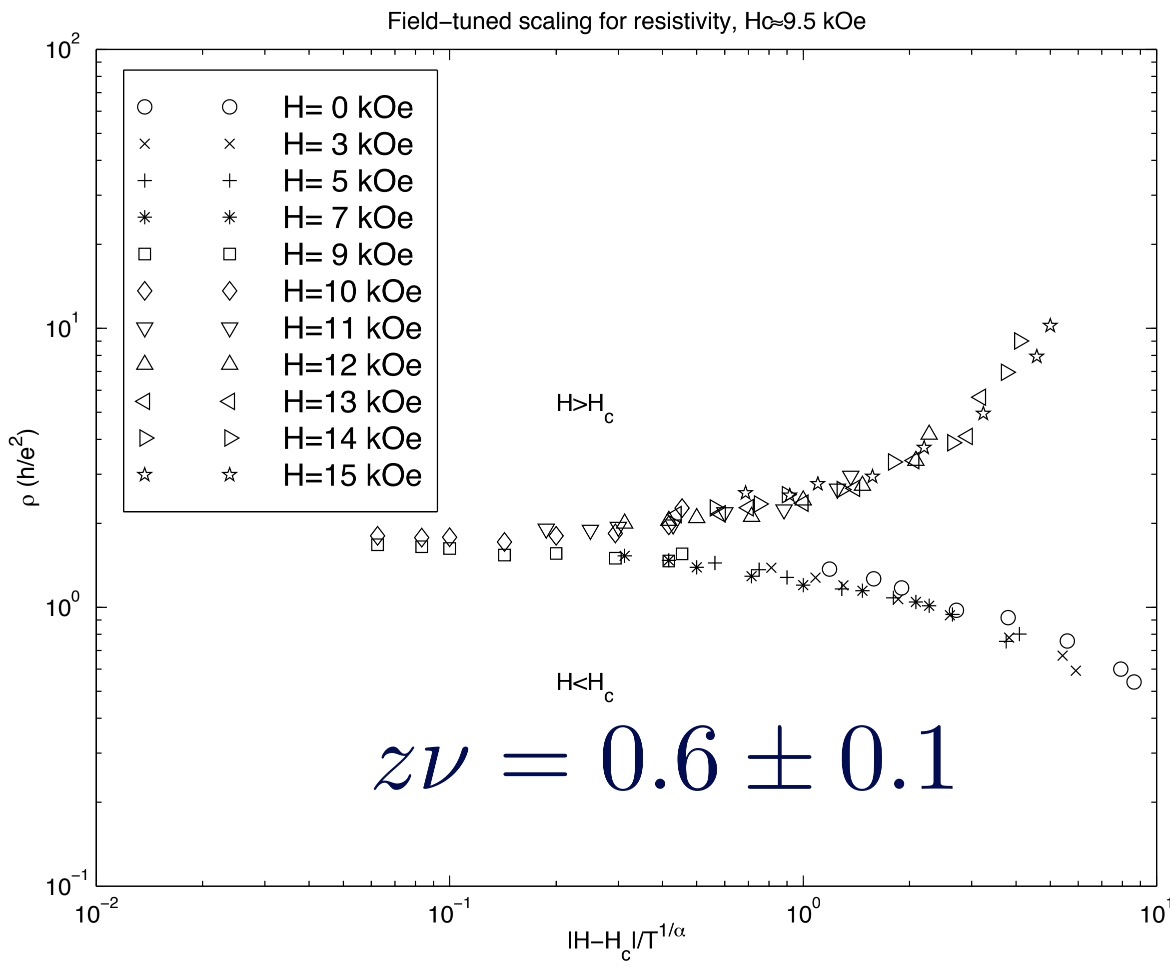
Bose Glass and Commensurate Bosons: Unhappy Marriage

Thanks to :
Frank Kruger and Jiansheng Wu

arXiv:0904.4480

Interacting electrons in a random potential

than the Fermi energy. It is interesting to note that experiments on the IST in bismuth films (N. Markovic, Christiansen, and A. M. Goldman, personal communication) show that $z_B \nu_B = 0.7 \pm 0.2$, which is remarkably close to the value, $\alpha = 0.6 \pm 0.1$, obtained in the scaling plot in Fig. (1).



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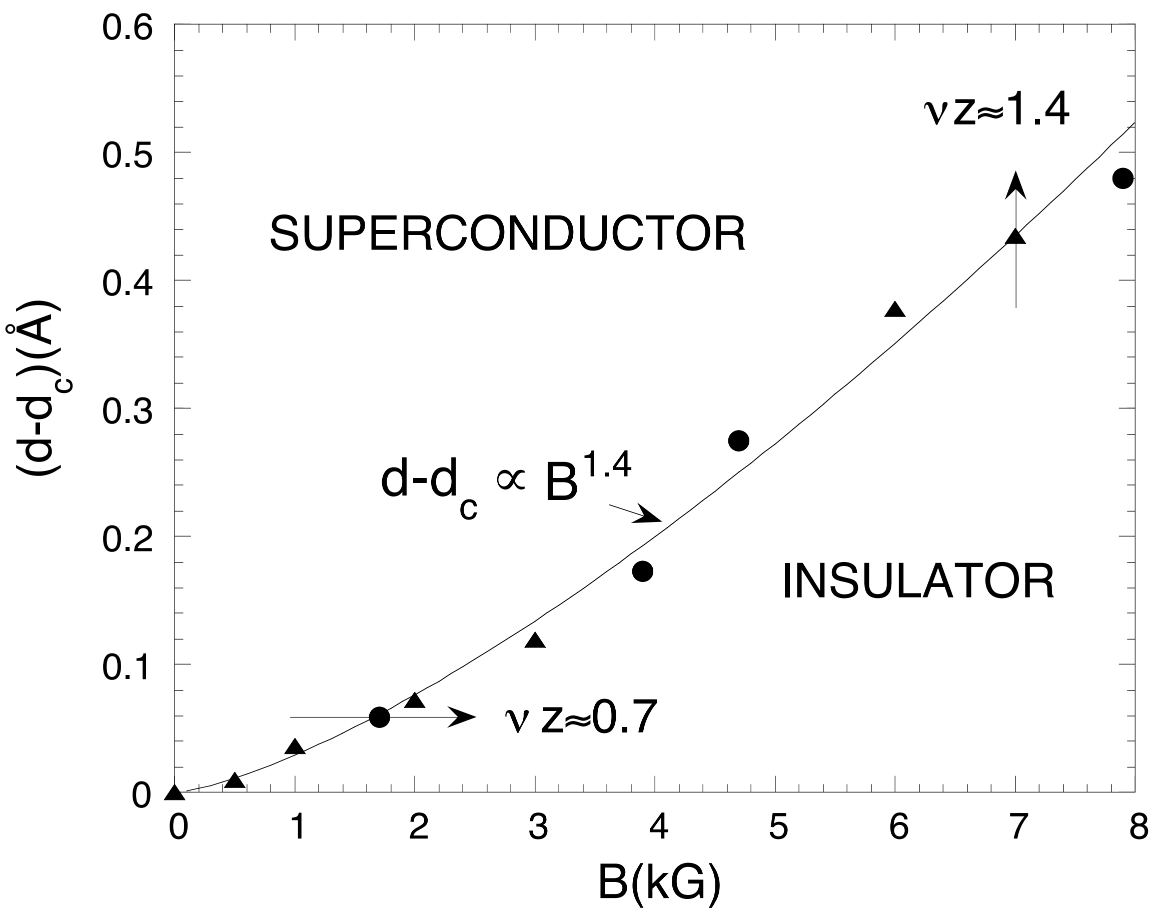

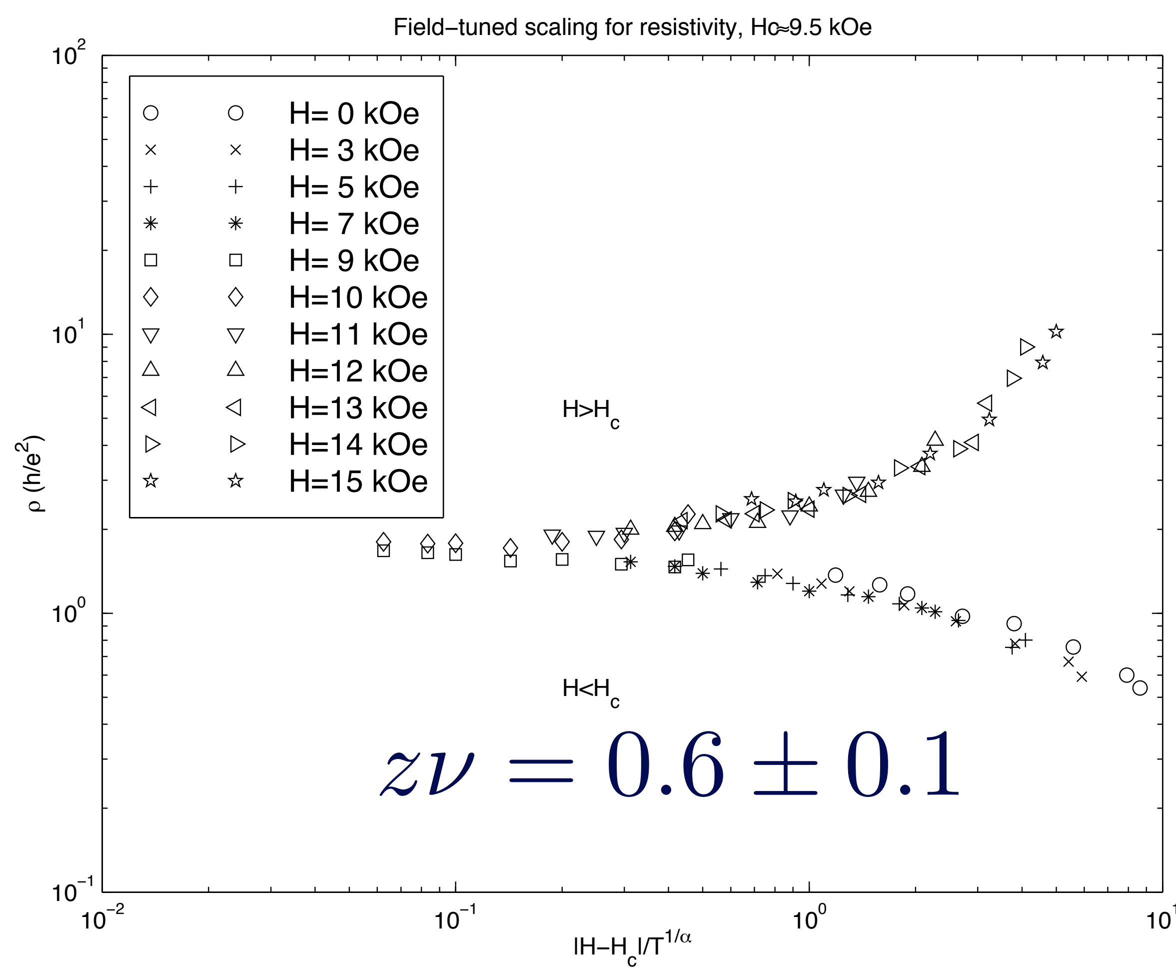

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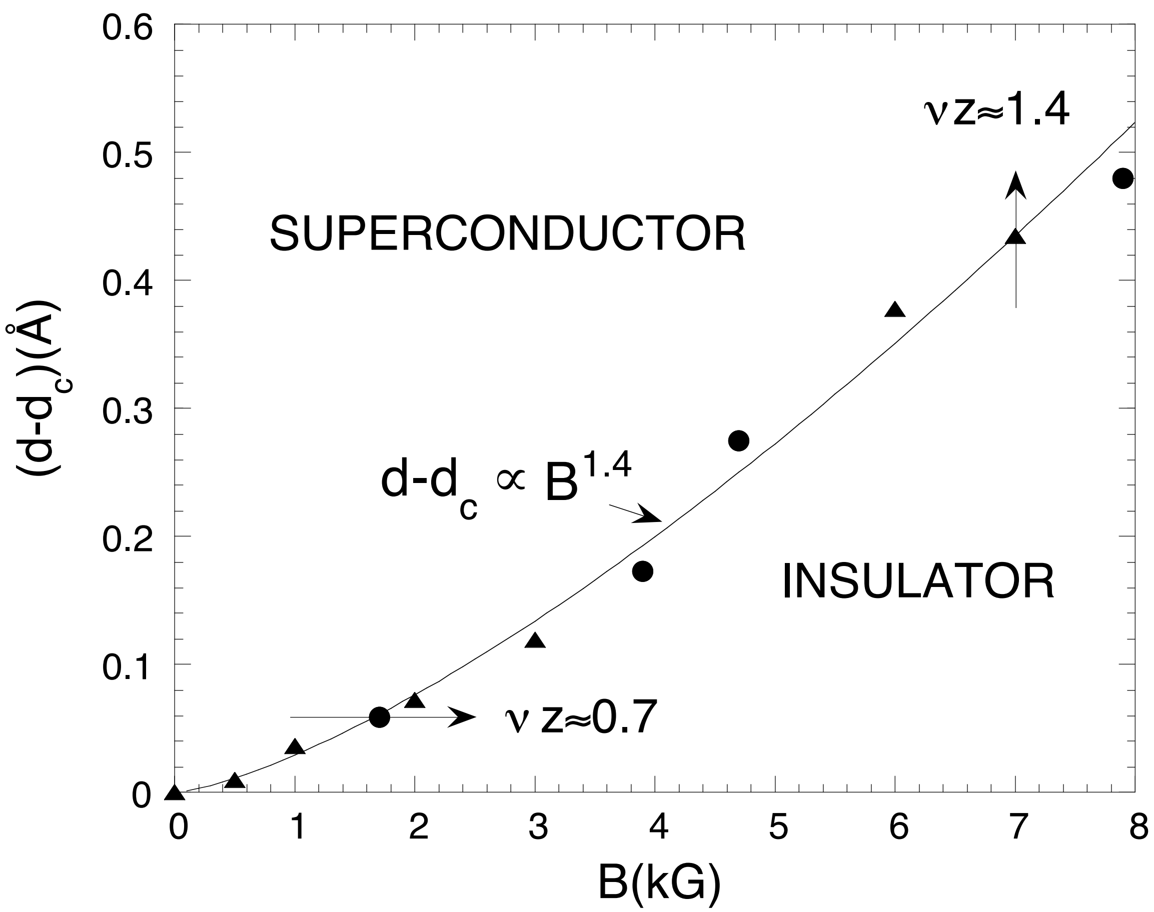

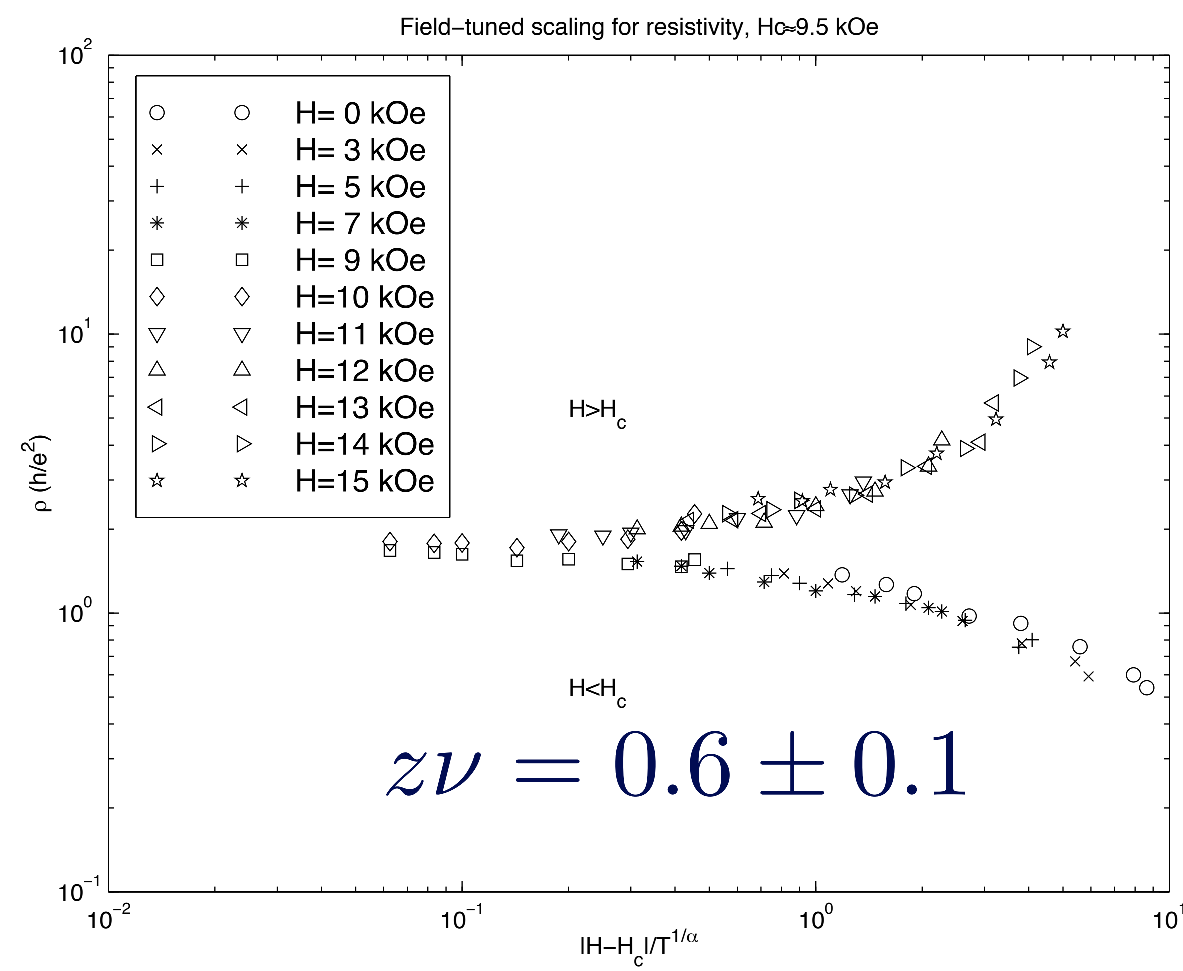

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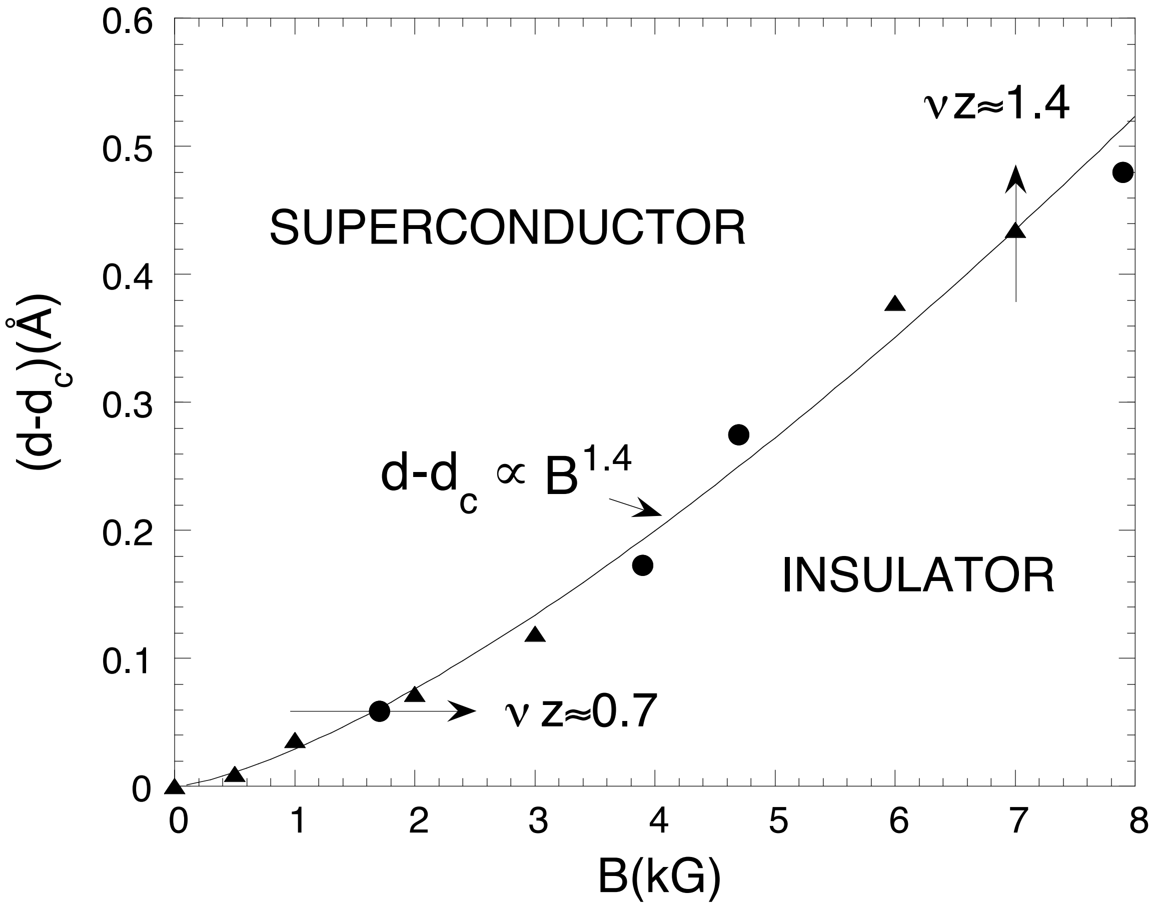
Interacting bosons in a random potential



