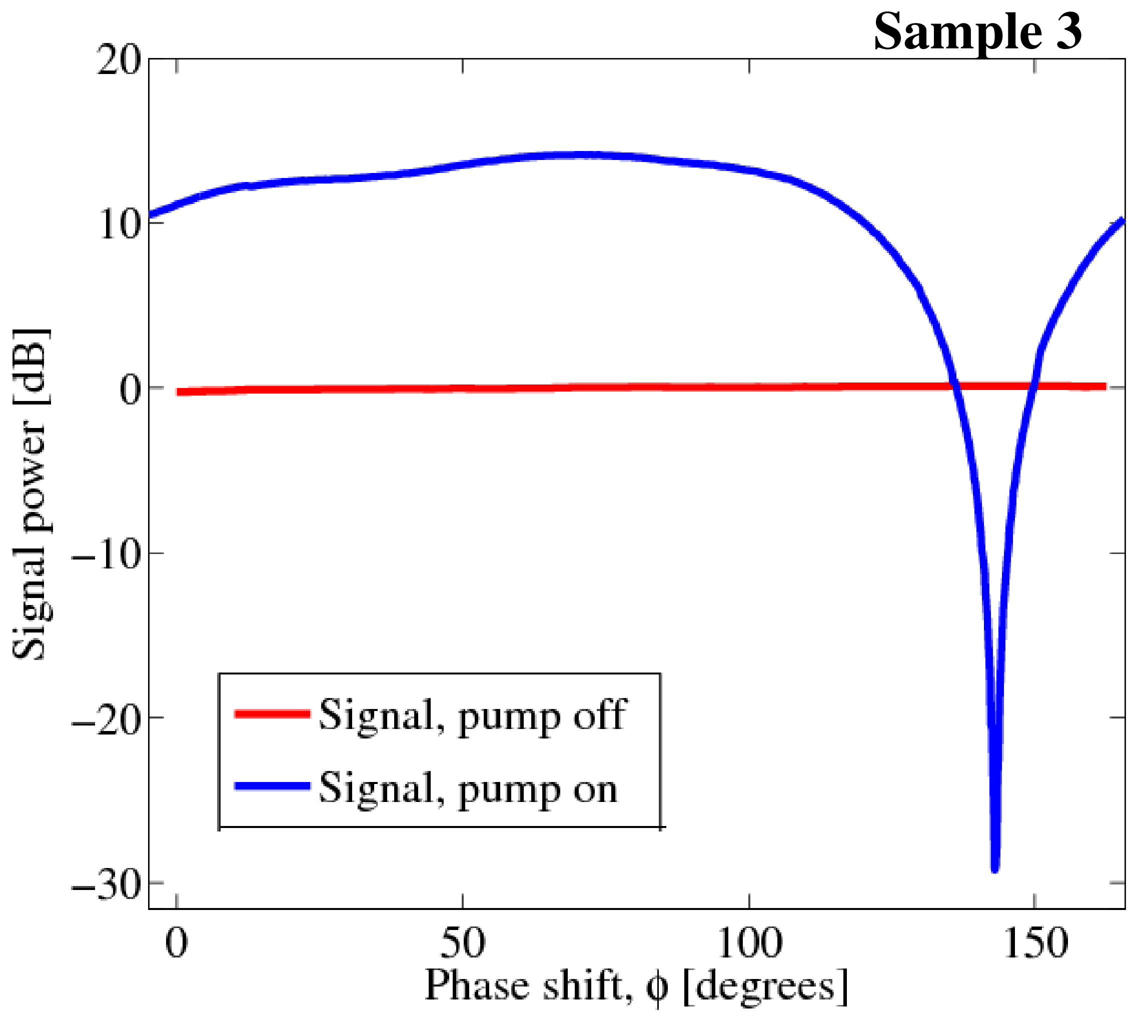




-30dB signal deamplification

•Deamplify Noise – noise squeezing

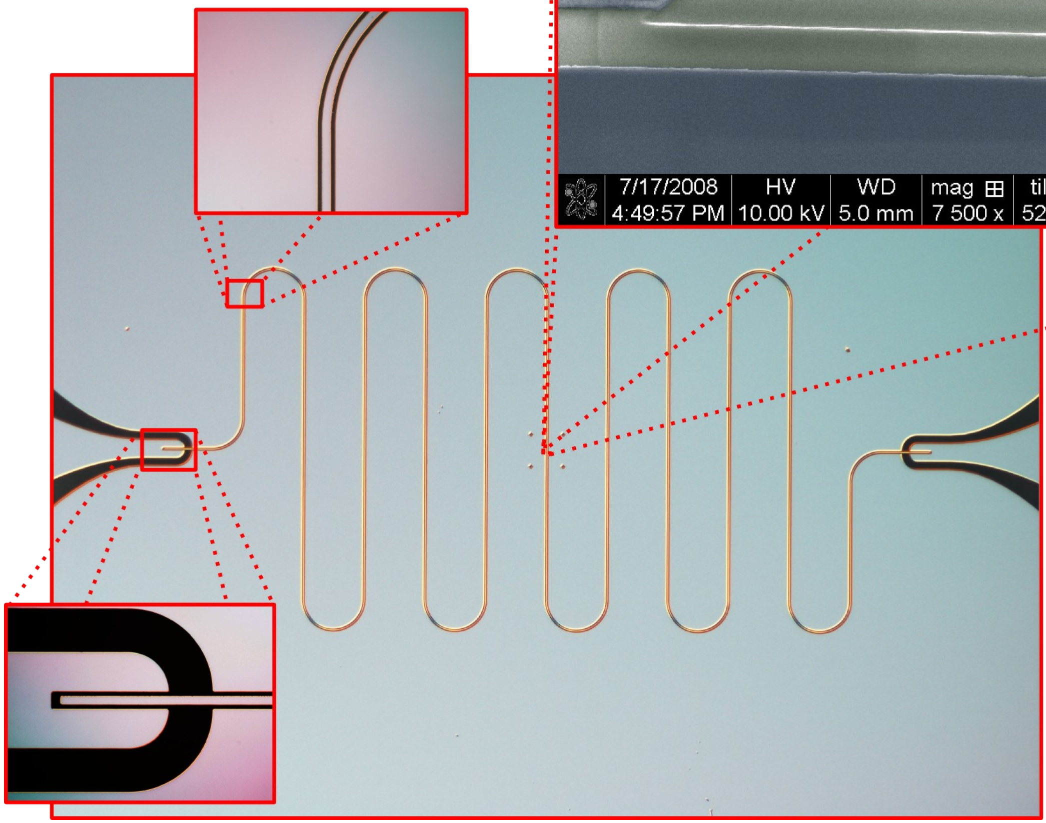
•Deamplify quantum zero point fluctuations
⇒Squeezed States

