The Many Faces of the Heterotic Vortex

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CROSSING THE BOUNDARIES --- May 14, 2009

Plan of the Talk

- Intro on non-Abelian Vortex
- $\mathcal{N}=2$ & $\mathcal{N}=1$ in 3+1
- Results of arxiv/0903.3422
- Work in progress with M.S.

  & M.S. & M.V. arxiv/0905.1664

What is the non-Abelian vortex

BASIC INGREDIENTS

- $U(n) = \dfrac{SU(n) \times U(1)}{\mathbb{Z}_n}$

- $n_f \geq n_c$ quarks

- CF phase:

$$\langle q \rangle = \begin{pmatrix} \sqrt{\xi} & & 0 \\ & \ddots & \\ & & \sqrt{\xi} \end{pmatrix}$$

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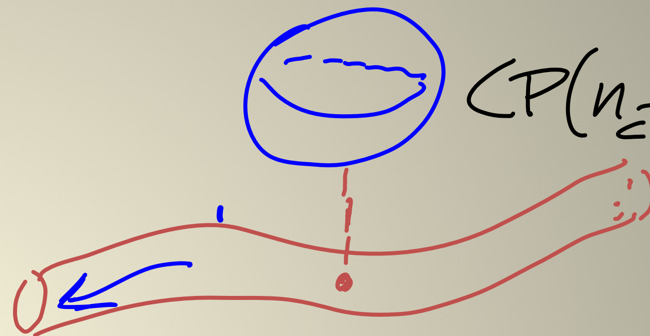


$CP(n_c-1)$

Generalization of ordinary ANO

- $SO(n)$ mag flux
- Moduli Space

Why bother about it …

* Is there . . .
* Confines non-Abelian monopoles
* Squeezing 3+1 ⟶ 1+1
* QCD dual superconductor
.
.
.

The theory: N=2 Super-QCD

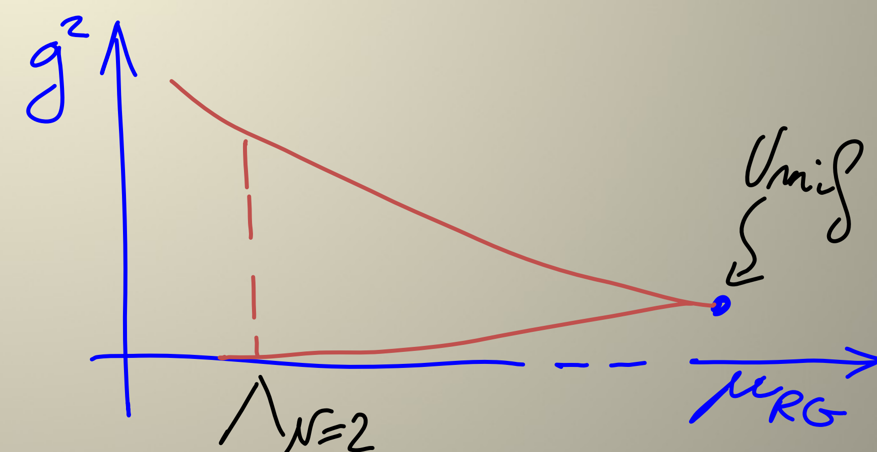
$U(n_c)$ gauge

$$\tau = \frac{4\pi i}{g^2} + \frac{\theta}{2\pi}$$

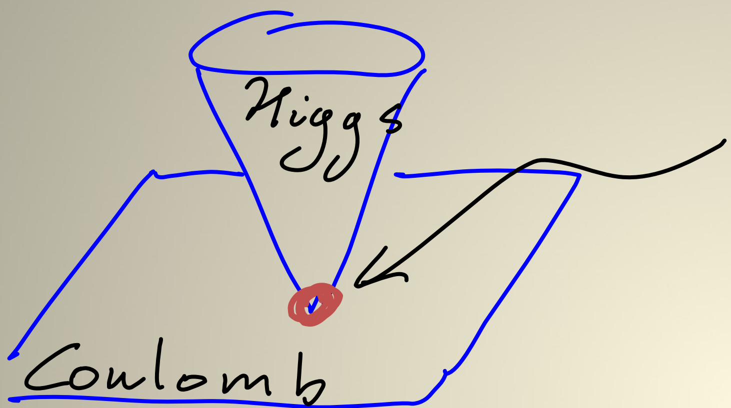
$\lambda \quad A_\mu \quad \psi \quad \phi$

$+m_f \quad q \quad \psi_q \quad \tilde{q} \quad \psi_{\tilde{q}}^{+}$

$$\Lambda_{N=2}^{2n_c-n_f} = \mu_{RG}^{2n_c-n_f}\, e^{-2\pi i \tau(\mu_{RG})}$$



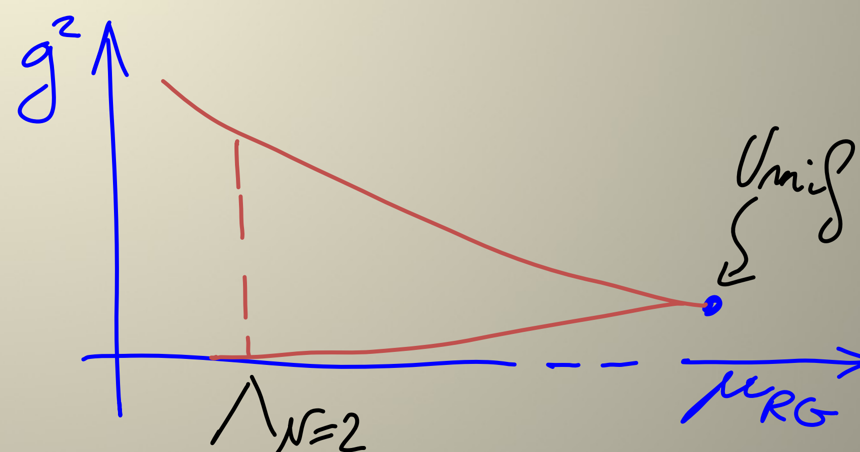
The theory: N=2 Super-QCD



root of b.b.

Moduli Space

$$\Lambda_{N=2}^{2n_c-n_f} = \mu_{RG}^{2n_c-n_f}\, e^{-2\pi i \tau(\mu_{RG})}$$



Add FI term, what happens …

for $U(1)$ $\qquad -2\xi \int d^2\theta\, d^2\bar{\theta}\; \mathrm{tr}\, V$

$\mathcal{N}=2$ UNBROKEN! but $SU(2)_R \longrightarrow U(1)_J$

Add FI term, what happens …

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$\mathcal{N}=2$ UNBROKEN! but $SU(2)_R \longrightarrow U(1)_J$

$$\langle q \rangle = \begin{pmatrix} \sqrt{2\xi} & & 0 \\ & \ddots & \\ & & \sqrt{2\xi} \end{pmatrix} \qquad \langle \tilde q \rangle = 0$$

Color-Flavor locking phase

$SU(n_c) \times SU(n_f) \longrightarrow SU(n_c)_{c+f} + \ldots$

Non-Abelian vortex here …

WEAK COUPLING $\sqrt{\xi} \gg \Lambda_{\mathcal{N}=2}$ $\qquad n_c = n_f = n$

$$\mathcal{L} = -\frac{1}{2g^2}\,\mathrm{tr}\,(FF) + (Dq)^\dagger Dq - \frac{g^2}{4}\,\mathrm{tr}\left(qq^\dagger - 2\xi\,\mathbb{1}\right)^2$$

Non-Abelian vortex here …

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TOPOLOGY $\pi_1(U(1)) = \mathbb{Z}$ = n times $\pi_1\left(\frac{SU(n)\times U(1)}{\mathbb{Z}_n}\right)$

