Phase Transitions in Hot and Dense QCD at Large N

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I. Introduction

- At large $T \gg \Lambda_{QCD}$ the system in the deconfined phase
- At small $T \ll \Lambda_{QCD}$ the system in the confined (hadronic) phase
- At large $\mu \gg \Lambda_{QCD}$ the system in the deconfined phase
- At small $\mu \ll \Lambda_{QCD}$ the system in the confined (hadronic) phase

It is clear:
something drastic must be happening on the way when temperature (chemical potential) varies

Question we want to address:
what are the most important vacuum configurations which are responsible for the transitions when $\mu(T)$ *varies ?*

2- Basic technique and methods:

- Main object: Large N QCD $(N_f \ll N)$
- Main technique-1: dual representation
- Main technique-2: holographic description
- Crucial element: Θ -parameter

The basic **Conjecture**:

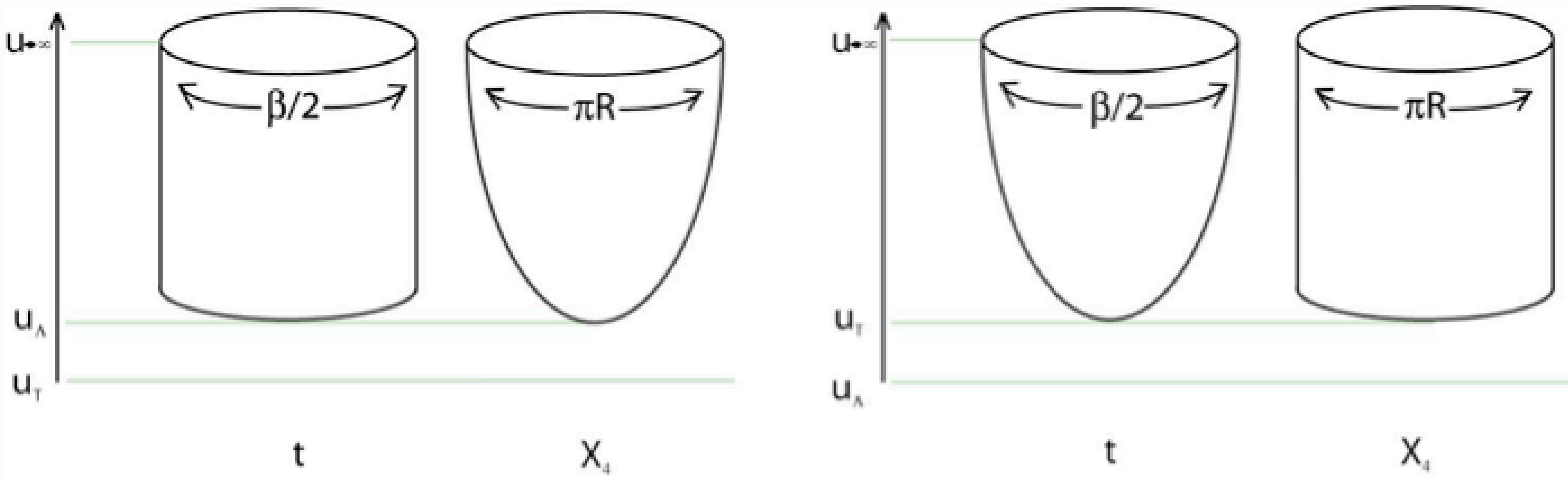
The Θ parameter suddenly changes its behavior precisely at the same point T_c where the phase transition happens

$$E_{vac} = N^2 \min_k \; h\left(\frac{\theta + 2\pi k}{N}\right), \quad T < T_c$$

$$E_{vac} \sim \cos\theta \cdot \exp\left(-N\right), \quad T > T_c$$

3. Support for the ***CONJECTURE*** from the holographic model of QCD

- The large N QCD is known to have a holographic description;
- Confined / deconfined phases in the holographic description can be studied in the standard way by analyzing the Polyakov's loop;
- Transition from confined to deconfined phase corresponds to the transition *from one background metric to another* at temperature T_c;
- The Θ behavior has been also studied in both phases with the result: the confinement- deconfinement *phase transition takes place precisely at T_c where Θ dependence drastically changes.*
- $$\chi(T) \sim \frac{\partial^2 E_{vac}}{\partial\theta^2} \sim 1, \qquad T < T_c$$
  $$\chi(T) \sim \frac{\partial^2 E_{vac}}{\partial\theta^2} \sim 0, \qquad T > T_c$$



