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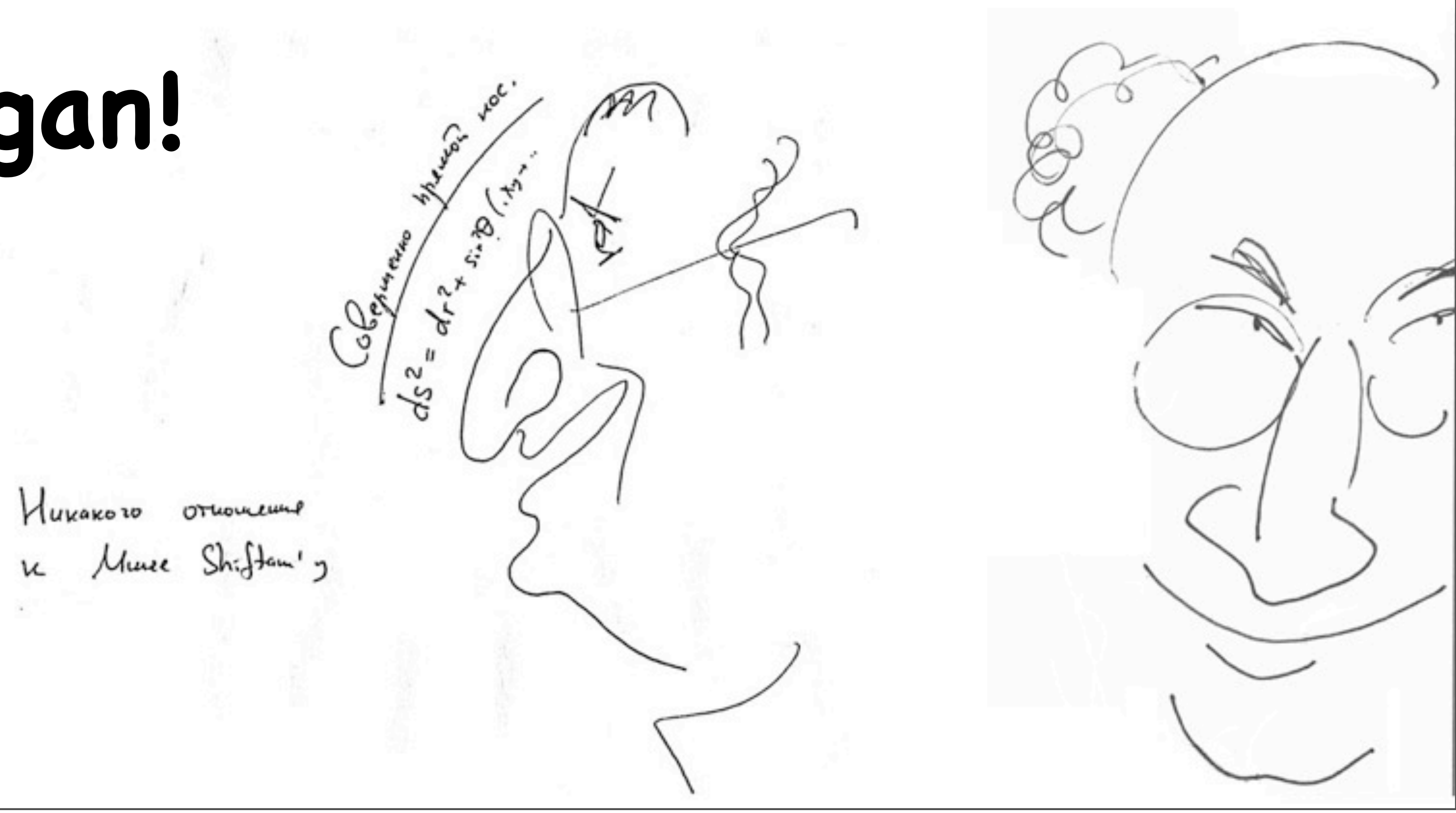

Still

in

Light-Cone Superspace

P. Ramond

Institute for Fundamental Theory

University of Florida

(with S. Ananth, D. Belyaev, L. Brink and S.-S. Kim)

light-cone superspaces with maximal Supersymmetry

light-cone superspaces with maximal Supersymmetry

8 Grassmann variables

eleven dimensions

SuperGravity

light-cone superspaces with maximal Supersymmetry

8 Grassmann variables

eleven dimensions

SuperGravity

4 Grassmann variables

ten dimensions

Super Yang-Mills

SuperConformal Theories

(W. Nahm, 1978)

SuperConformal Theories

(W. Nahm, 1978)

D=3: $OSp(2,2|n)$ $Sp(2,2)[SO(3,2)]\times SO(n)$

SuperConformal Theories

(W. Nahm, 1978)

D=3: OSp(2,2|n) Sp(2,2)[SO(3,2)]xSO(n)

D=4: SSU(2,2|n) SO(4,2)xSU(n)xU(1)

SuperConformal Theories

(W. Nahm, 1978)

D=3: OSp(2,2|n) Sp(2,2)[SO(3,2)]xSO(n)

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D=4: PSU(2,2|4) SO(4,2)xSU(4)

SuperConformal Theories

(W. Nahm, 1978)

D=3: $OSp(2,2|n)$ $Sp(2,2)[SO(3,2)] \times SO(n)$

D=4: $SSU(2,2|n)$ $SO(4,2) \times SU(n) \times U(1)$

D=4: $PSU(2,2|4)$ $SO(4,2) \times SU(4)$

D=5: $F[4]$ $SO(5,2) \times SU(2)$

SuperConformal Theories

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D=5: $F[4]$ $SO(5,2) \times SU(2)$