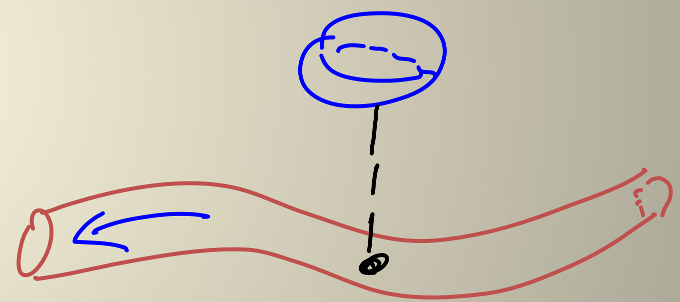
$$q = \begin{pmatrix} e^{i\vartheta} q(r)\sqrt{2\xi} & & 0 \\ & \ddots & \\ & & \sqrt{2\xi} \end{pmatrix} \qquad A_k = \begin{pmatrix} -\epsilon_{k\ell}\frac{\hat{r}_\ell}{r} f(r) & & \\ & 0 & \\ & & \ddots \\ & & & 0 \end{pmatrix}$$

Moduli space & Effective action

$SU(n)_{c+f} \longrightarrow SU(n-1) \times U(1)$

$\text{Coset} = CP(n-1)$

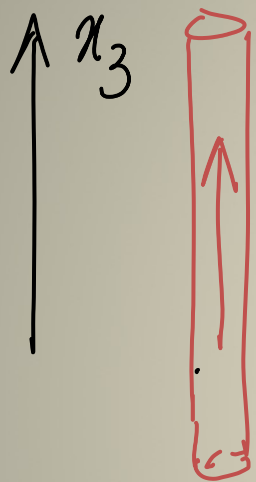
* INTERNAL MODULI
* $\overline{B}$ flux $SU(n)$



$$\beta = \frac{2\pi}{g^2} = \frac{n}{4\pi} \log \frac{\sqrt{3}}{\Lambda_{3+1}}$$

$$\Lambda_{CP} = \Lambda_{3+1}$$

SUSY N=(2,2) in 1+1



$$Q^1 = \left(Q^1_L \,,\; Q^1_R \right)$$

$$Q^2 = \left(Q^2_L \,,\; Q^2_R \right)$$

BROKEN BY THE VORTEX

$$\mathcal{N} = (2,2) = \left(Q^2_L \,,\; Q^1_R \right)$$

$$z \qquad\qquad \varphi^\ell$$

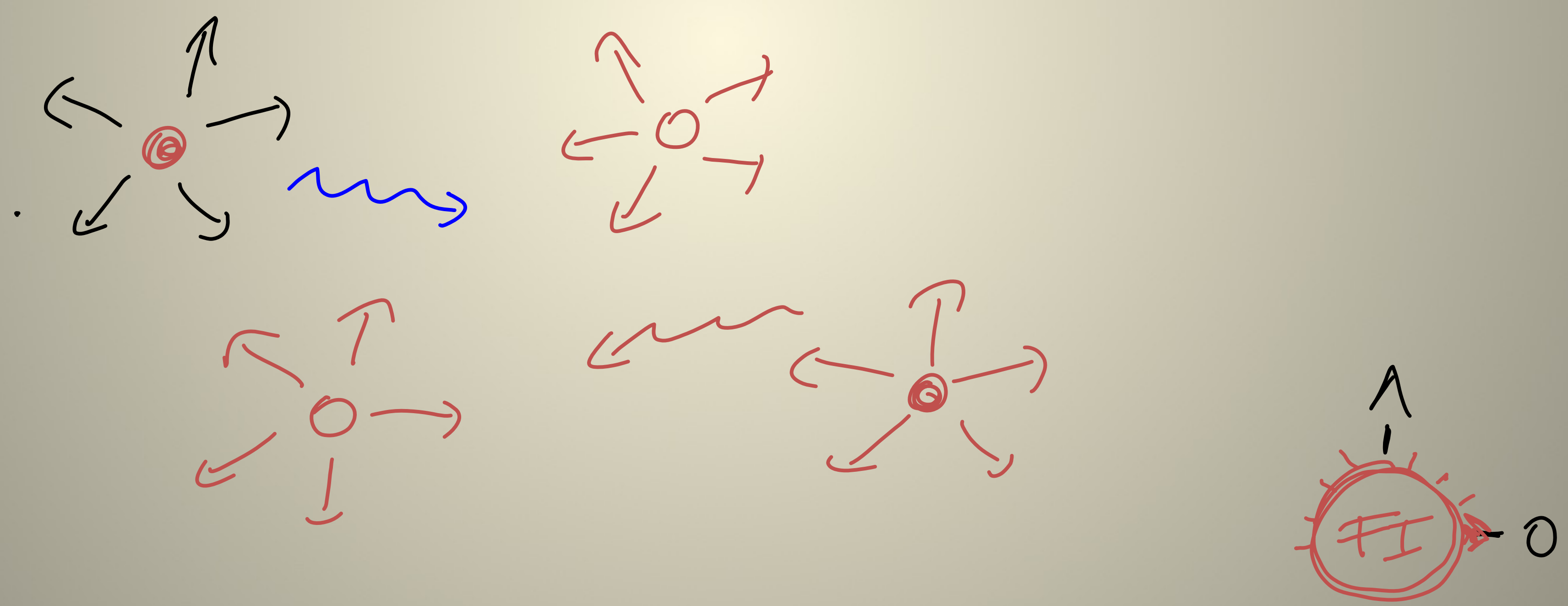
$$\xi_L \quad \xi_R \qquad\qquad \xi^\ell_L \quad \xi^\ell_R$$