low temperature phase

high temperature phase

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} \left((dx_\mu)^2 + f(U)(dx^4)^2\right) + \left(\frac{U}{R}\right)^{-\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} \left((dx_i)^2 + f_T(U)dt_E^2 + (dx^4)^2\right) + \left(\frac{U}{R}\right)^{-\frac{3}{2}} \left(\frac{dU^2}{f_T(U)} + U^2 d\Omega_4^2\right)$$

$$f_T \;=\; 1 - U_T^3/U^3$$

- At non zero temperature there are two backgrounds with similar asymptotic topology

$$R^3 \times S^1_\tau \times D(x_4, U) \times S^4$$

- At low temperature the first background dominates.
- At high temperature the black hole metric dominates which corresponds $\tau \leftrightarrow x_4$ interchange.
- The θ dependence correlates with the wrapping around x_4 coordinate. It is well defined at $T > T_c$.
- D-0 brane wrapped around x_4 is identified with instanton. It is well defined object at $T > T_c$.
- Drastic changes in θ corresponds to changes in metric in holographic description.

- The holographic model gives us a hint: θ dependence experiences some drastic changes exactly at the same point where phase transition takes place T_c

- In QFT we can do much better than that: we can compute the loop corrections at $T > T_c$ along with classical action $\exp(-N)$ which would gives us an estimation of the point T_c where the transition happens. In QFT we see precisely how these drastic changes are happening at given large N in terms of Λ_{QCD}

- The same logic can be applied to the transition at large chemical potential μ where holographic construction is not known (yet). lattice computations also do not help us in this regime of large μ .