D=6: $OSp(2n|6,2)$ $SO(6,2) \times Sp(2n)$

N=1 Super Yang-Mills in D=10

(4 Grassmann variables)

N=1 Super Yang-Mills in D=10

(4 Grassmann variables)

8 bosons & 8 fermions

N=1 Super Yang-Mills in D=10

(4 Grassmann variables)

8 bosons & 8 fermions

On the light-cone

$SO(n,2) \rightarrow SO(n-2)$

(little group)

D=3

D=3

$$OSp(2,2|8) \rightarrow SO(8)$$

(R-symmetry)

D=3

$OSp(2,2|8) \rightarrow SO(8)$

(R-symmetry)

bosons $\sim 8_v$; fermions $\sim 8_s$

(Bagger, Lambert, Gustavsson)

D=4

D=4

$$SO(8) \supset SO(2) \times SU(4)$$

D=4

$$SO(8) \supset SO(2) \times SU(4)$$

$$8_v = 1_1 + 6_0 + 1_{-1}\ ; \quad 8_s = 4_{1/2} + \bar{4}_{-1/2}$$

D=5

D=5

$$F[4] \to SU(2) \times SU(2)$$

D=5

$$F[4] \rightarrow SU(2) \times SU(2)$$

$$SO(8) \supset SU(2) \times Sp(4)$$

D=5

$$F[4] \rightarrow SU(2) \times SU(2)$$

$$SO(8) \supset SU(2) \times Sp(4)$$

$$Sp(4) \supset SU(2)$$

D=5

$$F[4] \rightarrow SU(2) \times SU(2)$$

$$SO(8) \supset SU(2) \times Sp(4)$$

$$Sp(4) \supset SU(2)$$

$$8 = (3,1) + (1,5) \quad ; \; 8 = (2,4)$$

(R-Spin 2) (R-Spin 3/2)

D=6

D=6

$$OSp(5|4,2) \rightarrow SU(2) \times SU(2) \times Sp(4)$$

D=6

$$OSp(5|4,2) \rightarrow SU(2)\times SU(2)\times Sp(4)$$

$$SO(8) \supset SU(2) \times Sp(4)$$

D=6

$$OSp(5|4,2) \to SU(2) \times SU(2) \times Sp(4)$$

$$SO(8) \supset SU(2) \times Sp(4)$$

$$8 = (3,1) + (1,5) \quad ; \quad 8 = (2,4)$$

(Arvidsson, Flink, Henningson)

Welcome to Superspace

LC2 Formalism

Light-Cone Coordinates:

$$x^{\pm} = \frac{1}{\sqrt{2}}(x^0 \pm x^3) \qquad \partial^{\pm} = \frac{1}{\sqrt{2}}(-\partial^0 \pm \partial^3)$$

$$x = \frac{1}{\sqrt{2}}(x^1 + ix^2) \qquad \overline{x} = \frac{1}{\sqrt{2}}(x^1 - ix^2) \qquad \partial = \frac{1}{\sqrt{2}}(\partial^1 + i\partial^2) \qquad \overline{\partial} = \frac{1}{\sqrt{2}}(\partial^1 - i\partial^2)$$

Light-Cone Gauge:

$$A^+ = \frac{1}{\sqrt{2}}(A^0 + A^3) = 0$$

$$A^- = \frac{1}{\sqrt{2}}(A^0 - A^3)$$ is replaced through eqs. of motions

Physical Fields:

$$\bar{A} = \frac{1}{\sqrt{2}}(A^1 + iA^2) \qquad A = \frac{1}{\sqrt{2}}(A^1 - iA^2)$$

Four Complex Grassmann Variables

Chiral derivatives

$$d^m \;\equiv\; -\frac{\partial}{\partial\bar\theta_m} - \frac{i}{\sqrt{2}}\theta^m\partial^+\,,\quad \bar d_m \;\equiv\; \frac{\partial}{\partial\theta^m} + \frac{i}{\sqrt{2}}\bar\theta_m\partial^+\;,$$

$$\left\{d^m\,,\,\bar d_n\right\} \;=\; -\,i\sqrt{2}\delta^m{}_n\partial^+$$

The Constrained Chiral Superfield

$$\varphi\,(y) \;=\; \frac{1}{\partial^+}\,A\,(y) \,+\, \frac{i}{\sqrt{2}}\,\theta^m\,\theta^n\,\overline{C}_{mn}\,(y) \,+\, \frac{1}{12}\,\theta^m\,\theta^n\,\theta^p\,\theta^q\,\epsilon_{mnpq}\,\partial^+\,\bar{A}\,(y)$$
$$+\, \frac{i}{\partial^+}\,\theta^m\,\bar{\chi}_m(y) + \frac{\sqrt{2}}{6}\theta^m\,\theta^n\,\theta^p\,\epsilon_{mnpq}\,\chi^q(y)$$

$$y \;=\; (\,x,\,\bar{x},\,x^+,\,y^- \equiv x^- - \frac{i}{\sqrt{2}}\,\theta^m\,\bar{\theta}_m\,)$$

[Brink, Lindgren and Nilsson '82]

The Constrained Chiral Superfield

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Chiral Constraint: $\quad d^m\,\varphi\,(y) \;=\; 0$

