Comments on susy gauge theories

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Great to be here, thank you Arkady and Happy Birthday Misha!

M theory

(explore one little boundary)

susy
multiplet
of anomalies

Instanton
Calculus
..4/5…

solitons

Sum rules

penguins!

Outline

- Review: multiplet of anomalies, a-maximization, a conjectured diagnostic.
- Report on somewhat related recent things:

  Shapere and Tachikawa: death report of Cardy's a-theorem conjecture.

  Hofman and Maldacena: a/c inequalities.
- Comments on sign of a-c.

Susy multiplet of anomalies and the NSVZ beta function

$$\bar{D}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} = D_\alpha X, \quad \mathcal{J}_\mu = J^R_\mu + \ldots + (\bar{\theta}\theta)^\nu T_{\mu\nu} + \ldots$$

$$T^\mu_\mu + i\partial^\mu j^R_\mu = X|_{\theta^2}$$

$$\Delta(Q_i) \equiv 1 + \frac{1}{2}\gamma_i(g) = \frac{3}{2}R(Q_i)$$

Dimension of chiral fields ~ their running R-charges.

Exact beta functions ~ linear combinations of R-charges of fields:

$$\beta_g^{NSVZ} \sim -g^3 \mathrm{Tr} R G^2$$

$U(1)_R$ ABJ anomaly.

$$W = h\mathcal{O} \qquad \beta_h = \frac{3}{2}h(R(\mathcal{O}) - 2)$$

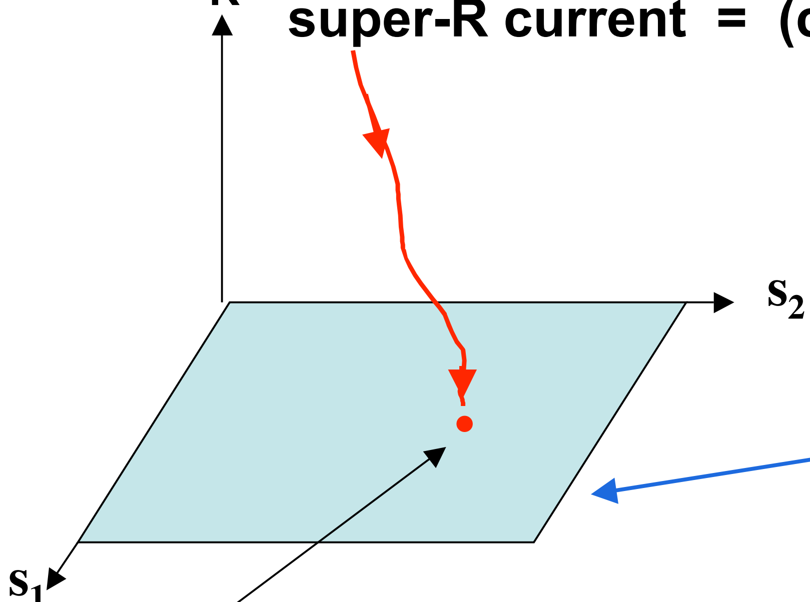
W's R-violation.

RG flowing R-charges

violated J_R

super-R current = (conserved part) + (violated part)

Picks up anom. dim. irrelevant in IR.



Space of all conserved R-symmetries

$$R = R_0 + \sum_I s_I F_I$$

Superconformal R symmetry of SCFT:

