Probing Universality in AdS/CFT

Adam Ritz

University of Victoria

with P. Kovtun [0801.2785, 0806.0110]
and J. Ward [0811.4195]

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Happy Birthday

Misha!

Oops...some fine print:

- this talk contains no strong gauge dynamics.........just the weakly-coupled dual
- this talk crosses no (phase) boundaries.........just sticks to the critical endpoint!

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Are all fixed points alike?

Which d-dimensional fixed points behave like classical GR in AdS$_{d+1}$?

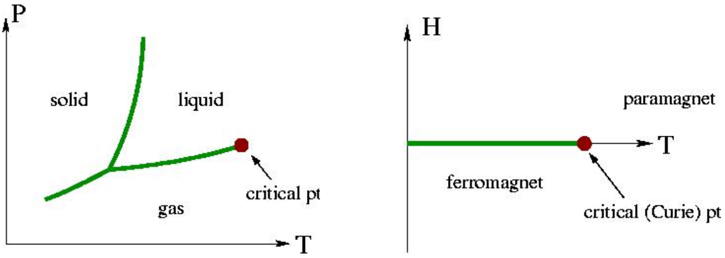
For example, when is AdS$_4$/CFT$_3$ most applicable to phenomena in condensed matter physics [Herzog et al. '07, Hartnoll et al. '07,...]

- There are strongly-interacting near-critical regimes for which AdS/CFT may provide a novel (and unique) toolkit
- Reverse engineering may allow CM physics to unlock some generic aspects of gravity, and the AdS/CFT correspondence

Static Universality

Static universality class criteria

- spatial dimension
- symmetry of the order paramater

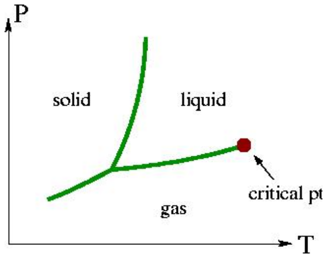


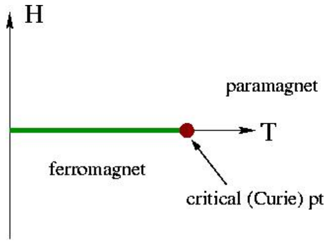
Both critical points are described by the same Ising CFT in d=3.

Dynamic Universality

Dynamic universality class criteria [Hohenberg & Halperin '77]

- spatial dimension
- symmetry of the order parameter
- conserved currents, ...

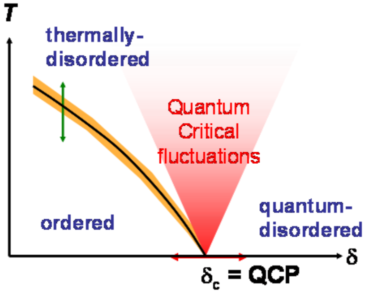




The dynamic universality classes of these critical points are different!

Quantum Critical Universality

When are these notions of universality “the same”?



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NB: this implies taking a single dominant scale (temperature)

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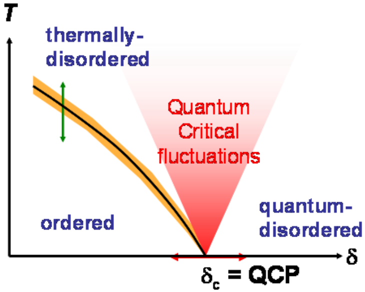

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Outline

1. Thermodynamics from central charges
2. Hydrodynamics from central charges
3. Conclusions

Outline

Vacuum CFTs and central charges

For a CFT vacuum state, symmetry determines the correlators of conserved currents:

$$\langle T_{\mu\nu}(x)T_{\alpha\beta}(0)\rangle = \frac{c}{x^{2d}}\Pi_{\mu\nu\alpha\beta}$$

- c – "measures" the total degrees of freedom

$$\langle J_\mu(x)J_\nu(0)\rangle = \frac{k}{x^{2(d-1)}}\Pi_{\mu\nu}$$

- k – "measures" the charged degrees of freedom

Thermal CFTs

For a CFT, with temperature the only scale:

$$p(T) = \frac{T}{V} \ln \mathrm{Tr}\left[e^{-\beta(H-\mu Q)}\right]\Big|_{\mu\to 0} = c' T^d + \frac{1}{2} k' \mu^2 T^{d-2} + \cdots$$

- c' measures entropy density $s = c' d T^{d-1}$
- k' measures charge susceptibility $\chi = k' T^{d-2}$

Claim

- In $d = 2$, conformal symmetry relates the vacuum and thermal states [Bloete et al. '86, Affleck et al. '86]:

$$d = 2 \qquad \longrightarrow \qquad c' = \frac{\pi}{6} c \qquad k' = \frac{1}{2\pi} k$$

so thermodynamics is uniquely fixed by the central charges.