The Constrained Chiral Superfield

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$$y \;=\; (\,x,\,\bar{x},\,x^+,\,y^- \equiv x^- - \frac{i}{\sqrt{2}}\,\theta^m\,\bar{\theta}_m\,)$$

[Brink, Lindgren and Nilsson '82]

Chiral Constraint: $$d^m\,\varphi\,(y) \;=\; 0$$

Inside-out Constraint: $$\overline{d}_m\,\overline{d}_n\,\varphi \;=\; \frac{1}{2}\,\epsilon_{mnpq}\,d^p\,d^q\,\overline{\varphi}$$

The Constrained Chiral Superfield

$$\varphi\,(y)^a \;=\; \frac{1}{\partial^+}\,A\,(y)^a + \frac{i}{\sqrt{2}}\,\theta^m\,\theta^n\,\overline{C}^{\,a}_{mn}\,(y) \;+\; \frac{1}{12}\,\theta^m\,\theta^n\,\theta^p\,\theta^q\,\epsilon_{mnpq}\,\partial^+\,\bar{A}\,(y)^a$$
$$+\;\frac{i}{\partial^+}\,\theta^m\,\bar{\chi}^a_m(y) + \frac{\sqrt{2}}{6}\theta^m\,\theta^n\,\theta^p\,\epsilon_{mnpq}\,\chi^q(y)^a$$

$$y \;=\; (\,x,\,\bar{x},\,x^+,\,y^- \equiv x^- - \frac{i}{\sqrt{2}}\,\theta^m\,\bar{\theta}_m\,)$$

[Brink, Lindgren and Nilsson '82]

Chiral Constraint: $\quad d^m\,\varphi\,(y) \;=\; 0$

Inside-out Constraint: $\quad \bar{d}_m\,\bar{d}_n\,\varphi \;=\; \frac{1}{2}\,\epsilon_{mnpq}\,d^p\,d^q\,\overline{\varphi}$

Kinematical || Dynamical

$$\text{Conformal Group} \quad \begin{cases} \text{Lorentz Group}: & J^{+-}\,, J^{+} \quad || \quad \mathcal{J}^{-} \\ \text{Translations}: & P\,, P^{+} \quad || \quad \mathcal{P}^{-} \\ \text{Dilatation}: & D \quad || \\ \text{Conformal}: & K\,, K^{+} \quad || \quad \mathcal{K}^{-} \end{cases}$$

$$\text{Supers} \quad \begin{cases} \text{Supersymmetry}: & q\,, \bar{q} \quad || \quad \mathcal{Q}\,, \overline{\mathcal{Q}} \\ \text{Superconformal}: & s\,, \bar{s} \quad || \quad \mathcal{S}\,, \overline{\mathcal{S}} \end{cases}$$

Super-Poincaré Group

Kinematical Generators

Translations, SO(2) rotations

$$p^+ \;=\; -\,i\,\partial^+\,,\qquad p \;=\; -\,i\,\partial\,,\qquad \bar{p} \;=\; -\,i\,\bar{\partial}\,,$$

$$j \;=\; x\,\bar{\partial} - \bar{x}\,\partial + \frac{1}{2}\,(\,\theta^m\,\frac{\partial}{\partial\theta^m} \;-\; \bar{\theta}_m\,\frac{\partial}{\partial\bar{\theta}_m}\,) - \lambda$$

$$j^+ \;=\; i\,x\,\partial^+\,,\qquad \bar{j}^+ \;=\; i\,\bar{x}\,\partial^+\,,\qquad j^{+-} \;=\; i\,x^-\,\partial^+ - \frac{i}{2}\,(\,\theta^p\bar{\partial}_p + \bar{\theta}_p\,\partial^p\,) + i\,.$$

Kinematical Susy

$$q^m \;=\; -\,\frac{\partial}{\partial\bar{\theta}} + \frac{i}{\sqrt{2}}\theta^m\,\partial^+ \qquad\qquad \bar{q}_m \;=\; \frac{\partial}{\partial\theta} - \frac{i}{\sqrt{2}}\bar{\theta}_m\,\partial^+$$

$$\{\,q^m\,,\,\bar{q}_n\,\} \;=\; i\sqrt{2}\,\delta^m_n\,\partial^+$$

(respect chirality) $$\{\,d^m\,,\,\bar{q}_n\,\} \;=\; \{\,q^m\,,\,\bar{d}_n\,\} \;=\; 0$$

temperature?

Kinematical Generators act linearly on the fields

$$\delta^{\text{kin}}_{\mathcal{O}}\,\varphi \;=\; \mathcal{O}\,\varphi$$

$$[\,\delta^{\text{kin}}_{\mathcal{O}}\,,\,\delta^{\text{kin}}_{\mathcal{O}'}\,]\,\varphi \;=\; \delta^{\text{kin}}_{[\mathcal{O},\mathcal{O}']}\,\varphi$$

example: kinematical Susy

$$\delta^{\text{kin}}_{s}\,\varphi \;=\; \epsilon^{m}\bar{q}_{m}\,\varphi$$

$$\delta^{\text{kin}}_{\bar{s}}\,\varphi \;=\; \bar{\epsilon}_{m}q^{m}\,\varphi$$

$$[\,\delta^{\text{kin}}_{s}\,,\,\delta^{\text{kin}}_{\bar{s}}\,]\,\varphi \;=\; \frac{i}{\sqrt{2}}\epsilon^{m}\bar{\epsilon}_{m}\partial^{+}\varphi \;=\; \frac{i}{\sqrt{2}}\,\delta^{\text{kin}}_{t^{-}}\,\varphi$$