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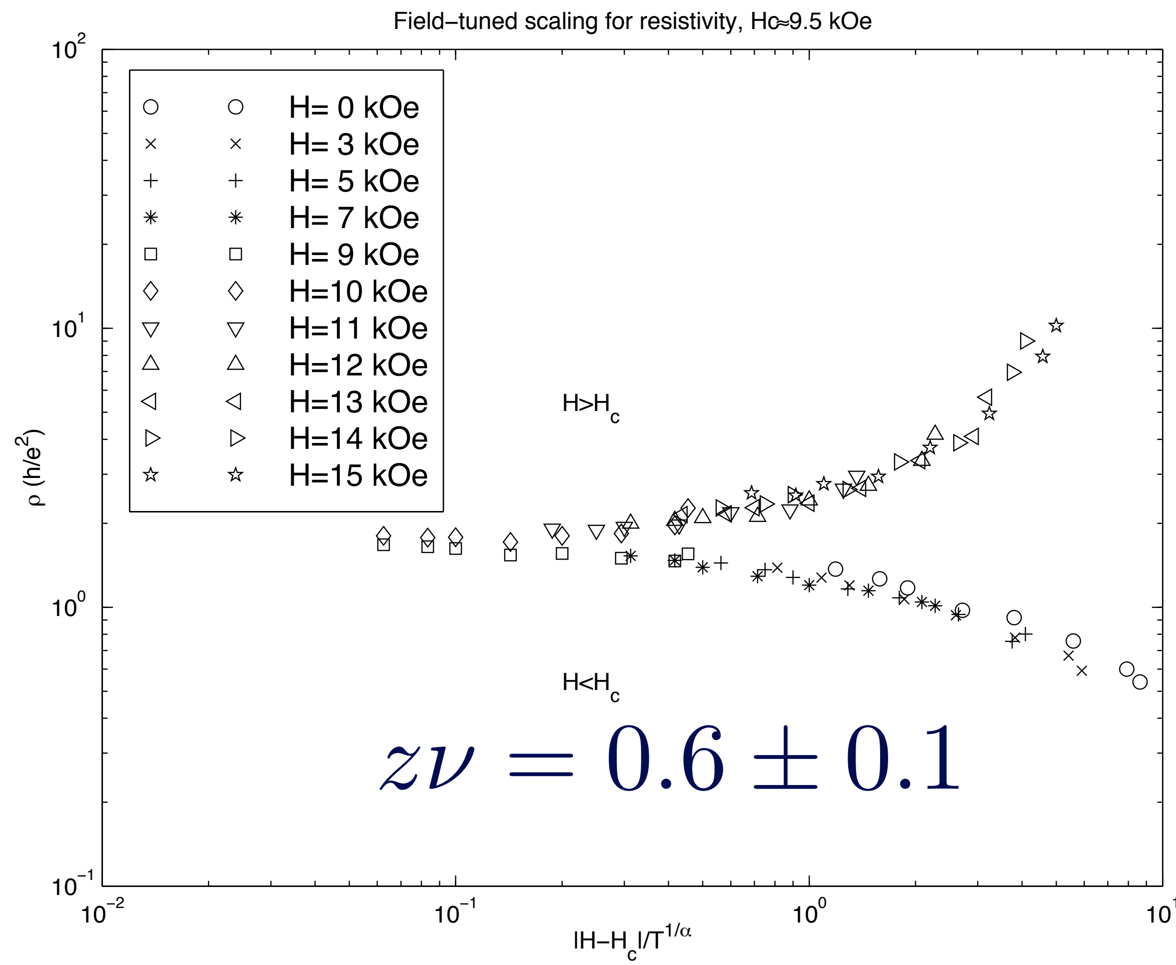

$$z\nu = 0.6 \pm 0.1$$

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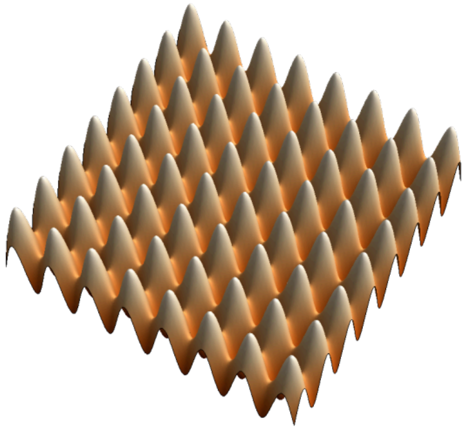
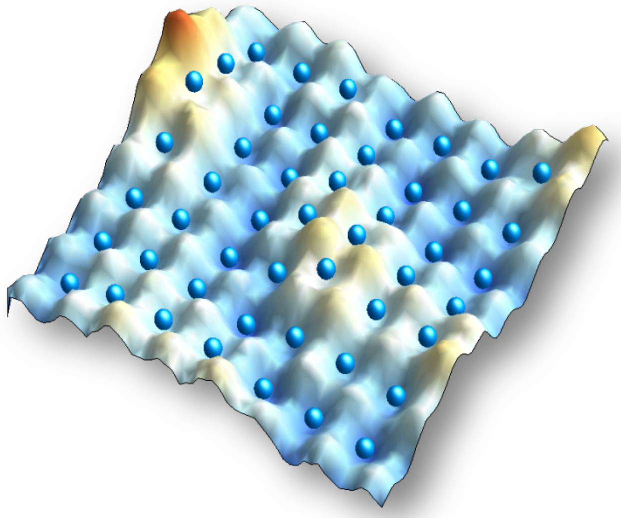
How is superfluitidy destroyed at commensurate boson fillings in the disordered Bose Hubbard model?

Interacting bosons in a random potential



Interacting bosons in a random potential



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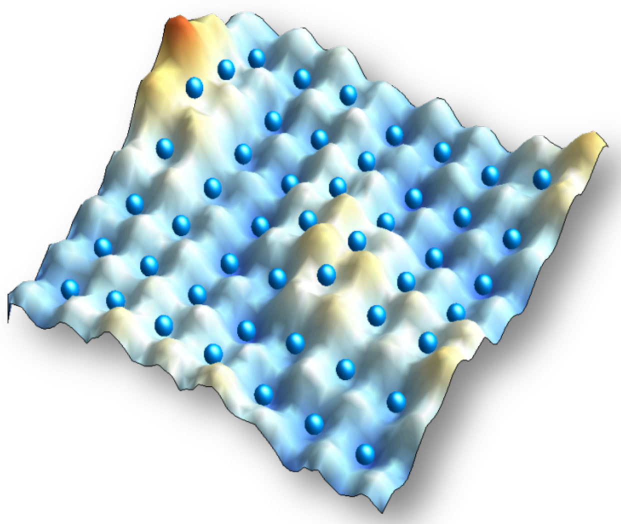
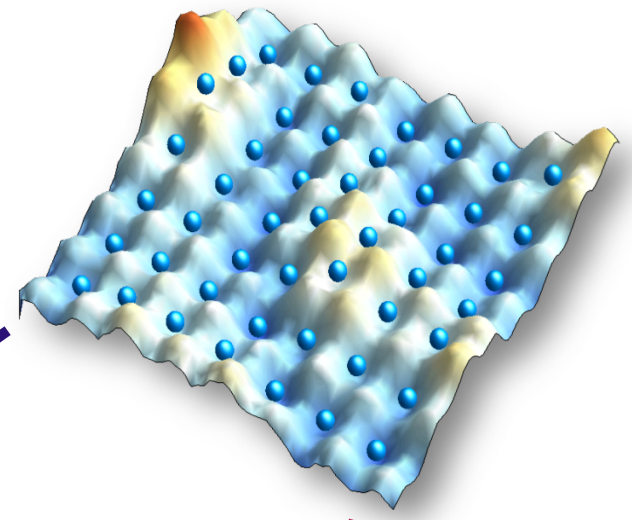


large filling

phase-only model J_{ij}

phase glass Bose metal

Interacting bosons in a random potential



large filling

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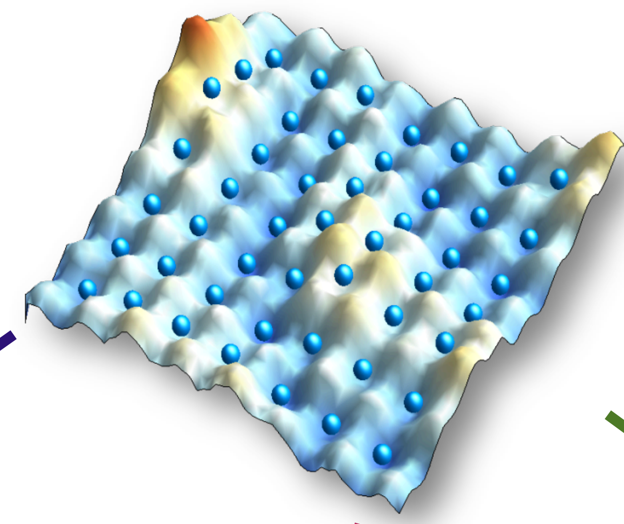
phase glass
Bose metal

ϕ^4+ random mass

^4He in Vycor

Interacting bosons in a random potential



large filling

phase-only model J_{ij}

phase glass
Bose metal

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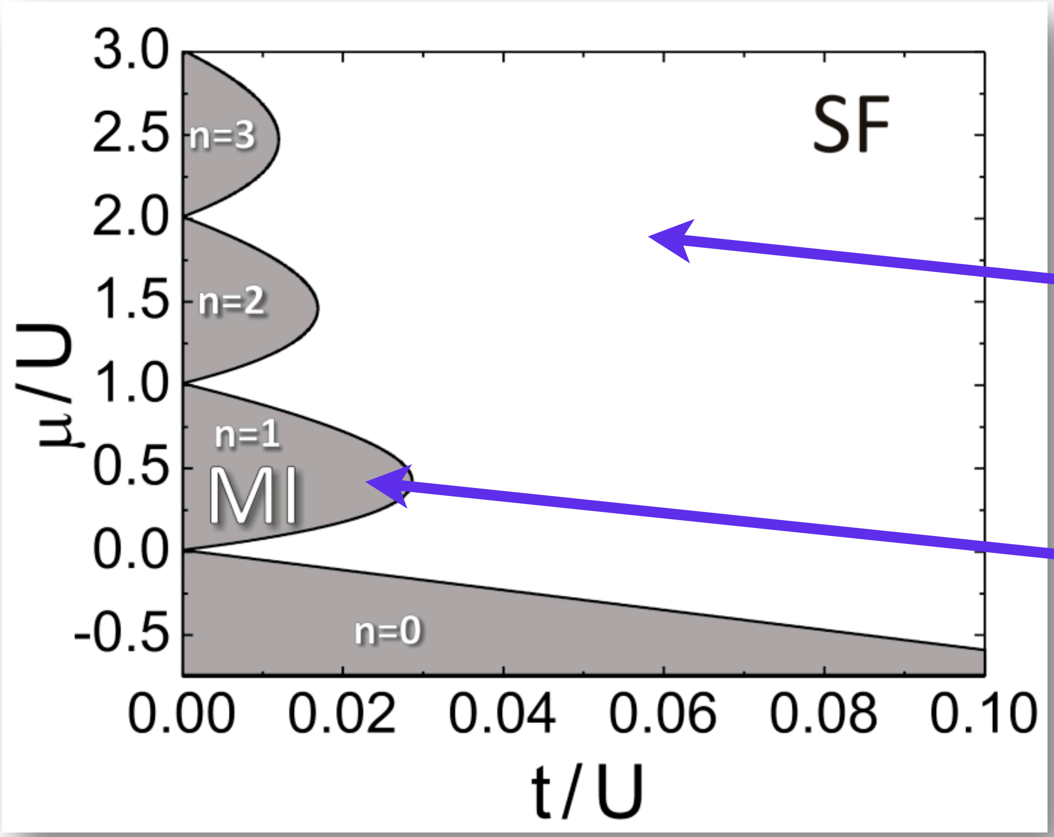
^4He in Vycor

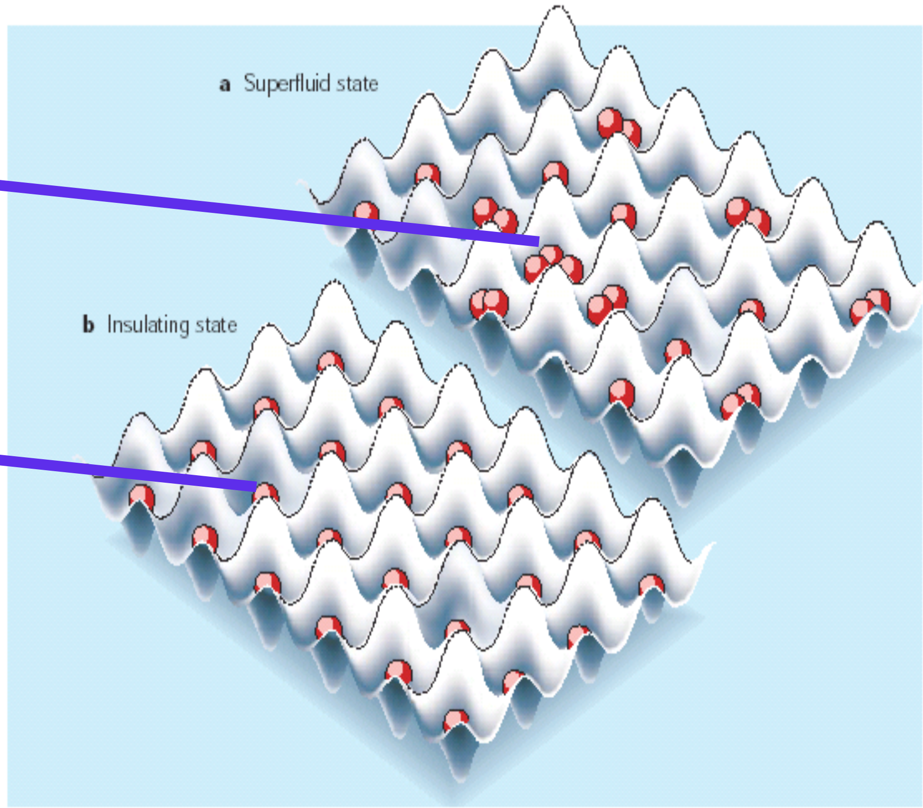
Bose Hubbard model+disorder

Bose glass

Clean Bose-Hubbard Model



$$\Delta\theta\Delta n \geq \hbar$$



t/U

Clean Bose-Hubbard Model

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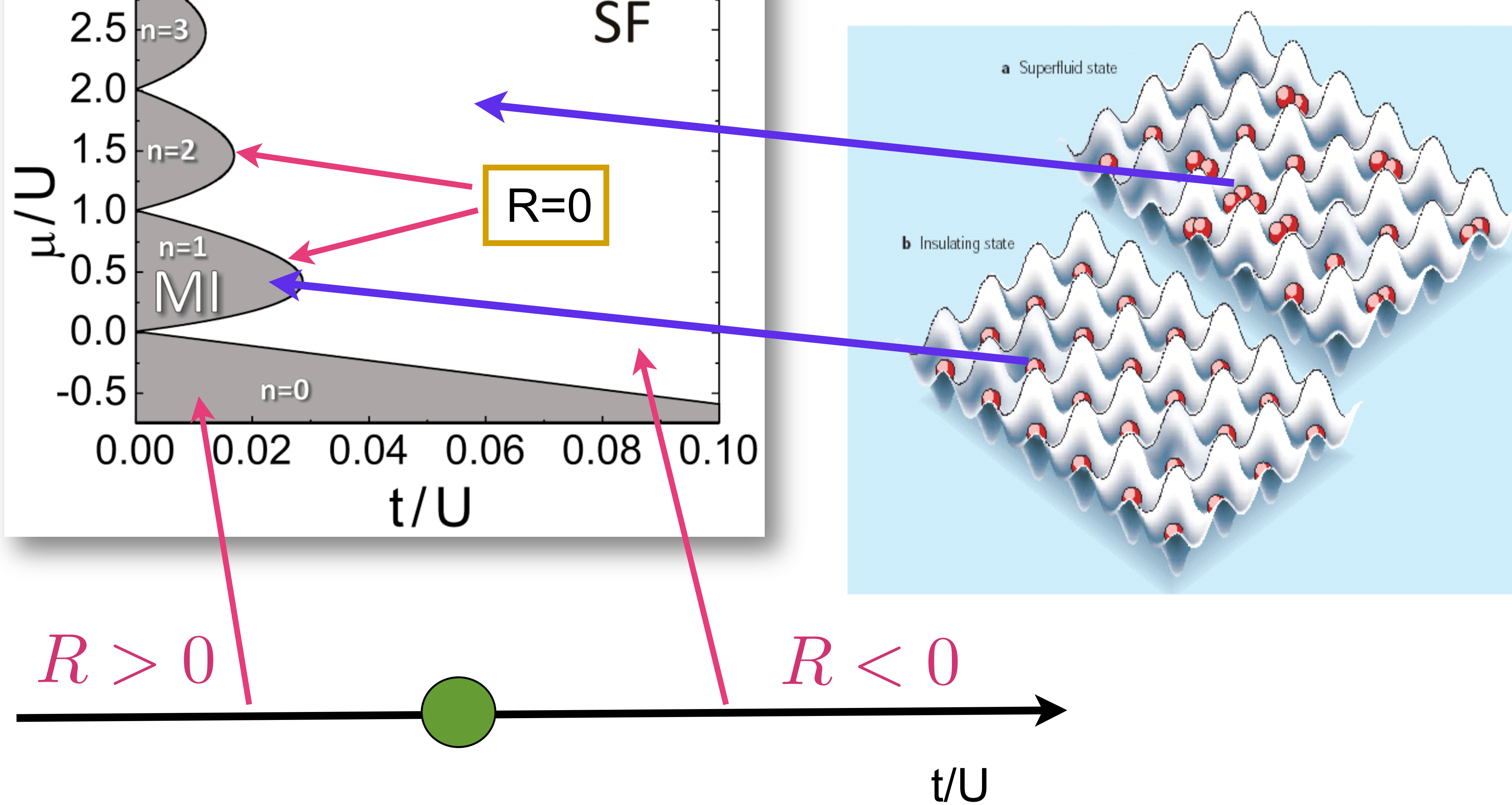


$$R = \frac{1}{zt} - \left[\frac{n}{\epsilon_-} + \frac{n+1}{\epsilon_+}\right]$$

$$\epsilon_+ = nU - \mu$$

$$\epsilon_- = (1-n)U + \mu$$

Disordered Bose-Hubbard Model

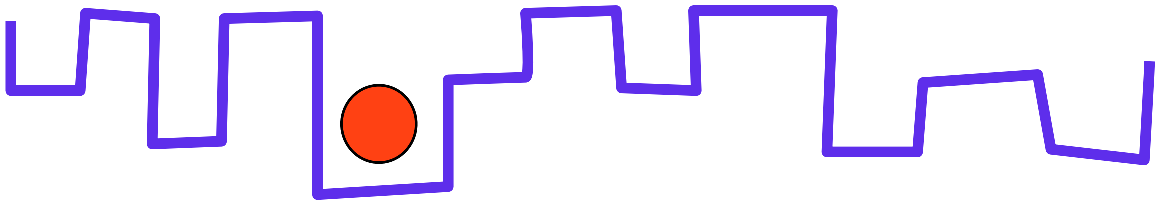
$\Delta > E_g/2$

No Mott insulator

Bose glass

Bose glass

perturb around atomic limit (t=0)



$$(t/U)^{r_{ij}}$$

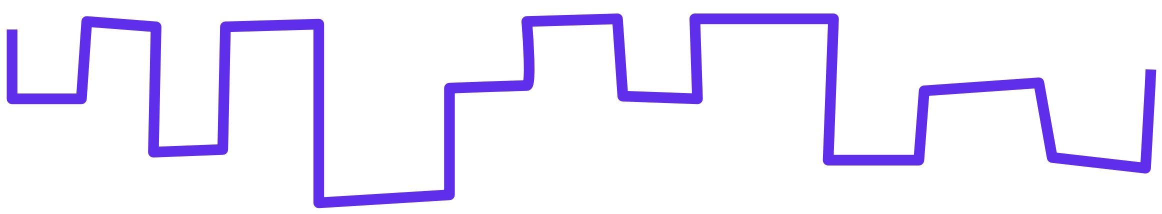
$$\xi \propto 1/\ln(t/U)$$

$$G \propto e^{-r_{ij}/\xi}$$

disorder-localised insulator

Bose glass

perturb around atomic limit (t=0)



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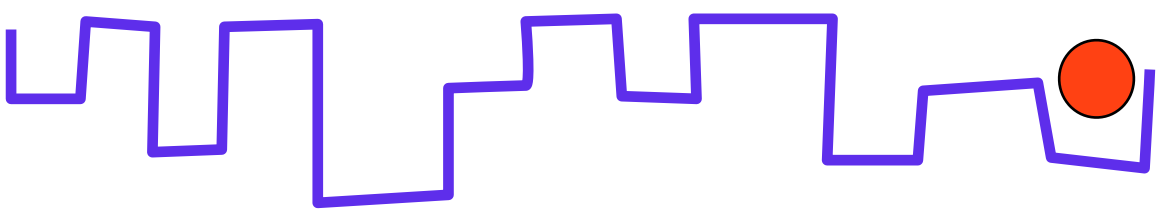

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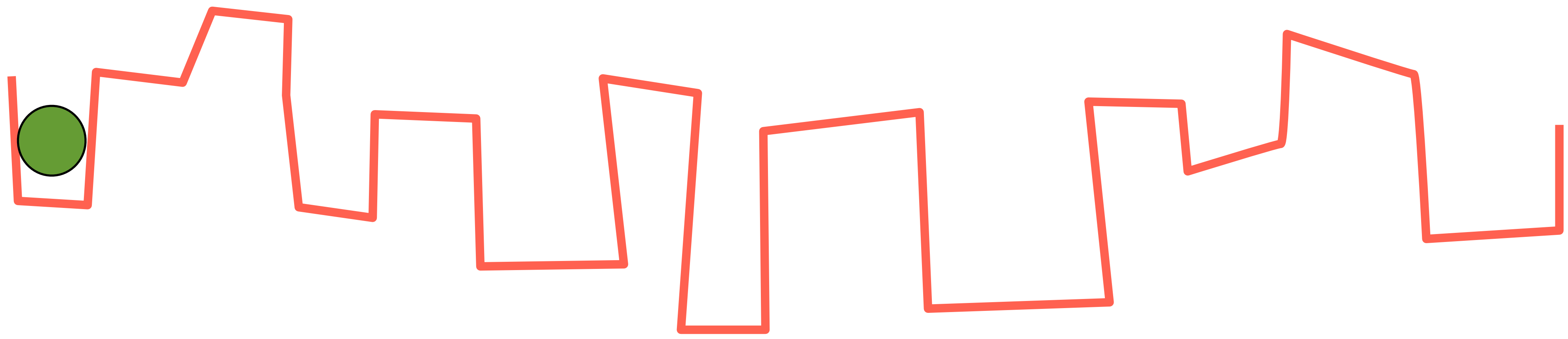
disorder-localised insulator

infinite-range hopping



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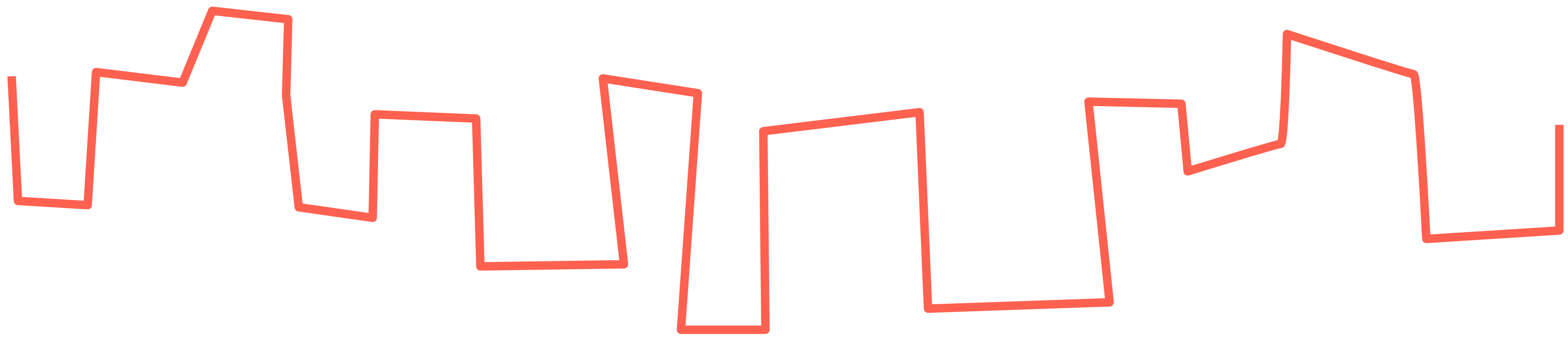