An experiment …

$\xi = 0$

Coulomb Phase

$$M_{BPS} = |Z|$$



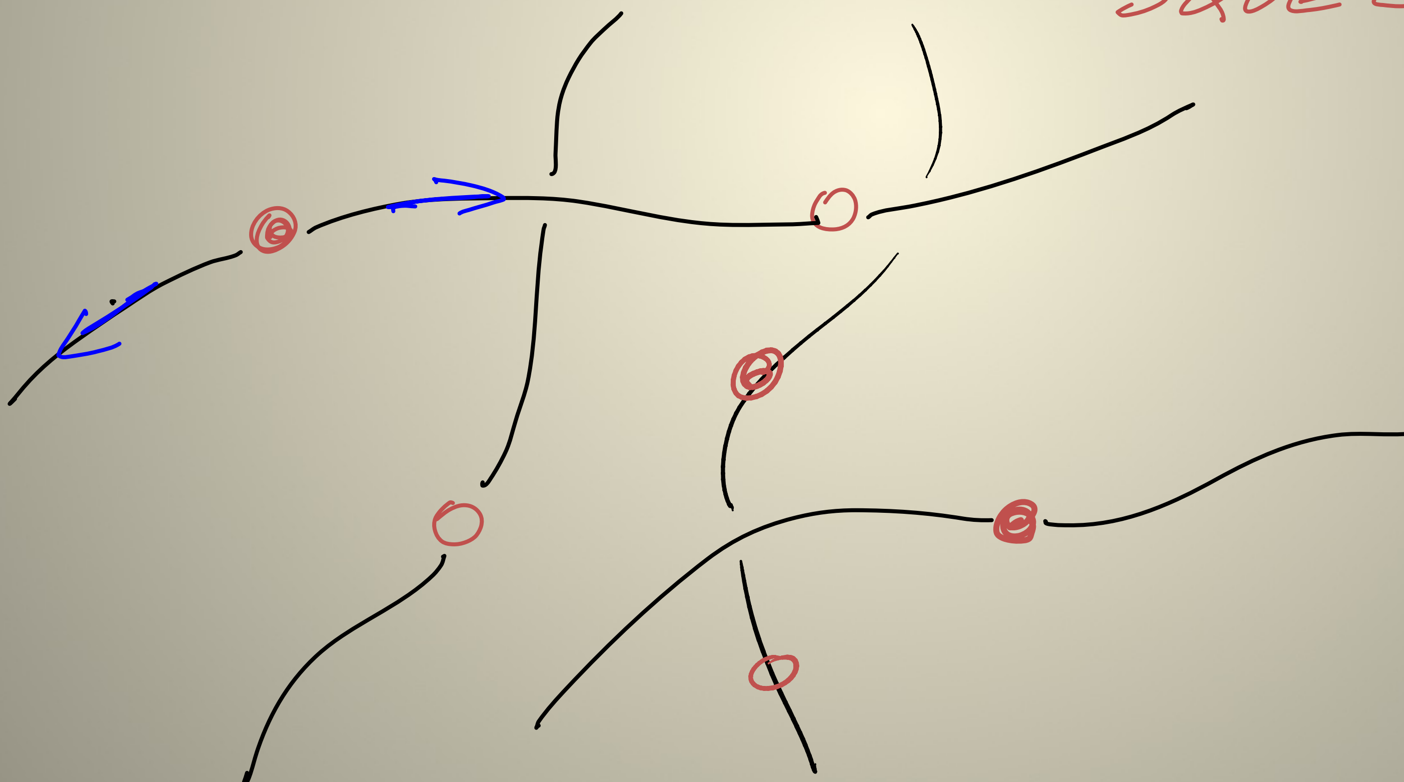
An experiment …

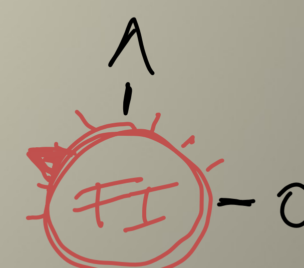
$\xi \gg \Lambda$

Higgs Phase

$$M^{3+1}_{BPS} = M^{1+1}_{BPS}$$

SQUEEZING!





N=1 mass deformation

$$\int d^2\theta \; \underbrace{\sqrt{2}\,\mu\, \mathrm{tr}_2\, \Phi^2}_{W(\Phi)} + h.c$$

COINCIDENCE

$W'(m) = 0$

$$\Lambda_{\mathcal{N}=1}^{3n_c-n_f} = \mu^{n_c}\, \Lambda_{\mathcal{N}=2}^{2n_c-n_f}$$

N=1 mass deformation

$$\int d^2\theta \, \underbrace{\sqrt{2}\,\mu \, \mathrm{tr}_2 \, \Phi^2}_{W(\Phi)} + h.c$$

COINCIDENCE

$$W'(m) = 0$$

SPECTRUM @ weak coupling

$$m_{A_\mu} = g\sqrt{2\xi} \qquad \underbrace{m_h = g\mu \qquad m_\ell = \xi/\mu}_{\text{large } \mu}$$

Φ

?

The Coincidence …

WITHOUT FI

$$W = t_2 W(\phi) + \tilde{Q}(\Phi - m) Q$$

$$W'(a) = 0$$

- Higgs $\tilde{q} q = W'(m)$ $\phi = m$
- Coulomb $q = \tilde{q} = 0$ $\phi = a$

$m \to a$ COULOMB

$+ Q, \tilde{Q}$ massless

The Coincidence …

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COULOMB

$m \to a$

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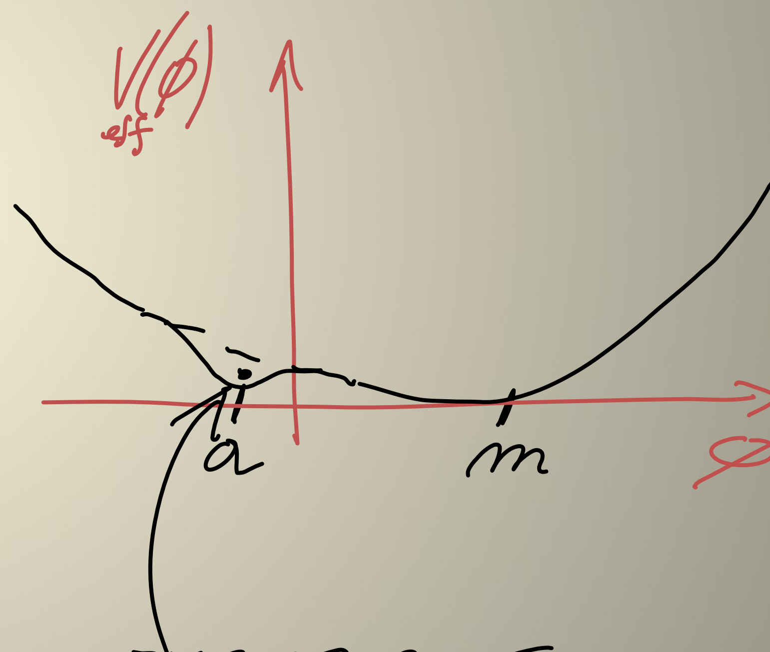
WITH FI



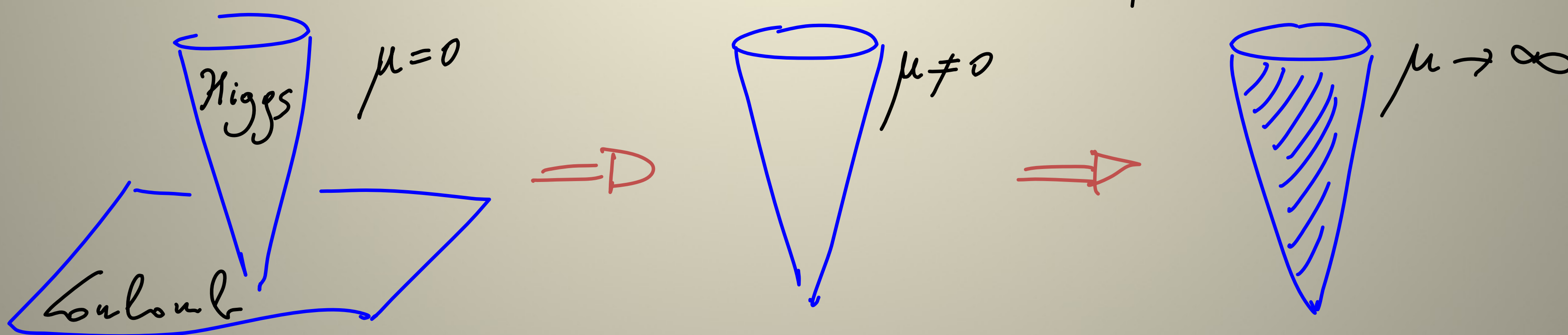
METASTABLE

Moduli space enhancement

$\mu \to \infty \qquad m_\ell = \xi/\mu \to 0$

$$W_{eff} = \frac{1}{\mu}\, \mathrm{tr}\, Q\tilde{Q}Q\tilde{Q}$$

"Pure" $\mathcal{N}=1$ SQCD has NO F_ϕ term



Heterotic N=(0,2) deformation

* String remain BPS if $\boxed{W'(m)=0}$

S.Y.

* FIRT Hyp. enhancement of SUSY
  No $N=(0,2)$ of $CP(N-1)$

* H.T. def

$$\int d^2x \, d^2\theta_R \; \mathcal{B} \, W'_{1+1}(\Sigma_1) + h.c.$$

$$W_{1+1} \propto \frac{1}{\sqrt{3}} W_{3+1}$$

Fermionic zero modes (1)

x_3

SUPERCHARGES

$$\begin{array}{cc} Q^1_L & Q^2_L \\ Q^1_R & Q^2_R \end{array}$$

μ $\quad$ $v_0\delta$

Fermionic zero modes (1)

x_3

SUPERCHARGES

	Q^1_L	Q^2_L
	Q^1_R	Q^2_R
		μ vort

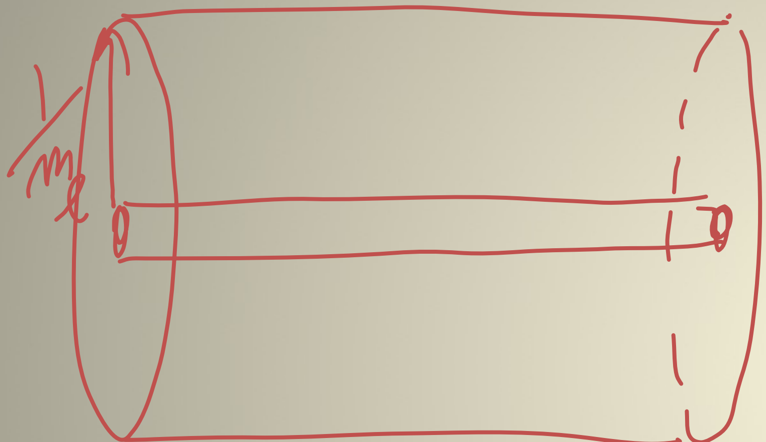
$$\mathcal{N}=(2,2) \equiv (Q^2_L, Q^1_R)$$

$$\Downarrow$$

$$\mathcal{N}=(0,2) \equiv (0, Q^1_R)$$

τ		φ	
ζ_L	ζ_R	ζ_L	ζ_R

Fermionic zero modes (2)



z: $\xi_L \quad \xi_R$

φ: $\xi_L \quad \xi_R$

DO <u>NOT</u> DISAPPEAR!