Dynamical Generators (linear only for free theories)

Hamiltonian

$$p^- \;=\; -\,i\frac{\partial\bar\partial}{\partial^+}$$

Boosts

$$j^- \;=\; i\,x\,\frac{\partial\bar\partial}{\partial^+} \;-\; i\,x^-\,\partial \;+\; i\left(\theta^p\bar\partial_p \;-\lambda-1\right)\frac{\partial}{\partial^+}\;,$$

$$\bar j^- \;=\; i\,\bar x\,\frac{\partial\bar\partial}{\partial^+} \;-\; i\,x^-\,\bar\partial \;+\; i\left(\bar\theta_p\partial^p+\lambda-1\right)\frac{\bar\partial}{\partial^+}\;.$$

Dynamical Susy (square root of the Hamiltonian)

$$q^m_- \;\equiv\; i\,[\,\bar j^-\,,\,q^{\,m}\,] \;=\; \frac{\bar\partial}{\partial^+}\,q^{\,m}\,, \qquad \bar q_{\,-\,m} \;\equiv\; i\,[\,j^-\,,\,\bar q_{\,m}\,] \;=\; \frac{\partial}{\partial^+}\,\bar q_{\,m}$$

$$\{\,q^m_-\,,\,\bar q_{\,-\,n}\,\} \;=\; i\,\sqrt{2}\,\delta^{\,m}{}_{n}\,\frac{\partial\bar\partial}{\partial^+}$$

Interactions:
Dynamical Generators act non-linearly

$$\delta_{\mathcal{O}}^{\mathrm{dyn}}\,\varphi \;=\; \delta_{\mathcal{O}}^{\mathrm{dyn,free}}\,\varphi + \delta_{\mathcal{O}}^{\mathrm{dyn,int}}\,\varphi \;=\; \mathcal{O}\,\varphi + \mathrm{non-linear}$$

$$[\,\delta_{\mathcal{O}}^{\mathrm{kin}}\,,\,\delta_{\mathcal{O}'}^{\mathrm{dyn}}\,]\,\varphi \;=\; \delta_{[\mathcal{O},\mathcal{O}']}^{\mathrm{dyn}}\,\varphi$$

SuperConformal Kinematics

$$K^+ = 2i\, x\, \bar{x}\, \partial^+ \qquad [\, K^+ , \, p^- \,] = -2i\, D + 2i\, j^{+-}$$

SuperConformal Susy

$$[\, K^+ \, , \, \bar{q}_{-\,n} \,] = \sqrt{2}\, \bar{s}_{+\,n}$$

$$s^m_- = i\, [\, j^- \, , \, s^m_+ \,]$$

$$\{q^m \, , \, \bar{s}_{-\,n}\} = -i\delta^m_n (D + j^{+-} + ij) + 2J^m{}_n$$

SU(4) R-symmetry

$$[\, J^m{}_n \, , \, J^p{}_q \,] = i\, \delta^m{}_q\, J^p{}_n - i\, \delta^p{}_q\, J^m{}_n - i\, \delta^m{}_n\, J^p{}_q + i\, \delta^p{}_n\, J^m{}_q$$

SuperConformal Dynamics

Dynamical Susy generates ALL dynamics

$$[\,\delta_s^{\rm dyn}\,,\,\delta_{\bar s}^{\rm dyn}\,]\,\varphi^a \;\;=\;\; -\sqrt{2}\,\delta_{p^-}^{\rm dyn}\,\varphi^a$$

$$[\,\delta_{\overline{K}}^{\rm kin}\,,\,\delta_{p^-}^{\rm dyn}\,]\,\varphi^a \;\;=\;\; -\,2\,i\,\delta_{\bar{j}^-}^{\rm dyn}\,\varphi^a$$

Commutation relations determine

unique dynamical Susy

SOLE Effect of Interactions

in N=4 Super Yang-Mills:

SOLE Effect of Interactions

in N=4 Super Yang-Mills:

$$\bar{\partial}\,\delta^{ab} \quad \rightarrow \quad (\bar{\partial}\,\delta^{ab} - g f^{abc} \partial^{+} \varphi^{c}\,) \;\equiv\; \mathcal{D}^{ab}$$

Transverse Covariant Derivative

SOLE Effect of Interactions

in N=4 Super Yang-Mills:

$$\bar\partial\,\delta^{ab} \quad \rightarrow \quad (\bar\partial\,\delta^{ab} - g f^{abc}\partial^+\varphi^c\,) \;\equiv\; \mathcal{D}^{ab}$$

Transverse Covariant Derivative

$$\delta_{\bar s}^{\rm dyn}\,\varphi^a \;=\; \frac{1}{\partial^+}\,\mathcal{D}^{ab}\,\epsilon^m\,\bar q_m\,\varphi^c$$

SOLE Effect of Interactions

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$$\bar\partial\,\delta^{ab} \quad \rightarrow \quad (\bar\partial\,\delta^{ab} - g f^{abc}\partial^{+}\,\varphi^{c}\,) \;\equiv\; \mathcal{D}^{ab}$$

Transverse Covariant Derivative

$$\delta_{\bar s}^{\rm dyn}\,\varphi^{a} \;=\; \frac{1}{\partial^{+}}\,\mathcal{D}^{ab}\,\epsilon^{m}\,\bar q_{m}\,\varphi^{c}$$

Incredibly Simple!