- In $d > 2$, conformal symmetry does not imply this constraint

BUT: it is true in general for CFTs with gravity duals

$$d = 3 \qquad \longrightarrow \qquad c' = \frac{\pi^3}{162} c \qquad k' = \frac{\pi}{24} k$$

$$d = 4 \qquad \longrightarrow \qquad c' = \frac{\pi^2}{80} c \qquad k' = \frac{1}{12} k$$

AdS gravity dual

The dual bulk geometry describing the CFT state, and the fluctuations which determine correlators of the conserved currents, follow at leading order in EFT from the Einstein-Maxwell action:

$$S = \frac{1}{16\pi G_{d+1}} \int \mathrm{d}^{d+1}x \sqrt{-g} \left[R + \frac{d(d-1)}{L^2} \right] - \frac{1}{4g_{d+1}^2} \int \mathrm{d}^{d+1}x \sqrt{-g}\, F^2 + \cdots$$

where G_{d+1} and g_{d+1}^2 encode details of the CFT.

Vacuum state $\leftrightarrow$ AdS$_{d+1}$: $ds^2 = z^{-2}(dz^2 + d\mathbf{x}^2)$

Large volume thermal state $\leftrightarrow$ RNAdS$_{d+1}$ BH: $T = T_{\mathrm{BH}}$, $\mu = A_t|_{z\to 0}$

Central charges and thermodynamics

- Perturbing the vacuum state, we obtain the correlators:

$$\langle T_{xy}(x_1)T_{xy}(x_2)\rangle \sim \frac{\partial^2 S^{\text{on-shell}}}{\partial h^{xy}(x_1)h^{xy}(x_2)} \quad \Longrightarrow \quad c = \text{const}(d) \times \frac{L^{d-1}}{G_{d+1}}$$

$$\langle J_x(x_1)J_x(x_2)\rangle \sim \frac{\partial^2 S^{\text{on-shell}}}{\partial A^x(x_1)A^x(x_2)} \quad \Longrightarrow \quad k = \text{const}(d) \times \frac{L^{d-3}}{g^2_{d+1}}$$

- From the thermal state dual to the black hole (with $\mu \to 0$):

$$s = \frac{A_{D-2}}{4G_{d+1}V} = c' d T^{d-1} \quad \Longrightarrow \quad c' = \text{const}(d)\frac{L^{d-1}}{G_{d+1}}$$

$$\chi|_{\mu=0} = \frac{\rho}{\mu} = k' T^{d-2} \quad \Longrightarrow \quad k' = \text{const}(d)\frac{L^{d-3}}{g^2_{d+1}}$$

Thermodynamics from central charges

Central charges (c, k) determine (s, χ) in CFTs with AdS gravity duals

$$\begin{aligned} \frac{c'}{c} &= \frac{1}{4\pi^{d/2}} \left(\frac{4\pi}{d}\right)^{d} \frac{\Gamma(d/2)^3}{\Gamma(d)} \frac{(d-1)}{d(d+1)} \\ \frac{k'}{k} &= \frac{1}{2\pi^{d/2}} \left(\frac{4\pi}{d}\right)^{d-2} \frac{\Gamma(d/2)^3}{\Gamma(d)} \end{aligned}$$

In the most relevant cases of $d = 3, 4$:

$$\begin{array}{llll} d = 3 & \longrightarrow & c' = \dfrac{\pi^3}{162} c & k' = \dfrac{\pi}{24} k \\ d = 4 & \longrightarrow & c' = \dfrac{\pi^2}{80} c & k' = \dfrac{1}{12} k \end{array}$$

Outline

Hydrodynamics

Assuming hydrodynamics controls the IR dynamics near the critical point:

$$\partial_\mu T^{\mu\nu} = \partial_\mu J^\mu = 0,$$

where (for a conformal fluid)

$$\begin{aligned} T^{\mu\nu} &= \frac{\epsilon}{3}(4u^\mu u^\nu + \eta^{\mu\nu}) - 2\,\eta\,\partial_{\langle\mu}u_{\nu\rangle} + \cdots \\ J^\mu &= \rho u^\mu - D\,\Pi^\mu_\nu \partial_\nu \rho + \cdots, \end{aligned}$$

- $D = \sigma/\chi$ is the diffusion const
- η is the shear viscosity

[NB: Strictly we require $J_\mu \to e J_\mu$ so σ and χ are implicitly proportional to e^2]