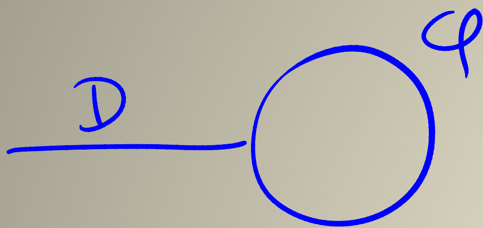
S.Y.

$$\psi_L \propto \frac{e^{-m_\ell r}}{\sqrt{r}}$$

$\mu \to \infty$

THEY BECOME NON NORMALIZABLE!

Large n solution & DSB

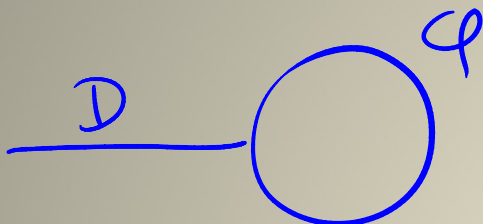


$$i\beta + n\int \frac{d^4k}{4\pi^2}\,\frac{1}{k^2+D-2|\phi|^2} = 0$$

RG running

$$2|\phi|^2 - D = \Lambda^2$$

Large n solution & DSB



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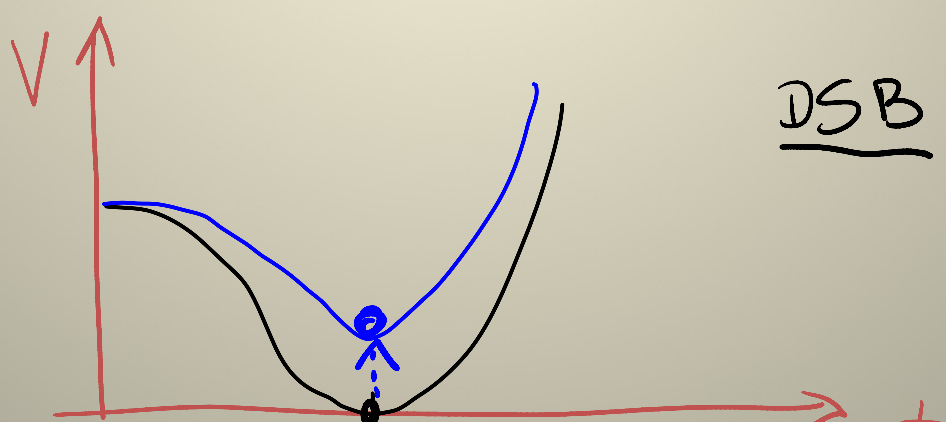
$$V_{eff}(\phi) = \frac{n}{8\pi}\left\{\Lambda^2 + 2|\phi|^2\left(\log\frac{2|\phi|^2}{\Lambda^2} - 1\right) + 8|\phi|^2 u\right\}$$

$\langle \xi\xi \rangle$



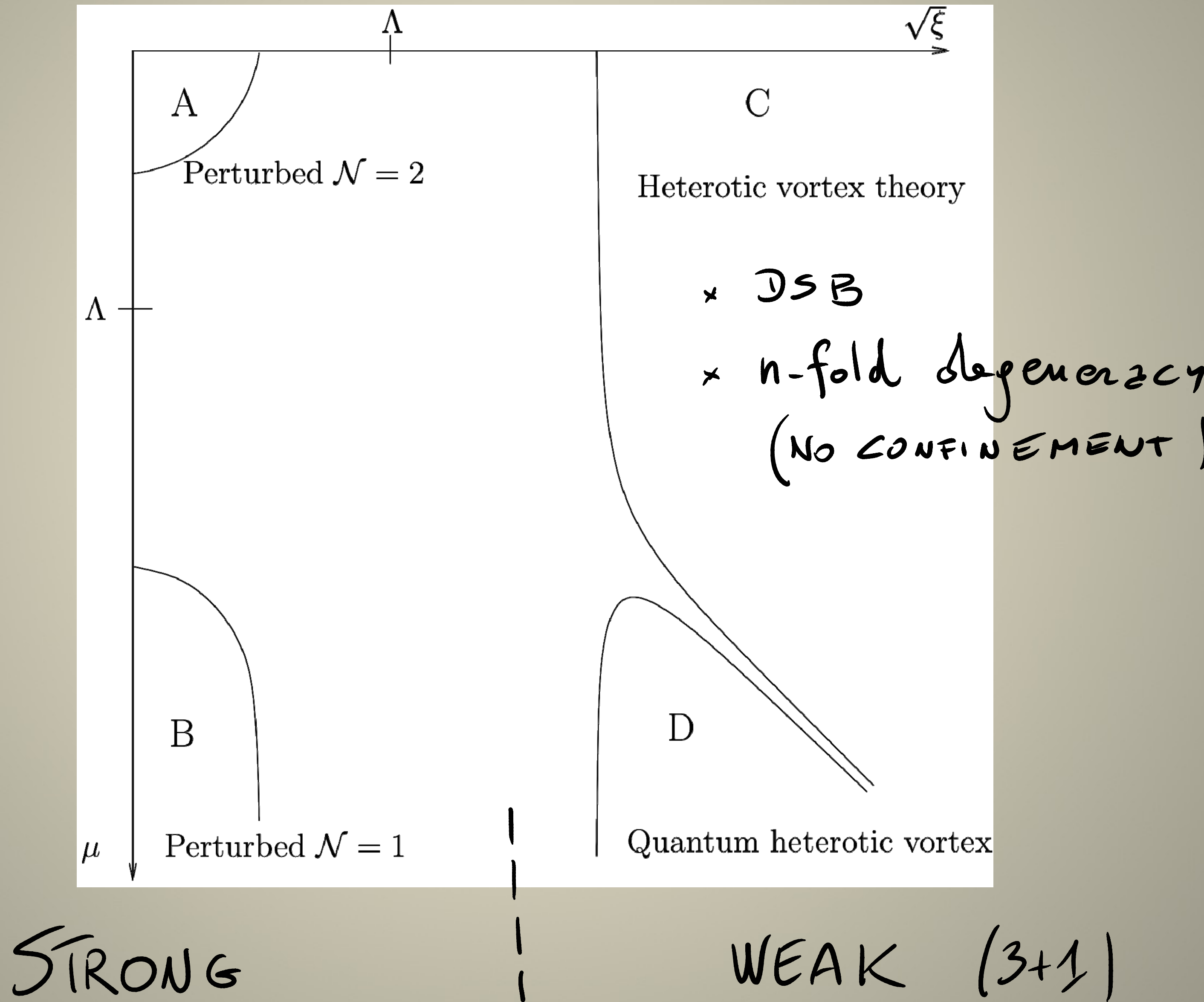
<u>DSB</u>

Shifman
-Yung

New stuff …

- Explor Paramete Space (μ, ξ) $\left\{ \begin{array}{l} \text{DSB} \ldots \\ \text{degeneracy} \ldots \end{array} \right.$
- $\mu \, \mathrm{tr} \left(\frac{\Phi^2}{2} - a \, \Phi \right)$ LINEAR TERM
- Conjectured NEW CFT as $\sqrt{\xi} \gg 1, \ \mu \to \infty$