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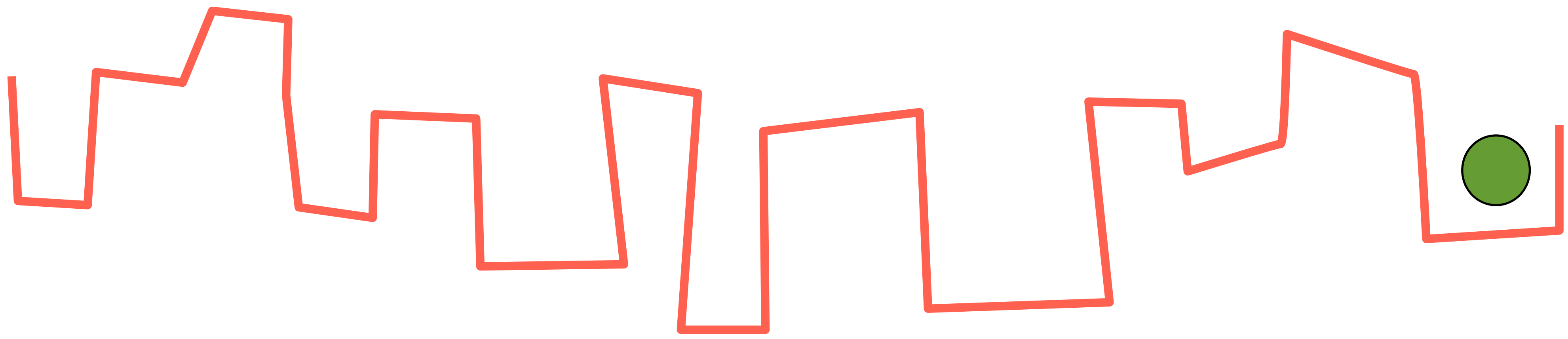

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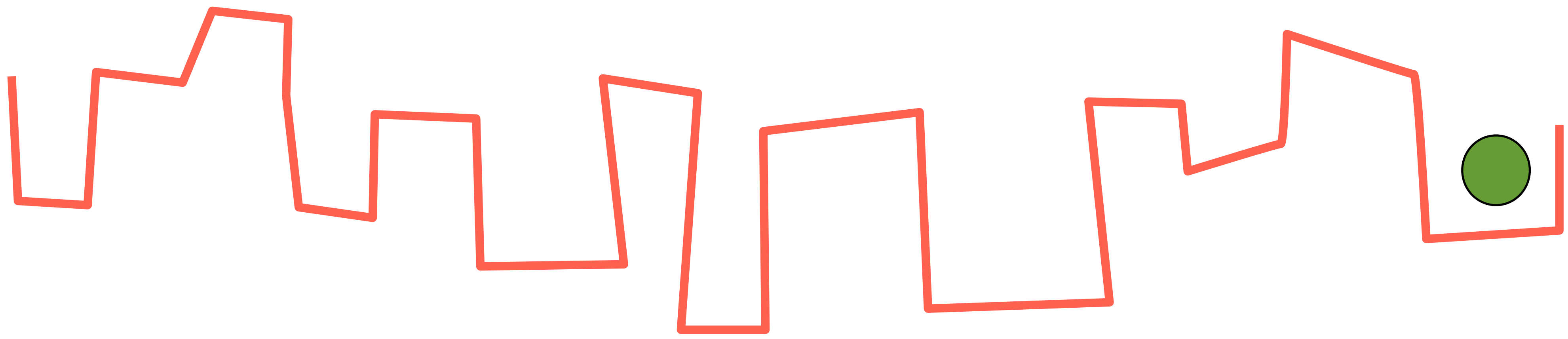
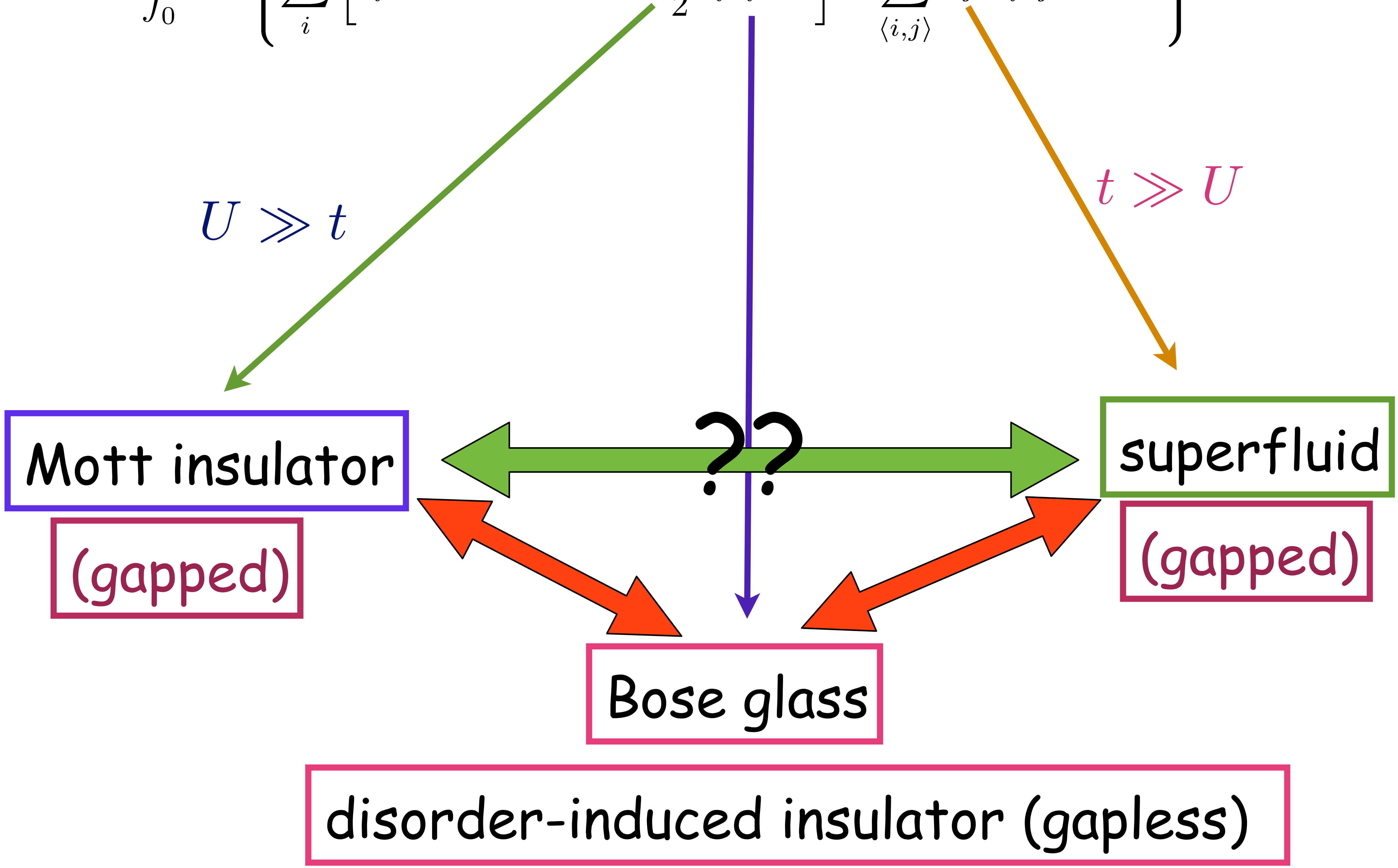


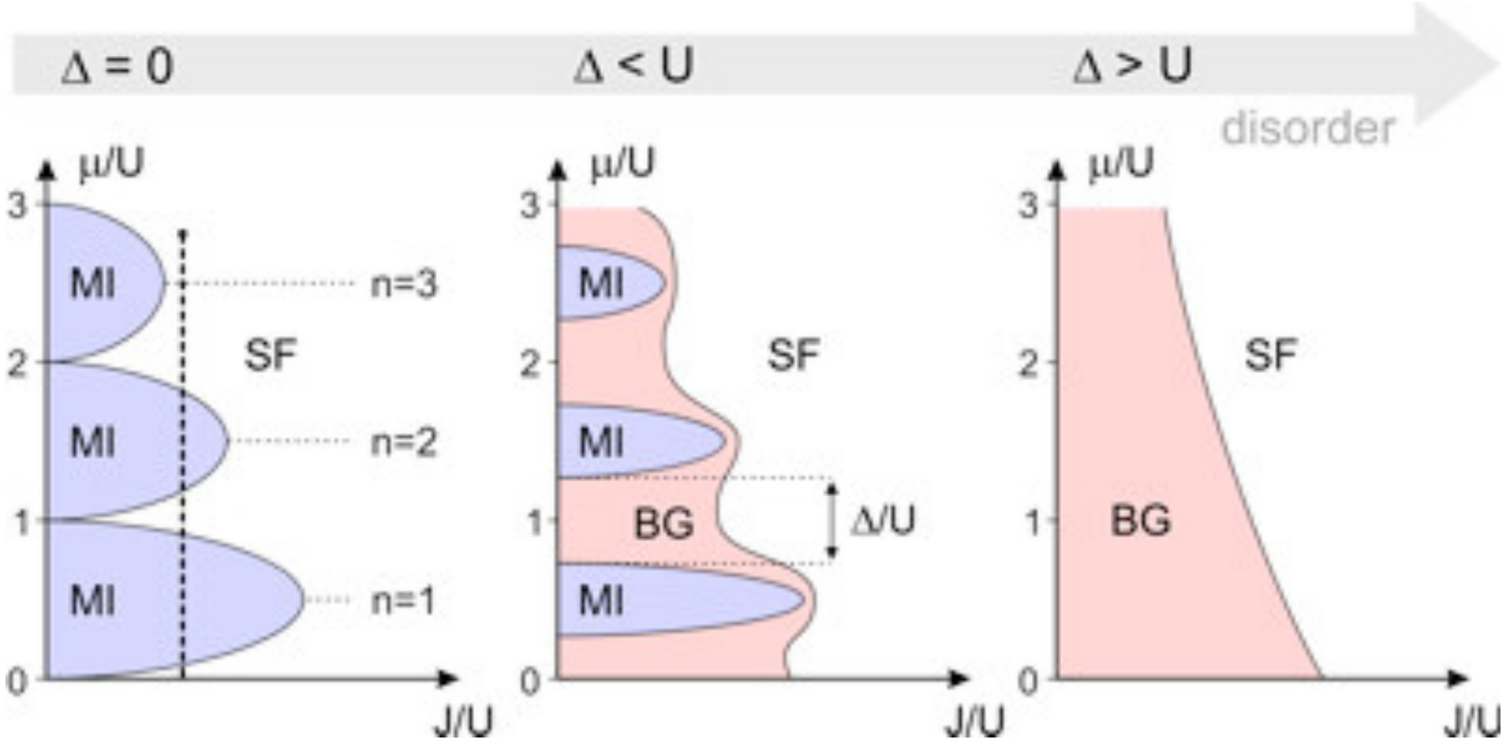
resonances abound

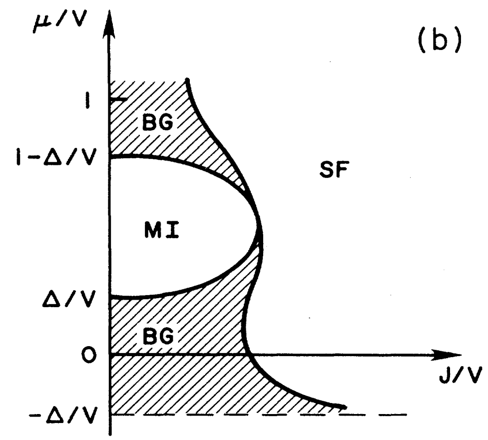
no Bose glass

$$S = \int_0^\beta d\tau \left\{ \sum_i \left[b_i^\dagger(\partial_\tau - \mu + \epsilon_i) b_i + \frac{U}{2} b_i^\dagger b_i^\dagger b_i b_i \right] - \sum_{\langle i,j \rangle} t_{ij} (b_i^\dagger b_j + c.c.) \right\},$$



phase diagram





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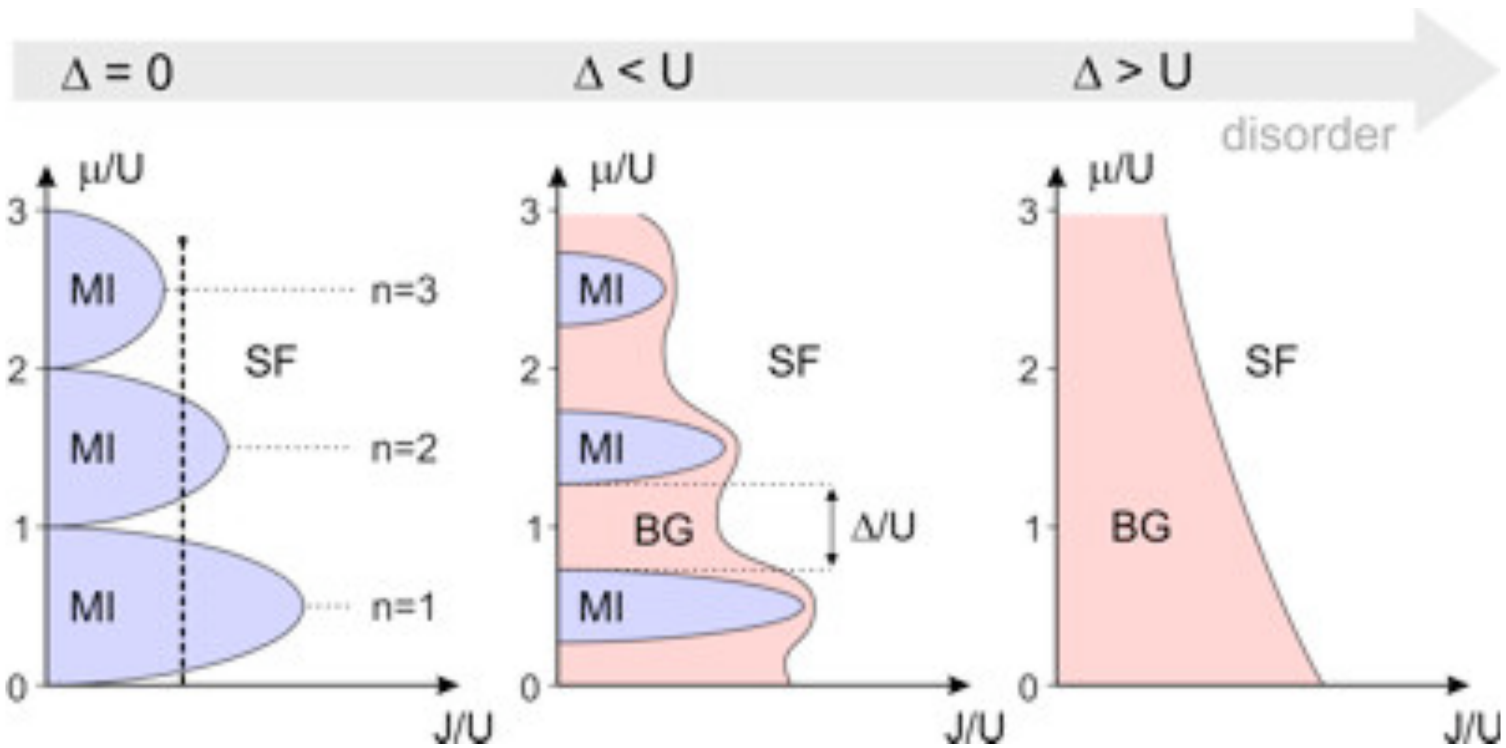
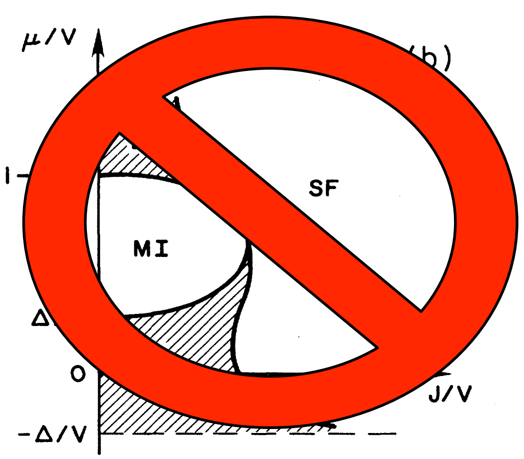


In the case of Fig. 2(b), the only direct Mott insulator-superfluid transitions occur at the tips, so it is tempting to hypothesize that the MHL theory would likewise apply to these transitions. Since the Bose glass states in Fig. 2(b) extend right up to the tips, however, it could well be that a more complicated multicritical fixed point describes the transition in this case. Since expressions like (4.24) do not easily allow for a description of both the Mott and Bose glass insulators, it is difficult to say anything definitive about this possibility. Furthermore, as we have argued in Sec. II, we believe that both phase diagrams shown in Figs. 2(b) and 2(c) are unlikely to obtain.



No calculation

Calculations

MI-SF at integer filling

[13] R.T. Scalettar, G.G. Batrouni, and G.T. Zimanyi, Phys. Rev. Lett. **66**, 3144 (1991).
[14] W. Krauth, N. Trivedi, and D. Ceperley, Phys. Rev. Lett. **67**, 2307 (1991).
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No MI-SF

Absence of a Direct Superfluid to Mott Insulator Transition in Disordered B Systems

L. Pollet,[1] N.V. Prokof'ev,[1,2] B.V. Svistunov,[1,2] and M. Troyer[3]

arXiv:0903.3867

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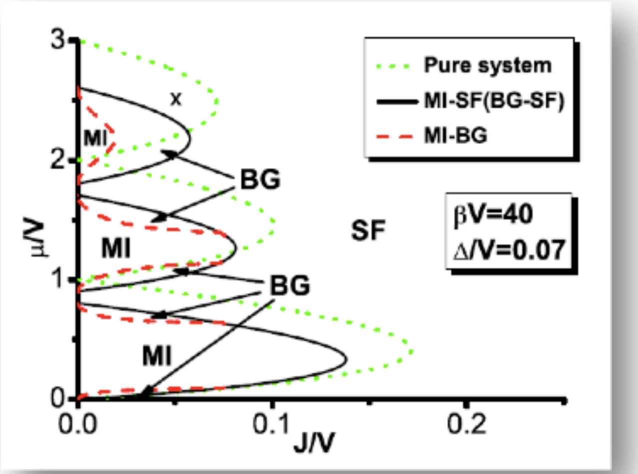

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No MI-SF

What is the phase diagram for the disordered Bose- Hubbard Model??