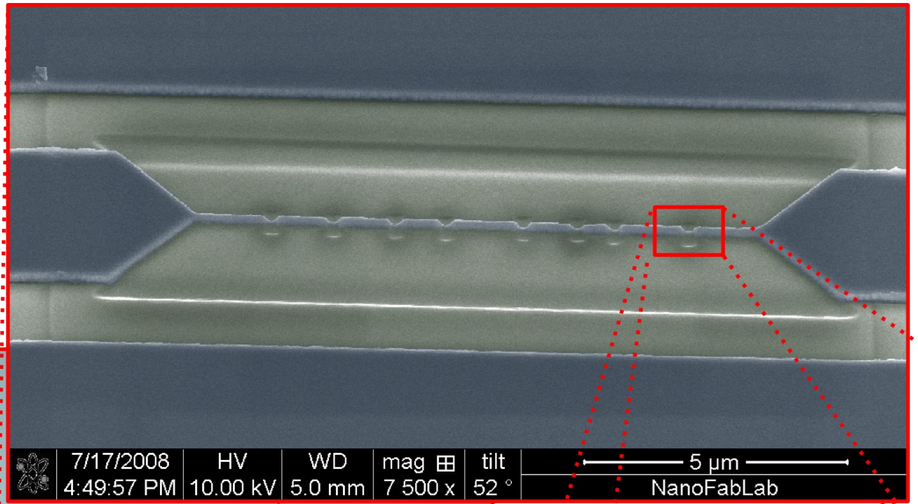


E. Tholen *et al.* unpublished

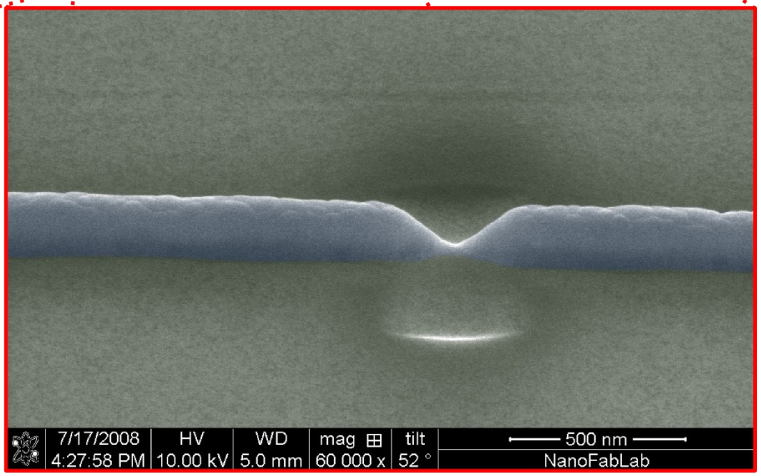
Weak link array resonator center

Resonator Fabricated by *Robin Cantor* Starcryo.com

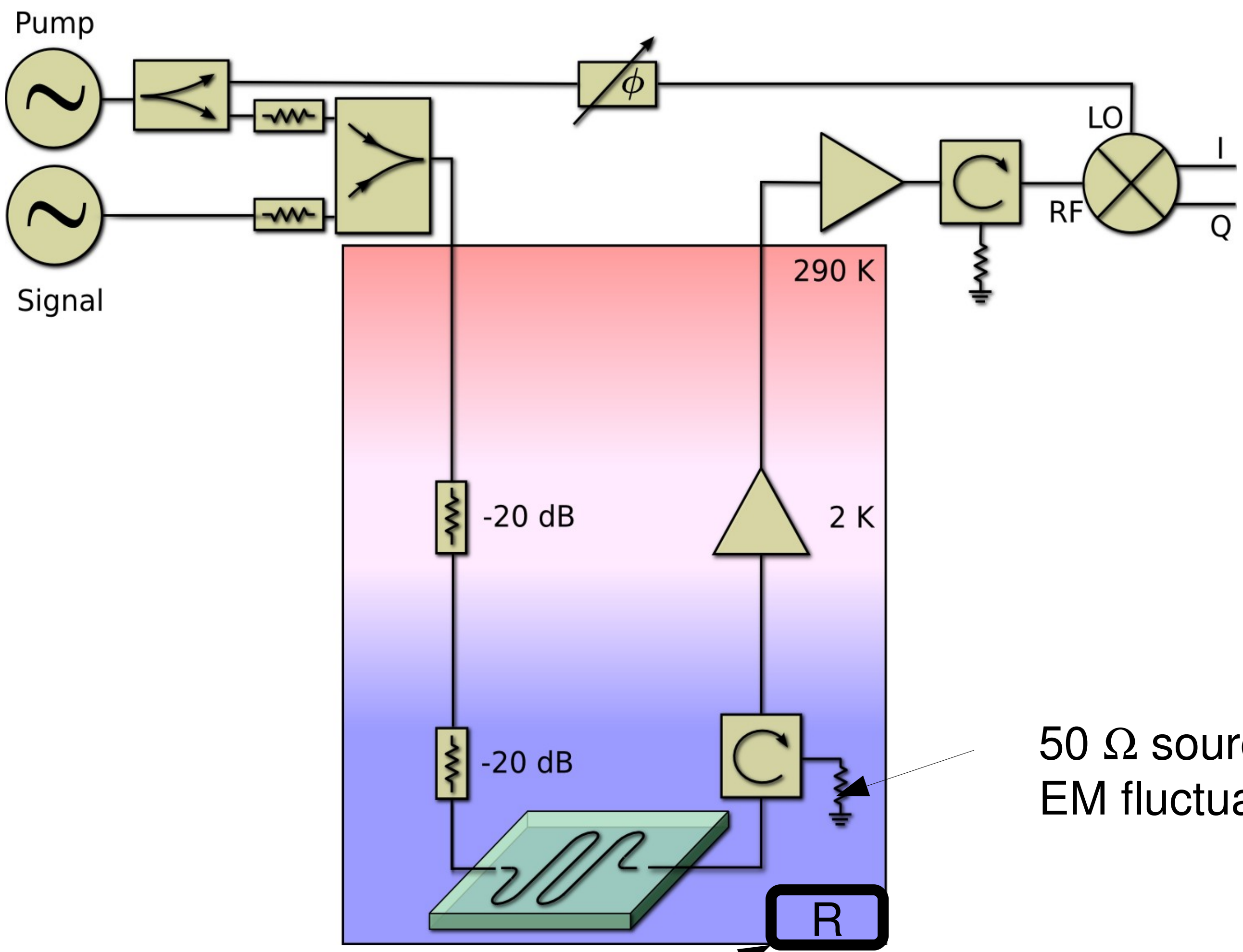




FIB weak link done at KTH



Calibrating noise measurement



50 Ω source of EM fluctuations

Resistive thermometer

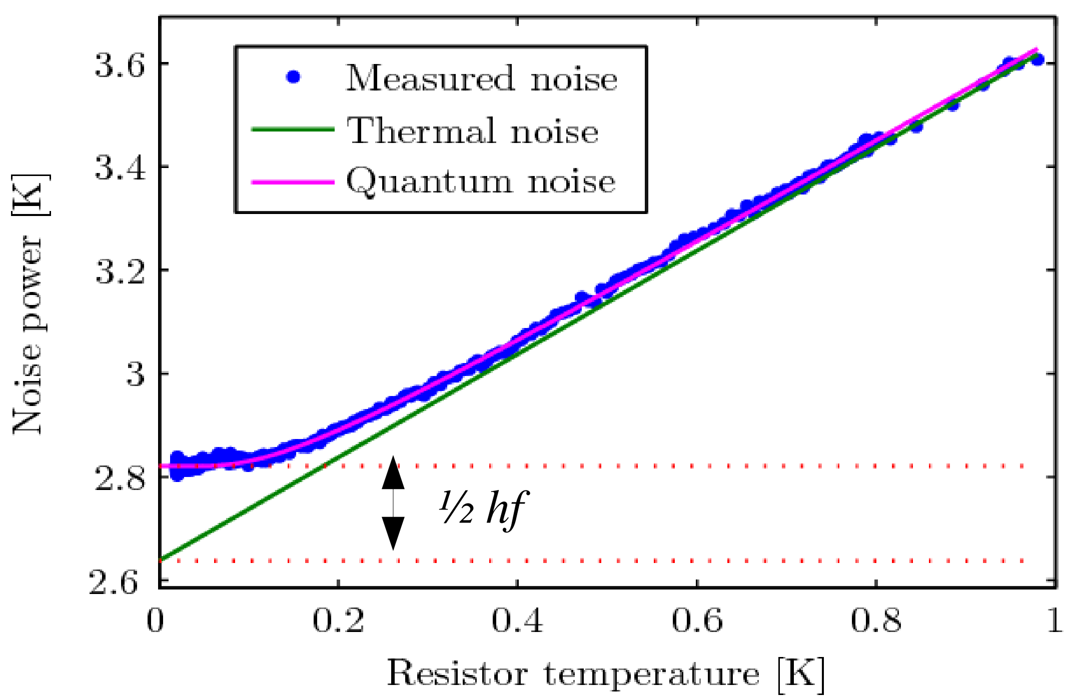
Calibrating noise measurement

$$\langle E \rangle = \hbar\omega \left(\frac{1}{2} + \langle n \rangle \right)$$

In thermal equilibrium photons are Bose-Einstein distributed, i.e.