$$R_* = R_0 + \sum_I s_I^* F_I$$

Determine via a-maximization

KI and B. Wecht

a-maximization

$$T^{\mu}_{\mu} + i\partial_\mu J^\mu_R = X|_{\theta^2}$$ Curved background

$$X \supset \widehat{\beta}_{NSVZ}(R_i) W^2 + c\mathcal{W}^2 - a\Xi$$ Euler

Weyl^2

$$a = \frac{3}{32}(3\mathrm{Tr}R^3 - \mathrm{Tr}R)$$

$$c = \frac{1}{32}(9\mathrm{Tr}R^3 - 5\mathrm{Tr}R)$$

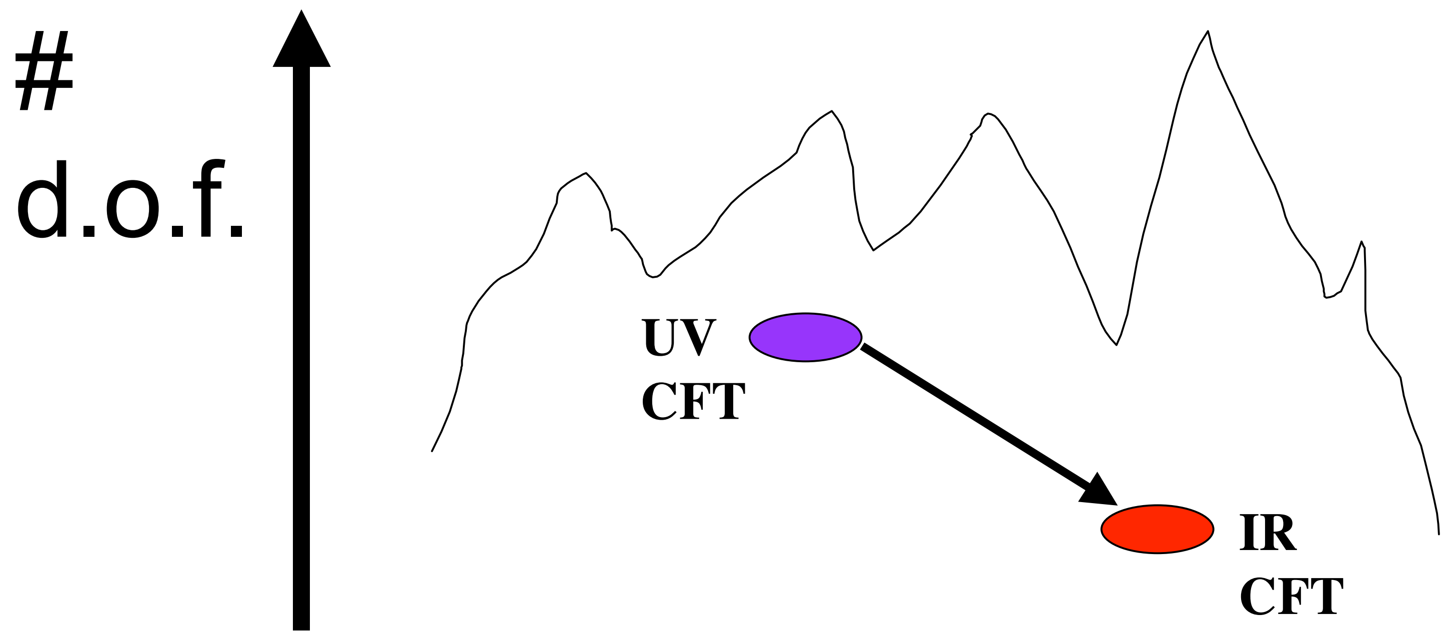
Anselmi, Freedman, Grisaru, Johansen.

Now vary $R \to R + \epsilon F, \quad X \to X + \epsilon \bar{D}^2 J_F,$ find

$$\bar{D}^2 J_F = k_{FFF} W_F^2 + k_F \mathcal{W}^2 \qquad a \to a$$

The correct R-symmetry locally maximizes a. KI, Wecht

Recall *a* in **Cardy conjecture** '88:



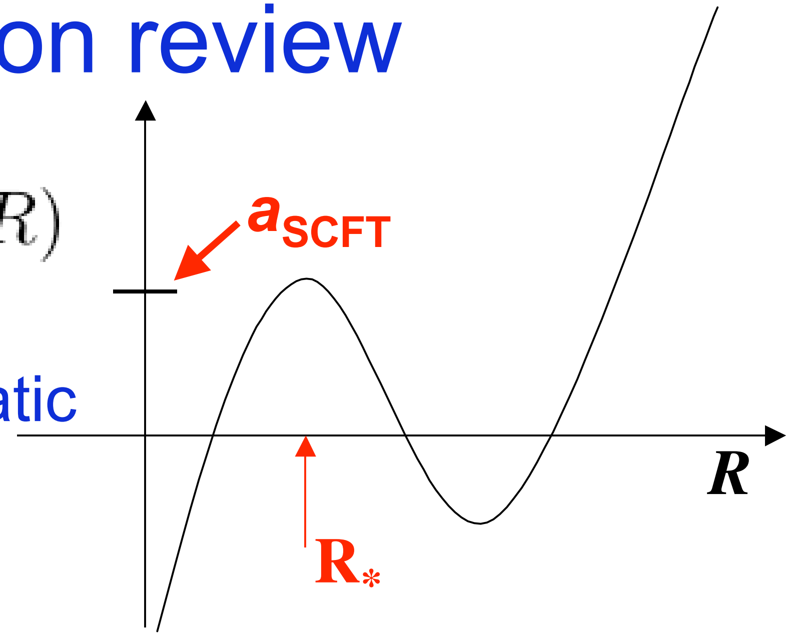
" # d.o.f. ~ ***a***"   So conjecture

$a_{UV} > a_{IR}$ for all (unitary) RG flows.

Conjectured 4d analog of Zamolodchikov's c-theorem.

a-maximization review

$$a_{trial}(R) = \frac{3}{32}(3TrR^3 - TrR)$$



Exact R-charges = roots of quadratic eqns with integer coeffs. So R_i and a and c, are necessarily **Algebraic numbers.**
Can't continuously vary.
a- max also "almost proves" the Cardy conjecture, modulo possible violations by unseen accidental symmetries (accidental symmetries required by unitarity bound are OK).

Also, can define $a(R, g_I)$ along RG flow, with a-gradients $\partial_{g_I} a(R, g_I) \sim \beta_I$ ~ exact beta functions (e.g. $\widehat{\beta}_{NSVZ}$).
Kutasov; Barnes, KI, Wecht, Wright; Kutasov, Schwimmer

IR free or interacting? Proposed diagnostic

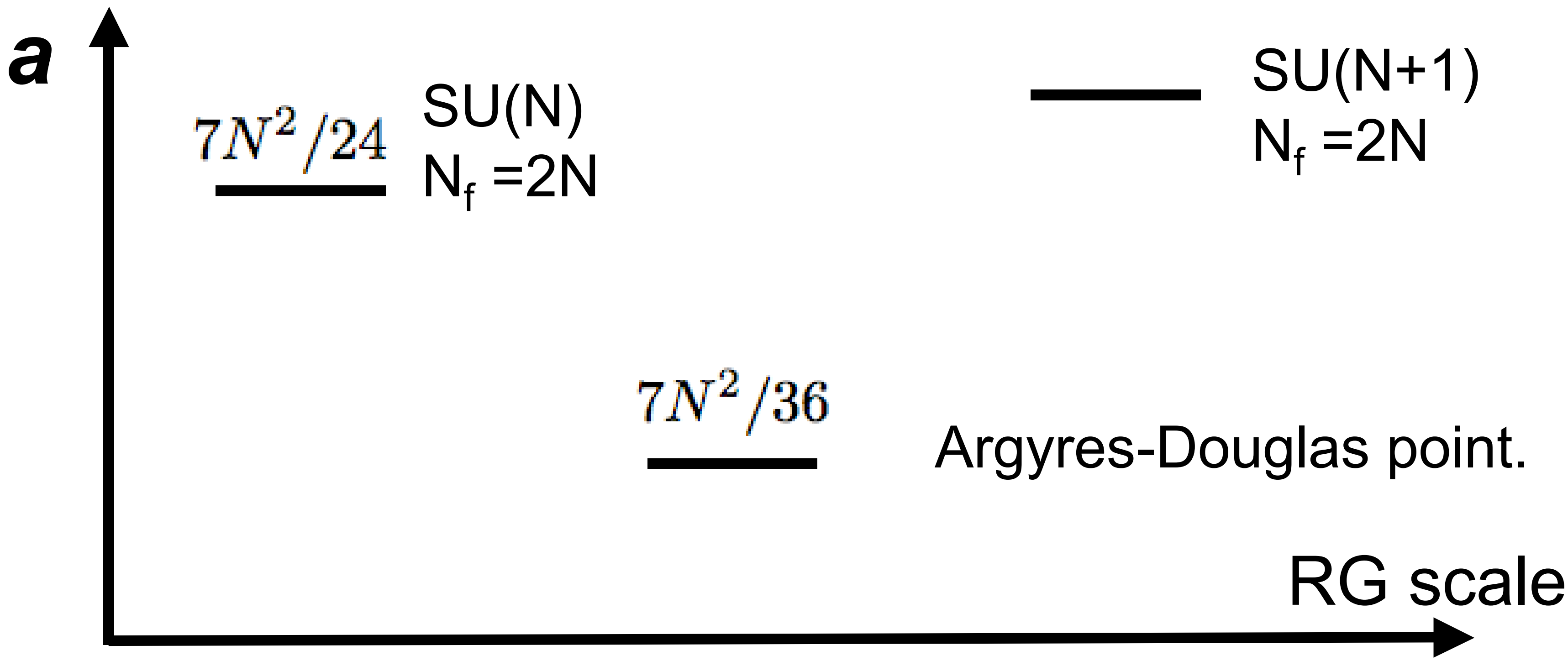
How to know if correct phase of theory is IR free, like pions with chiral symmetry breaking, or interacting, as with the conformal window? Conjecture: correct phase has the larger a_{IR}. Intuition: "Shorter RG flow is correct."