$$\mathcal{W}^a_m \;=\; (\bar\partial\,\delta^{ab} - g f^{abc}\partial^+\,\varphi^c\,)\,\bar q_{+\,m}\,\varphi^b$$

$$\begin{aligned} H &= (\,\mathcal{W}^a\,,\,\mathcal{W}^a\,) \\ &= i\int d^4x\,d^4\overline{\theta}\,d^4\theta\,\overline{\mathcal{W}}^{a\,m}\,\frac{1}{\partial^{+\,3}}\mathcal{W}^a_m \end{aligned}$$

H=0: first order diff. equations!

$$\mathcal{W}^a_m \;=\; (\bar\partial\,\delta^{ab} - g f^{abc}\partial^+\,\varphi^c\,)\,\bar q_{+\,m}\,\varphi^b$$

Hamiltonian is a quadratic form

$$\begin{aligned} H &= (\,\mathcal{W}^a\,,\,\mathcal{W}^a\,) \\ &= i\int d^4x\,d^4\overline{\theta}\,d^4\theta\,\overline{\mathcal{W}}^{a\,m}\,\frac{1}{\partial^{+\,3}}\mathcal{W}^a_m \end{aligned}$$

H=0: first order diff. equations!

(BPS)

Apply Same Algebraic Techniques
to find the
Light-Cone Superspace Formulation
of the

D=3 Super-Conformal Theory

Bagger, Lambert, Gustavsson,
B. Nilsson

Use the N=4 Superfield

$$
\begin{aligned}
\varphi\,(y) \quad = \quad & \frac{1}{\partial^+}\,A\,(y)\;+\;\frac{i}{\sqrt{2}}\,\theta^m\,\theta^n\,\overline{C}_{mn}\,(y)\;+\;\frac{1}{12}\,\theta^m\,\theta^n\,\theta^p\,\theta^q\,\epsilon_{mnpq}\,\partial^+\,\bar{A}\,(y) \\
& +\;\frac{i}{\partial^+}\,\theta^m\,\bar{\chi}_m(y) + \frac{\sqrt{2}}{6}\theta^m\,\theta^n\,\theta^p\,\epsilon_{mnpq}\,\chi^q(y)
\end{aligned}
$$

Use the N=4 Superfield

$$
\begin{aligned}
\varphi\,(y) \;\; &= \;\; \frac{1}{\partial^+}\,A\,(y) + \frac{i}{\sqrt{2}}\,\theta^m\,\theta^n\,\overline{C}_{mn}\,(y) + \frac{1}{12}\,\theta^m\,\theta^n\,\theta^p\,\theta^q\,\epsilon_{mnpq}\,\partial^+\,\bar{A}\,(y) \\
&+ \;\frac{i}{\partial^+}\,\theta^m\,\bar{\chi}_m(y) + \frac{\sqrt{2}}{6}\theta^m\,\theta^n\,\theta^p\,\epsilon_{mnpq}\,\chi^q(y)
\end{aligned}
$$

since

OSp(2,2|8)

closes linearly (free theory)

on φ(y)

Use the N=4 Superfield

$$\varphi\,(y)^{A} = \frac{1}{\partial^+}\,A\,(y)^{A} + \frac{i}{\sqrt{2}}\,\theta^m\,\theta^n\,\overline{C}_{mn}^{\;A}\,(y) + \frac{1}{12}\,\theta^m\,\theta^n\,\theta^p\,\theta^q\,\epsilon_{mnpq}\,\partial^+\,\bar{A}\,(y)^{A}$$
$$+\ \frac{i}{\partial^+}\,\theta^m\,\bar{\chi}_m^{\,A}(y) + \frac{\sqrt{2}}{6}\theta^m\,\theta^n\,\theta^p\,\epsilon_{mnpq}\,\chi^q(y)^{A}$$

since

OSp(2,2|8)

closes linearly (free theory)

on φ(y)

(A is some label)

Use Same Tactics

$$[\,\boldsymbol{\delta}_{\epsilon\overline{\mathcal{Q}}}\,,\,\boldsymbol{\delta}_{\bar{\epsilon}\mathcal{Q}}\,]\,\varphi^a \;=\; \sqrt{2}\,\bar{\epsilon}_m\epsilon^m\,\boldsymbol{\delta}_{\mathcal{P}-}\,\varphi^a \quad\rightarrow\quad \boldsymbol{\delta}_{\mathcal{P}-}\,\varphi^a$$

$$[\,\delta_K\,,\,\boldsymbol{\delta}_{\mathcal{P}-}\,]\,\varphi^a \;=\; 2i\,\boldsymbol{\delta}_{\mathcal{J}-}\,\varphi^a \quad\rightarrow\quad \boldsymbol{\delta}_{\mathcal{J}-}\,\varphi^a$$

$$[\,\delta_K\,,\,\boldsymbol{\delta}_{\mathcal{J}-}\,]\,\varphi^a \;=-i\,\boldsymbol{\delta}_{\mathcal{K}-}\,\varphi^a \quad\rightarrow\quad \boldsymbol{\delta}_{\mathcal{K}-}\,\varphi^a$$

$$[\,\delta_K\,,\,\delta_{\epsilon\overline{\mathcal{Q}}}\,]\,\varphi^a \;=\; \sqrt{2}\,\boldsymbol{\delta}_{\epsilon\overline{\mathcal{S}}}\,\varphi^a \quad\rightarrow\quad \boldsymbol{\delta}_{\epsilon\overline{\mathcal{S}}}\,\varphi^a$$