$$\langle n \rangle = \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1}$$

$$\langle E \rangle = \hbar\omega \left(\frac{1}{2} + \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1} \right)$$



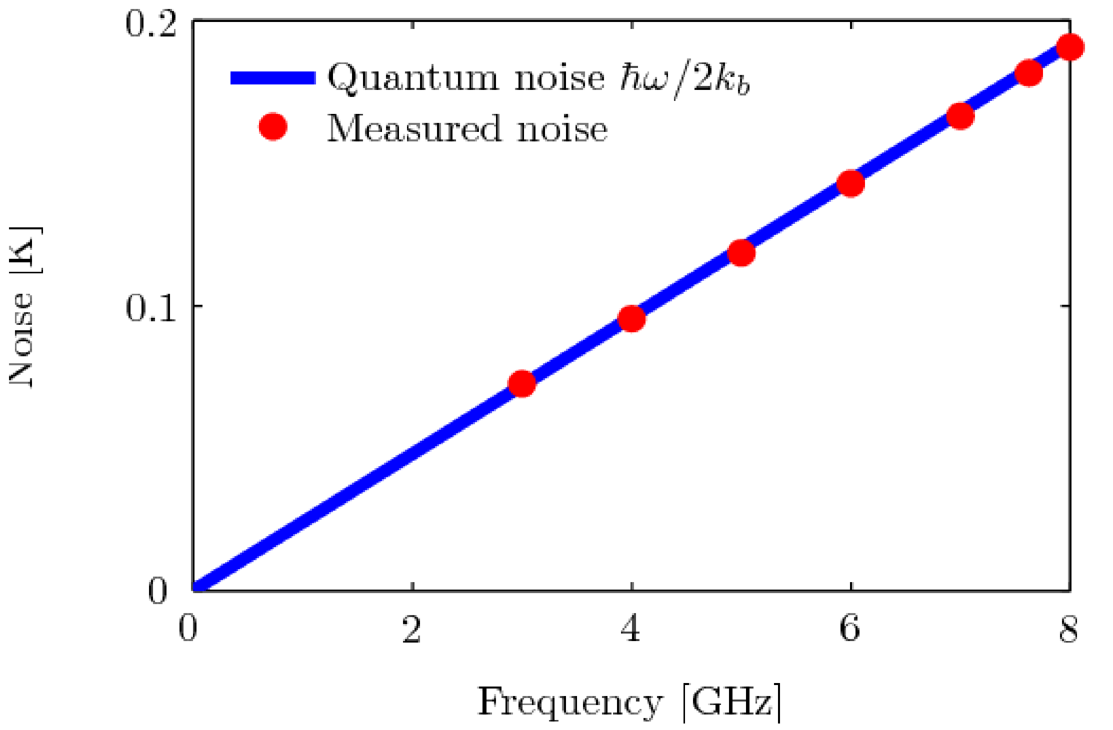
Measuring quantum noise

$$\langle E \rangle = \hbar\omega \left(\frac{1}{2} + \langle n \rangle \right)$$

In thermal equilibrium photons are Bose-Einstein distributed, i.e.

$$\langle n \rangle = \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1}$$

$$\langle E \rangle = \hbar\omega \left(\frac{1}{2} + \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1} \right)$$