4. Support for the ***CONJECTURE*** from the lattices:

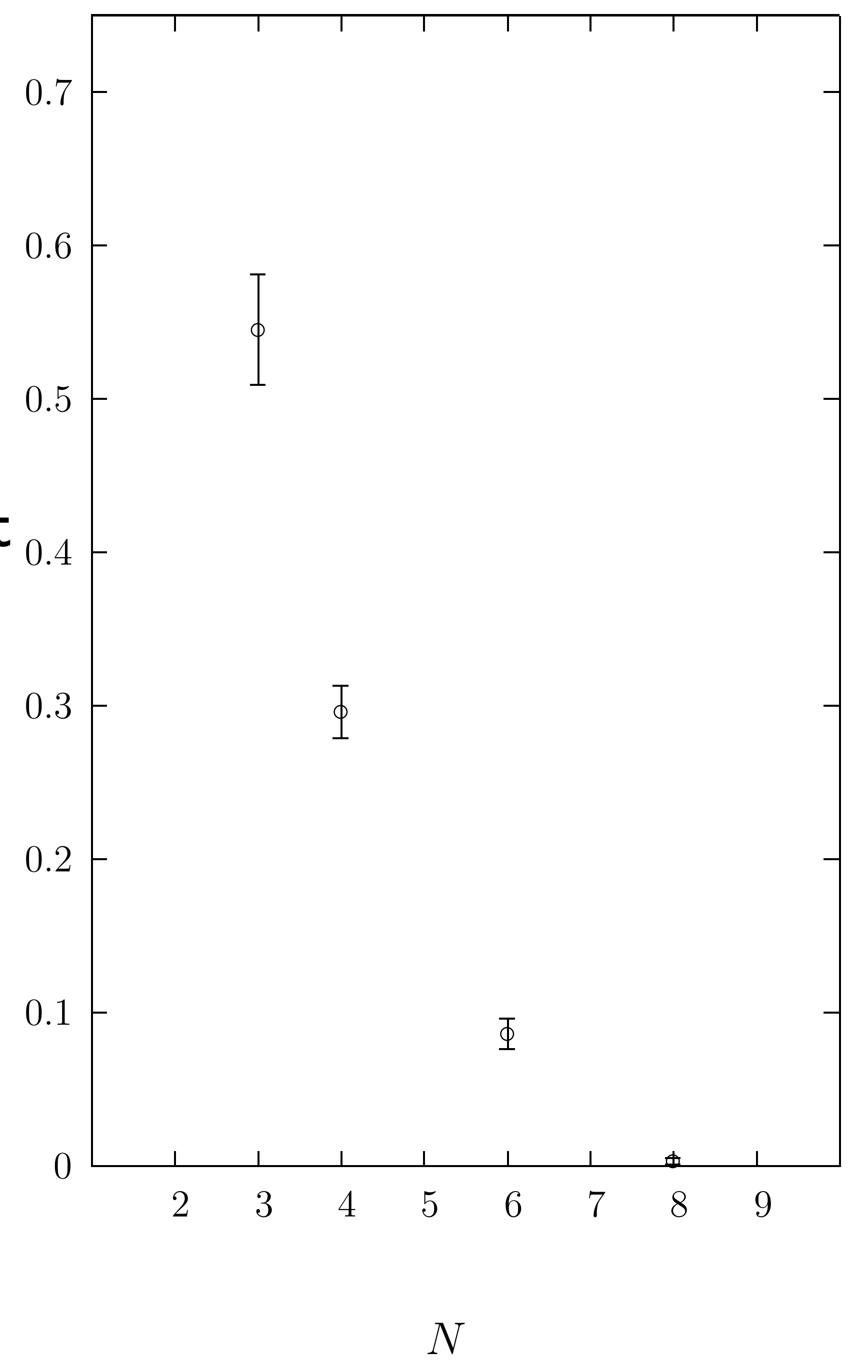
the ratio

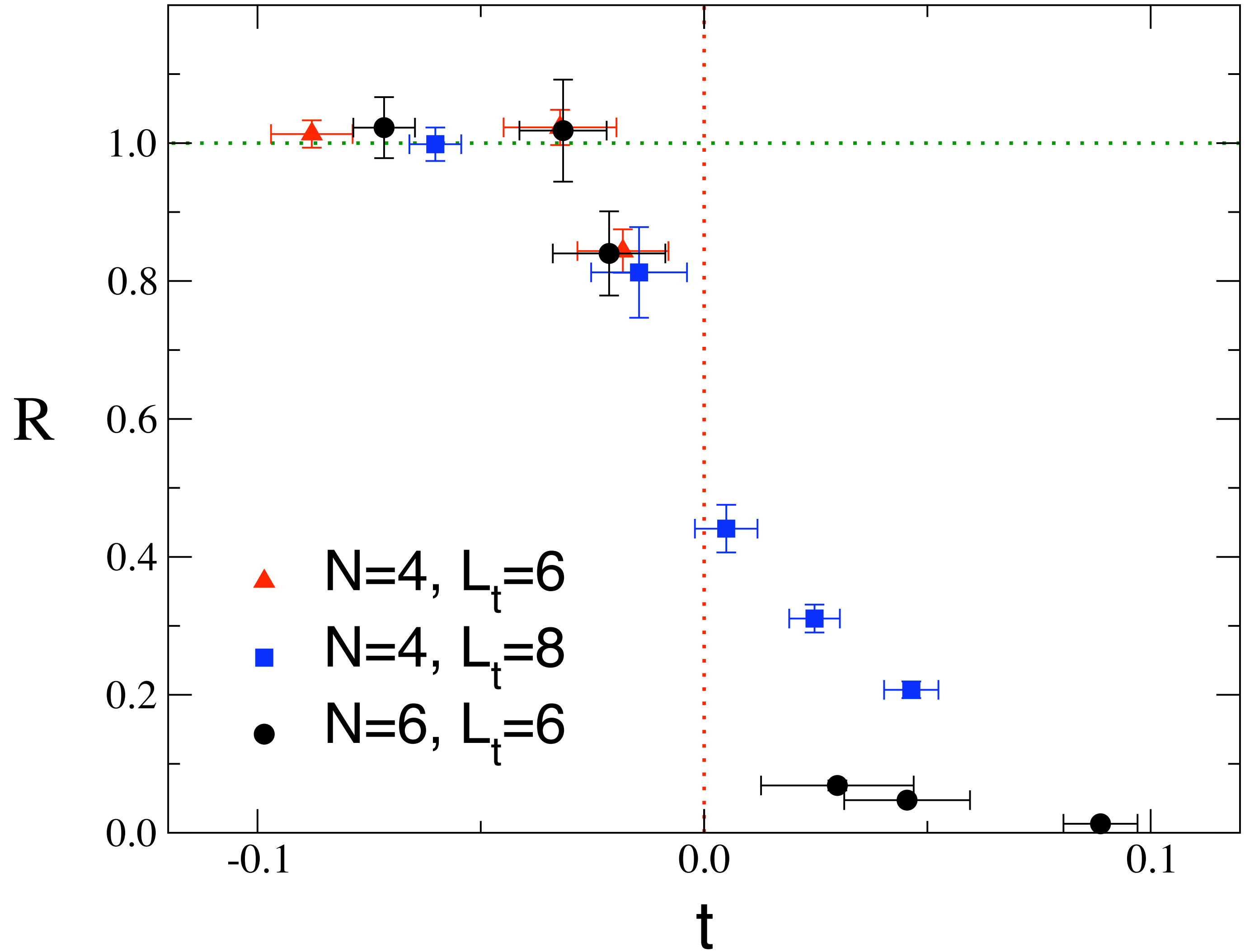
$$R \equiv \chi(T = T_c + \epsilon)/\chi(T = T_c - \epsilon)$$