SO(8) ⊃ SU(4)XU(1)

$$\mathrm{SO}(8) \quad \begin{cases} \mathrm{SO}(6): & J^m{}_n \\ \mathrm{U}(1): & J \\ \text{Coset } \mathrm{SO}(8)/[\mathrm{SO}(6)\times \mathrm{U}(1)]: & J^{mn}\ , J_{mn} \end{cases}$$

U(1) generator

$$J \;=\; -\,\frac{i}{4\sqrt{2}}\,(\,q^l\,\bar{q}_l \;-\; \bar{q}_l\,q^l\,)\,\frac{1}{\partial^+}$$

(was helicity)

Coset generators

$$J^{mn} \;=\; \{\,q^m_-\,,\,s^n_+\,\} \;=\; \frac{i}{\sqrt{2}}\,q^m\,q^n\,\frac{1}{\partial^+}$$

$$\bar{J}_{mn} \;=\; \{\,\bar{q}_{-m}\,,\,\bar{s}_{+n}\,\} \;=\; \frac{i}{\sqrt{2}}\,\bar{q}_m\,\bar{q}_n\,\frac{1}{\partial^+}$$

Action on the superfield

$$\delta_{U(1)}\,\varphi^a \;=\; \omega J\,\varphi^a$$

$$\delta_{coset}\,\varphi^a \;=\; \frac{i}{2\sqrt{2}}\,\omega^{mn}\bar{q}_m\,\bar{q}_n\,\frac{1}{\partial^+}\,\varphi^a$$

Constraints on Susy transformations

$$\delta^{\rm dyn}_{\bar{s}}\,\varphi^a \;=\; \frac{i}{\sqrt{2}}\frac{\partial}{\partial^+}\epsilon^m\bar{q}_m\,\varphi^a + \delta^{\rm int}_{\bar{s}}\,\varphi^a$$

$$[\,\delta_{U(1)}\,,\,\delta_s\,]\,\varphi^a \;=\; \frac{\omega}{2}\,\delta_s\varphi^a$$

SO(8) vector:

$$[\,\delta_{coset}\,,\,\delta_s\,]\,\varphi^a \;=\; \delta^{\prime}_{\bar{s}}\,\varphi^a$$

$$\epsilon^{m'} \;=\; \omega^{mn}\,\bar{\epsilon}_n$$

$$[\,\delta_{\overline{coset}}\,,\,\delta_s\,]\,\varphi^a \;=\; 0$$

Desperately Seeking Susy

$$\delta_{\bar{s}}^{\mathrm{dyn}}\,\varphi^a \;=\; \frac{i}{\sqrt{2}}\frac{\partial}{\partial^+}\epsilon^m\bar{q}_m\,\varphi^a$$

(free theory)

Desperately Seeking Susy

$$\delta_{\bar{s}}^{\mathrm{dyn}}\,\varphi^a \;=\; \frac{i}{\sqrt{2}}\frac{\partial}{\partial^+}\epsilon^m\,\bar{q}_m\,\varphi^a \qquad + \;\delta^{\mathrm{int}}_{\mathrm{susy}}\varphi^a\,?$$

(free theory)

Desperately Seeking Susy

$$\delta_{\bar{s}}^{\mathrm{dyn}}\,\varphi^a \;=\; \frac{i}{\sqrt{2}}\frac{\partial}{\partial^+}\epsilon^m\,\bar{q}_m\,\varphi^a \quad + \;\delta_{\mathrm{susy}}^{\mathrm{int}}\varphi^a\,?$$

(free theory)

In D=3 Dim φ is half-odd integer

Desperately Seeking Susy

$$\delta_{\bar{s}}^{\mathrm{dyn}}\,\varphi^a \;=\; \frac{i}{\sqrt{2}}\frac{\partial}{\partial^+}\epsilon^m\,\bar{q}_m\,\varphi^a \quad + \; \delta_{susy}^{int}\varphi^a\,?$$

(free theory)

In D=3 Dim φ is half-odd integer

$$\delta_{susy}^{int}\varphi^a \quad \sim \quad \varphi^b\varphi^c\varphi^d$$

Desperately Seeking Susy

$$\delta^{\rm dyn}_{\bar s}\,\varphi^a \;=\; \frac{i}{\sqrt{2}}\,\frac{\partial}{\partial^+}\,\epsilon^m\,\bar q_m\,\varphi^a \quad + \;\delta^{\rm int}_{\rm susy}\varphi^a\,?$$

(free theory)

In D=3 Dim φ is half-odd integer

$$\delta^{\rm int}_{\rm susy}\varphi^a \;\sim\; \varphi^b\varphi^c\varphi^d$$

$$\longrightarrow \quad f^{abcd}$$

Technology Interlude

$$\widehat{\mathcal{O}} \to \frac{1}{\partial^+}\mathcal{O}$$

coherent states

$$E_\eta = e^{\eta\cdot\hat{\bar{d}}} \qquad E_\epsilon = e^{\epsilon\cdot\hat{\bar{q}}}$$

$$E_{\bar{\epsilon}\eta} = e^{\bar{\epsilon}\cdot\hat{q}}\, E_\eta \qquad E_z = e^{z\,\hat{\partial}}$$

d-eigenstates

$$d^m\, E_\eta\, \varphi = i\sqrt{2}\,\eta^m \varphi$$

chiral engineering

two chiral fields

$$Z^{bc(\eta)} \;=\; \frac{1}{\partial^{+A}} \left\{ E_\eta \partial^{+B} \varphi^b \, E_\eta^{-1} \partial^{+C} \varphi^c \right\}$$