Attempting to squeeze quantum noise

Summary weak link PA

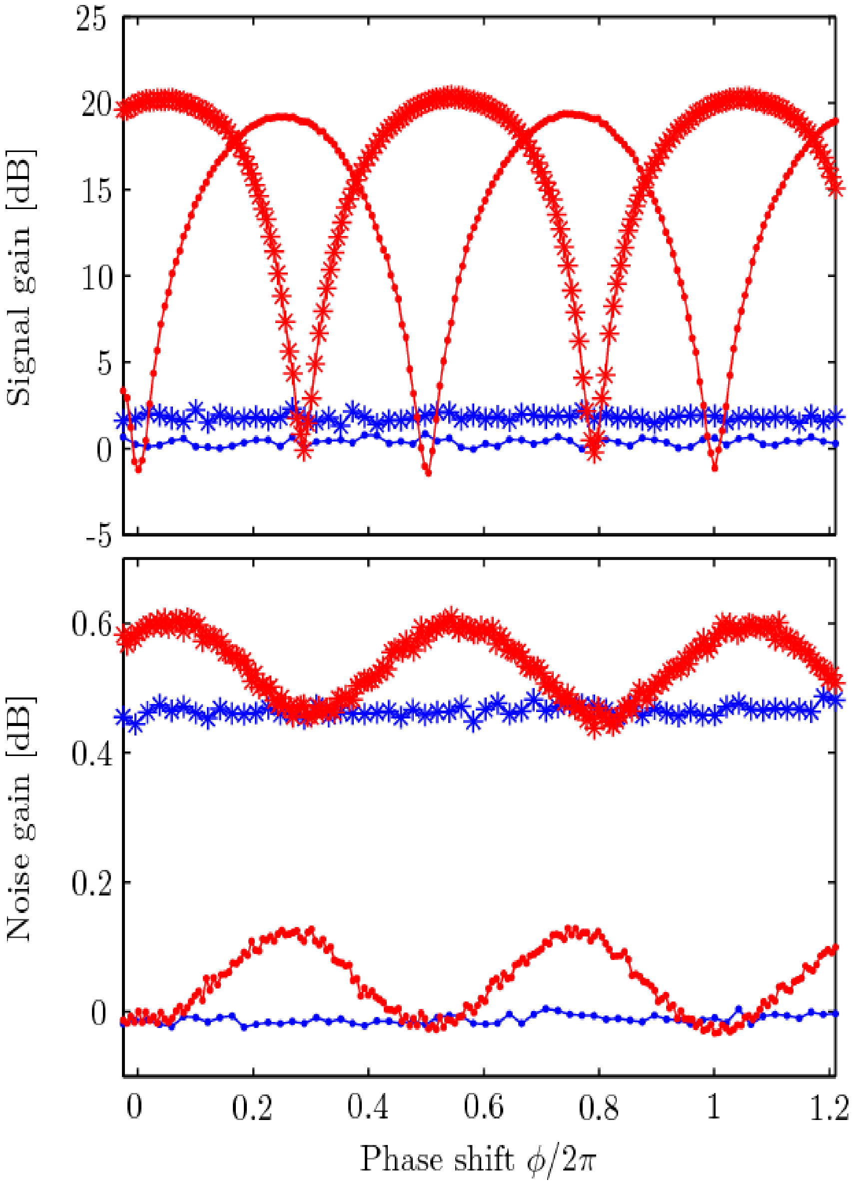
•Good Low Power Bifurcation

•Good signal gain

•Poor signal Squeezing

•No Quantum noise squeezing

•Is quantum noise squeezing possible with pair-breaking nonlinearity?



Observation of Zero-Point Noise Squeezing via a Josephson-Parametric Amplifier

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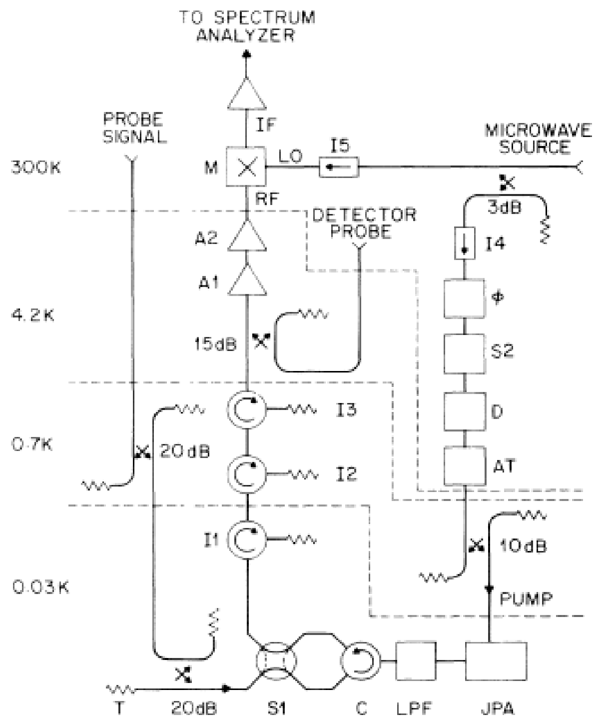
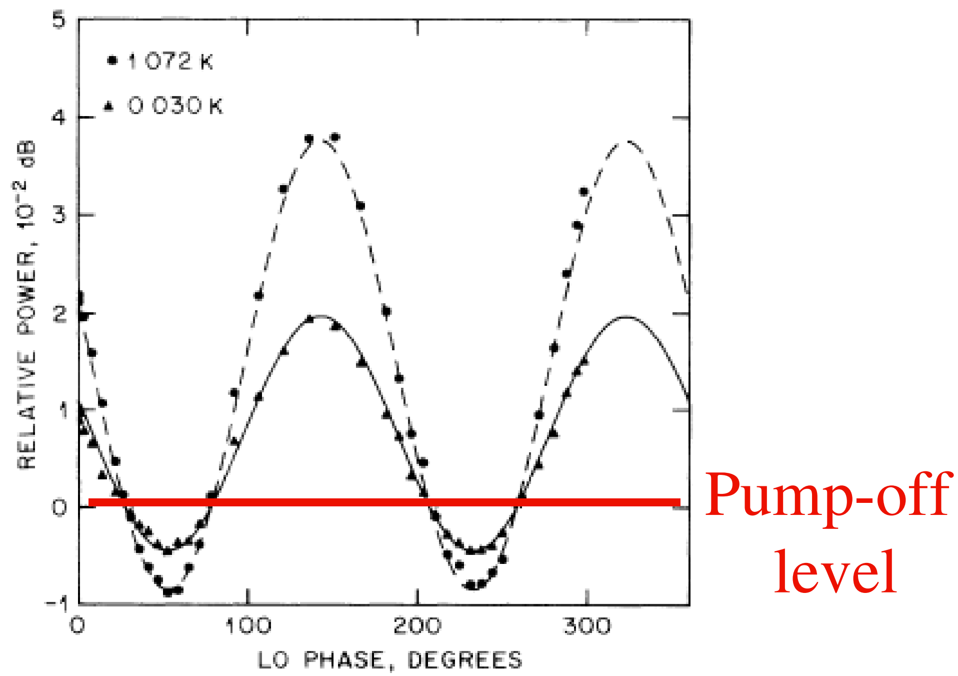
(Received 7 June 1990)





Amplification and squeezing of quantum noise with a tunable Josephson metamaterial