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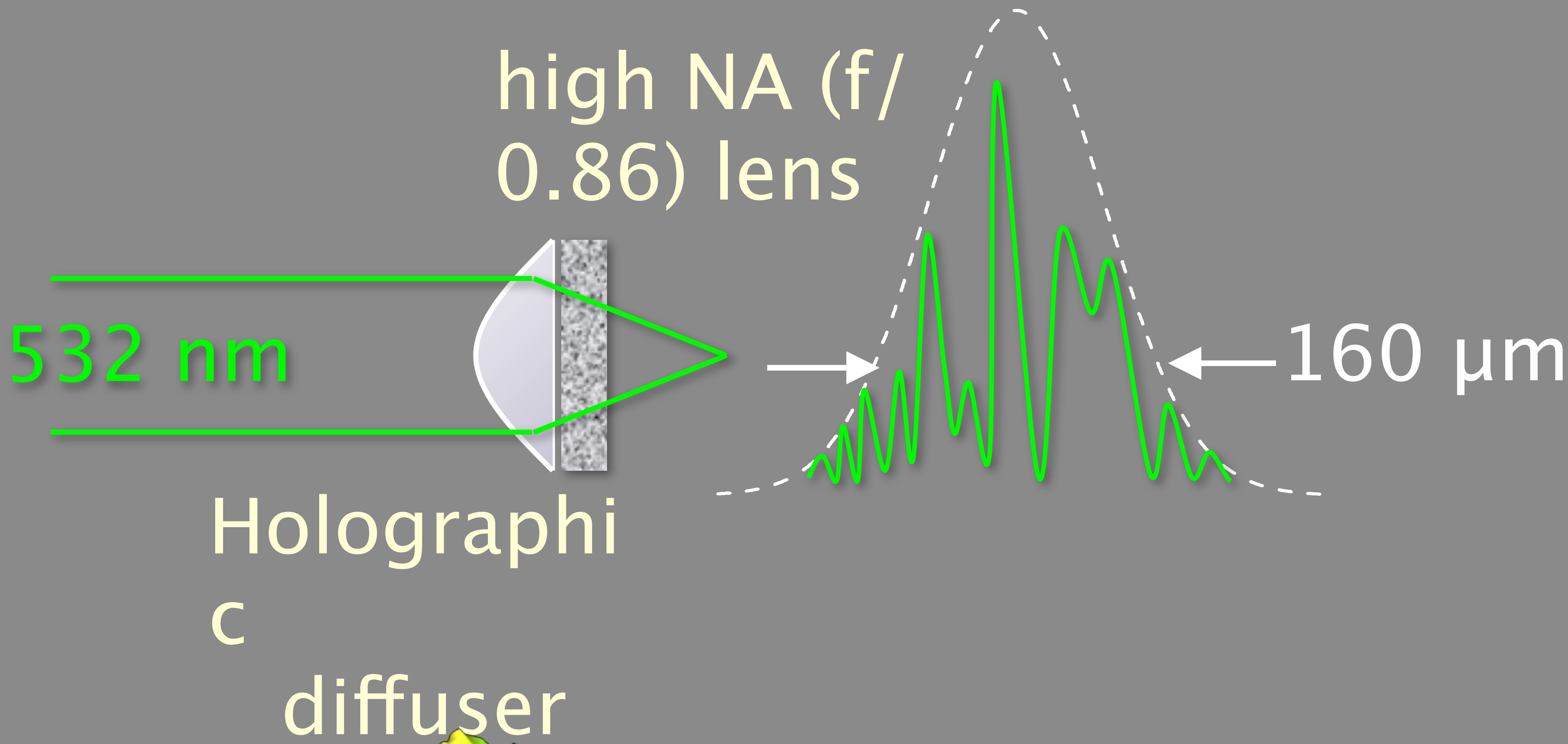
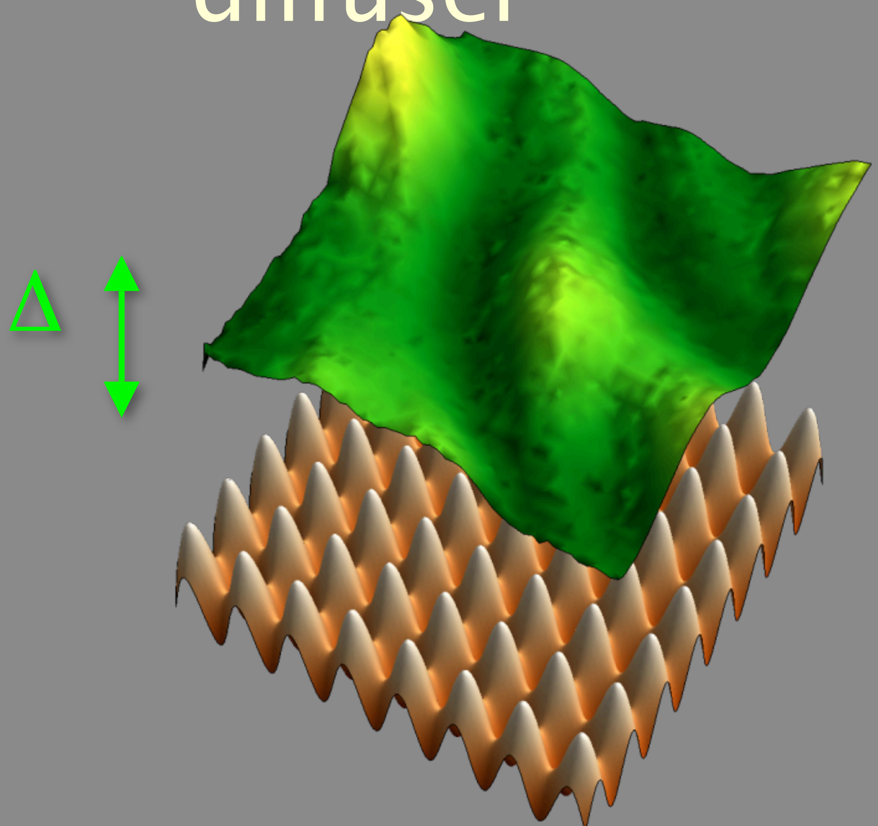
Optical Lattices

Controllable disorder: speckle

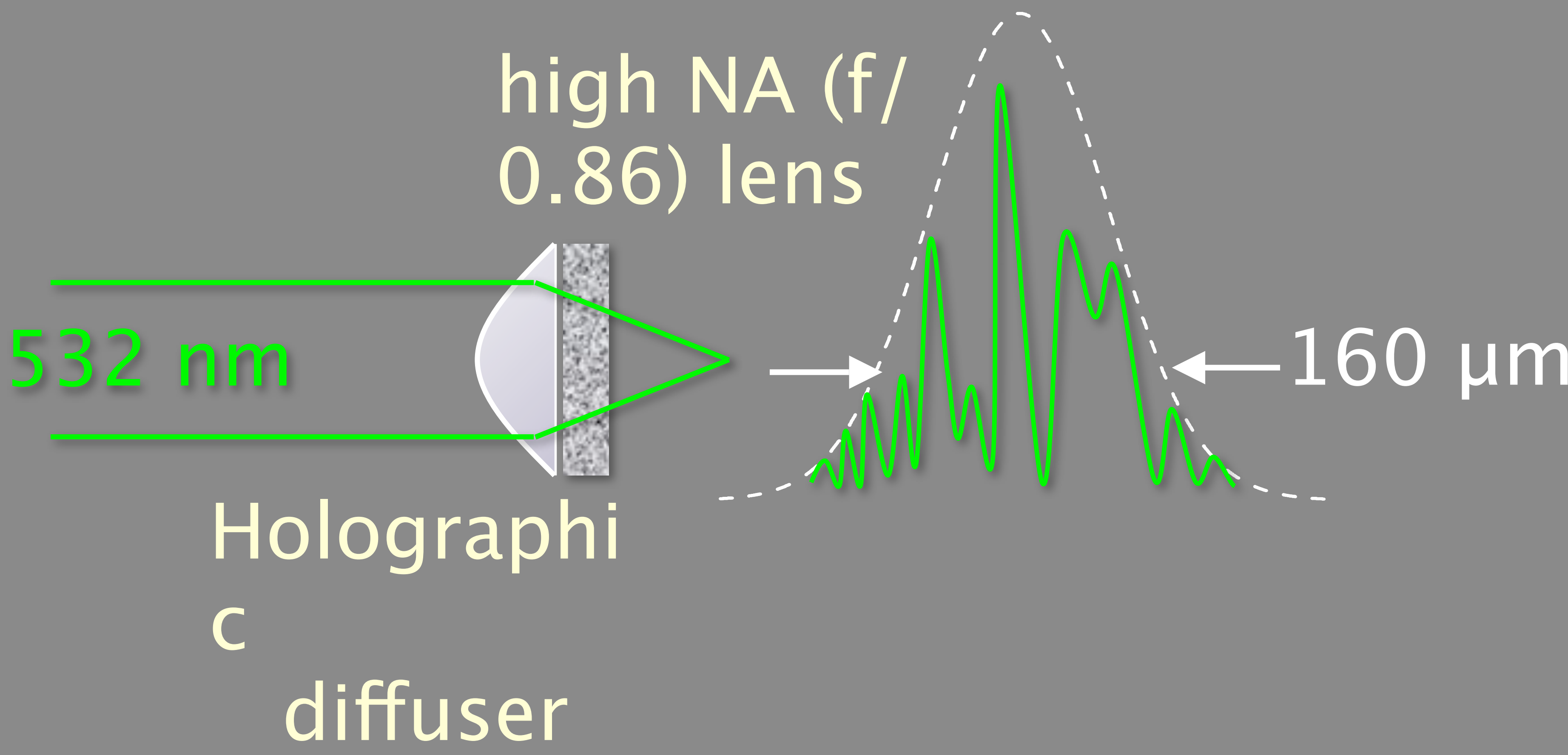


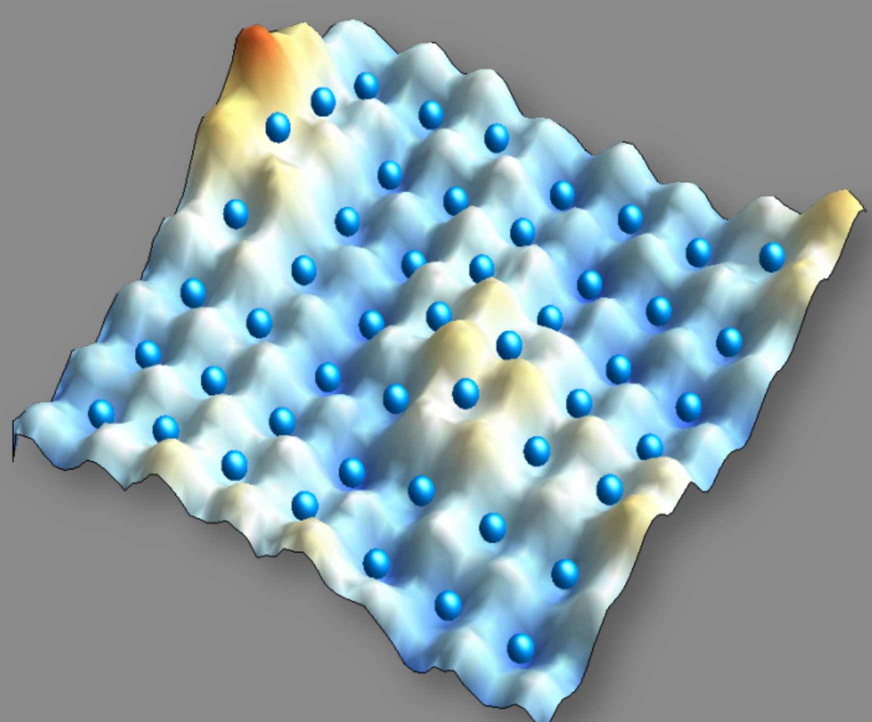


Potentials add to produce disordered lattice

B. DeMarco, M. White, ...

Controllable disorder: speckle





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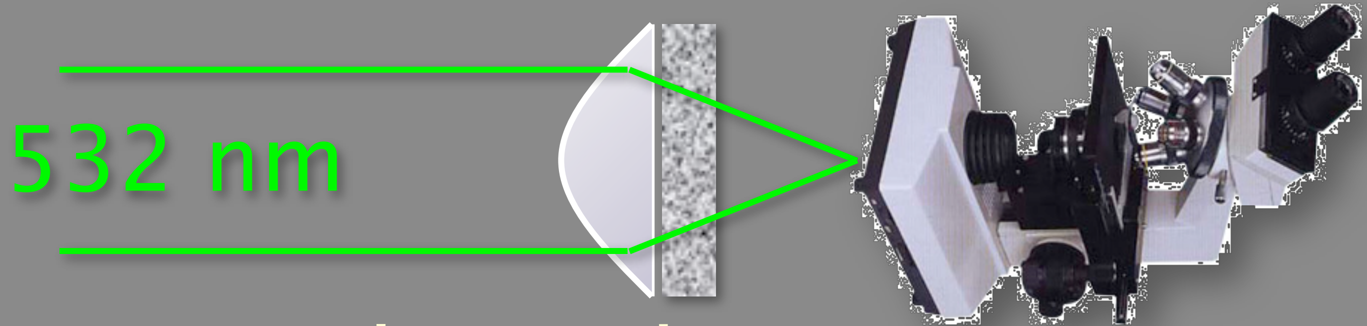
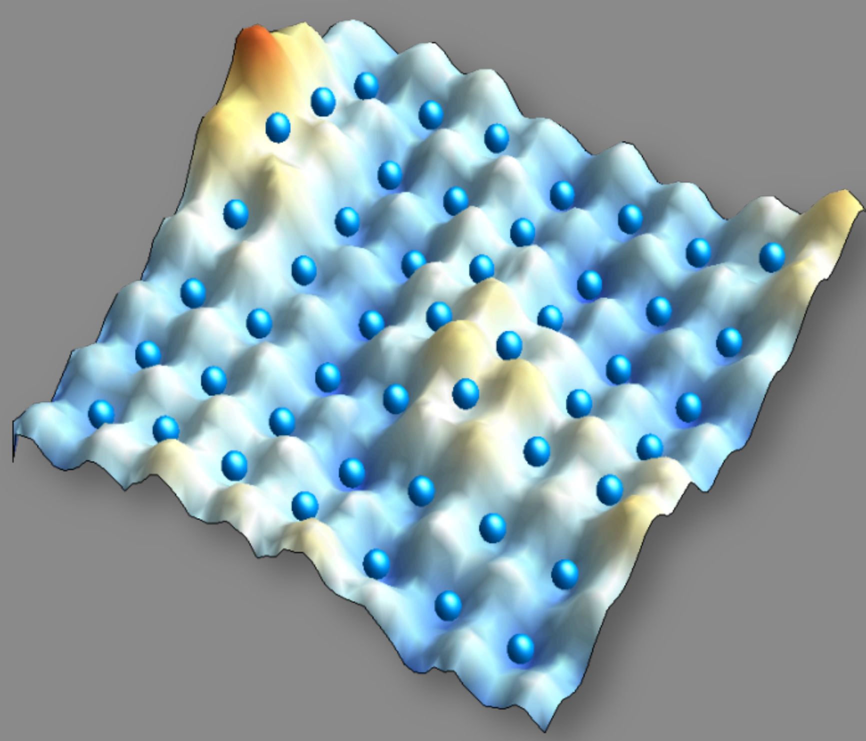


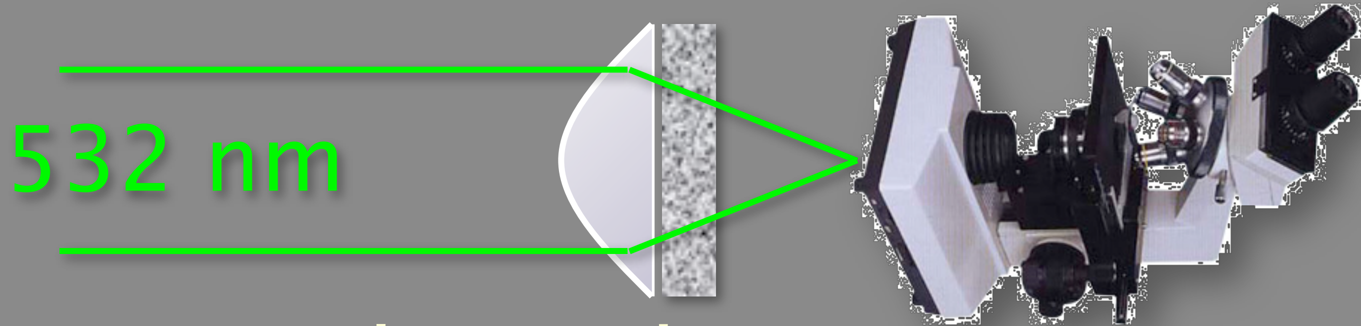


Potentials add to produce disordered lattice

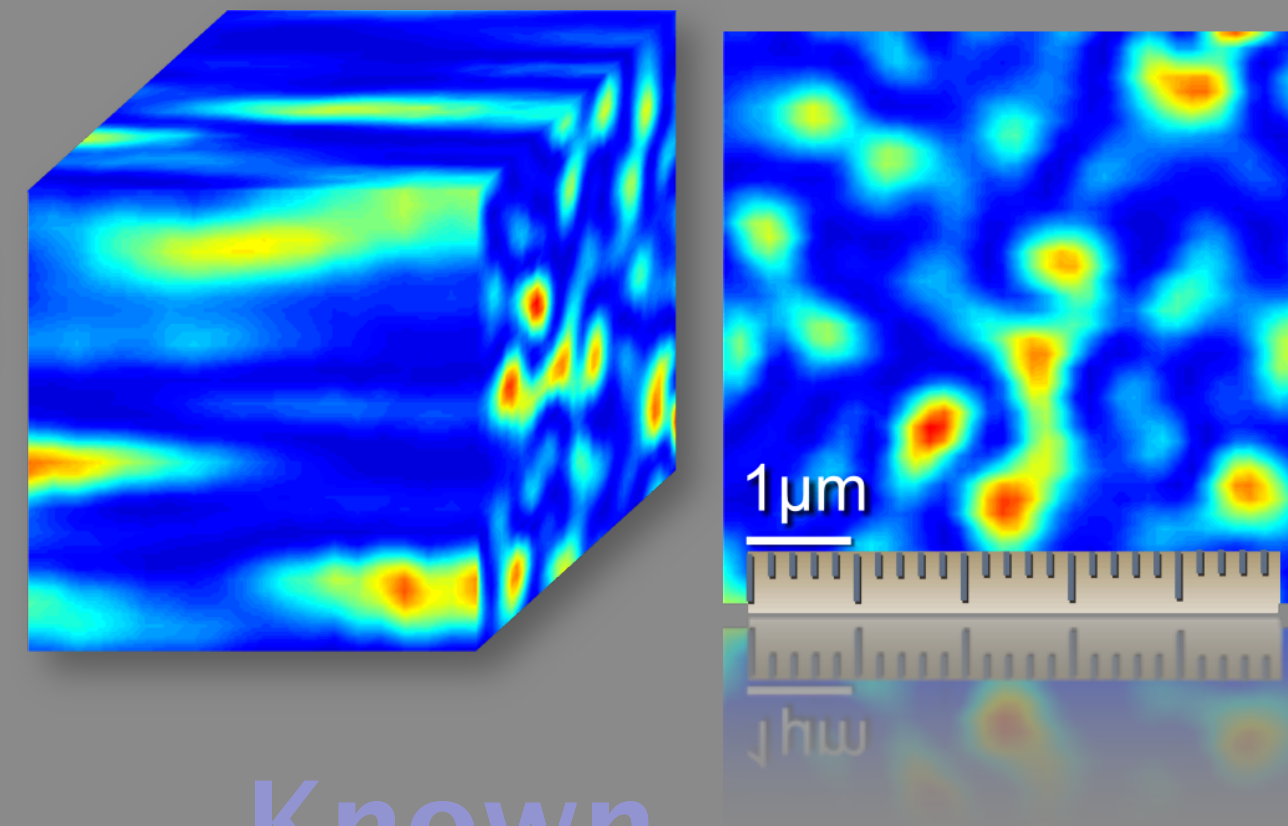
B. DeMarco, M. White, ...

Controllable disorder: speckle

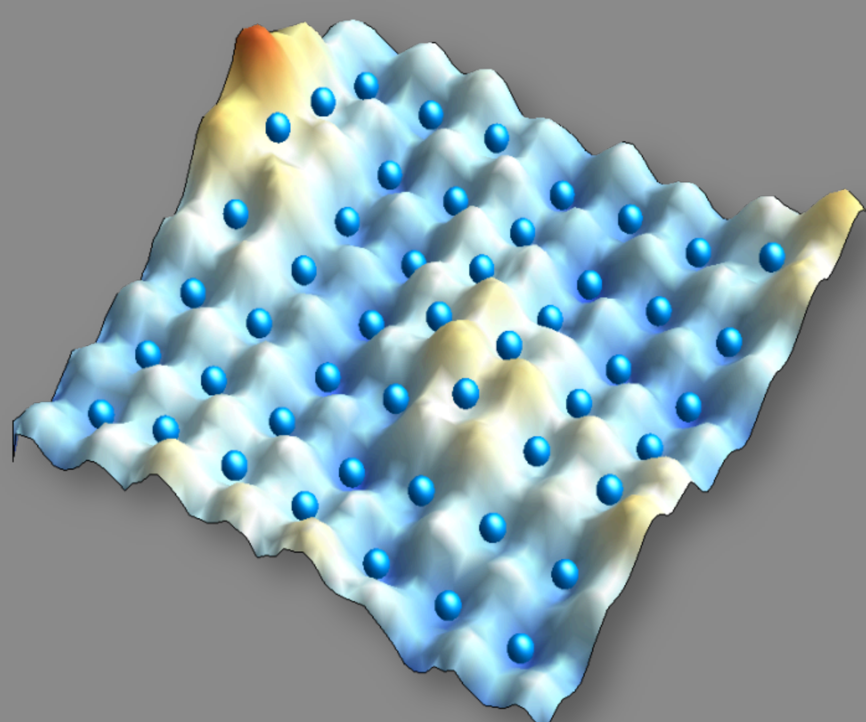
high NA (f/0.86) lens



Holographic diffuser



Known disorder!

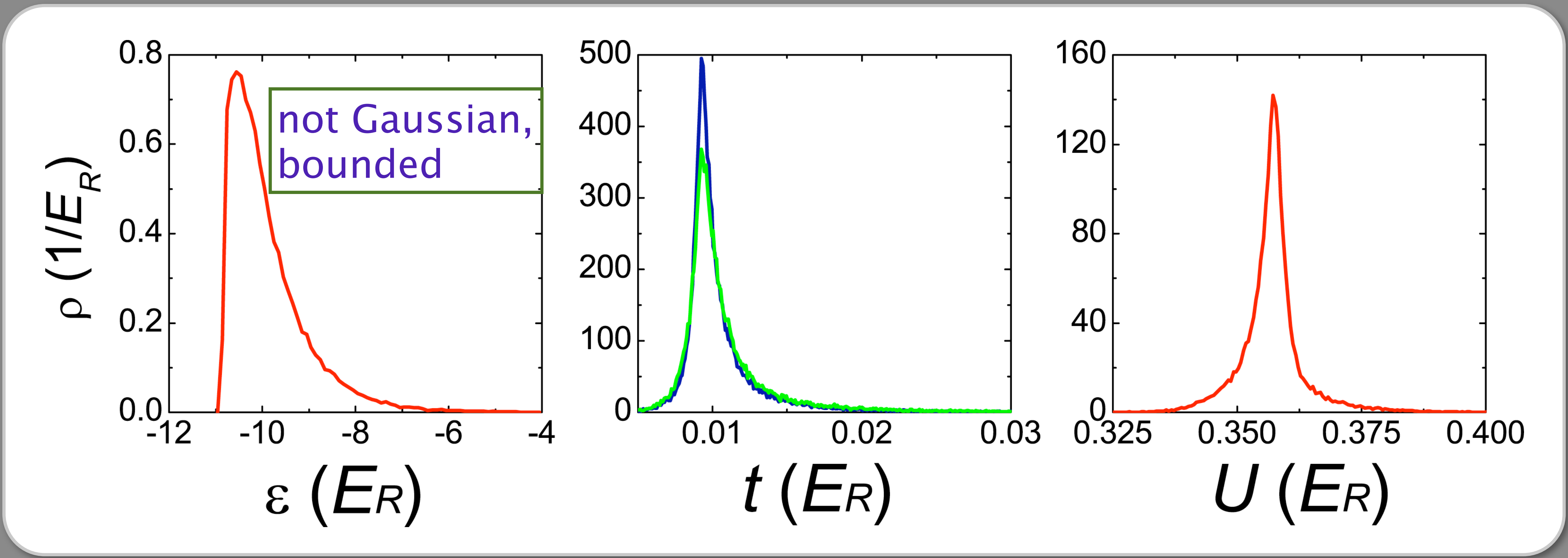


Potentials add to produce disordered lattice

B. DeMarco, M. White, ...

BH parameters

Calculation by Ceperley's group using known po



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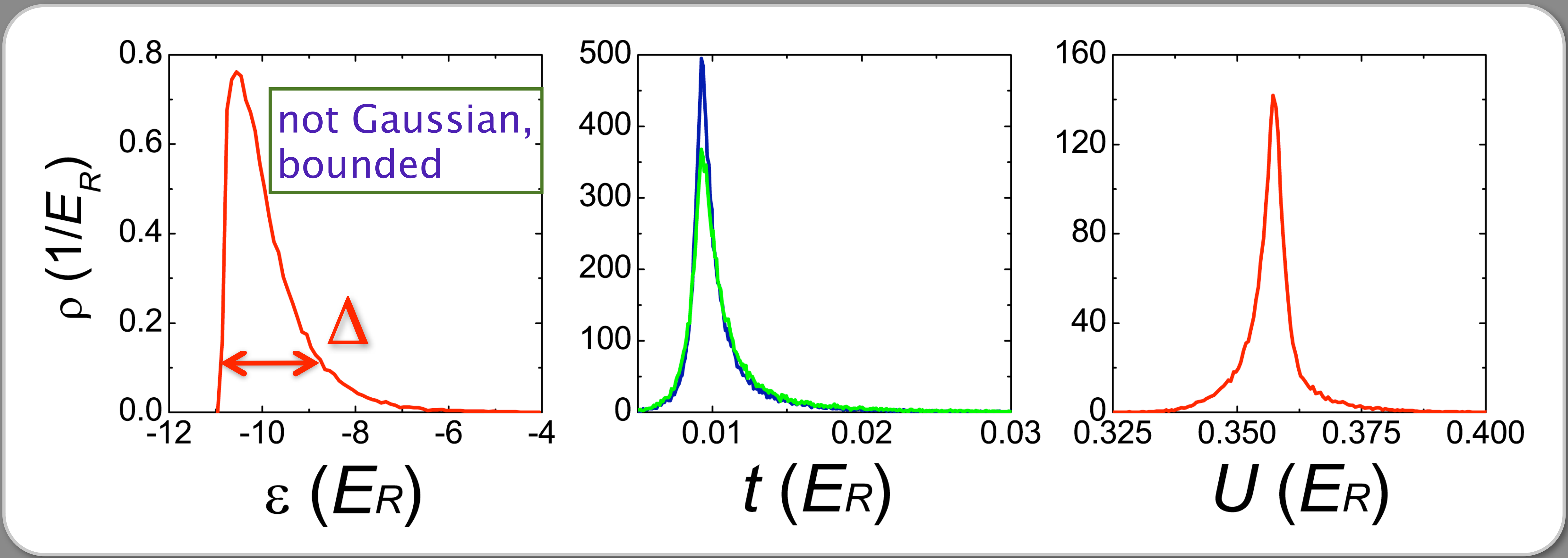