Transport from thermodynamics

Using Kubo relations, and the AdS/CFT prescription [Policastro et al '02]:

$$\eta = -\lim_{\omega\to 0} \mathrm{Im}\,\frac{1}{\omega}\langle T_{xy}T_{xy}\rangle(\omega,\mathbf{0})_{\mathrm{R}} = \frac{1}{16\pi G_{d+1}}\left(\frac{4\pi L}{d}\right)^{d-1} T^{d-1}$$

$$\sigma = -\lim_{\omega\to 0} \mathrm{Im}\,\frac{1}{\omega}\langle J_x J_x\rangle(\omega,\mathbf{0})_{\mathrm{R}} = \frac{1}{g_{d+1}^2}\left(\frac{4\pi L}{d}\right)^{d-3} T^{d-3}\,.$$

It follows that CFTs with gravity duals have transport coefficients determined by thermodynamics!

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}\,, \qquad \frac{\sigma}{\chi} = D = \frac{1}{4\pi T}\frac{d}{d-2}\,.}$$

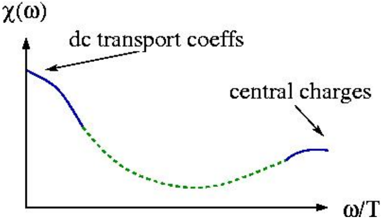
Transport from central charges

Putting the pieces together [Kovtun & AR '08] :

$$\sigma = \frac{\chi}{4\pi T}\frac{d}{d-2} = \left[\frac{1}{8\pi^{d/2+1}}\frac{d}{d-2}\left(\frac{4\pi}{d}\right)^{d-2}\frac{\Gamma(d/2)^3}{\Gamma(d)}\right] kT^{d-3}.$$

$$\eta = \frac{s}{4\pi} = \left[\frac{1}{16\pi^{d/2+1}}\frac{d-1}{d+1}\left(\frac{4\pi}{d}\right)^{d}\frac{\Gamma(d/2)^3}{\Gamma(d)}\right] cT^{d-1}.$$

General relations, independent of field content, symmetries, etc!



The two limits $\omega/T \to 0$ and $\omega/T \to \infty$ are generally determined by different physics [Sachdev '99]

Central charges and symmetry breaking

A corrollary (cf. weak gavity conjecture [Arkani-Hamed et al '06])

$$\frac{\eta}{\sigma} = \left[\frac{8\pi^2(d-1)(d-2)}{d^3(d+1)}\right] \frac{c}{k}\, T^2 \propto \frac{g_{d+1}^2}{G_{d+1}}$$

The ratio c/k also determines when the U(1) symmetry is unstable to condensation forming a superfluid (or superconducting) phase.
In $d = 3$, the instability occurs when [Denef & Hartnoll '09]:

$$q^2 \geqslant (3 + 2\Delta(\Delta - 3))\frac{k}{c}$$

with q (Δ) the charge (dimension) of the operator breaking U(1).

Comparisons

- *d=4*: The AdS value is possible if $2n_s + n_f = 8n_v$, and in general:

$$\frac{3}{8} \leqslant \frac{c/c'|_{\text{free}}}{c/c'|_{\text{AdS}}} \leqslant \frac{9}{4}.$$

- *d=3*: In this case $(c/c')^{\text{free}} \propto 1/\zeta(3)$ so the AdS value is not possible at weak coupling.

  For the O(N) model at large N [Sachdev '93, Chubukov et al '94]

$$\frac{c'/c|_{\text{O(N)}}}{c'/c|_{\text{AdS}}} \approx 1.07, \qquad \frac{k'/k|_{\text{O(N)}}}{k'/k|_{\text{AdS}}} \approx 1.31, \qquad \text{BUT} \quad \sigma/\chi|_{\text{O(N)}} \sim \mathcal{O}(N).$$

  [Cf. The proposed higher-spin AdS dual [Klebanov & Polyakov '02]]

Outline

Conclusions

- CFTs with AdS (gravity) duals are a precise subset with linked vacuum, thermodynamic, and hydrodynamic properties
- Are there physical quantum critical regimes with these features?

Bulk higher dimension operators generically correct these universal relations:

- $R^{n>1}$ terms correct η/s [Buchel et al '04; Brigante et al '07, Kats and Petrov '07]
- RF^2 corrects σ/χ [AR & Ward '08]
- Can symmetries protect these relations beyond the classical gravity (large N, λ) limit? Are some regimes in the swampland?

Are there other universal relations of this type, associated with perturbing by other "control" paramaters, or for NR quantum critical pts?

- finite chemical potential or magnetic field (and zero T) - extremal BHs
- the confinement or χSB scales?