$$d^m \, Z^{bc(\eta)} \;=\; 0$$

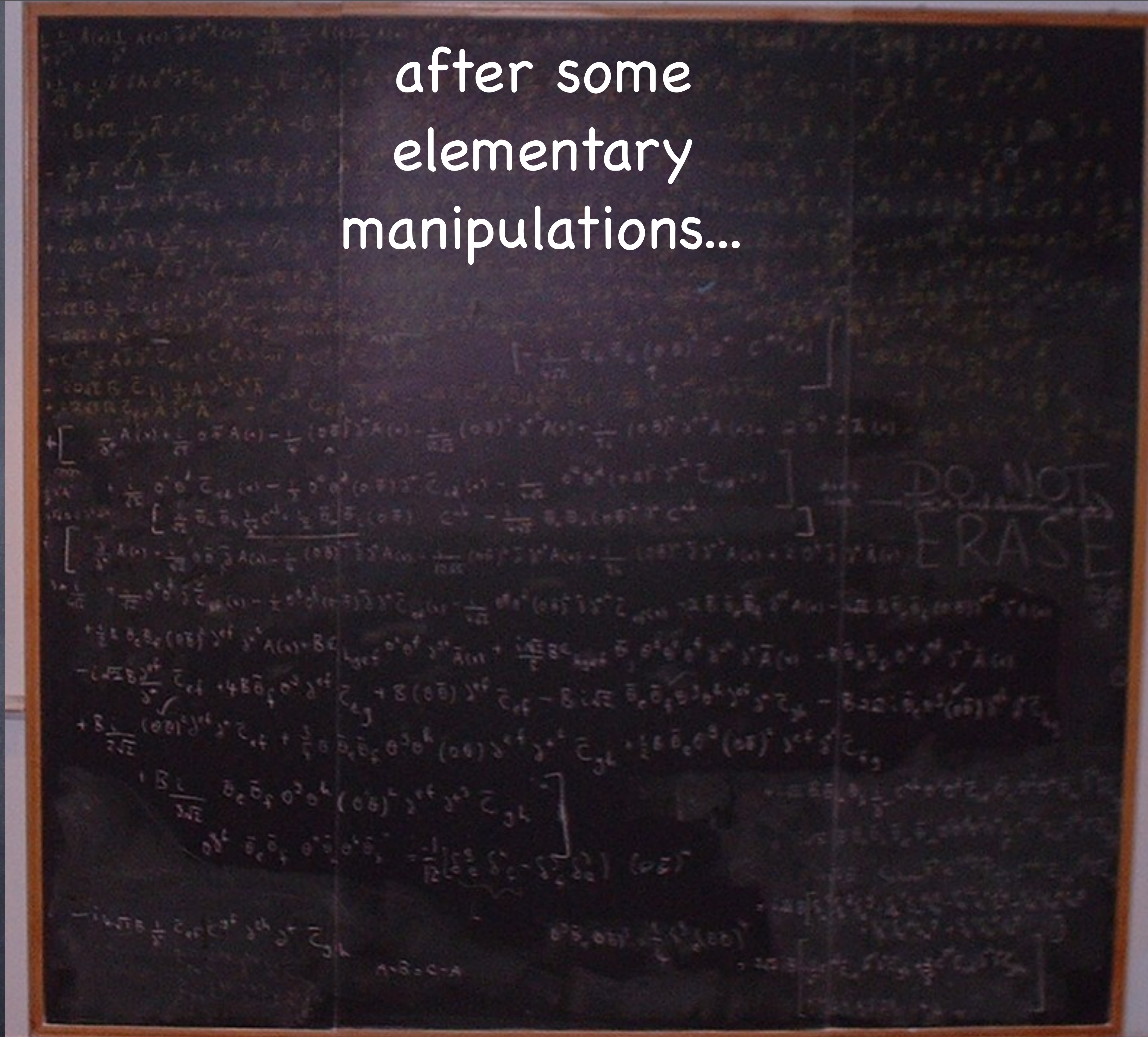
three chiral fields: nested construction

$$\Delta_\alpha^{a\,(\bar\epsilon\eta,\zeta)} \;=\; \frac{f^{abcd}}{\partial^{+A_\alpha}} \left\{ E_{\bar\epsilon\,\eta} \partial^{+B_\alpha} \varphi^b \, E_{\bar\epsilon\,\eta}^{-1} \frac{1}{\partial^{+M_\alpha}} \left[E_\zeta \partial^{+C_\alpha} \varphi^c \, E_\zeta^{-1} \partial^{+D_\alpha} \varphi^d \right] \right\}$$

$$d^m \, \Delta_\alpha^{a\,(\bar\epsilon\eta,\zeta)} \;=\; 0$$

after some
elementary
manipulations...

after some elementary manipulations...