in deconfined and confined phases at

$$T \simeq T_c$$

B. Lucini, M. Teper, U. Wenger, 2004





Support for the ***CONJECTURE*** from the lattices:

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the ratio $R(T) \equiv \chi(T)/\chi(T=0)$ as a function of reduced temperature $t = T/T_c - 1$ for N=4, 6, L.Del Debbio, et al.2004

5. Deconfined Phase, $T > T_c$

- According to the *Conjecture*, one can study the confinement -deconfinement phase transition by analyzing the θ dependence rather than Polyakov's loop.

- The θ dependence for $T > T_c$ is determined by instantons.

- Instanton expansion converges at $T > T_c$

$$V_{\rm inst}(\theta) \sim e^{-\gamma N} \cos\theta, \qquad \gamma = \left[\frac{11}{3} \ln\left(\frac{\pi T}{\Lambda_{QCD}}\right) - 1.86\right],$$

- Critical temperature is determined by the condition

$$\gamma = \left[\frac{11}{3} \ln\left(\frac{\pi T_c}{\Lambda_{QCD}}\right) - 1.86\right] = 0 \quad \Rightarrow \quad T_c(N=\infty) \simeq 0.53\Lambda_{QCD},$$

- Identification of T_c with the phase transition point (and not with super cooled/superheated regions when two phases may coexist) is based on holographic picture when T_c is a true phase transition point at large N

Phase transition at large N. Basic observations at non-vanishing T

- There is an exponentially large "T -independent" contribution due to the entropy, number of embeddings SU(2) to SU(N), etc

$$e^{+1.86N}$$

- There is an exponentially large "T -dependent" contribution

$$\left(\frac{\Lambda_{QCD}}{\pi T}\right)^{\frac{11}{3}N} = \exp\Big[-\frac{11}{3}N\cdot\ln\Big(\frac{\pi T}{\Lambda_{QCD}}\Big)\Big].$$

- The fermion- related contributions lead to the sub-leading 1/N effects, as expected:

$$\sim \langle\bar{\psi}\psi\rangle^{N_f} \sim e^{N\cdot\left(\kappa\ln|\langle\bar{\psi}\psi\rangle|\right)}. \qquad \kappa\equiv\frac{N_f}{N}\rightarrow 0$$

Deconfined phase--continue

- For any positive $\gamma > 0$ the instanton density is parametrically small and calculations are under complete theoretical control even in close vicinity of T_c

$$V \sim \cos\theta \cdot e^{-\alpha N(\frac{T-T_c}{T_c})}, \quad 1 \gg \left(\frac{T-T_c}{T_c}\right) \gg 1/N.$$

- Topological susceptibility $\chi(T > T_c) \sim e^{-N} = 0$ obviously vanishes in agreement with results from holographic QCD