For susy theories, this conjecture implies that only chiral operators with $R(X)<5/3$ can become IR free.

Works in all understood examples. For SU(2) with $Q_{I=3/2}$ it suggests a CFT rather than an IR free theory. (If IR free, gives a simple model of DSB; if CFT it does not - KI, Seiberg, Shenker). Recent work of Poppitz and Unsal (last talk!) also suggests a 4d interacting CFT phase.

Shapere and Tachikawa '08: "Cardy's conjecture is dead."

Claimed counter-example based on strongly coupled N=2 SCFTs. Doesn't contradict a-maximization since R-symm is completely accidental. Compute ***a, c*** using TQFT results.



Hofman and Maldacena '08
Conformal collider physics

Consider correlation functions of energy flux ops

$$\mathcal{E}(\theta) = \int dt\; r^2 n^i T^0_i(t, r\vec{n})|_{r\to\infty}$$

Conjecture they're always positive. They show this implies

$$\frac{31}{18} \geq \frac{a}{c} \geq \frac{1}{3}, \qquad \boxed{\frac{3}{2} \geq \frac{a}{c} \geq \frac{1}{2},} \qquad \frac{5}{4} \geq \frac{a}{c} \geq \frac{1}{2}.$$

N=0 **N=1** **N=2**

Upper limit: **free** vector. Lower limit: **free** matter.

Hofman and Maldacena '08
Conformal collider physics, cont.

$$\langle J_R\cdot\epsilon|\mathcal{E}(\widehat{n})|J_R\cdot\epsilon\rangle = 1 + 3\frac{c-a}{c}(\cos^2\theta - \frac{1}{3}) \geq 0$$

$$\langle T\cdot\epsilon|\mathcal{E}(\widehat{n})|T\cdot\epsilon\rangle = 1 + 6\frac{c-a}{c}(\frac{\epsilon^*_{ij}\epsilon_{il}n_j n_l}{\epsilon^*_{ij}\epsilon_{ij}} - \frac{1}{3}) \geq 0$$

(We'll soon discuss the sign of *c*-a. Note the energy flux is peaked parallel to the polarization of the state if *c-a* >0, whereas for *c-a* < 0 it is peaked in the orthogonal direction.)

Stronger a/c bounds for **interacting** theories?

No **interacting** 4d CFTs near free matter or gauge fields.
Need mixture of gauge fields and matter; conformal window.