Exploring the parameter space



(A): SW solution

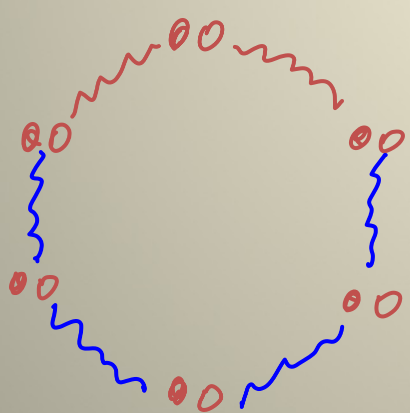
SW curve

$$y^2 = \frac{1}{4}\det(z-\phi)^2 - \Lambda^n z^n$$

$$= \frac{1}{4}\left(z^n - \Lambda^n\right)^2$$

$$\begin{array}{c|cccc}
 & U(1)_1 & - & - & U(1)_n \\ \hline
E_1 & 1 & & & 0 \\
\vdots & & \ddots & & \\
\vdots & & & \ddots & \\
E_n & 0 & & & 1
\end{array}$$

Roots



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\hline
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\vdots & & \ddots & \\
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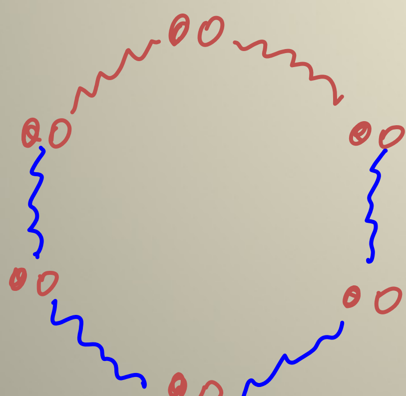
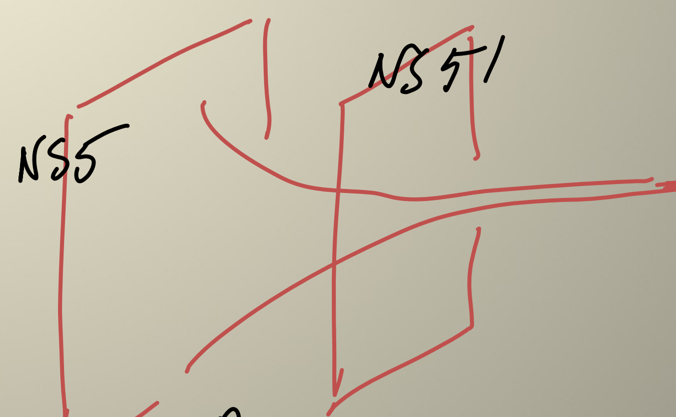
Roots



MQCD Curve

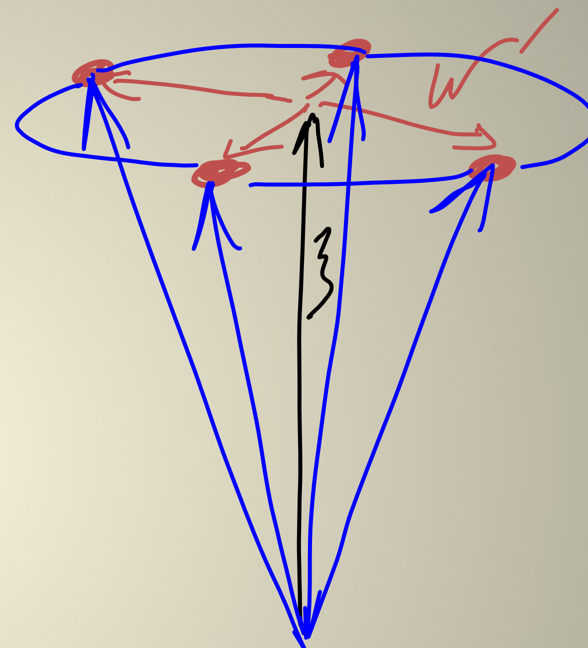


DISCONNECTED!

(A): Vortices here

$$T_J \propto \sqrt{|W'(z \infty t_J)|^2 + \xi^2}$$

$$\simeq \xi + \sim \frac{\mu^2 \Lambda^2}{\xi} + \dots$$

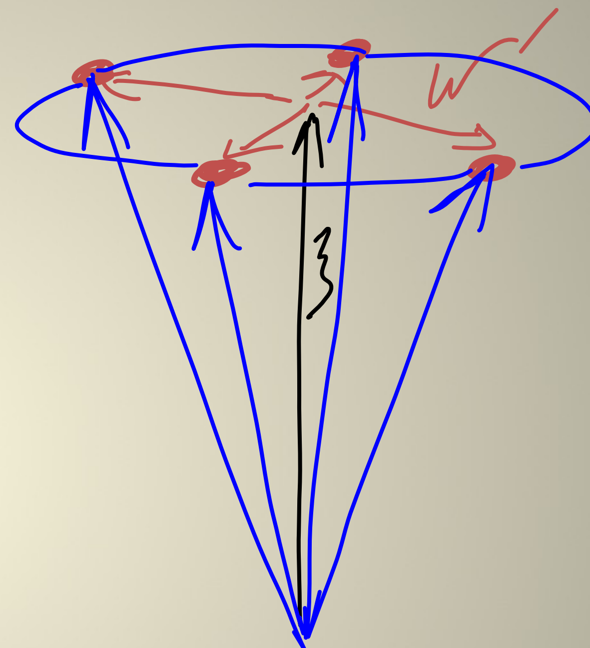


$$\underbrace{\mathrm{Re}\, W',\ \mathrm{Im}\, W',\ \xi}_{\text{TRIPLET of } SU(2)_R}$$

(A): Vortices here

$$T_J \propto \sqrt{|W'(z_{\mathrm{vort}_J})|^2 + \zeta^2}$$

$$\simeq \zeta + \sim \frac{\mu^2 \Lambda^2}{\zeta} + \dots$$



$$\underbrace{\mathrm{Re}\,W',\ \mathrm{Im}\,W',\ \zeta}_{\text{TRIPLET of } SU(2)_R}$$

Explain:

- DSB & degeneracy $(a = 0)$
- No degeneracy $a \neq 0$
- SUSY RESTORATION $a = \Lambda e^{\frac{i 2\pi k}{n}}$!