- One can compute $T_c(\mu)$ for small chemical potential μ

$$T_c(\mu) = T_c(\mu = 0)\left[1 - \frac{3N_f\mu^2}{4N\pi^2 T_c^2(\mu = 0)}\right], \quad \mu \ll \pi T_c, \quad N_f \ll N.$$

- This formula works amazingly well even for N=3, N_f=2 (lattice results from Philip de Forcrand et al, 2002):

$$T_c(\mu)^{lat} = T_c(\mu = 0)^{lat}\left[1 - 0.500(67)\frac{\mu^2}{\pi^2 T_c^2(\mu = 0)^{lat}}\right], \quad N_f = 2, \quad N = 3$$

6. Phase transition at large μ

- We use the same logic to study the phase transition at large μ

- We find the first order phase transition at very large μ

$$\gamma = \left[\frac{11}{6}\ln\left(\frac{N_f\bar{\mu}^2}{\Lambda_{QCD}^2}\right) - 1.1\right] = 0 \quad \Rightarrow \quad \mu_c(N=\infty) \simeq 1.4 \cdot \Lambda_{QCD}\sqrt{\frac{N}{N_f}}, \quad N_f \ll N$$

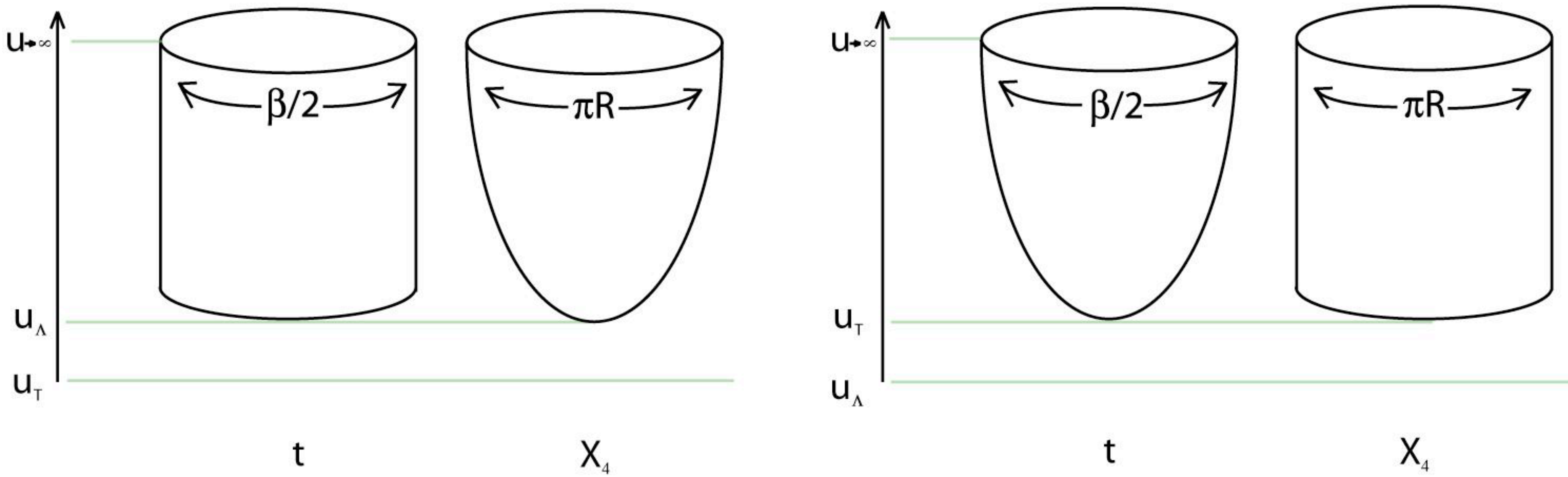
- The topological susceptibility vanishes in deconfined phase:

$$V_{\rm inst}(\theta) \sim \cos\theta \cdot e^{-\alpha N\left(\frac{\mu-\mu_c}{\mu_c}\right)}, \quad \frac{1}{N} \ll \left(\frac{\mu-\mu_c}{\mu_c}\right) \ll 1,$$

- The large scale $\mu_c \sim \sqrt{N}\Lambda_{QCD}$ is consistent with recent findings by McLerran and Pisarski