E.g. $\mathcal{N}$=1 Banks-Zaks type ($g_* << 1$) CFTs, with large N_c:

$$3T_2(G) \approx T_2(r) \qquad\qquad \frac{a}{c} \approx 1 - \frac{|r| - 3|G|}{6|G| + 2|r|} \le 1$$

Of large N_c reps, adjoints minimize $|r|/|G|$ for fixed $T_2(r)/T_2(G)$.

E.g. SQCD with $N_f = 3N_c$ $\quad \frac{a}{c} \approx \frac{5}{6}$.

Large N_c susy Banks-Zaks fixed points have $\frac{a}{c} \le 1.$

Aside on AdS/CFT and a/c

$S_{E.H.}$ in AdS leads to a=c. Henningson and Skenderis.

Get differing *a* and *c* by adding R^2 terms. Then find

$$\frac{\eta}{s} = \frac{1}{4\pi}(1 - \frac{c-a}{c} + \ldots)$$

Kats, Petrov; Buchel, Myers, Sinha, etc.

Kovtun, Son, Starinets conjectured bound suggests perhaps *a* > *c*. In fact, the opposite, *c* > *a* , is typical for interacting thys!

(Brigante, Liu, Myers, Shenker, Yaida microcausality bound Is equivalent to the Hofman-Maldacena lower bound on *a/c*)

Susy *a/c*, via the superconformal $U(1)_R$

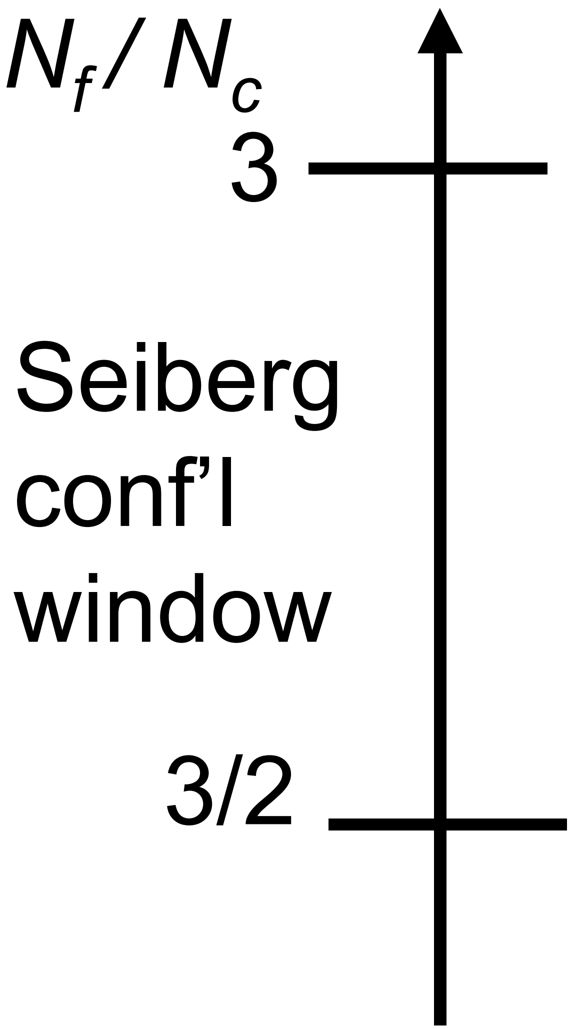
H.M. Upper bound (free vector) $\frac{3}{2} \geq \frac{a}{c} \quad \rightarrow \quad \mathrm{Tr} R^3 \geq \mathrm{Tr} R$ *

H.M. Lower bound (free matter) $\frac{a}{c} \geq \frac{1}{2} \quad \rightarrow \quad \mathrm{Tr} R^3 \geq \frac{1}{9} \mathrm{Tr} R$ **

$a - c = \frac{1}{16} \mathrm{Tr} R$ Typically negative for SCFTs.