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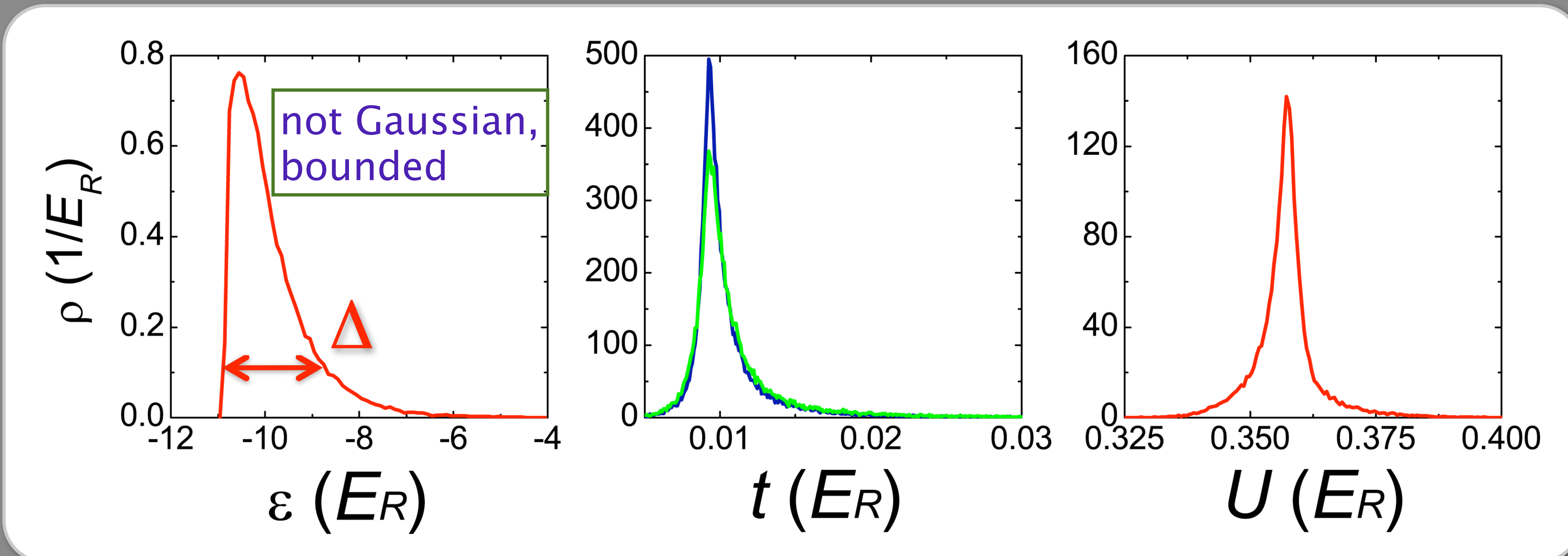
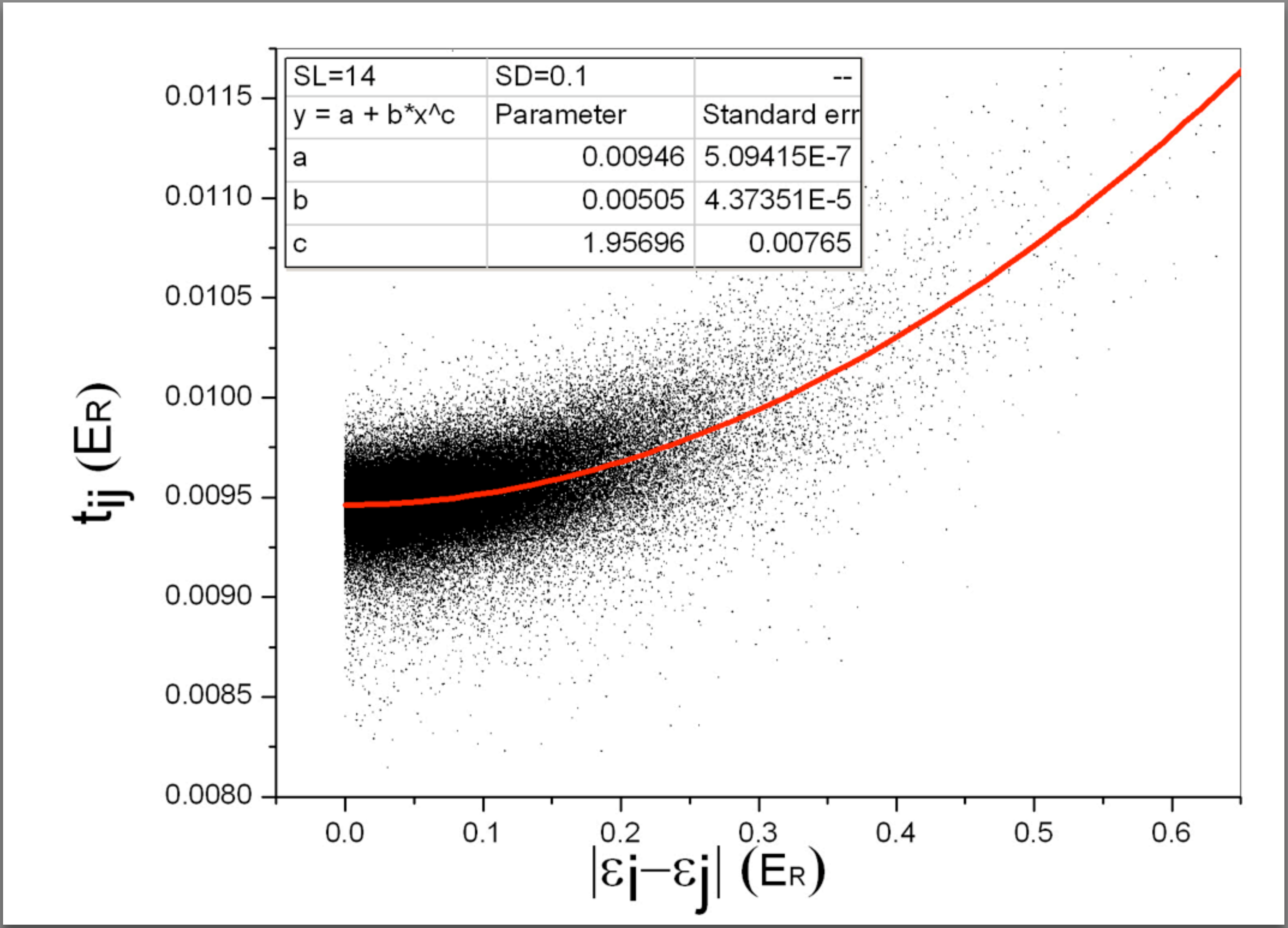
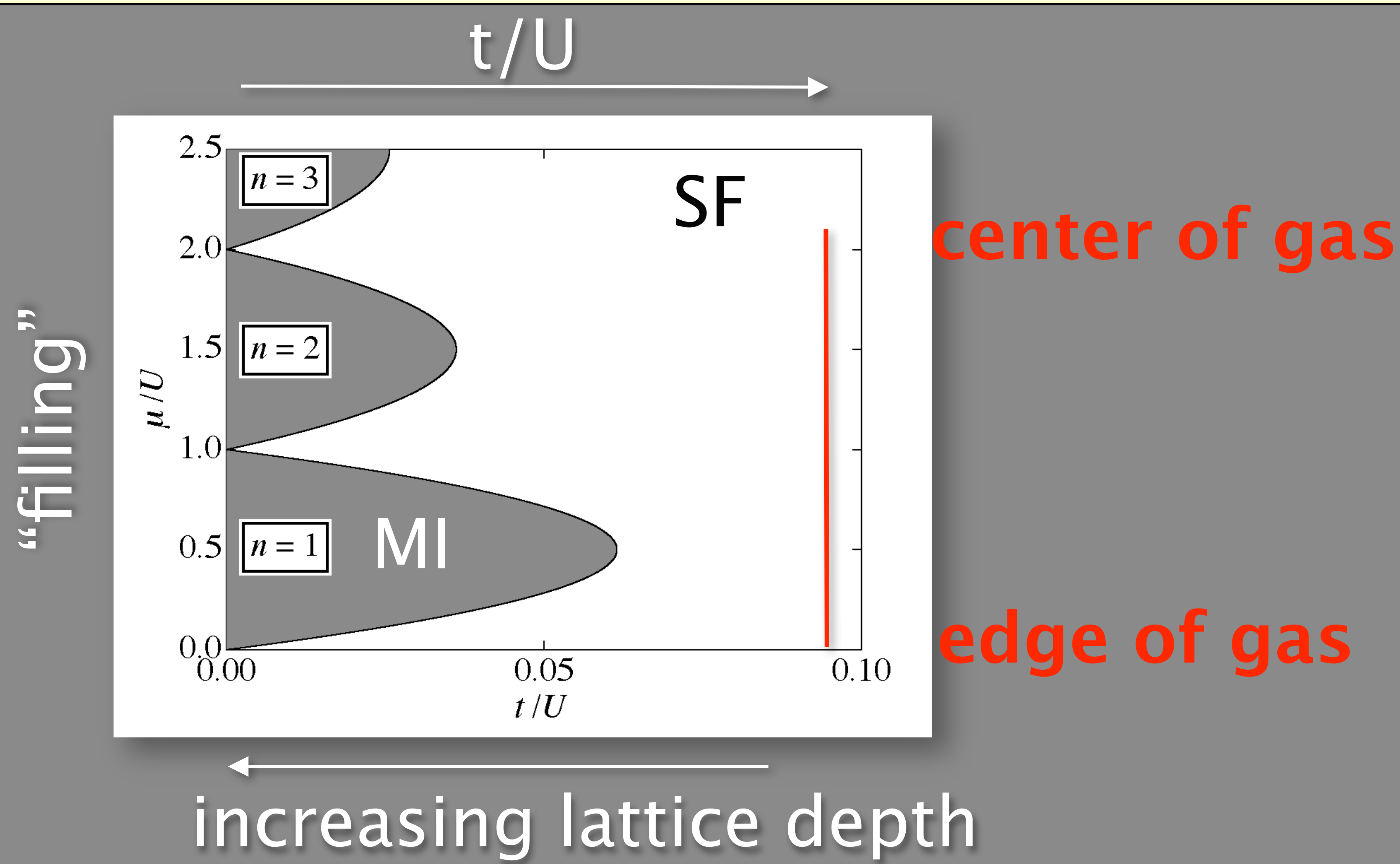


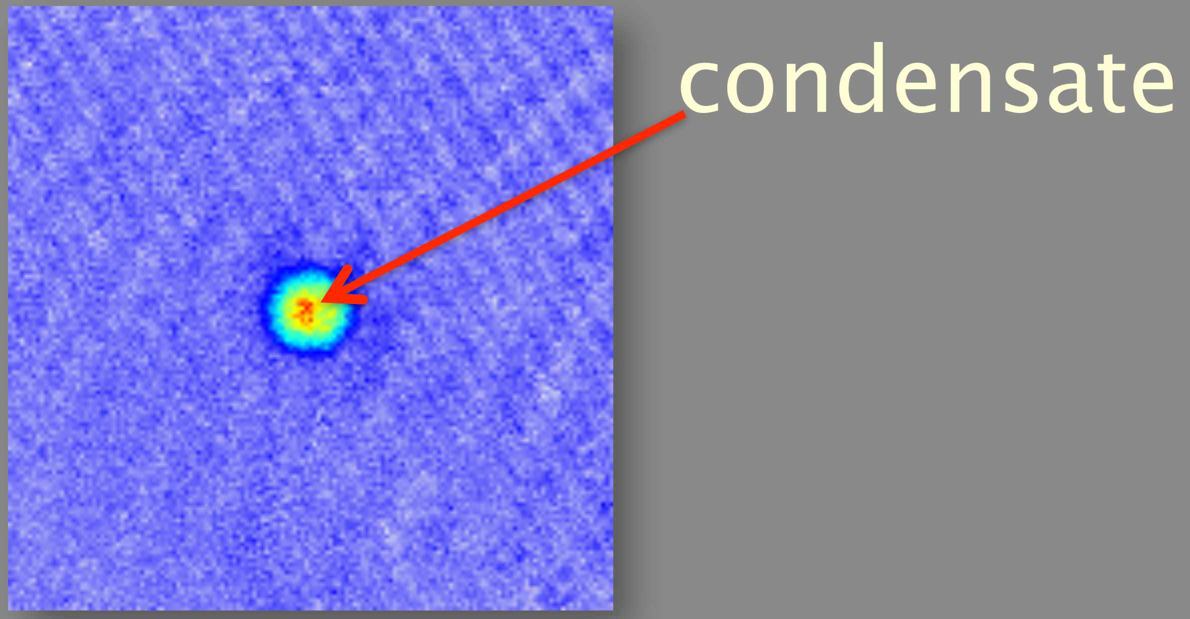
First disordered "material" in which microscopic disorder completely determined

BH parameters – correlations



Constrain theory using BEC fraction?

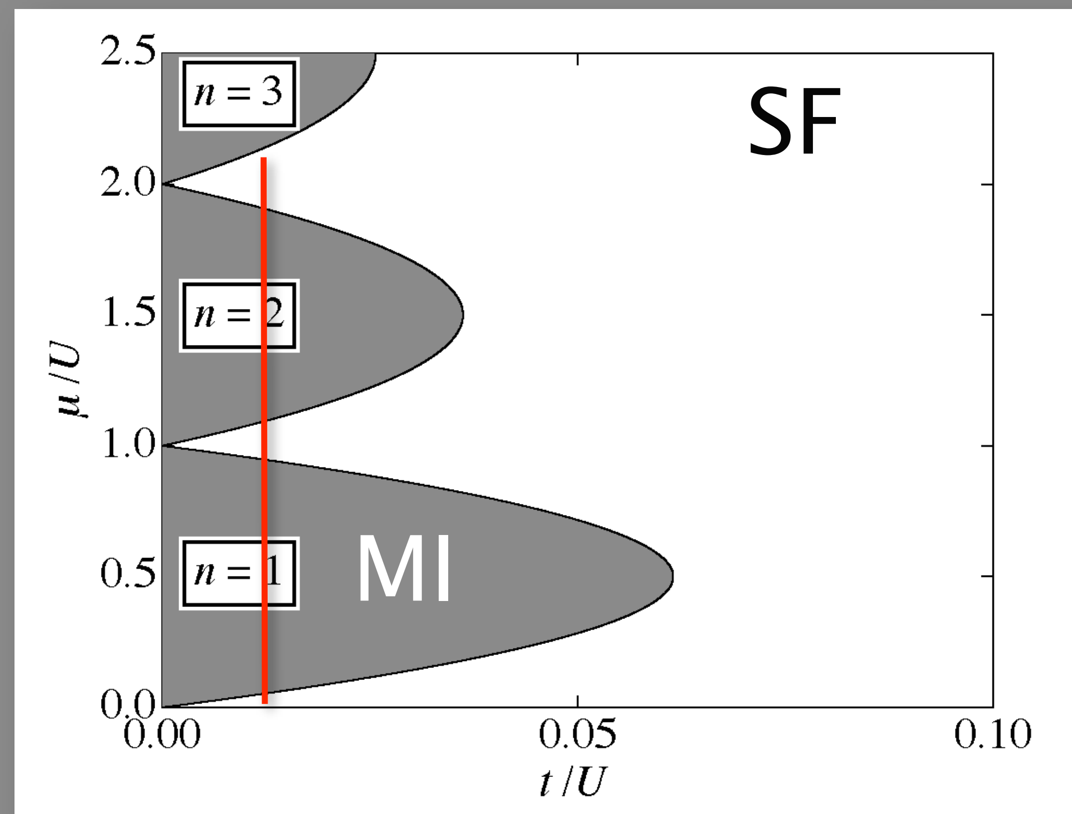




Constrain theory using BEC fraction?

t/U

"filling"



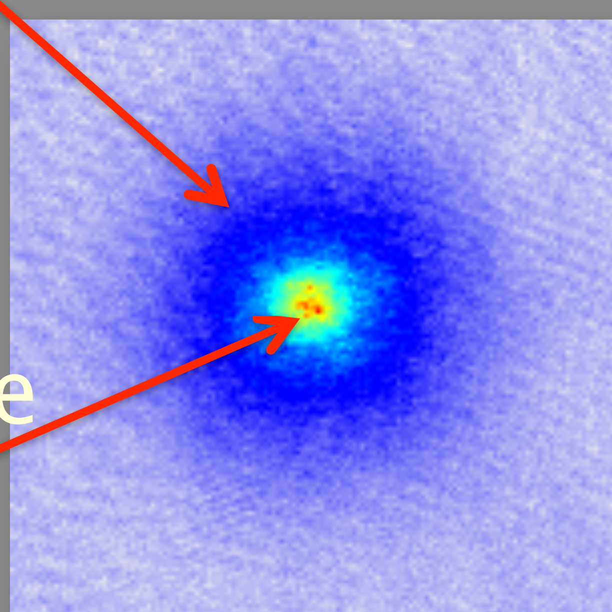
center of gas

edge of gas

increasing lattice depth

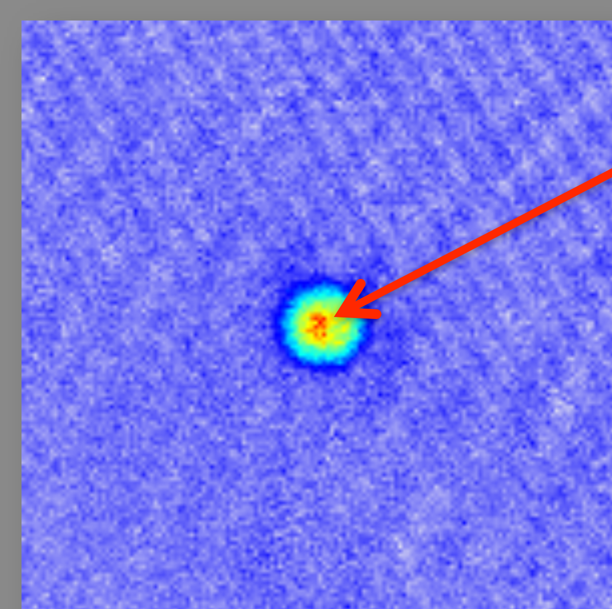
non-condensate (MI, BG, …)