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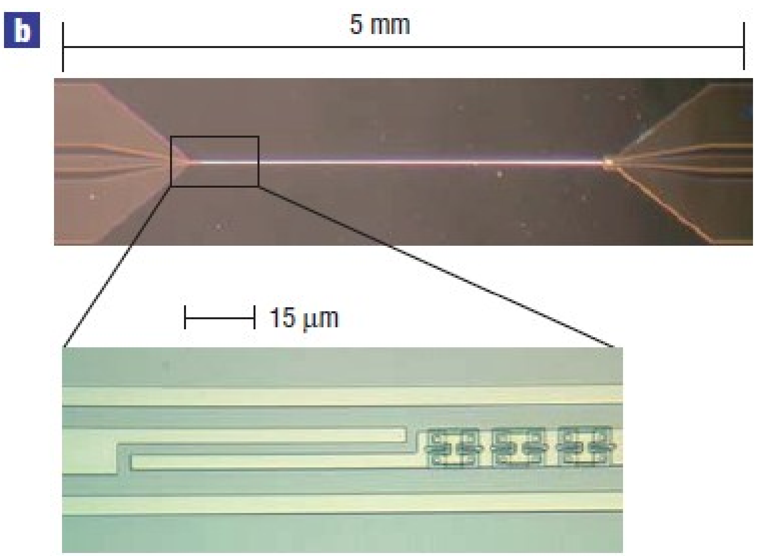
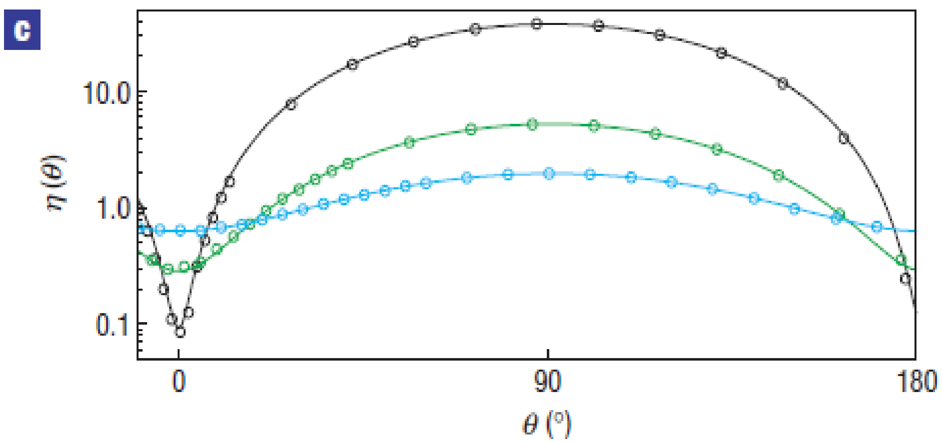
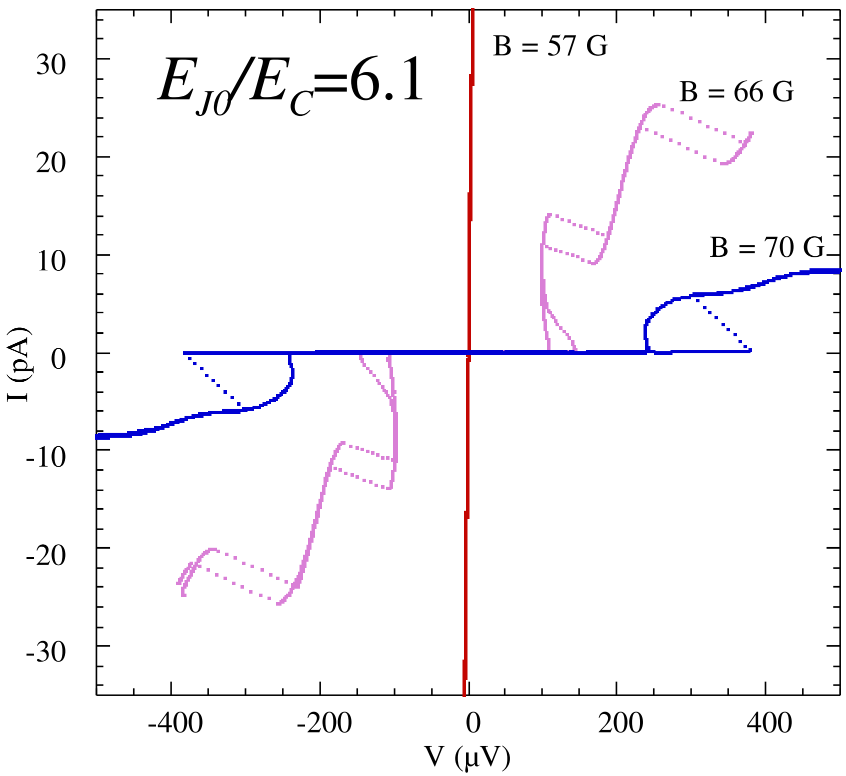