(B) Perturbed N=1

FIRST $\mathcal{N}=1$ SQCD

$$M = \tilde{Q} Q$$

$$B = \epsilon\, q \cdots q$$

$$\tilde{B} = \epsilon\, \tilde{q} \cdots q$$

$$\det M - B\tilde{B} = 0$$

$$\Downarrow$$

$$\det M - B\tilde{B} = \Lambda^{2n}$$

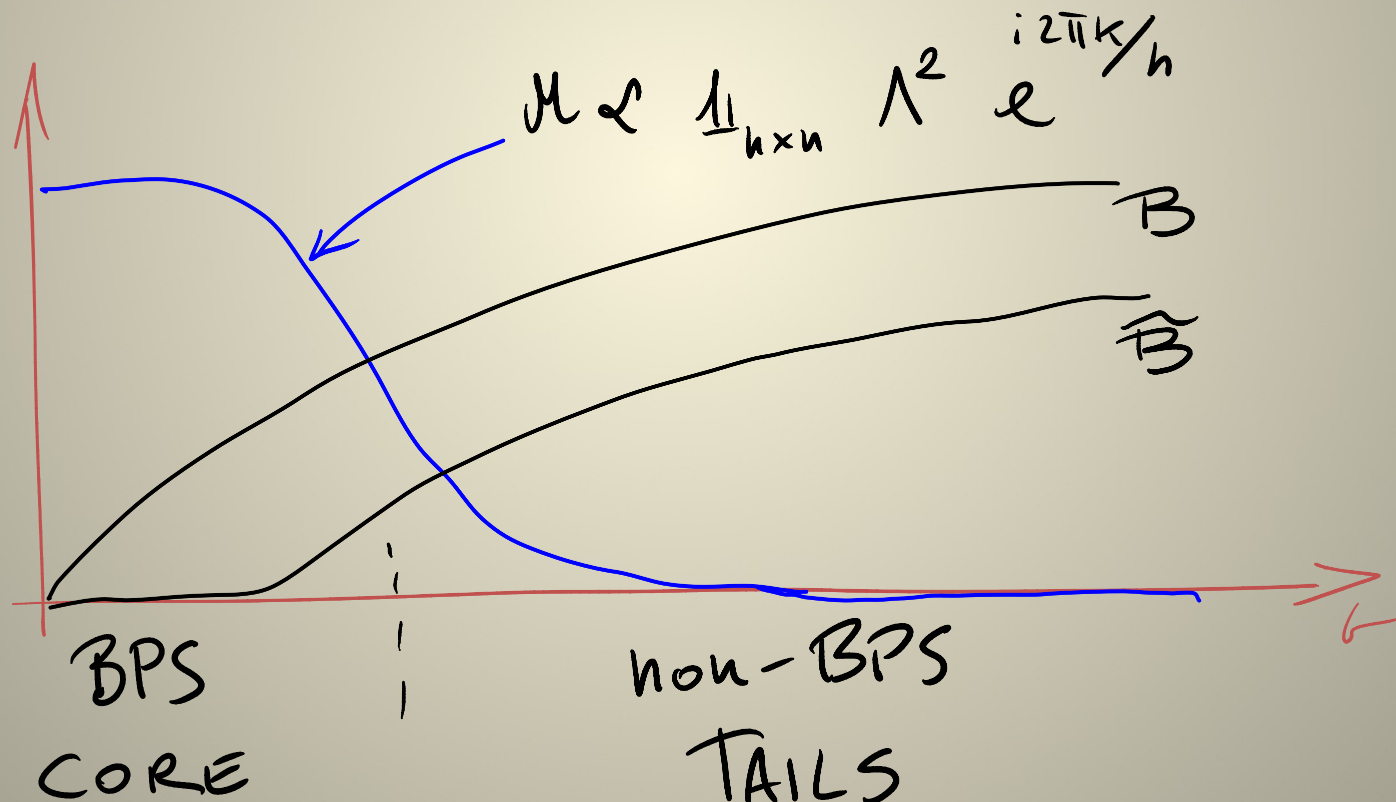
n "BASES"

Where $\tilde{B} = 0$

$$\boxed{M = \mathbb{1}_{n\times n}\, \Lambda^2\, e^{\frac{2\pi i k}{n}}}$$

(B): Effective superpotential

$$W = -\frac{1}{\mu}\left[\mathrm{tr}\, Q\tilde{Q}Q\tilde{Q} - a\mu\, \mathrm{tr}\, Q\tilde{Q}\right]$$



(C): Heterotic vortex theory

$$V_{eff}(\sigma) = \frac{m}{8\pi}\left\{\Lambda^2 + \sigma^2\left(\log\frac{\sigma^2}{\Lambda^2} - 1\right) + 8|\sigma - a|^2 u\right\}$$

- $a = \frac{\Lambda}{\sqrt{2}}$ SUSY IS <u>NOT</u> BROKEN

- SUSY IS ACTUALLY ENHANCED

$\mathcal{N} = (2,2)$ SPECTRUM !

(D): Quantum Heterotic Vortex

$\mu \to \infty$

z $\qquad$ φ^ℓ

$\cancel{\zeta_L}$ $\quad \zeta_R$ $\quad \cancel{\zeta_L^\ell}$ $\quad \zeta_R^\ell$

$\mathcal{N}=(0,2)$

(D): Quantum Heterotic Vortex

$\mu \to \infty$

$\cancel{\xi_L^z} \quad \xi_R \qquad \varphi^\ell \quad \cancel{\xi_L^\ell} \quad \xi_R^\ell$

$\mathcal{N} = (0,2)$

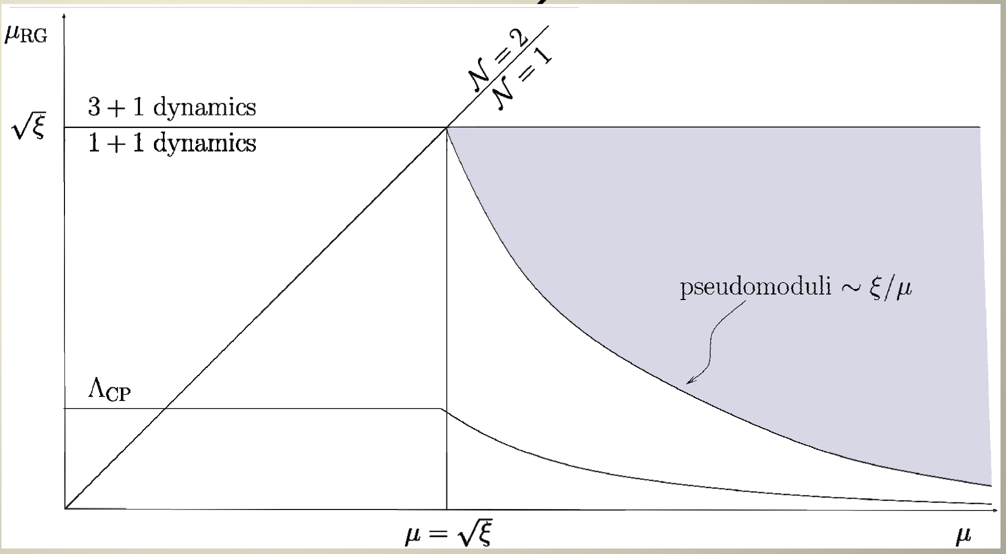
But there are $\underline{\underline{2}}$ problems

- ANOMALY ?
- α DEPENDENCE ?

(D): Peculiarities

Everything would be solved if

$\Lambda_{CP} \longrightarrow 0$ as $\mu \longrightarrow \infty$!



THERE IS ENTANGLEMENT BETWEEN 3+1 & 1+1

$$S_{tot} = S_{3+1} + S_{1+1} + S_{int}$$

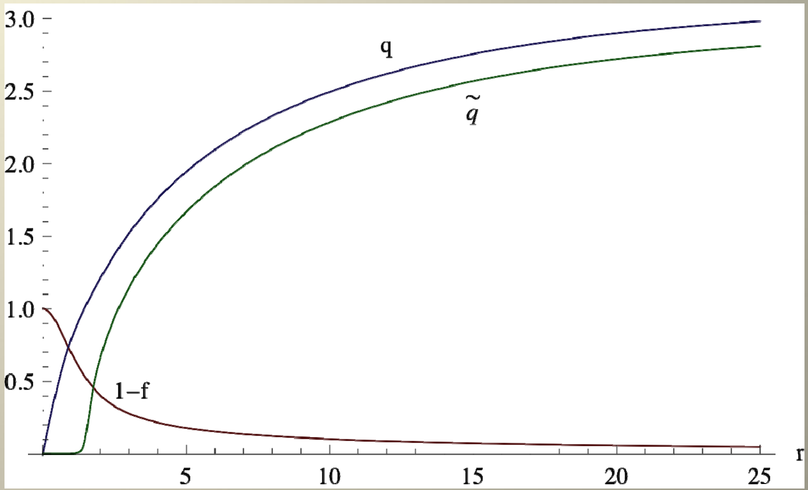
(D):Entanglement: 3+1 with1+1

Effective Action :

$$\begin{cases} q(r) = q^{(0)}(r) + \delta q(r) \\ \tilde{q}(r) = \tilde{q}^{(0)}(r) + \delta \tilde{q}(r) \\ A_k(r) = A_k^{(0)}(r) + \delta A_k(r) \end{cases}$$

$$S_{\text{int}} \sim \underbrace{\log\left(\frac{\sqrt{\xi}}{\lambda_\perp}\right) \cdot \frac{1}{\rho}}_{1/\beta_0} \cdot \int d^2x \, \left| q \cdot \delta\tilde{q} \right|^2$$

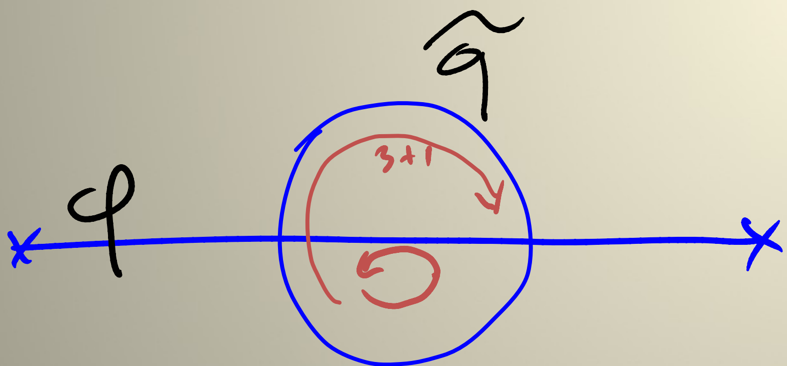
IF $\lambda_\perp \simeq \mu_{RG}$!