$N-N_0$



condensate

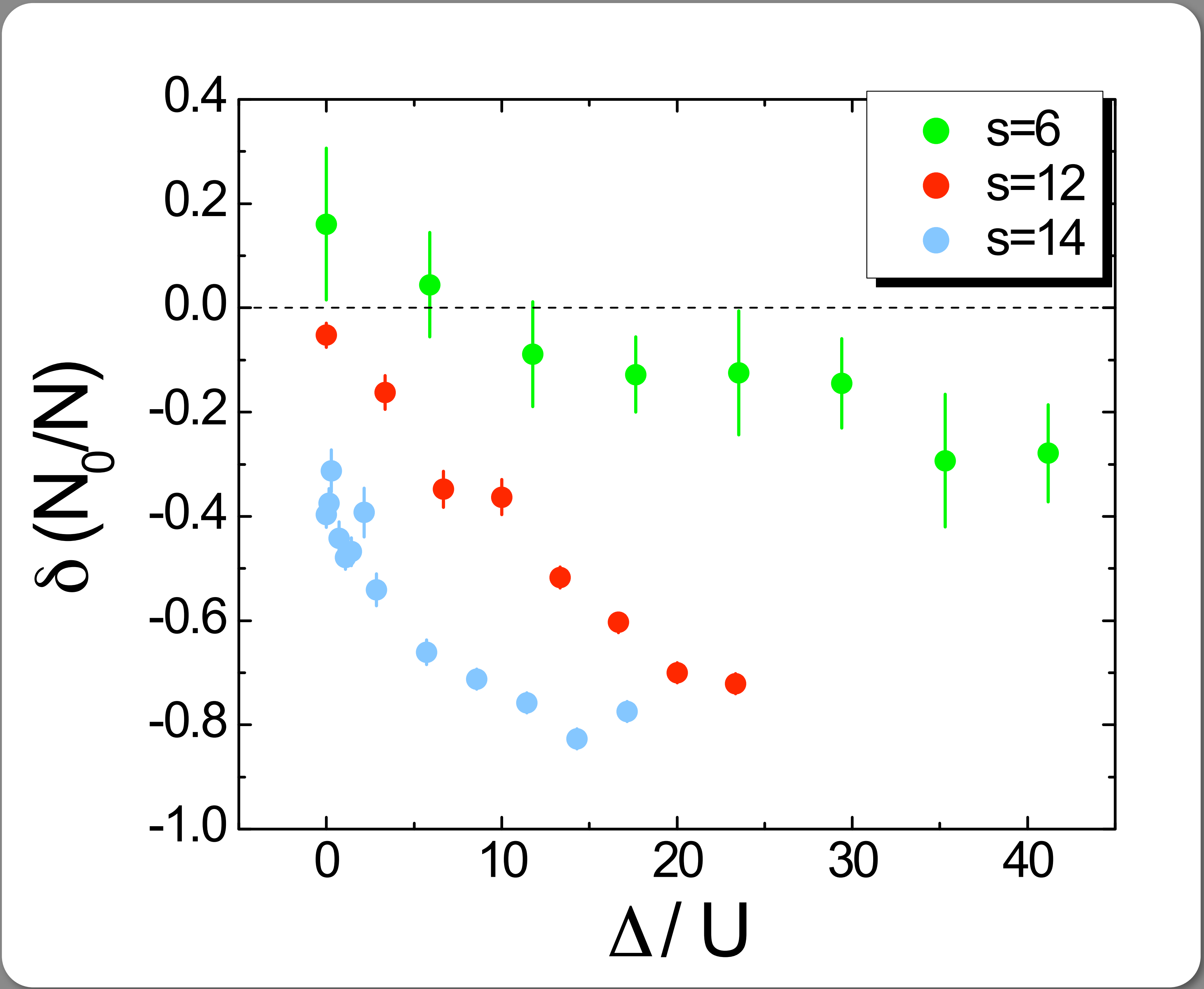
N_0

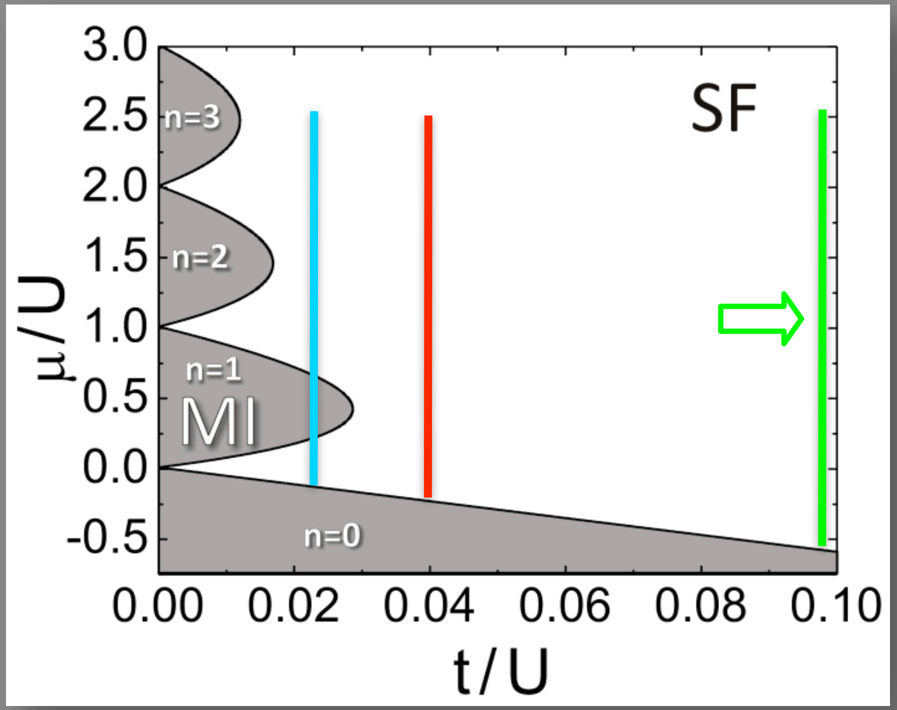


condensate

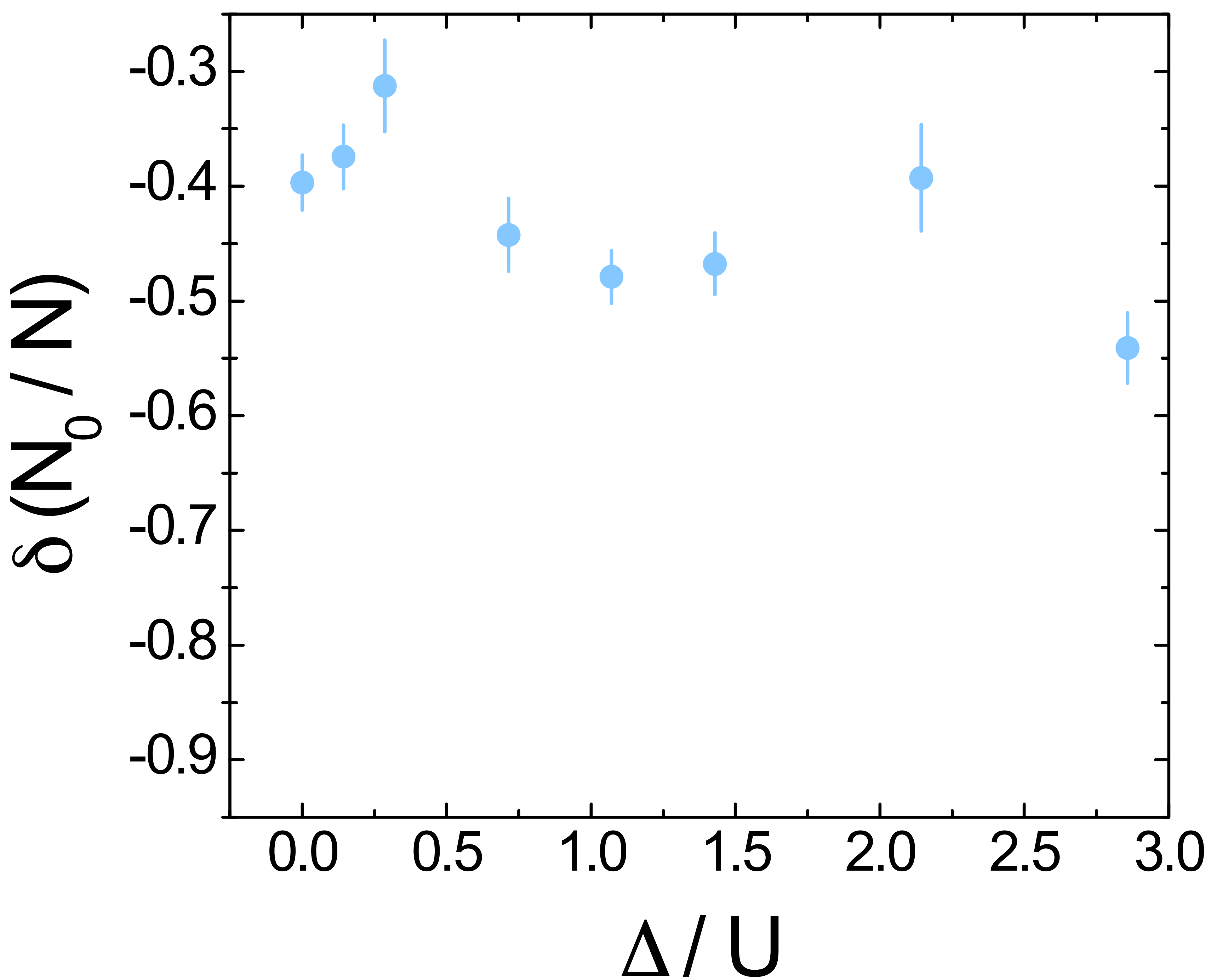
Effect of disordered lattice

$$\delta\left(N_0/N\right)=\left(N_0/N_{bandmap}-N_0/N_{adiabatic}\right)/N_0/N_{adiabatic}$$





MI+SF mixture (s=14), low disorder

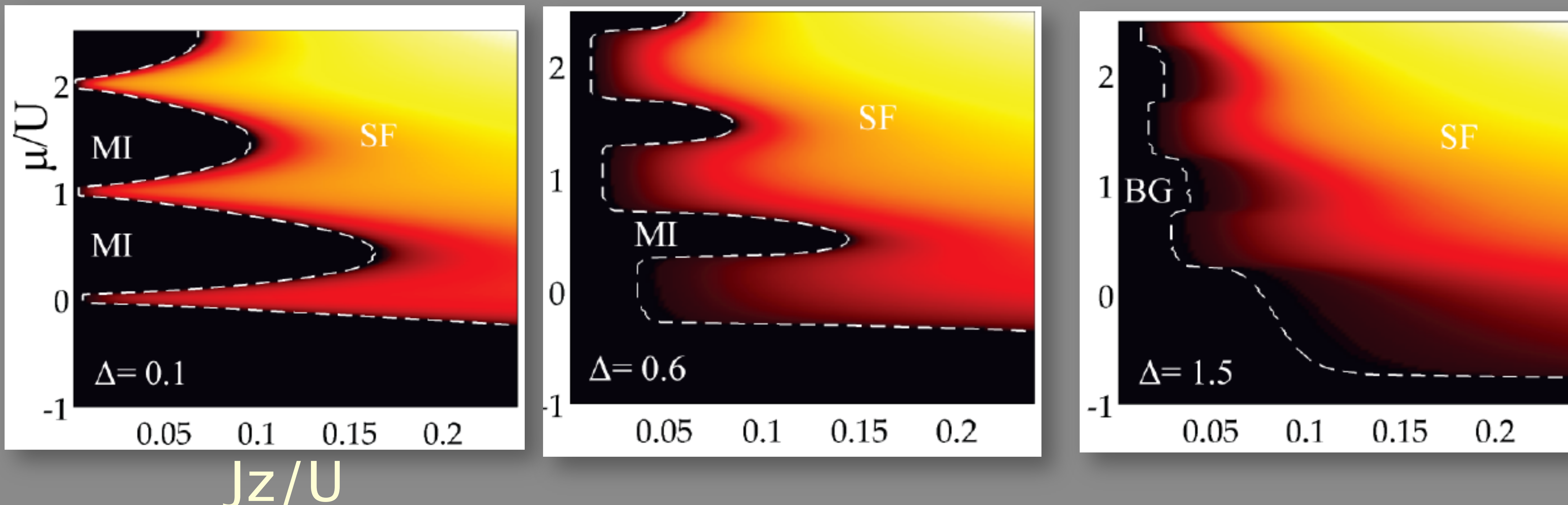


Phys. Rev. Lett. **102**, 055301 (2009)

A benchmark for theory!

Condensate fraction in the DBH model

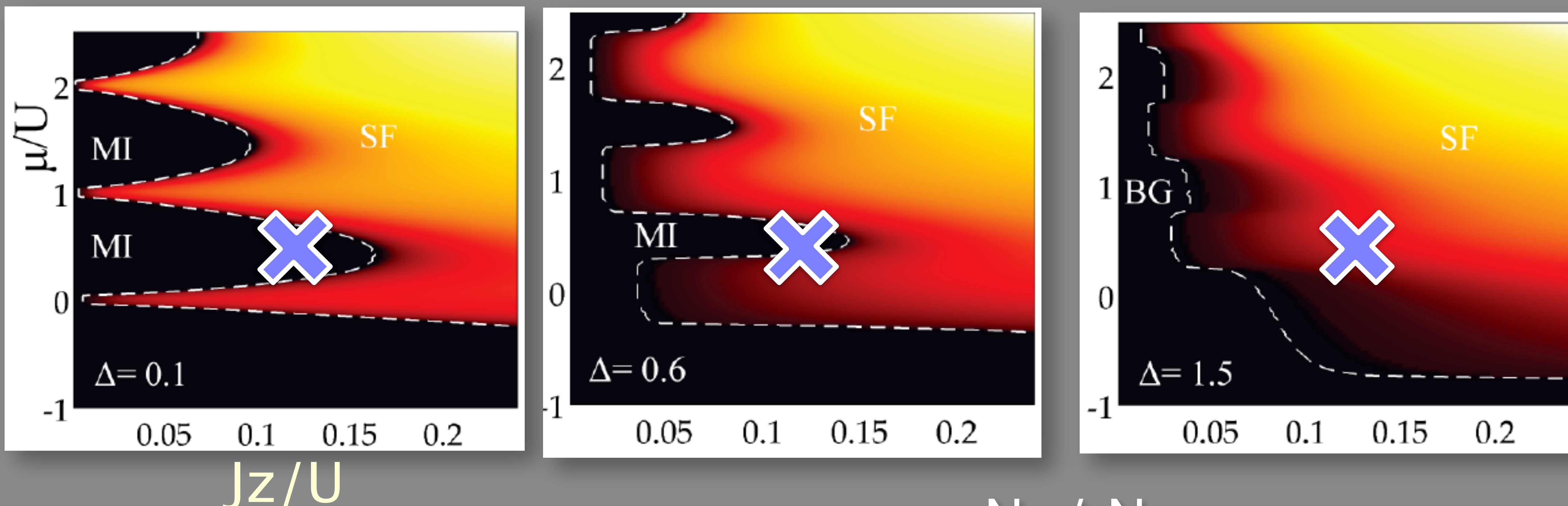
Benchmark for theory



- Stochastic Mean Field Theory
  Bissbort and Hofstetter
- Diagonal disorder, flat distribution

Condensate fraction in the DBH model

Benchmark for theory



- Stochastic Mean Field Theory
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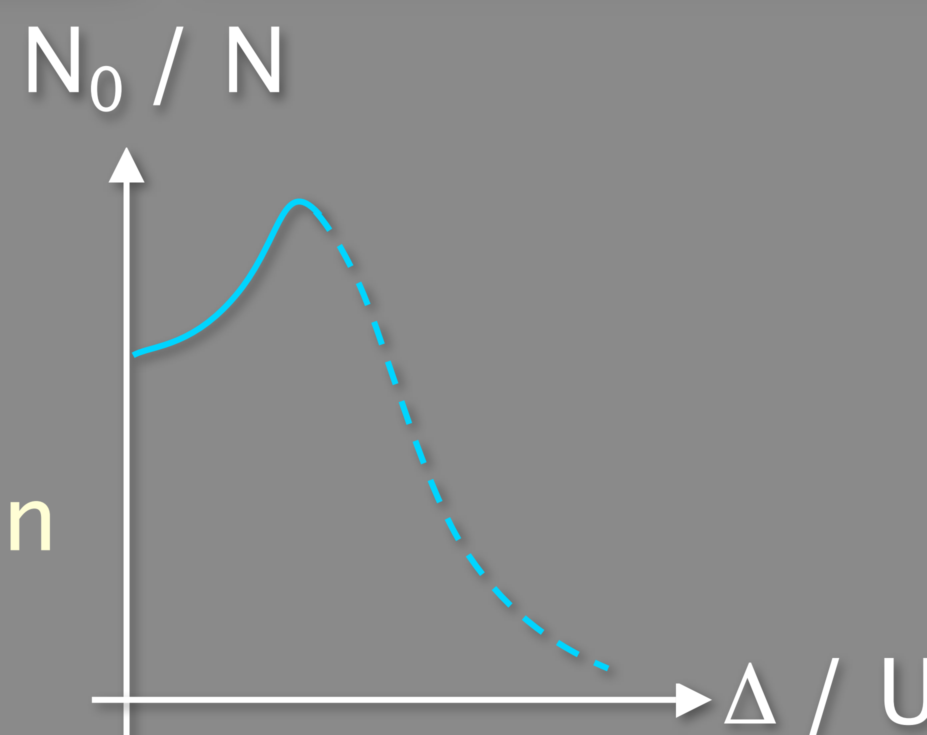
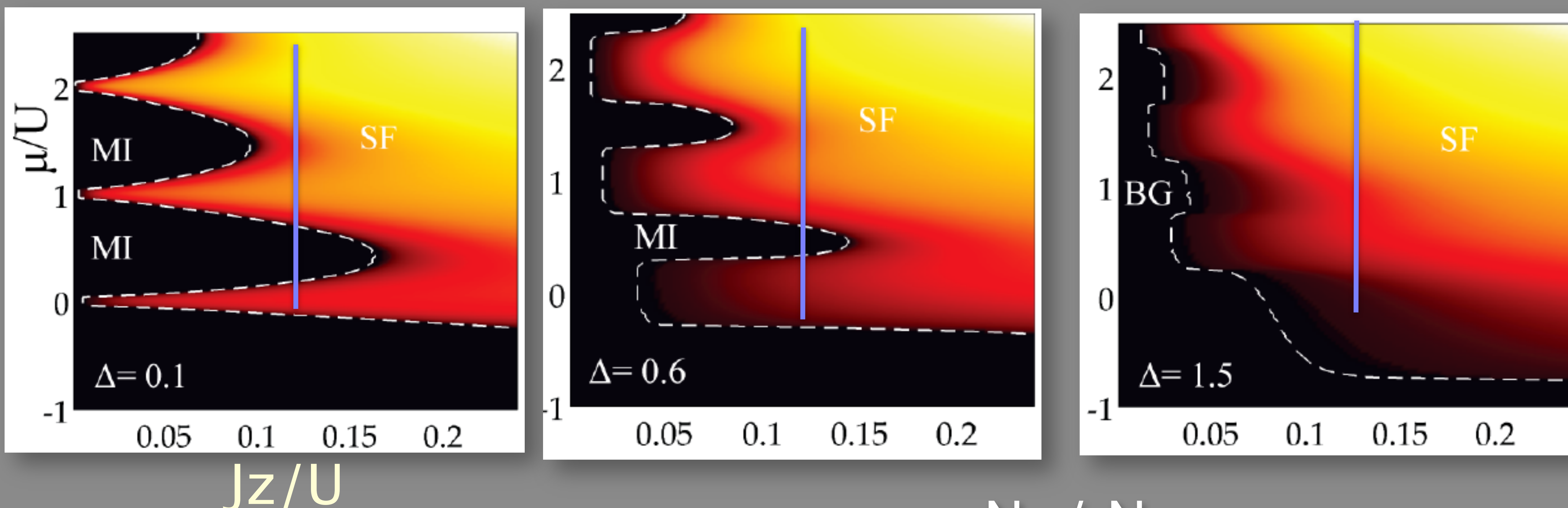
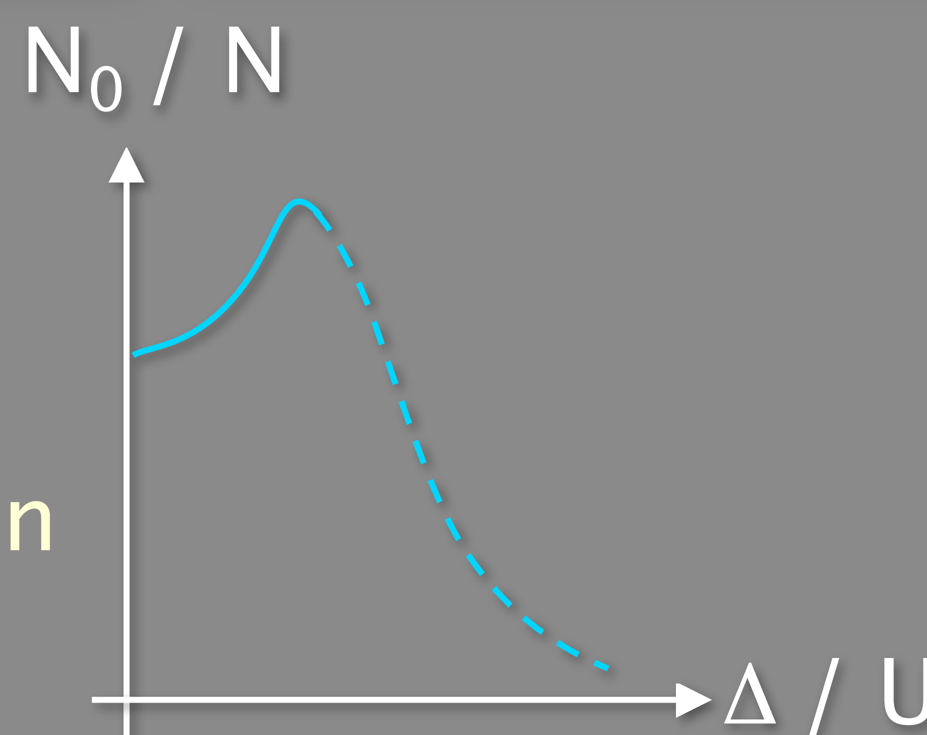


Condensate fraction in the DBH model

Benchmark for theory





- Stochastic Mean Field Theory
  Bissbort and Hofstetter
- Diagonal disorder, flat distribution

$\boxed{\text{correlated disorder: important??}}$

$\boxed{\text{Bose glass??}}$

RG on disordered Bose Hubbard model (one-loop order)

- diagonal and off-diagonal disorder (correlated)
- replicate action
- HS auxillary field to decouple off-diagonal hopping terms
- continuum limit and expand in spatial and temporal gradients

Effective Action

$$
\begin{aligned}
\mathcal{S}_{\text{eff}} \;=\; & \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^D r \left\{ \sum_a \left(K_1 \psi_a^* \partial_\tau \psi_a + K_2 |\partial_\tau \psi_a|^2 \right.\right. \\
& \left.\left. + K_3 |\nabla \psi_a|^2 + R|\psi_a|^2 + H|\psi_a|^4 \right\} \right. \\
& + G \sum_{ab} \int_{\tau\tau'} \int \mathrm{d}^D r\, |\psi_a(\tau)|^2 \, |\psi_b(\tau')|^2, \qquad (2)
\end{aligned}
$$

disorder coupling constant

Replica off-diagonal
non-local in time

$U(1) \rightarrow K_1 = \partial R / \partial \mu |_{\Delta=0}$

$K_1, K_2, R, H, G \quad \text{are} \quad \mathrm{f(t, U, m, \mu)}$

scale fields such that K_3=1

eliminate momenta in $e^{-\mathrm{d}l} \le |\boldsymbol{q}| \le 1$

mass

interactions

$$
\begin{aligned}
\frac{\mathrm{d}r}{\mathrm{d}l} &= 2r + 2f_2\overline{h} + f_1\overline{g} \\
\frac{\mathrm{d}\gamma_1}{\mathrm{d}l} &= -(d_z - 2)\gamma_1 - f_1^2\gamma_1\overline{g} \\
\frac{\mathrm{d}\gamma_2}{\mathrm{d}l} &= -2(d_z - 1)\gamma_2 - f_1^2(f_1\gamma_1^2 + \gamma_2)\overline{g} \\
\frac{\mathrm{d}\overline{h}}{\mathrm{d}l} &= -(D + d_z - 4)\overline{h} - 5f_3\overline{h}^2 - 6f_1^2\overline{g}\overline{h} \\
\frac{\mathrm{d}\overline{g}}{\mathrm{d}l} &= -(D - 4)\overline{g} - 2(f_1^2 + f_3)\overline{g}^2 - 4f_3\overline{g}\overline{h},
\end{aligned}
$$

disorder is always relevant for D<4

scale fields such that K_3=1

eliminate momenta in $e^{-\mathrm{d}l} \leq |\boldsymbol{q}| \leq 1$

iterate until

mass

interactions

$$\frac{\mathrm{d}r}{\mathrm{d}l} = 2r + 2f_2\overline{h} + f_1\overline{g}$$

$$\frac{\mathrm{d}\gamma_1}{\mathrm{d}l} = -(d_z-2)\gamma_1 - f_1^2\gamma_1\overline{g}$$

$$\frac{\mathrm{d}\gamma_2}{\mathrm{d}l} = -2(d_z-1)\gamma_2 - f_1^2(f_1\gamma_1^2+\gamma_2)\overline{g}$$

$$\frac{\mathrm{d}\overline{h}}{\mathrm{d}l} = -(D+d_z-4)\overline{h} - 5f_3\overline{h}^2 - 6f_1^2\overline{g}\overline{h}$$

$$\frac{\mathrm{d}\overline{g}}{\mathrm{d}l} = -(D-4)\overline{g} - 2(f_1^2+f_3)\overline{g}^2 - 4f_3\overline{g}\overline{h},$$