7. D-defects in Holographic model

- **The basic lessons from D0 instantons:**
- *the phase transition can be interpreted as Hawking Page phase transition when two metrics get interchanged.*
- *the wrapping around* x_4 *is stable* $T > T_c$ *and unstable* $T < T_c$
- *the wrapping around* τ *is stable* $T < T_c$ *and unstable* $T > T_c$
- *the* θ *dependence is linked to* x_4 *wrapping*
- *the instantons are identified with D0 branes wrapped around* x_4



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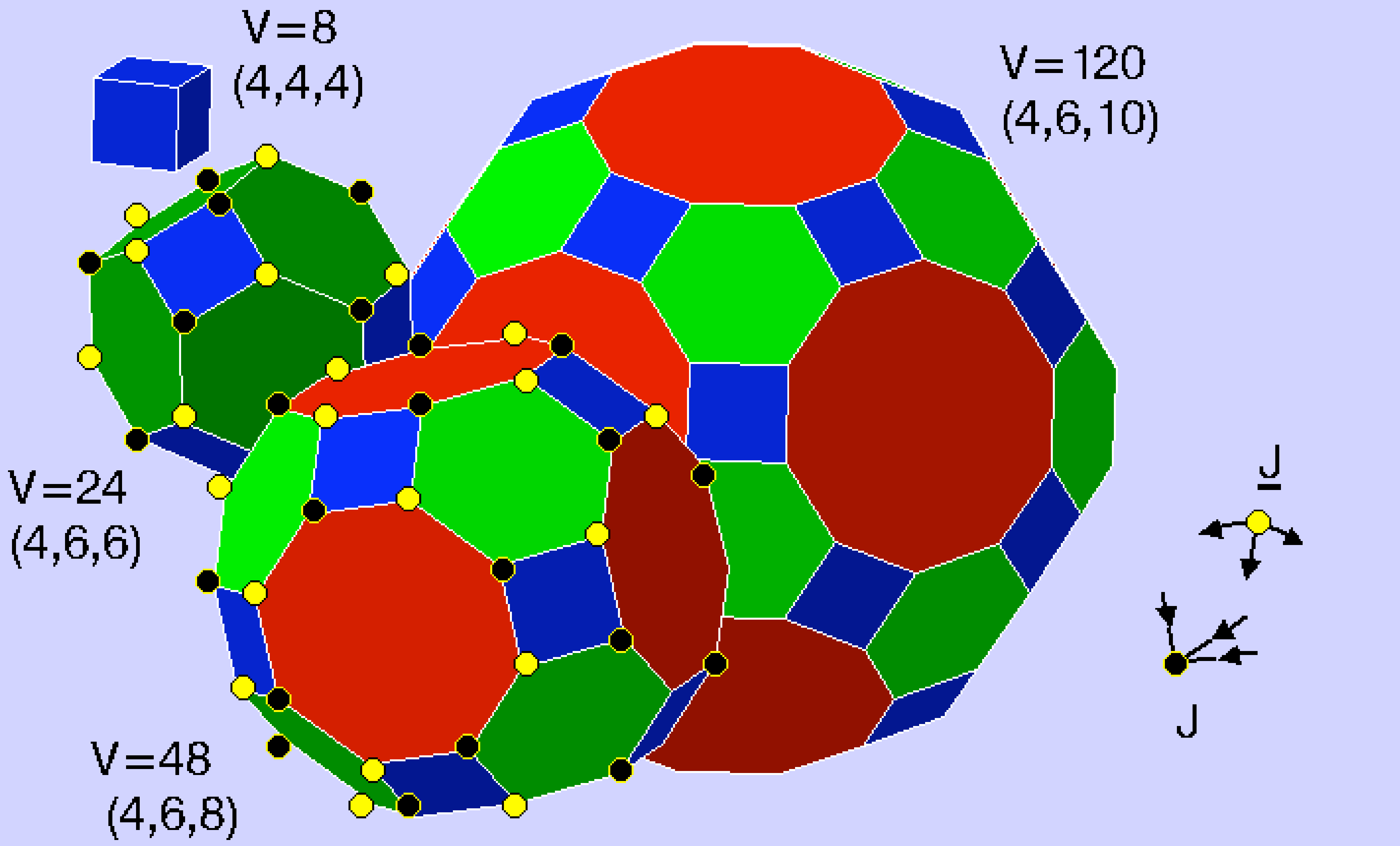
$$f_T = 1 - U_T^3/U^3$$

- There are many other D0, D2, D4, D6, D8 defects (and their superpositions) which may play an important role in physics (work with A. Gorsky, V.I. Zakharov, 2009).