In fact, using the results from a-maximization, the ineq. * is (more-or-less) clearly satisfied, whereas ** is not obvious. For negative Tr R, this fits with the fact that the lower bound is stronger. Positive Tr R would obviate the lower bound.

a/c for SQCD



$a/c = 5/6$

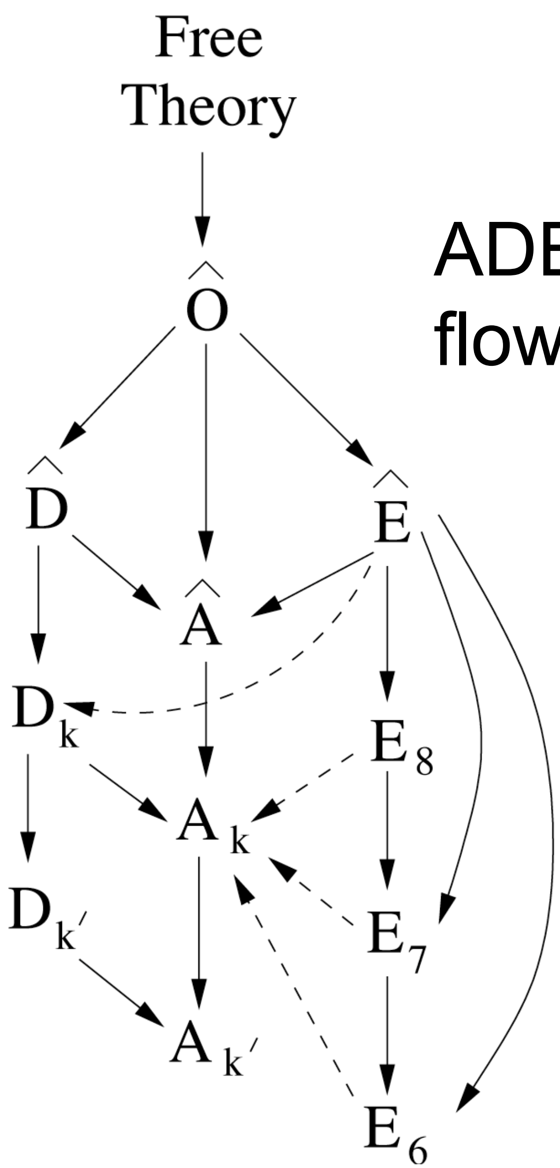
decreasing

$a/c = 2/3$

$a/c < 3/2$ comfortably satisfied. $a/c < 1$.

Free magnetic before $a/c > 1/2$ is violated. (Sum the IR free mag. matter and glue contributions then.)

a/c for generalizations of SQCD, with adjoints



ADE classification of SCFTs and RG flows (with $a_{IR} < a_{UV}$ for all), KI & Wecht.

Parnachev and Razamat recently verified that all SCFTs satisfy the H.M. bounds $1/2 < a/c < 3/2$.

In fact, all observed to have

$1/2 < a/c < 1$. $\longleftrightarrow$ $\mathrm{Tr}\, R < 0$

Can we find any *a/c*>1 examples??

Yes, it seems, but only in examples where we're not 100% certain…

SO(N) with symmetric tensor S. ‘t Hooft anomalies suggest it is IR free (*a/c*=1/2) . Misleading; actually Interacting (Brodie, Cho, KI). Assuming minimal accidental symmetries gives *a/c*=381/362=1.05249

$$\mathrm{TrR} \to \mathrm{TrR} + \sum_{free\ \mathcal{O}} (2/3 - R(\mathcal{O}))$$

Increases *a-c* for minimal accidental symmetries, decreases *a-c* if unseen accidental symmetries….

Last example: SU(2) with $Q_{I=3/2}$

$R(Q)=3/5.$ $R(X=Q^4)=12/5.$

t' Hooft anomalies: $\mathrm{Tr}\, R = 7/5$, $\mathrm{Tr}\, R^3 = (7/5)^3$

Matching, suggests IR free. (Or conformal, and it's a misleading fluke.) KI, Seiberg, Shenker.

If IR free, $a/c=1/2$ (lower limit of HM bound).

If CFT, $a/c = 183/158 =$ **1.16**. Perhaps the record for the **largest *a/c* in a fully interacting (?...) CFT**.

Conclude

Happy Birthday Misha!