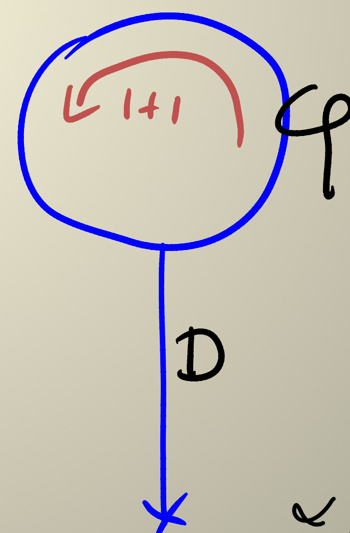
(D): New CFT mechanism

$$S = \int d^4x\, |\partial_\mu \tilde{\varphi}|^2 + \int d^2x \left\{ |\nabla \varphi|^2 + D\left(|\varphi|^2 - \beta\right) + \frac{1}{\beta}|\varphi \cdot \tilde{\varphi}|^2 \right\}$$

S_{int}

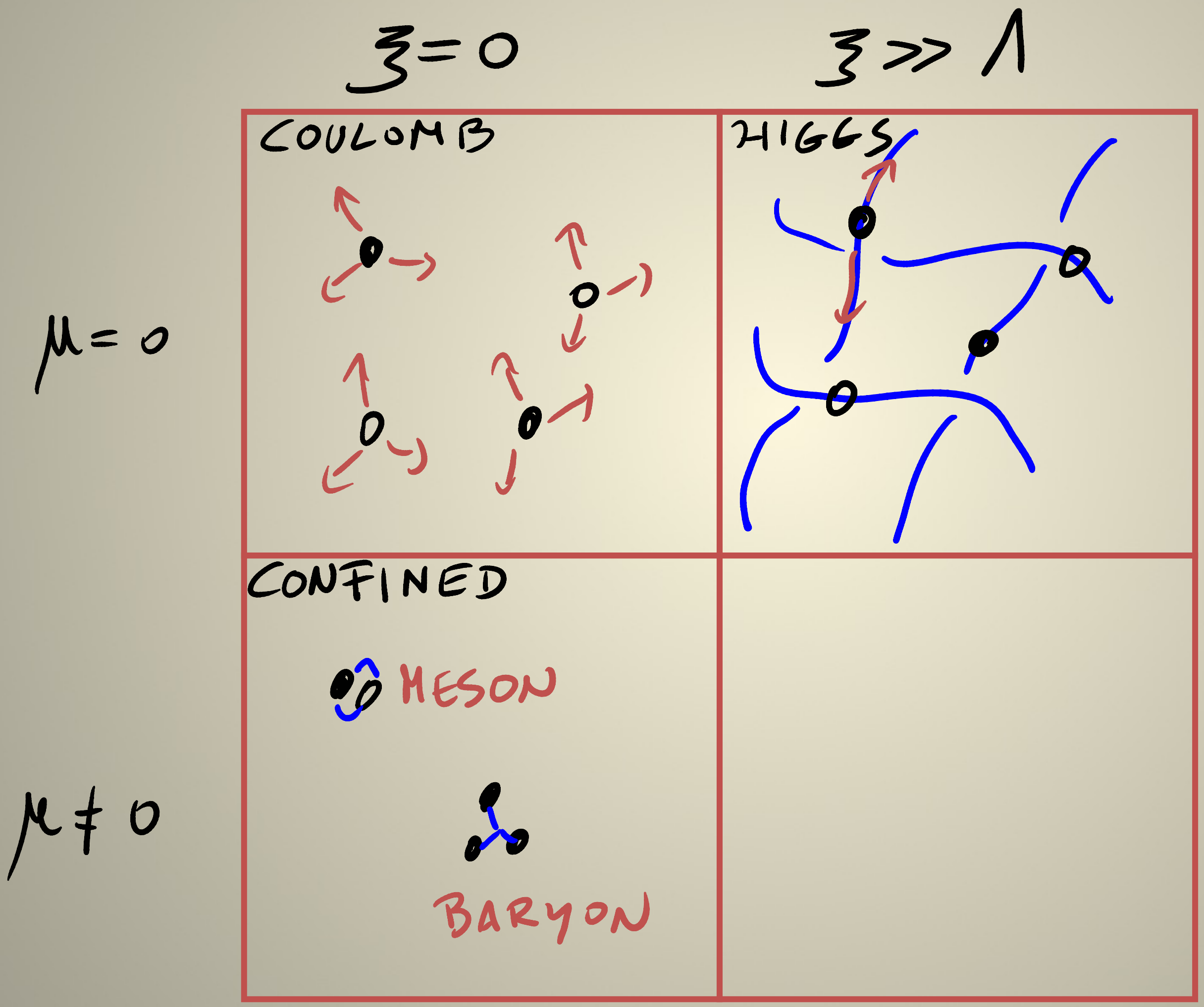


$$\propto \int d^6k\, \frac{1}{k^6} \sim \log \mu_{RG}$$

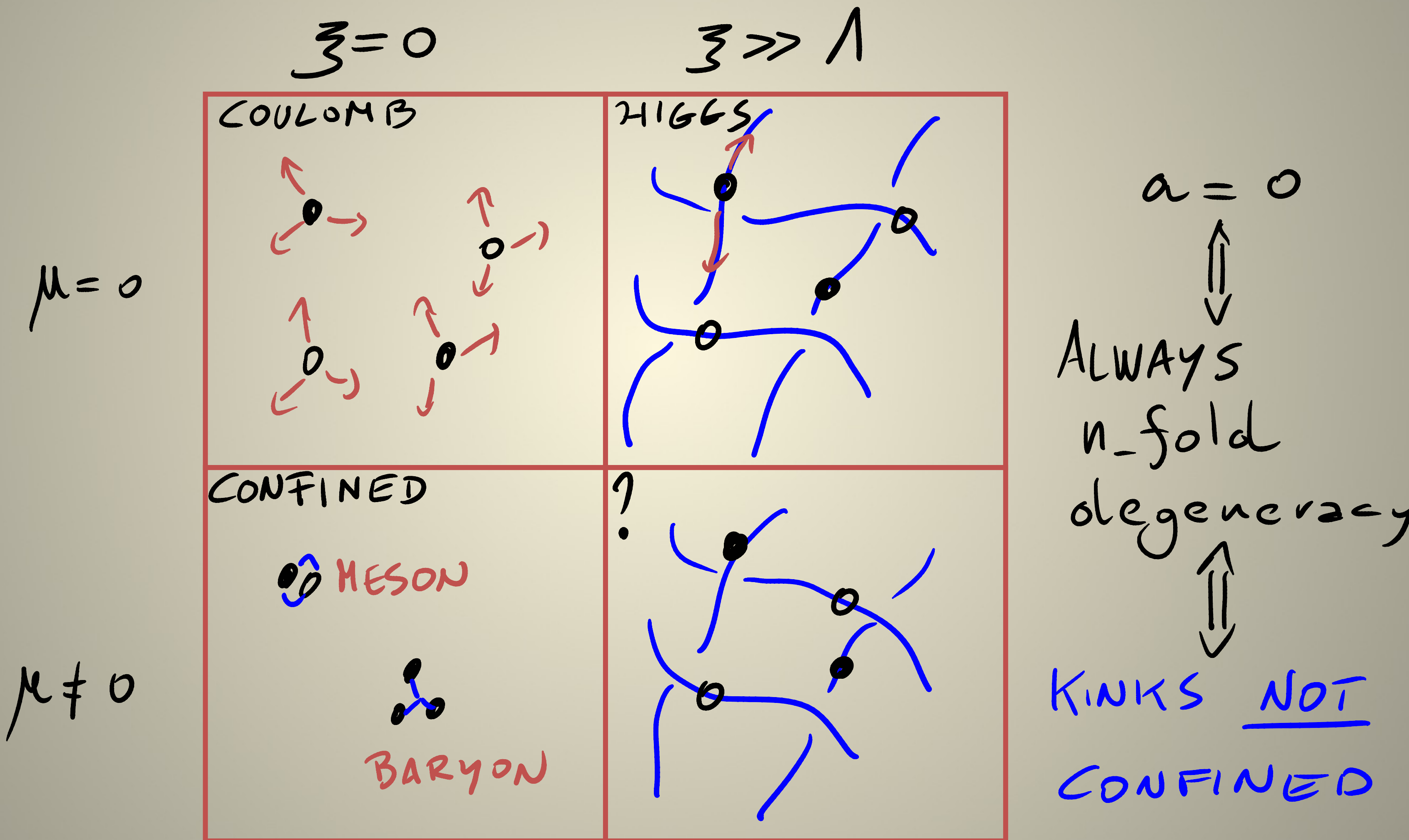


$$\propto \int d^2k\, \frac{1}{k^2} \sim \log \mu_{RG}$$

Work in progress …



Work in progress …



The Idea

$$
\begin{tikzcd}
 & \mu \arrow[dr, dash] & \\
\text{DSB} \arrow[ur, dash] \arrow[dr, dash] & & \text{Both are needed} \\
 & \Lambda_{CP} \arrow[ur, dash] &