- in particular, consider D4 wrapped around τ and S^4 . This is so-called N-Junction. It is N- vertex with N elementary strings attached. Similar (pure gauge) object has been previously discussed to construct baryons (if matter is present)-- Sakai-Sugimoto model.

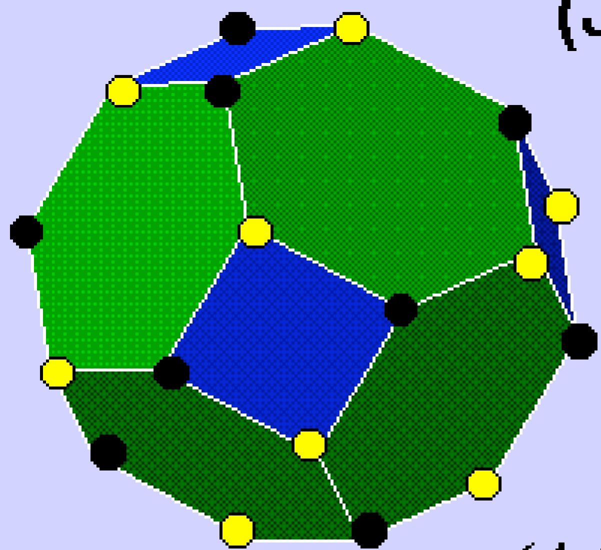
- In pure YM one can combine D4 particle with D4 antiparticle to form gauge invariant object. The mass of the object scales as N and it is made of gluons.

- In QFT such objects have been previously discussed (Csorgo, Gyulassy and Kharzeev, 2004) motivated by the discovery of the carbonic Fullerences C_{60}, C_{70} in 1985.



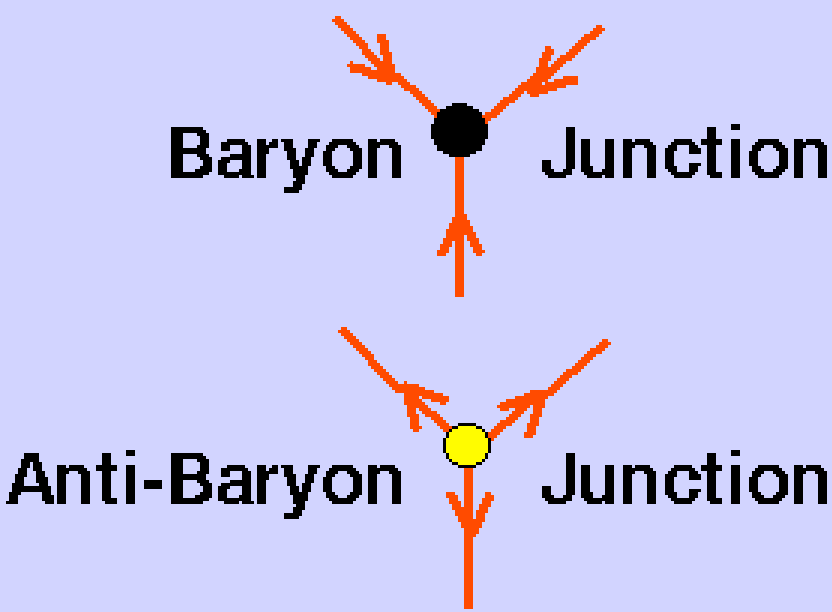
QCD Junction Buckyballs : $(J\underline{J})_{V/2}$