Superconductor to Insulator Transition

- When $Z_A >> R_Q = h/4e^2 =$ 6.45 kΩ
- Phase charge duality

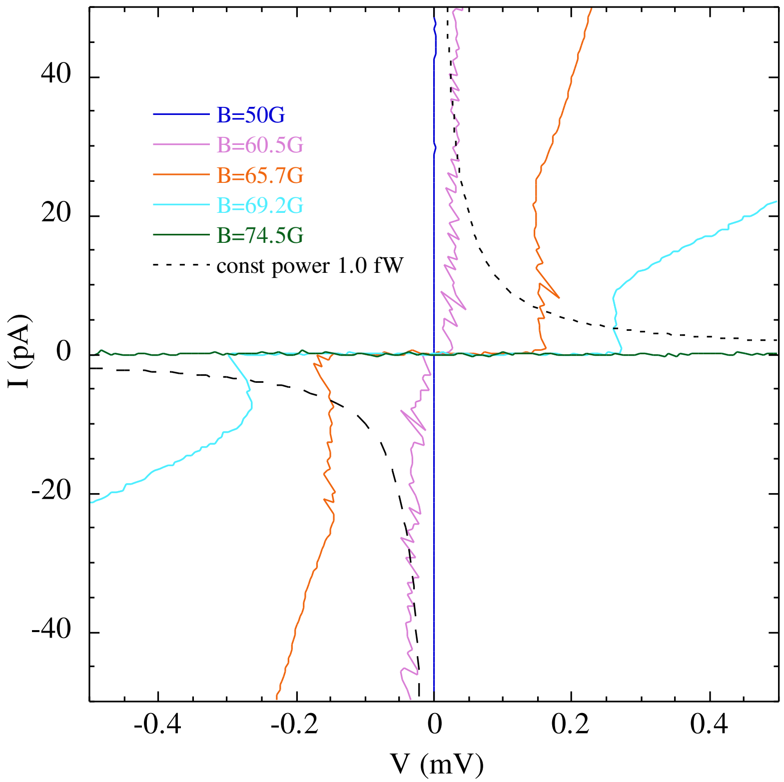


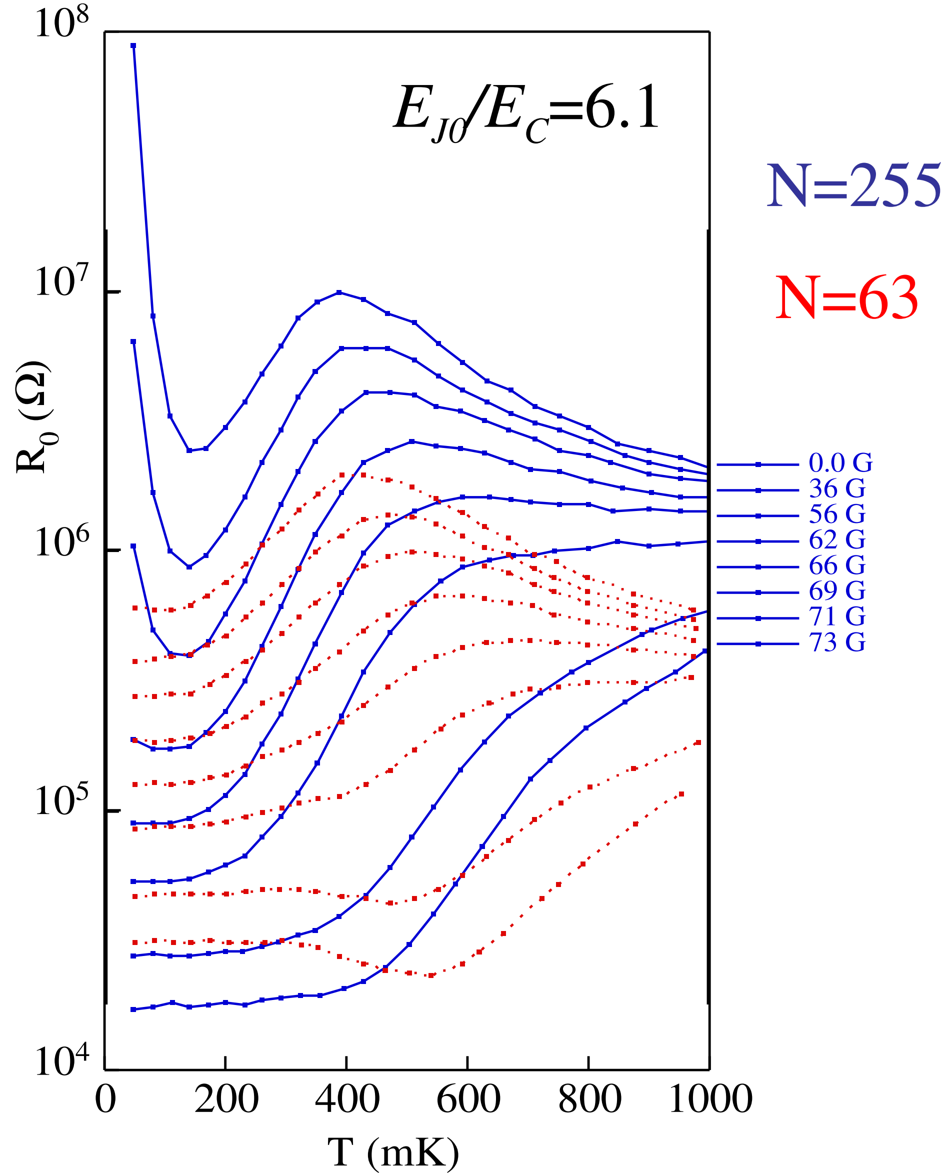
Quantum Phase Transition in 1D array

Quantum Phase Transition

Tune E_J with Φ_{ext}

Superconductor ⇒ Insulator

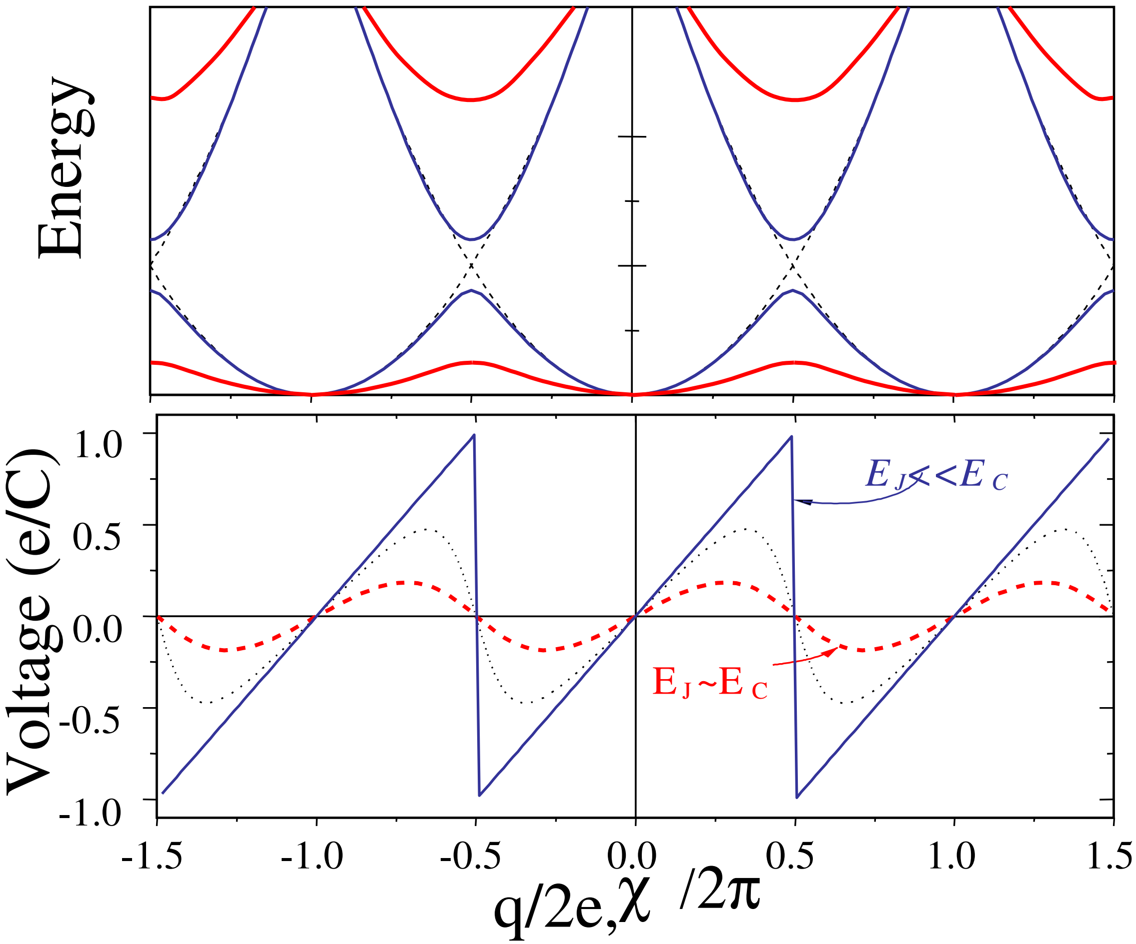




Haviland *et. al* Jour. Low Temp Phys, **118**, 73 (2000)