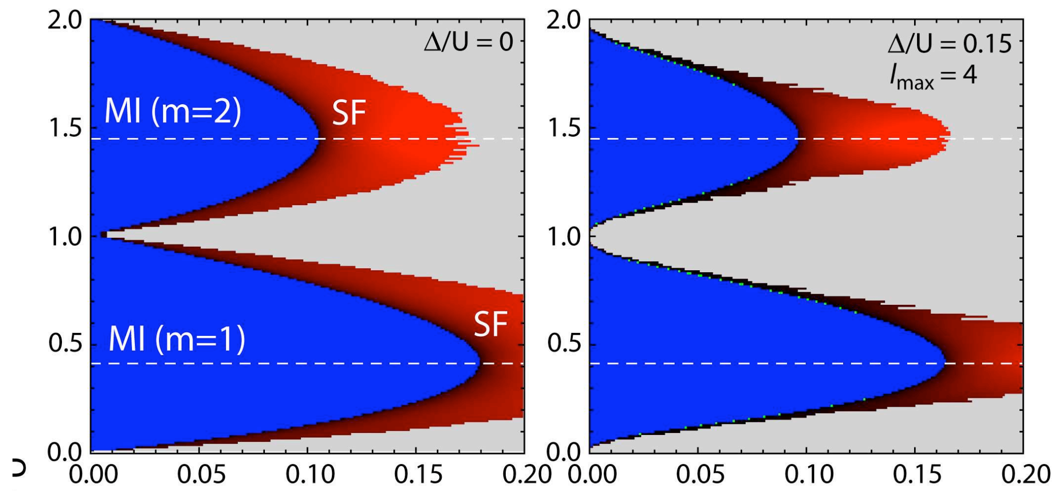
$$r(l_s)+1=0_+$$

$$g(l^*)=1$$

disorder is always relevant for D<4

Results

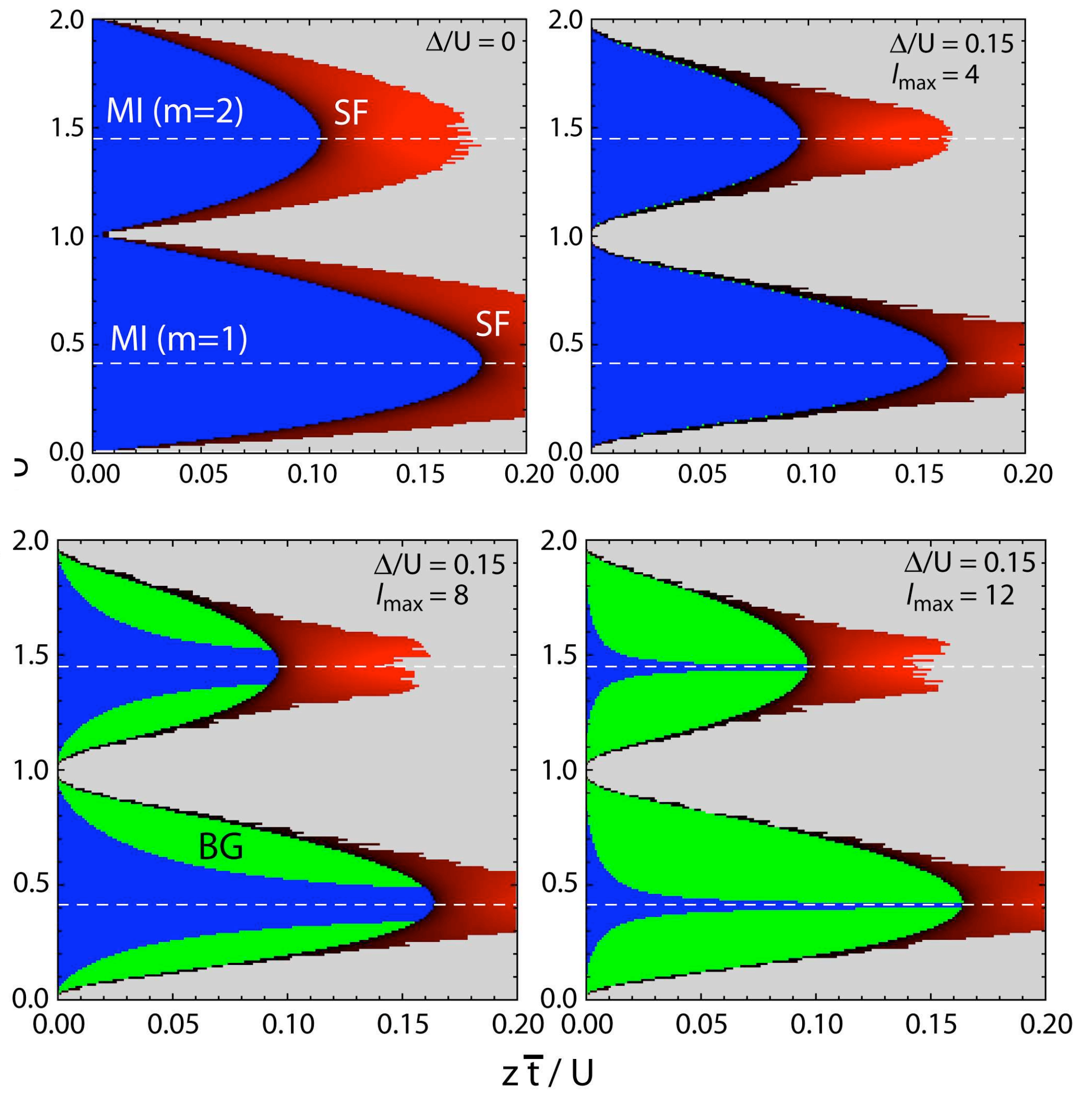
$$L = a\exp(l_{\max})$$



`No' Bose glass yet

Results

$$L = a \exp(l_{\max})$$

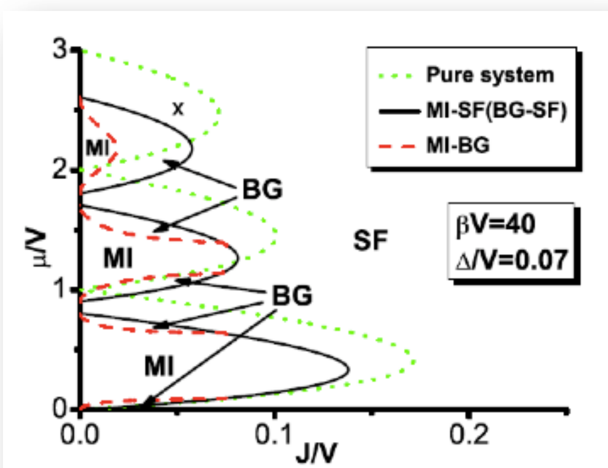


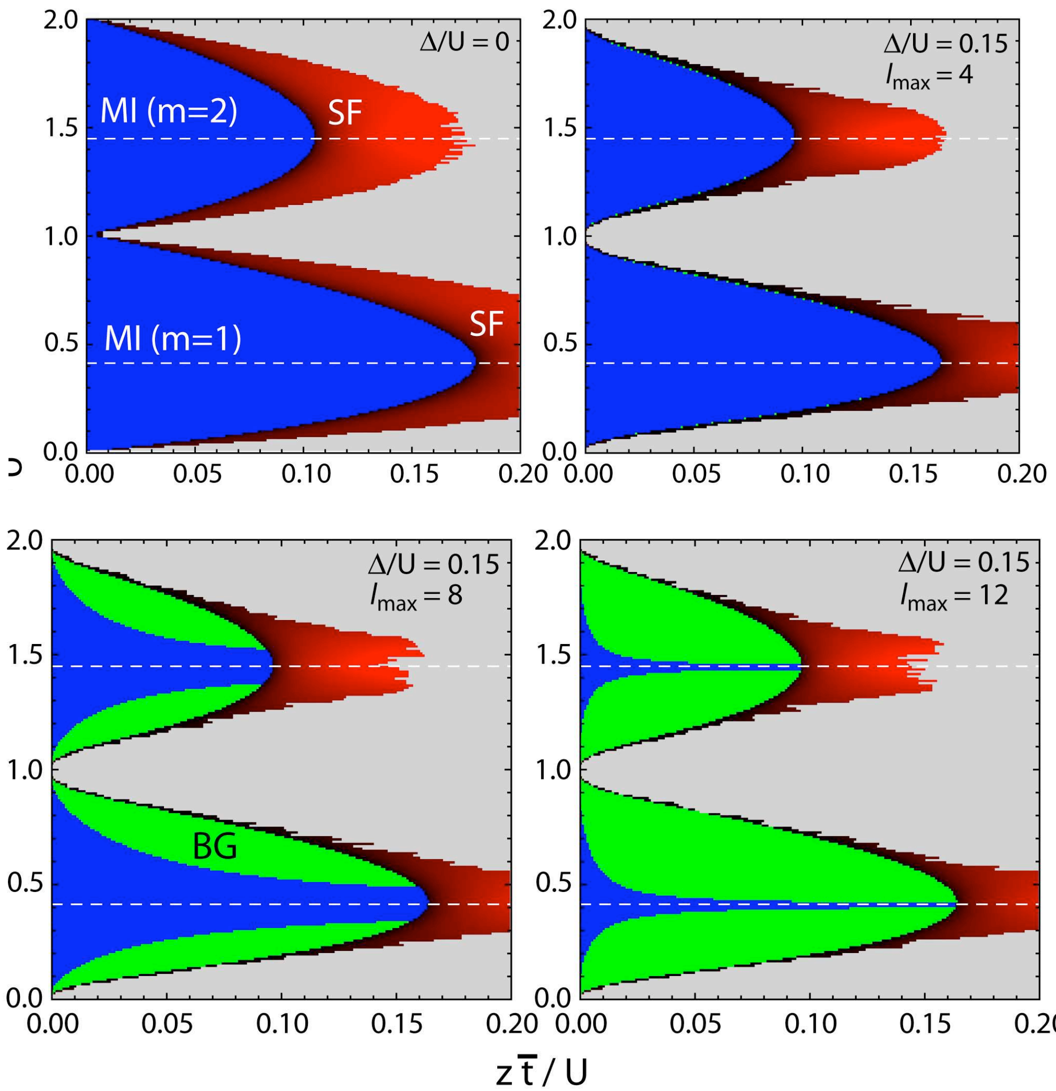
\`No' Bose glass yet

Results

$$L = a\exp(l_{\max})$$

earlier result





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Results

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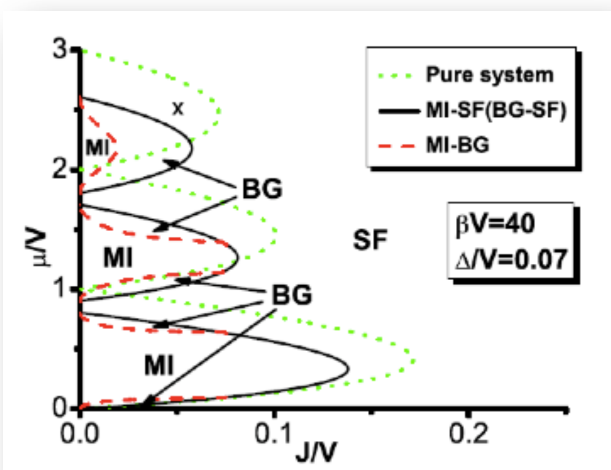
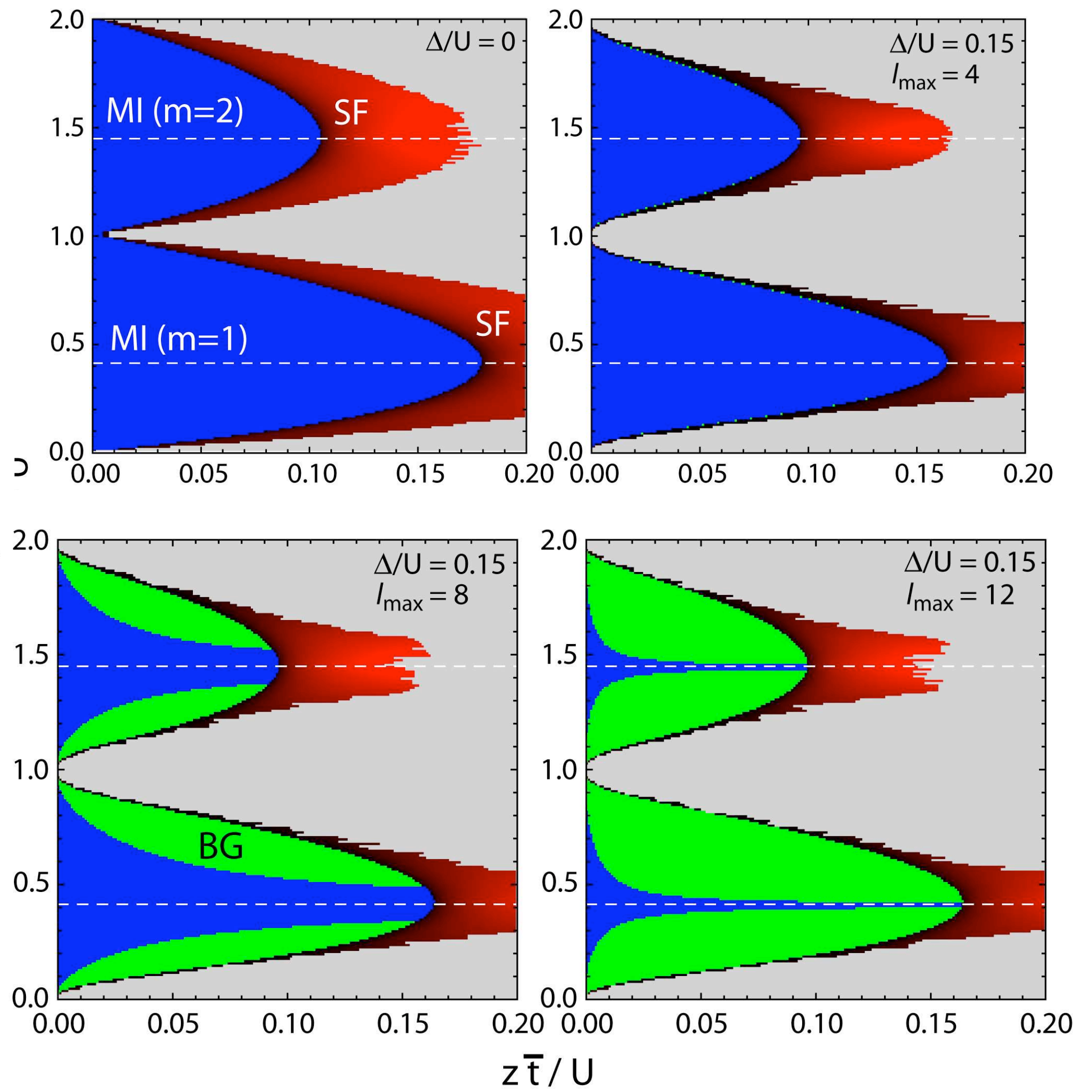
earlier result





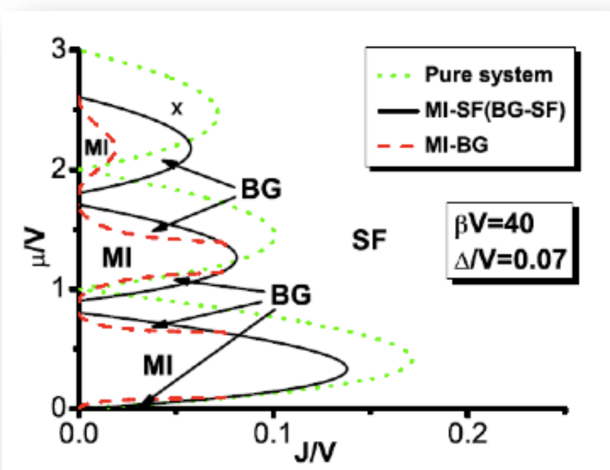
\`No' Bose glass yet

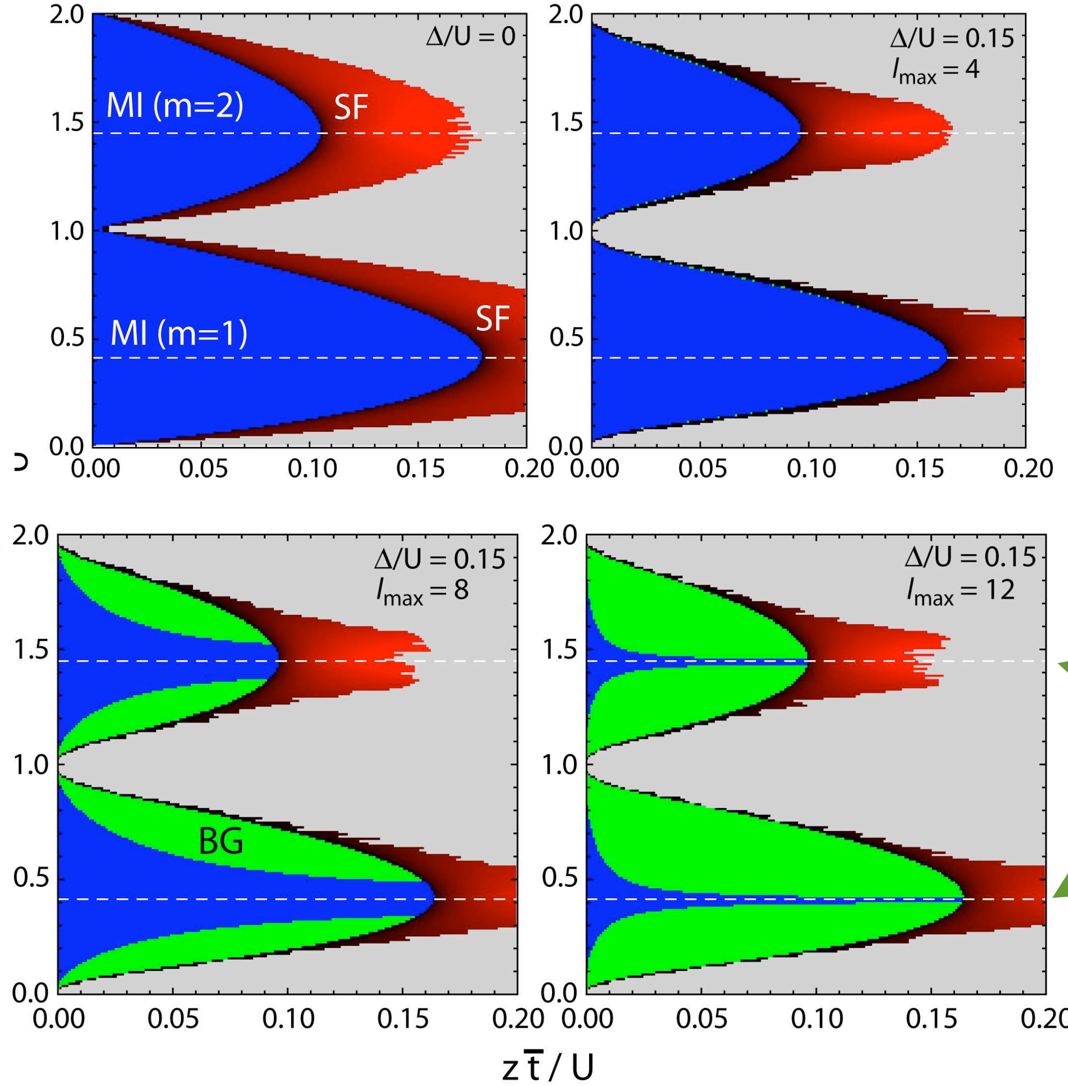
Bose Glass is size dependent (rare regions)

Results

$$L = a\exp(l_{\max})$$

earlier result





`No' Bose glass yet

G=0: no disorder

Bose Glass is size dependent (rare regions)

WHY?

Mottness

random mass couples linearly to chemical potential

$$S_{\text{eff}} = \int_0^\beta d\tau \int d^D r \, \{ .... + m_i |\psi|^2 + H|\psi|^4 \}$$

replicate,
cumulant expansion

$$\Delta^2 \psi_a(\tau)^2 \psi_b(\tau')$$

$$\Delta^2 = \overline{\epsilon_i^2} - \bar{\epsilon_i}^2 \neq 0$$

so what gives?

In BH model, the random mass does not couple linearly to the chemical potential

$$R_i = 1/(zt) - m/[(1-m)U + \mu + \epsilon_i] - (m+1)/[mU - \mu - \epsilon_i]$$

$$\Big\downarrow\; R + \delta R_i + \dots$$

$$(\delta R_i)^2 \psi_a(\tau)^2 \psi_b(\tau')^2$$

In BH model, the random mass does not couple linearly to the chemical potential

$$R_i = 1/(zt) - m/[(1-m)U + \mu + \epsilon_i] - (m+1)/[mU - \mu - \epsilon_i]$$

$$R + \delta R_i + \ldots$$

$$(\delta R_i)^2 \psi_a(\tau)^2 \psi_b(\tau')^2$$

$$(\partial R/\partial\mu)^2 \Delta^2$$

but $(\partial R/\partial\mu)_{\Delta=0} = 0$

at Mott lobe tip

General Result

$$G = -\frac{1}{2}\sum_{k,l=1}^{\infty} \frac{1}{k!l!}\, \frac{\partial^k R}{\partial \mu^k}\frac{\partial^l R}{\partial \mu^l}\bigg|_{\Delta=0} \left(\overline{\epsilon^{k+l}} - \overline{\epsilon^k}\;\overline{\epsilon^l}\right)$$

$\frac{\partial R}{\partial \mu}|_{\Delta=0} = 0$ at tip of Mott lobe $\longrightarrow G = 0$

one-loop order: no disorder at Mott tips

$\boxed{\text{Leading term}}$

$$\Delta^4$$

$\boxed{\text{Leading term}}$

Δ^4

$\boxed{\text{interactions (Mottness) suppresses disorder}}$

Leading term

Δ^4

interactions (Mottness) suppresses disorder

explains lack of consensus at commensuration

Leading term

Δ^4

$\Delta^2\psi_a^4(\tau)\psi_b^4(\bar{\tau})$

interactions (Mottness) suppresses disorder

explains lack of consensus at commensuration

BG-SF instability: at least 2-loops (Oh ...)