two independent ansätze

both transform as $\underset{\sim}{8}$ of SO(8)

$$\delta_{\bar{s}}^{\text{int}}\,\varphi^a \;=\; \sum_{\alpha=1/2,-1/2} (-1)^\alpha \,\frac{\partial}{\partial\eta^{[2-2\alpha]}}\,\frac{\partial}{\partial\zeta^{[2+2\alpha]}}\,\Delta_\alpha^{a\,(\bar{\epsilon}\eta,\zeta)}\,|_{\eta=\zeta=0}$$

triality

and/or

$$\sum_{\alpha=-1,0,1} (-1)^\alpha \,\frac{\partial}{\partial\eta^{[2-2\alpha]}}\,\frac{\partial}{\partial\zeta^{[2+2\alpha]}}\,\Delta_\alpha^{a\,(\bar{\epsilon}\eta,\zeta)}\,|_{\eta=\zeta=0}$$

$$\frac{\partial}{\partial\eta^{[2-2\alpha]}}\,\frac{\partial}{\partial\zeta^{[2+2\alpha]}} \;\equiv\; \frac{\epsilon^{i_1\cdots i_{2-2\alpha}\cdots i_4}}{(2+2\alpha)!(2-2\alpha)!}\,\frac{\partial}{\partial\eta^{i_1\cdots i_{2-2\alpha}}}\,\frac{\partial}{\partial\zeta^{i_{3-2\alpha}\cdots i_4}}$$

$$A_{\alpha-1} = A_\alpha + 1 \qquad B_{\alpha-1} = B_\alpha + 1 \qquad M_{\alpha-1} = M_\alpha - 2$$

$$C_{\alpha-1} = C_\alpha - 1 \qquad D_{\alpha-1} = D_\alpha - 1$$

Light-Cone Hamiltonian

$$[\,\delta_s^{\rm free} + \delta_s^{\rm int}\,,\,\delta_{\bar s}^{\rm free} + \delta_{\bar s}^{\rm int}\,]\varphi^a \;=\; \delta_{p^-}^{\rm dyn}\,\varphi^a$$

terms linear in f^{abcd}

$$[\,\delta_s^{\rm free}\,,\,\delta_{\bar s}^{\rm int}\,]\varphi^a + [\,\delta_s^{\rm int}\,,\,\delta_{\bar s}^{\rm free}\,]\varphi^a$$

$$\delta_{p^-}\,\varphi^a \;=$$

$$i\bar{\epsilon}\cdot\epsilon\frac{\partial}{\partial z}\left\{\sum_{\alpha=\frac{1}{2},-\frac{1}{2}}(-1)^\alpha\,\frac{\partial}{\partial\eta^{[2-2\alpha]}}\,\frac{\partial}{\partial\zeta^{[2+2\alpha]}}\Delta_\alpha^{a\,(z\eta,\zeta)}+\sum_{\alpha=-1,0,1}(-1)^\alpha\,\frac{\partial}{\partial\eta^{[2-2\alpha]}}\,\frac{\partial}{\partial\zeta^{[2+2\alpha]}}\Delta_{\frac{1}{2}+\alpha}^{a\,(\eta,z\zeta)}\right\}_{z=\eta=\zeta=0}$$

$$E_z \;=\; e^{\,z\,\hat{\partial}} \qquad\qquad E_{\,z\eta}\;=E_zE_\eta$$

Conformal generator

$$K \;=\; -\,2i\,x\,(\frac{1}{2}\,x\,\partial-x^-\,\partial^+ +\theta^m\,\frac{\partial}{\partial\theta^m}+\bar{\theta}_m\,\frac{\partial}{\partial\bar{\theta}_m})$$

From Hamiltonian to Boost

$$[\,K\,,\,p^-\,]\;=\;-\,2i\,j^-$$

$$[\,j^-\,,\,p^-\,]\;=\;0$$

$$\delta_{p^-}\,\varphi^a \;=$$

$$i\bar{\epsilon}\cdot\epsilon\frac{\partial}{\partial z}\left\{\sum_{\alpha=\frac{1}{2},-\frac{1}{2}}(-1)^\alpha\,\frac{\partial}{\partial\eta^{[2-2\alpha]}}\,\frac{\partial}{\partial\zeta^{[2+2\alpha]}}\Delta_\alpha^{a\,(z\eta,\zeta)}+\sum_{\alpha=-1,0,1}(-1)^\alpha\,\frac{\partial}{\partial\eta^{[2-2\alpha]}}\,\frac{\partial}{\partial\zeta^{[2+2\alpha]}}\Delta_{\frac{1}{2}+\alpha}^{a\,(\eta,z\zeta)}\right\}_{z=\eta=\zeta=0}$$

$$E_z \;=\; e^{\,z\,\hat{\partial}} \qquad\qquad E_{\,z\eta} \;= E_z E_\eta$$

Conformal generator

$$K \;=\; -\,2i\,x\,(\frac{1}{2}\,x\,\partial - x^-\,\partial^+ + \theta^m\,\frac{\partial}{\partial\theta^m} + \bar{\theta}_m\,\frac{\partial}{\partial\bar{\theta}_m})$$

From Hamiltonian to Boost

$$[\,K\,,\,p^-\,]\;=\;-\,2i\,j^-$$

$$[\,j^-\,,\,p^-\,]\;=\;0$$

square roots? $(\partial^+)^{1/2}$

Work in Progress

(with Belyaev, Brink and S-S Kim)

Check boost and Hamiltonian commute

Compute f-f term

Properties of f^{abcd} from commutation

Continue Program to d=5,6

stay tuned...

stay tuned...

Bon Anniversaire

Михаил Аркадьевич

N=8 Light-Cone Superspace

houses

D=11: N=1 SuperGravity SO(9); $F_4/SO(9)$

D=4: N=8 SuperGravity SO(2)x $E_{7(7)}$

$\mathbf{1}_{1}$ $\mathbf{4}_{1/2}$ $\mathbf{6}_{0}$ $\mathbf{4}_{-1/2}$ $\mathbf{1}_{-1}$

D=3: N=16 SuperGravity $E_{8(8)}$

D=2: N=16 Theory $E_{9(9)}$