Quasi Charge description of Josephson Junction

Averin, Likharev and Zorin 1984



$$H = \frac{Q^2}{2C} - E_J \cos\left(2\pi \frac{\Phi}{\Phi_0}\right)$$

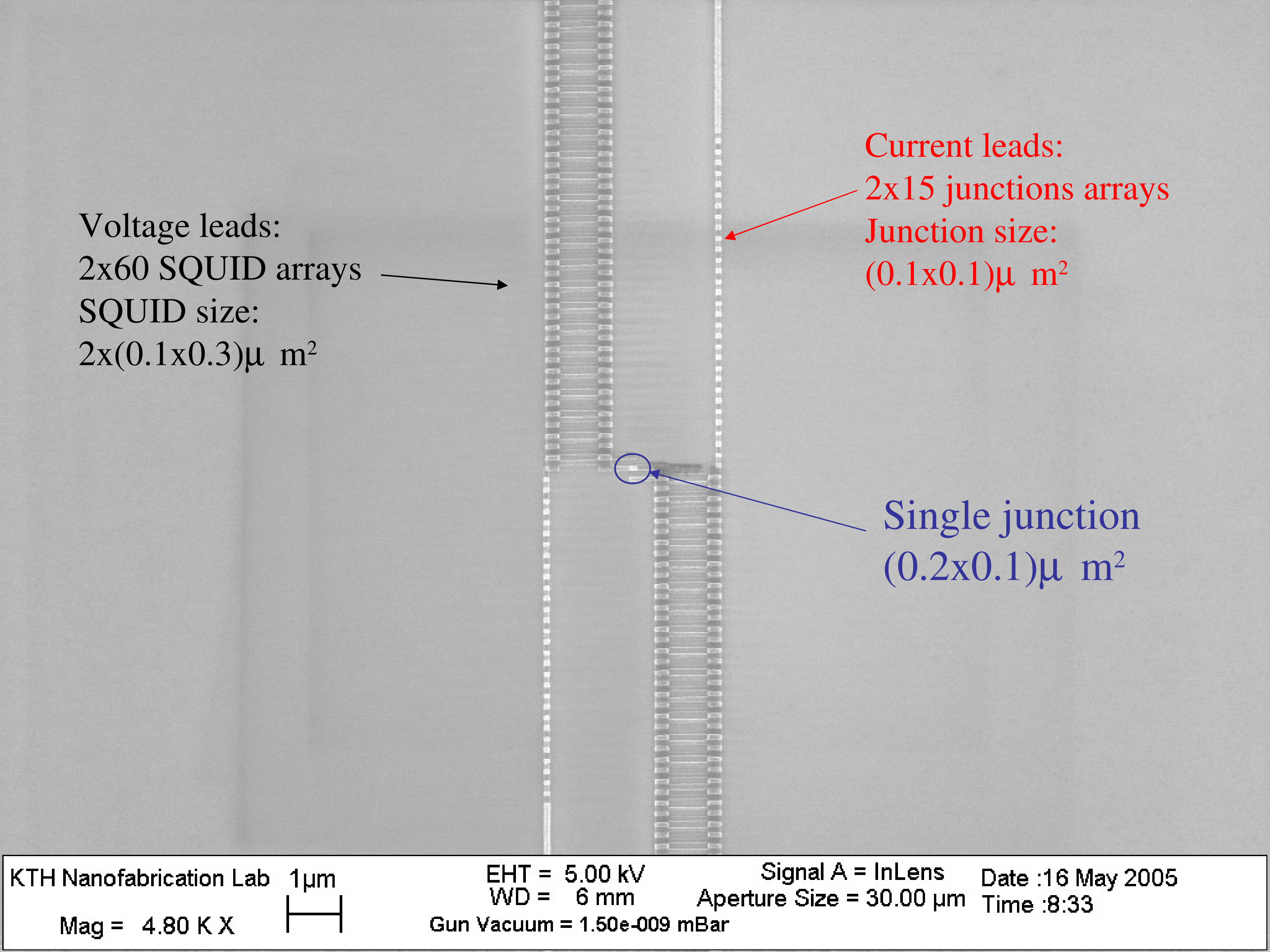
$$V = \frac{dE_0}{dq} = V_C \operatorname{saw} \chi$$

Critical Voltage: $V_C(E_J/E_C)$

dimensionless quasi-charge

$$\chi = \frac{2e}{2\pi} q$$

$$I = \frac{2e}{2\pi} \dot{\chi}$$


Voltage leads:
2x60 SQUID arrays
SQUID size:
2x(0.1x0.3)μ m²
Current leads:
2x15 junctions arrays
Junction size:
(0.1x0.1)μ m²
Single junction
(0.2x0.1)μ m²
KTH Nanofabrication Lab
1μm
Mag = 4.80 K X
EHT = 5.00 kV
WD = 6 mm
Gun Vacuum = 1.50e-009 mBar
Signal A = InLens
Aperture Size = 30.00 μm
Date :16 May 2005
Time :8:33

Temperature dependence of I-V curve

Solid lines Theory with:

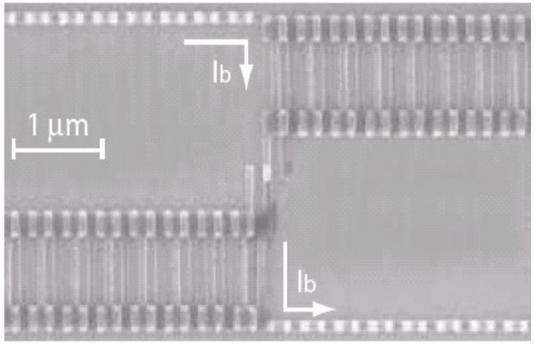
V_C (T=0) = 30 μ V

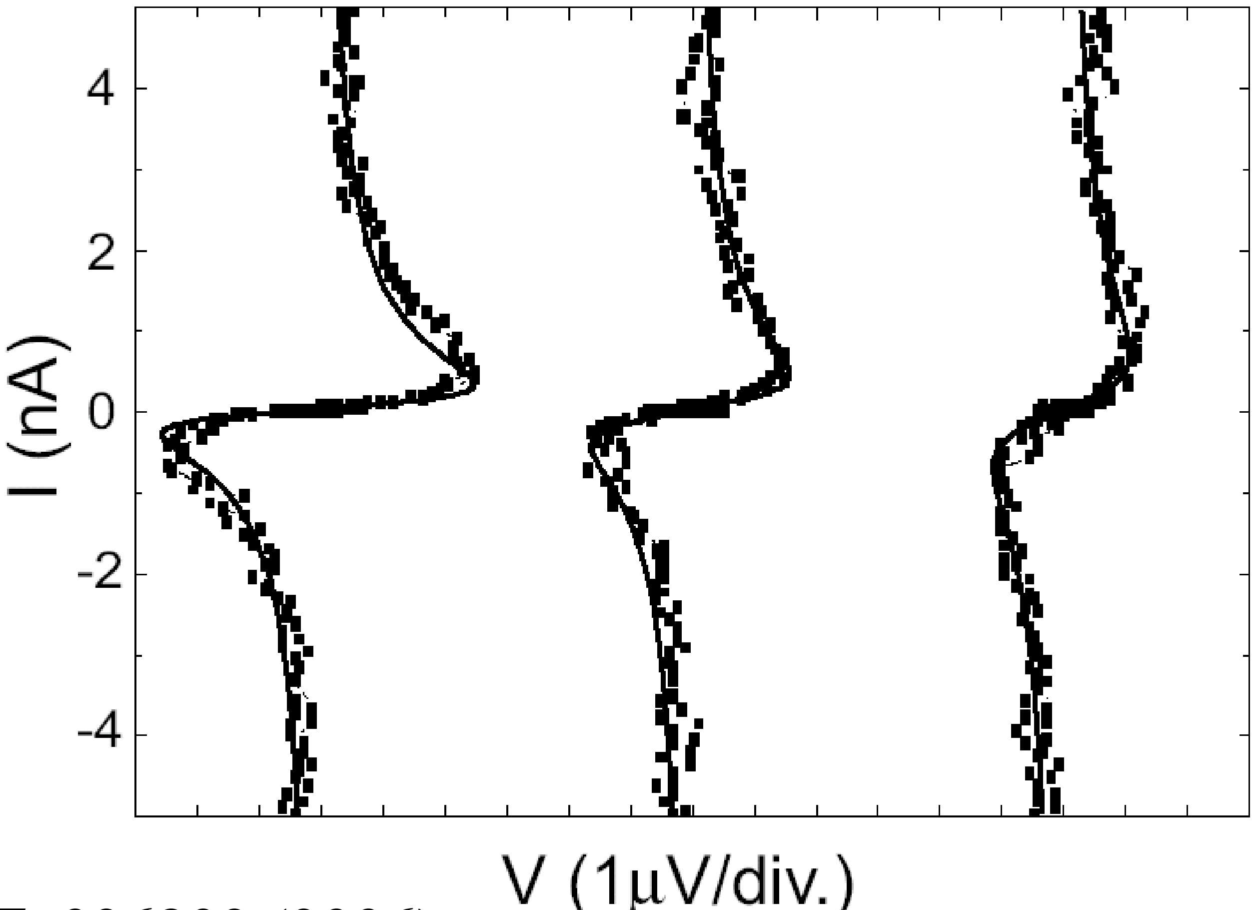
R_{fit} = 150 kΩ

T_{meas} = 50 mK
T_{fit} = 160 mK

T_{meas} = 250 mK
T_{fit} = 260 mK

T_{meas} = 300 mK
T_{fit} = 400 mK





E_J/E_C~3

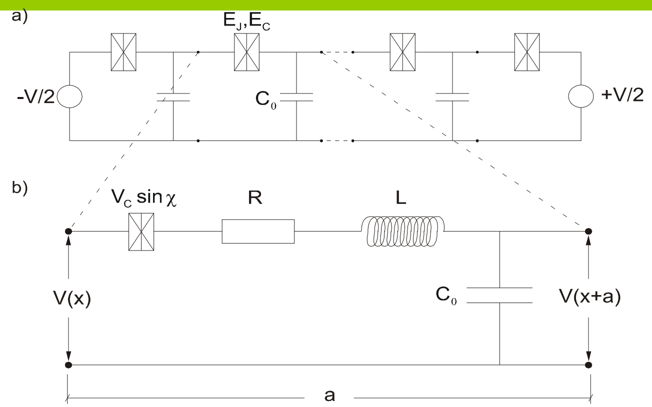
C ~0.9 fF

R_N ~ 2 kΩ

Corlevi *et al.* PRL **97,** 096802 (2006)

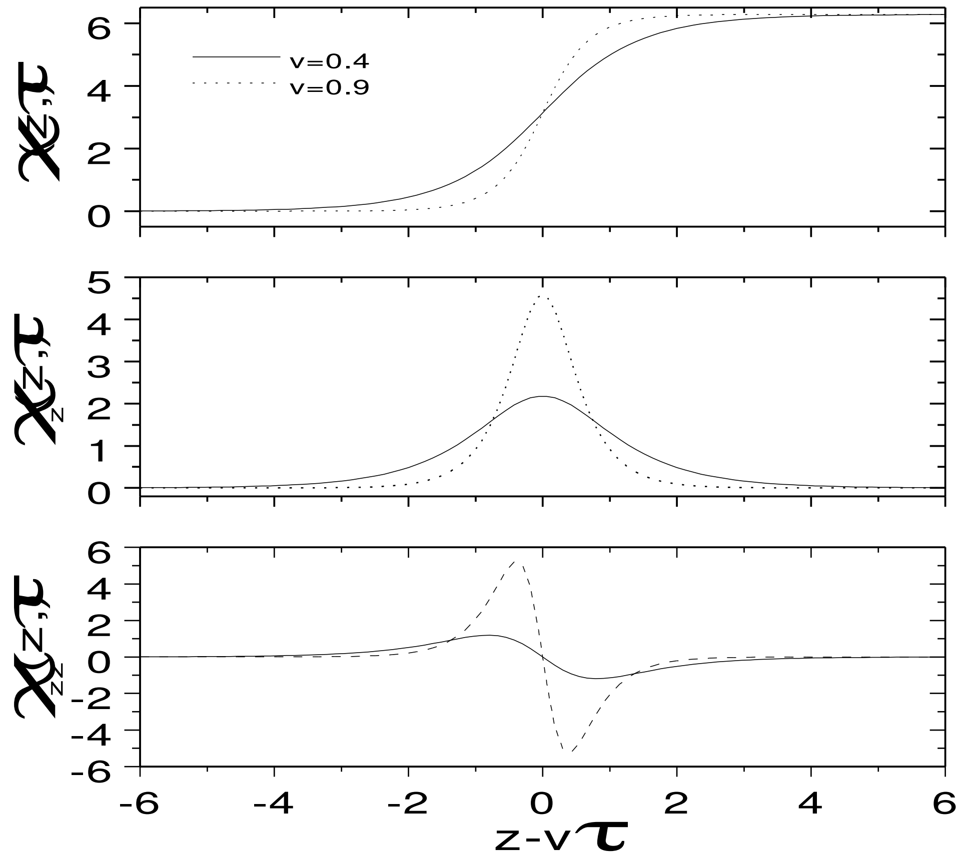
1D Series Array: Quasicharge Dynamics

Peter Ågren, Ph.D Thesis, KTH, 2001



$$\partial_{xx}\chi - \frac{1}{v_0^2}\partial_{tt}\chi - \frac{1}{\lambda_s^2}\operatorname{saw}\chi = rc_0\partial_t\chi$$

Phase	Quasicharge
Magnetic Flux	Potential
Super Current	Electric field



Conclusions

- Classical, linear Electrodynamics
  - microwave photonic band gap
  - impedance matching possible for stripline geometry

- Classical nonlinear Electrodyanics
  - parametric amplifier
  - Noise squeezing, vacuum squeezing

- Quantum limit
  - Coulomb blockade of Cooper pairs
  - Charge solitons – duality to fluxon

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Frank Hekking

Carsten Hutter

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