Leading term

$$\Delta^4$$

$$\Delta^2\psi_a^4(\tau)\psi_b^4(\bar{\tau})$$

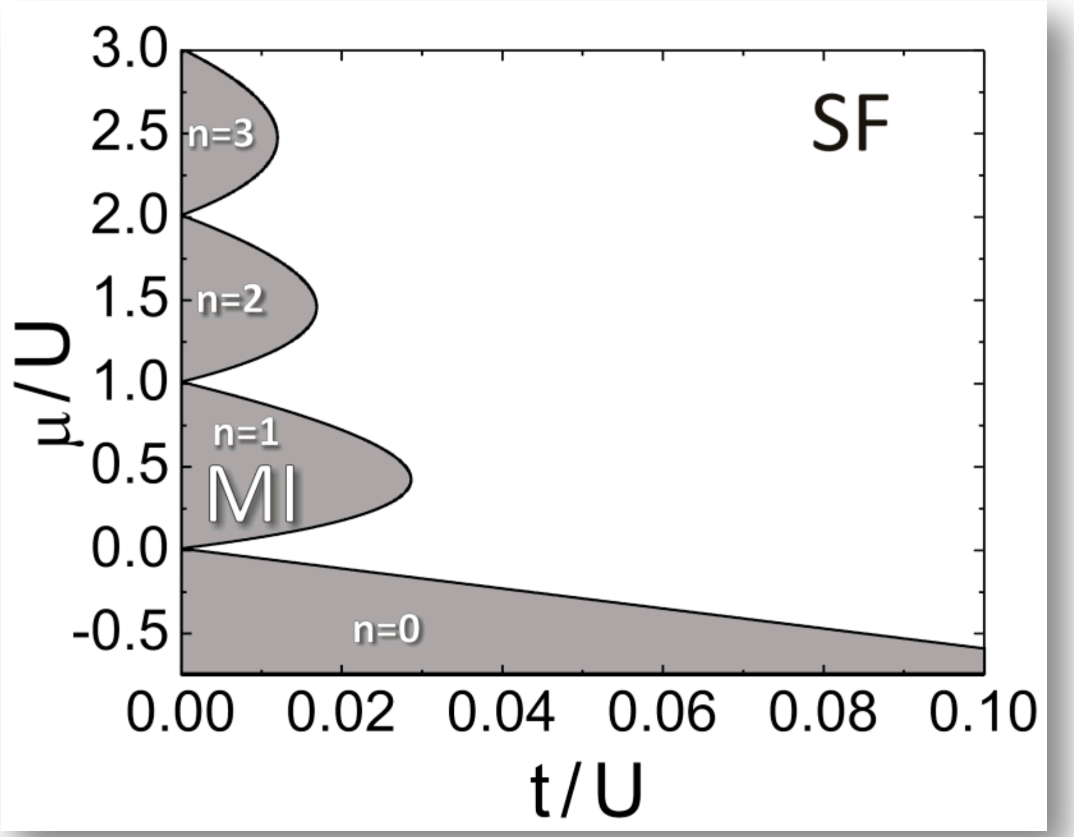
interactions (Mottness)
suppresses disorder

BG-SF instability:
at least 2-loops (Oh ...)

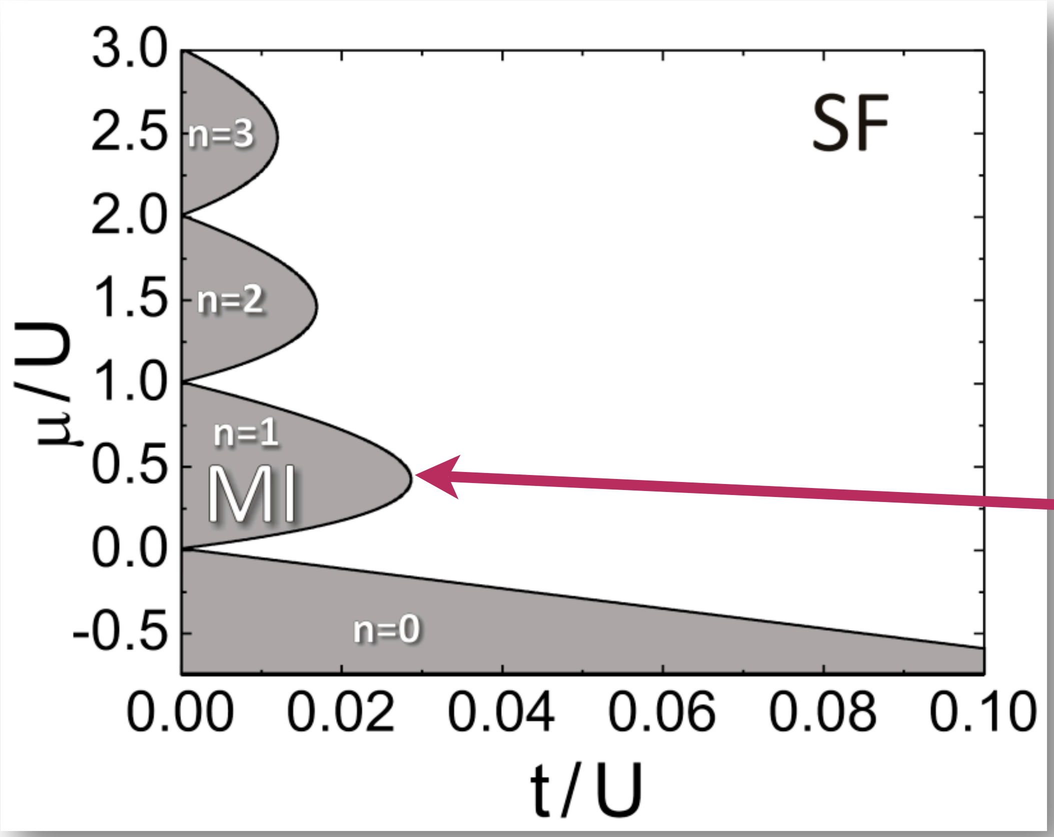
explains lack of consensus
at commensuration

universality class,
wavefunctions??

Physical argument

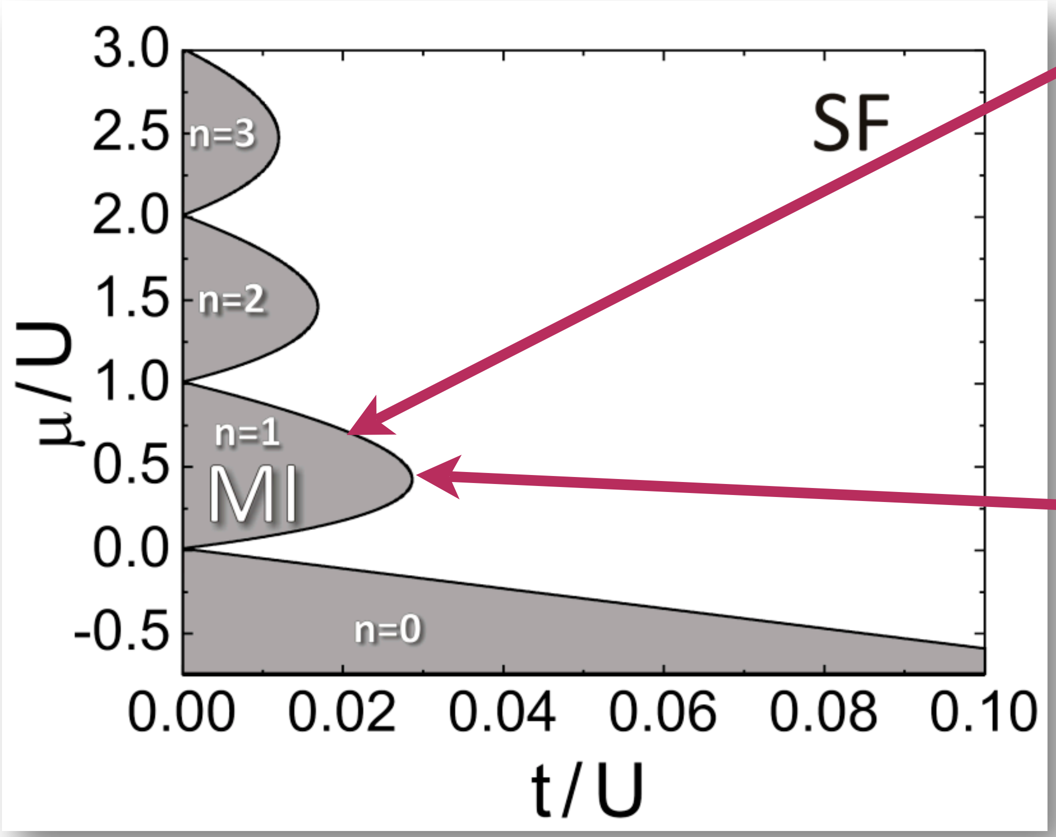


Physical argument



No number fluctuations:
MI-SF is driven by hopping
d+1 XY (KT)

Physical argument

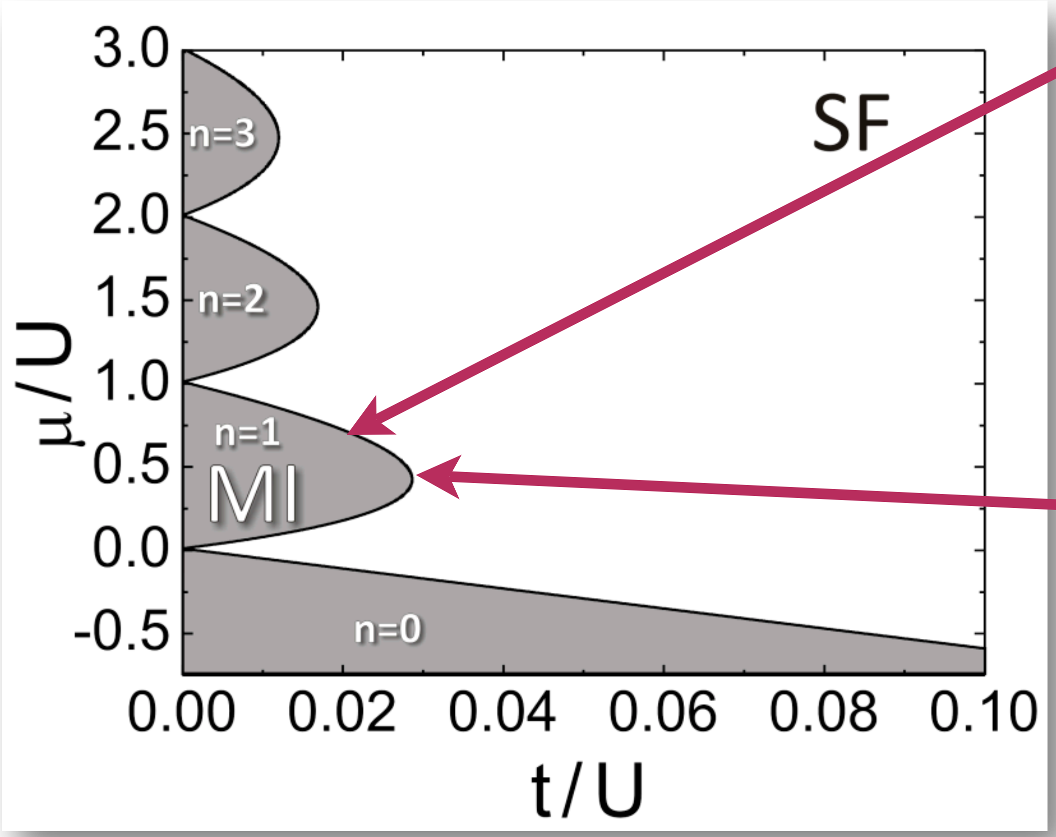


Density-driven MI-SF transition (mean-field)

No number fluctuations: MI-SF is driven by hopping d+1 XY (KT)

Physical argument

Density-driven MI-SF transition (mean-field)

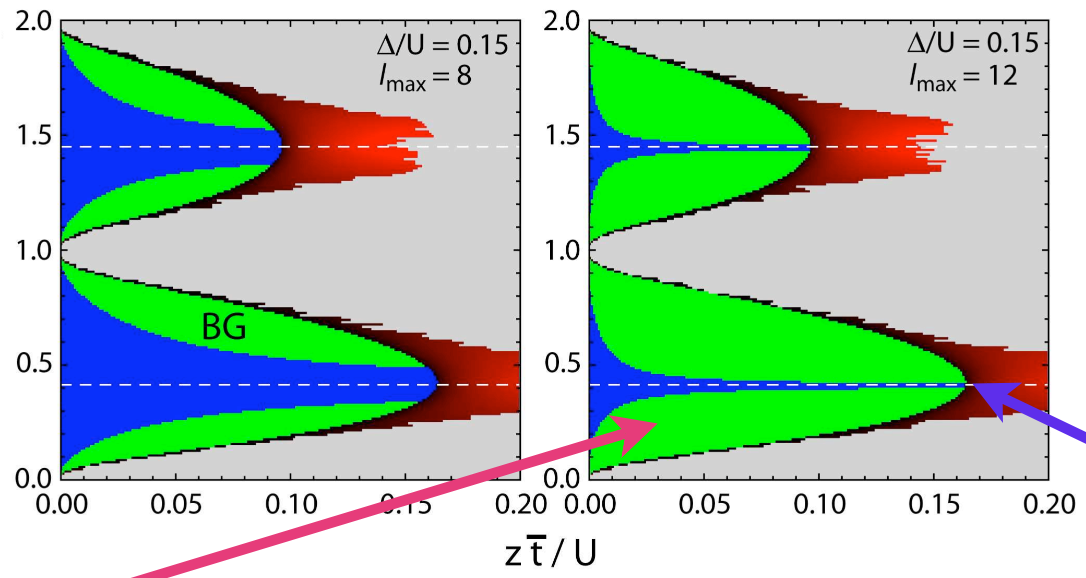


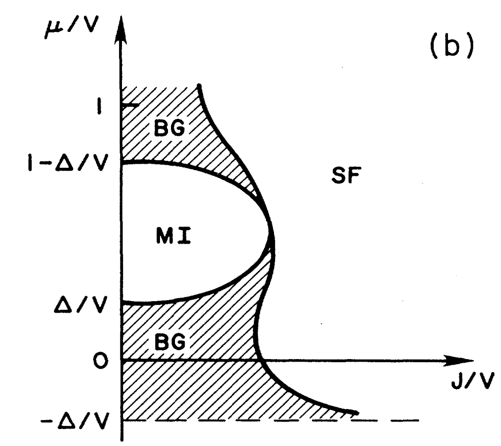
No number fluctuations: MI-SF is driven by hopping d+1 XY (KT)

Clean System: two transitions

How many transitions in dirty system?

RG: Two transitions





number fluctuations

$\Delta > \Delta_c$

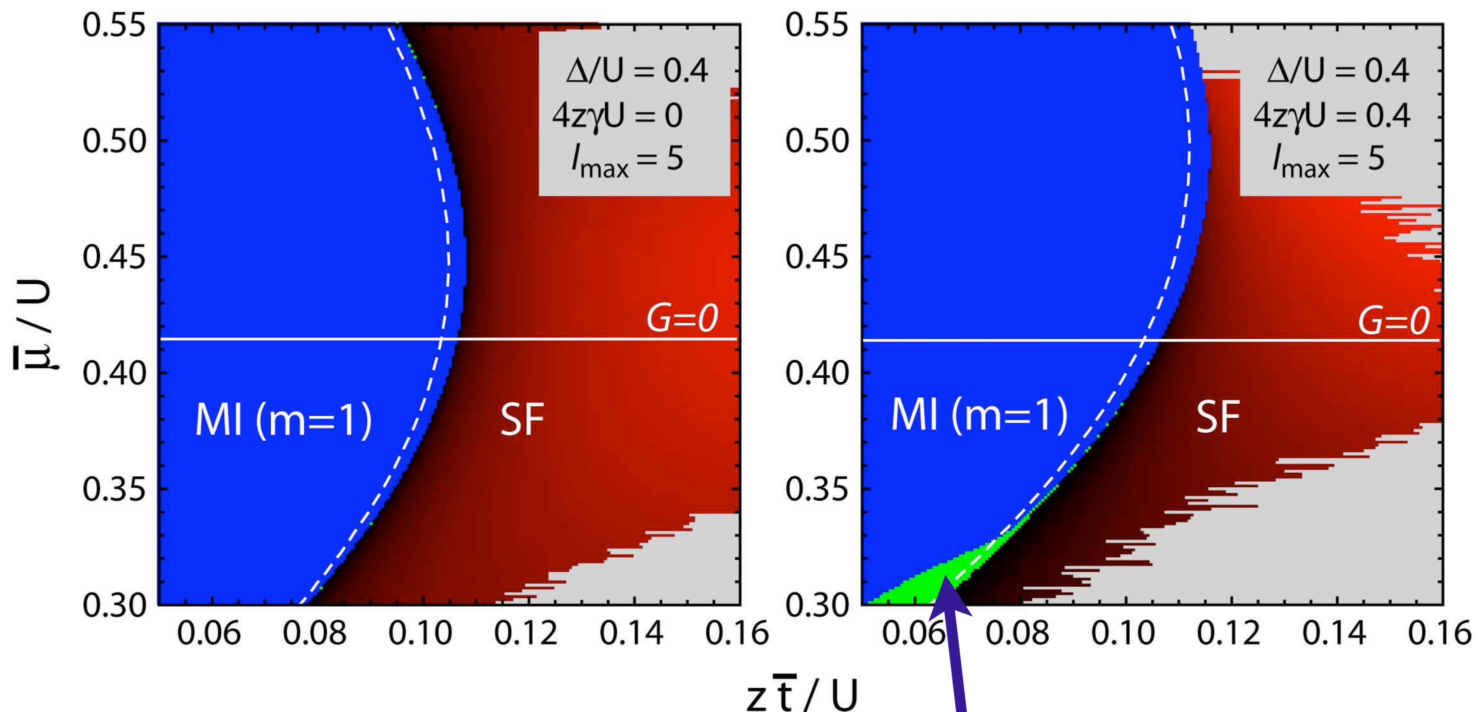
hopping dominates

one-loop

different exponents

two-loop

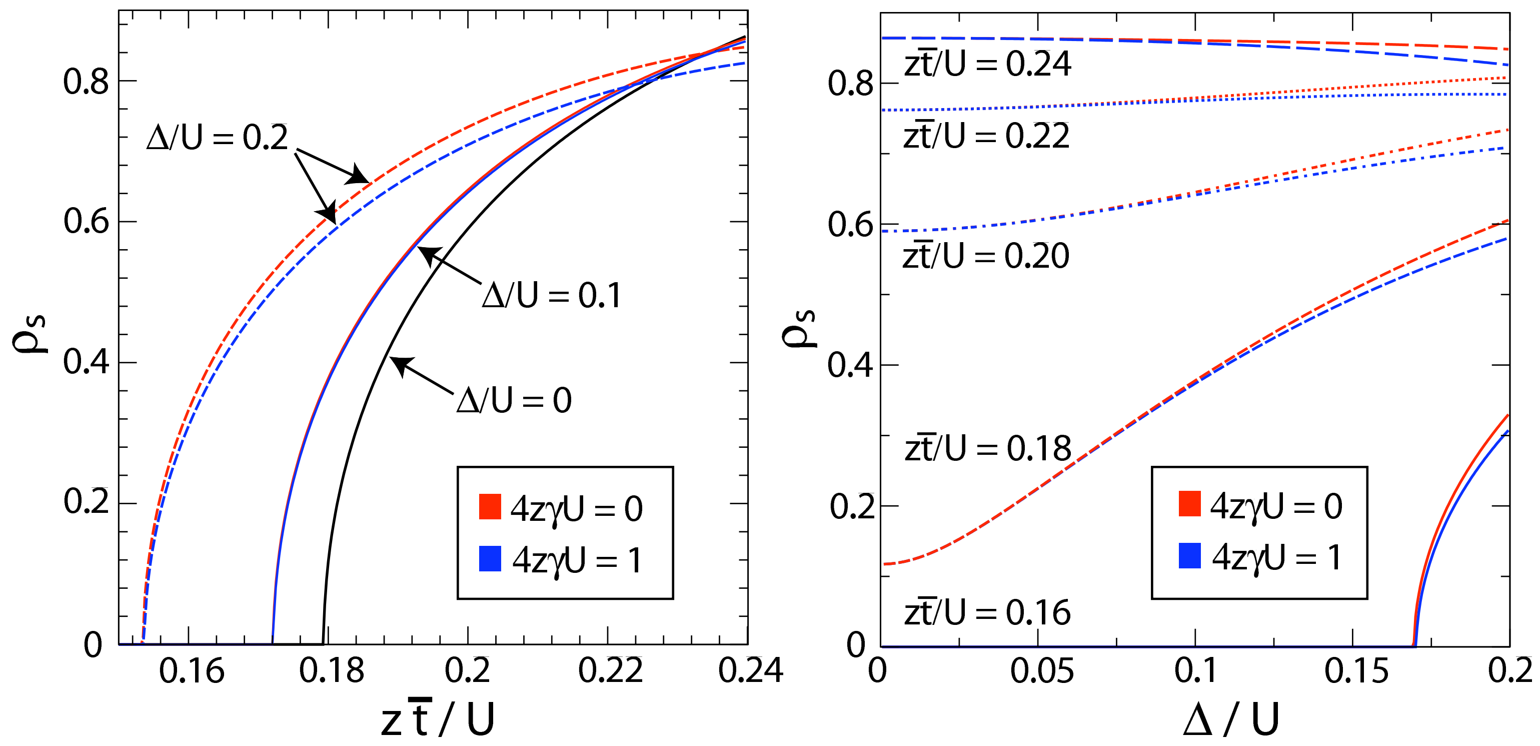
Correlations in the disorder



enhancement of Bose glass

superfluid density

$$(\rho_s^2 = r(l_s)/\bar{h}(l_s)\exp(-Dl_s))$$



correlations enhance downturn of superfluid density as seen experimentally something new in interactions+disorder

two MI-SF transitions

hopping disorder suppresses superfluid fraction

Thanks to:
F. Kruger, J. Wu
NSF/ACIF-DMR, ICMT (Urbana)