Femto-meter scale

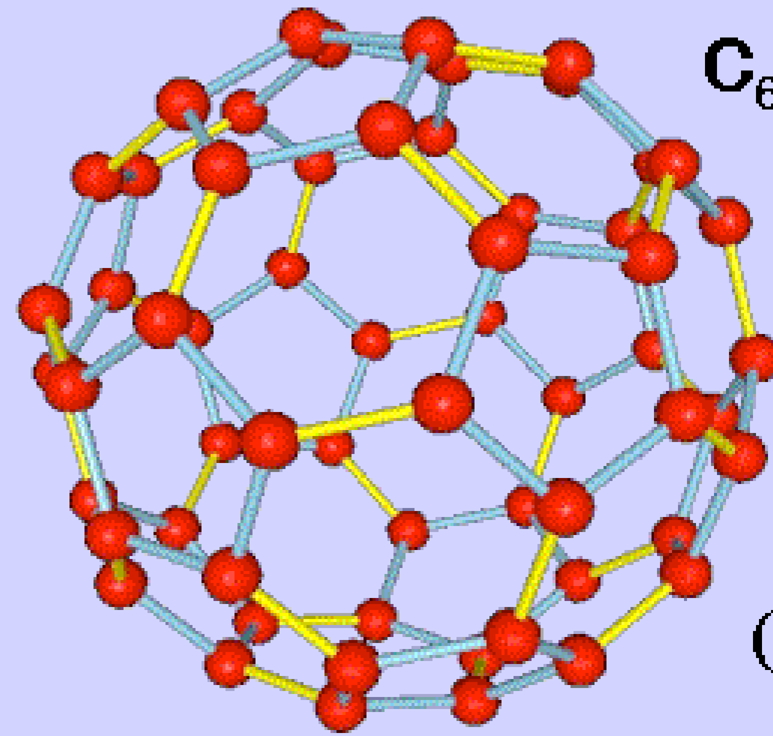


$(J\bar{J})_{12}$

(4,6,6)

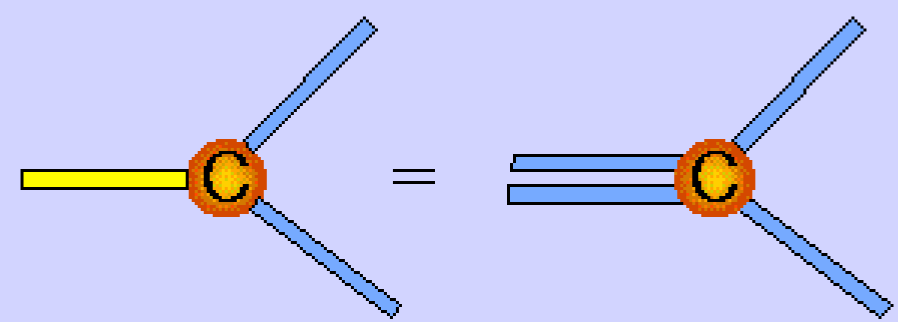


Nano-meter scale



C_{60}

(5,6,6)



Carbon Bonds

- Different wrapping: consider D4 wrapped around S^4 and x_4 . This is the same N-Junction with N elementary strings attached, but now it is 4d object sensitive to θ , well defined above $T > T_c$ (D4 instanton) rather than 3d object well defined below $T < T_c$ (D4 particle).

- One can identify D4 instanton with configuration of N different calorons(for $T > T_c$)with maximally nontrivial holonomies (pierre van Baal...). Such configuration indeed gives an infrared finite contribution to Z.

- Objects with fractional magnetic (topological) charges 1/N may also emerge in this classification, similar to well-known SUSY cases. They may play a crucial role in the phase transition (subject for a different talk). See also talks by D.Tong, K.Konishi where 1/N constituents also appear in different models.

Propaganda

- We presented the arguments that the sharp changes in θ happen at the point of the phase transition T_c (support comes from holographic QCD and lattices)

- The instanton calculus is justified in the region close to the phase transition point $(T-T_c)/T_c \gg 1/N$.

- The same conjecture (on sharp θ dependence at T_c) can be used in the region with very large chemical potential where no lattice results exist. We predict that $\mu_c \sim \sqrt{N}$ if $N_f \ll N$. This is consistent with recent McLerran, Pisarski results.

- New types of D-defects emerge in the holographic description (D4 instantons, D2 domain walls, fractional 1/N instanton quarks and many others).