\end{tikzcd}
$$

$$\delta \propto \frac{\mu^2 \Lambda^2}{3}$$

The Idea

DSB — μ — Both are needed

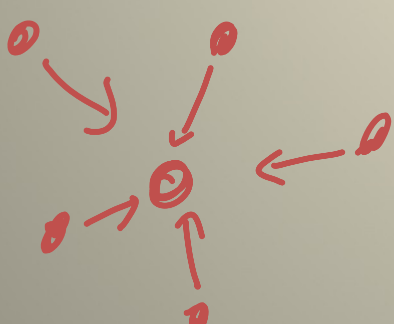
DSB — Λ_{CP} — Both are needed

$\delta \propto \frac{\mu^2 \Lambda^2}{3}$

ABELIAN VORTEX

Global $U(1)$

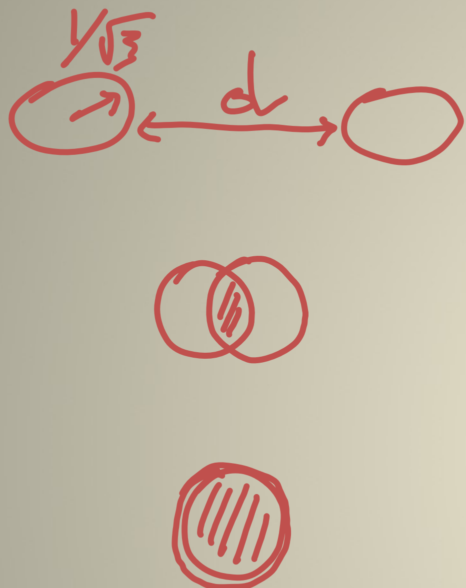
$q = \left(\circlearrowleft \right) \quad A = \left(\circlearrowleft \right)$

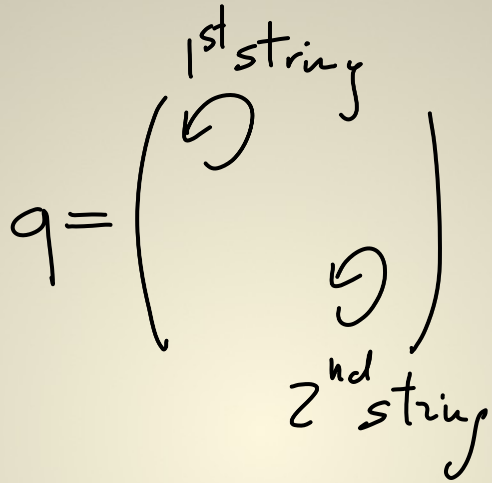


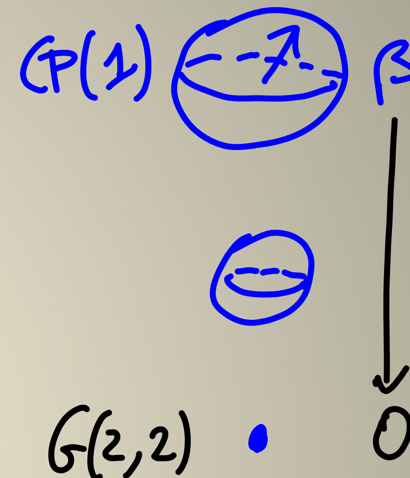
__NO__ INTERNAL M. S.

$\Longrightarrow$ NO DSB!

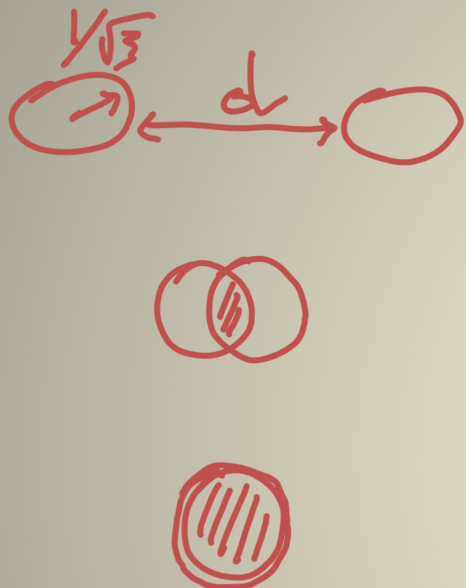
Potential

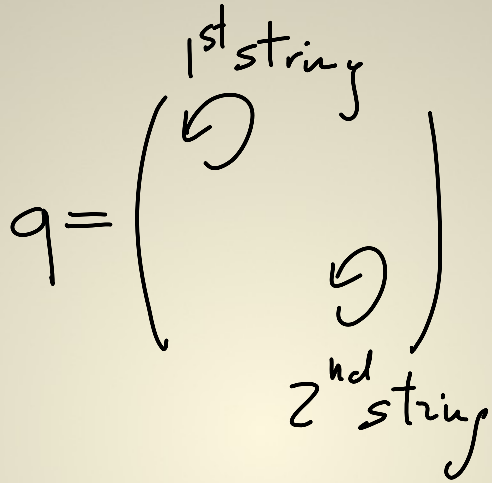


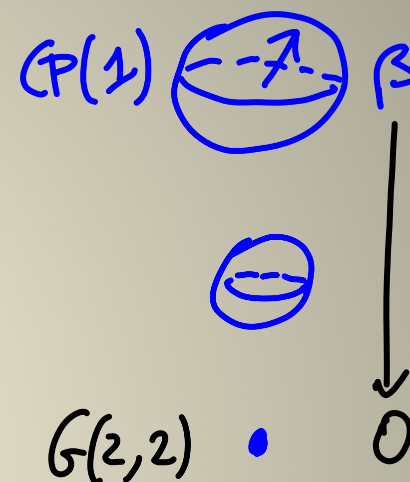




Potential

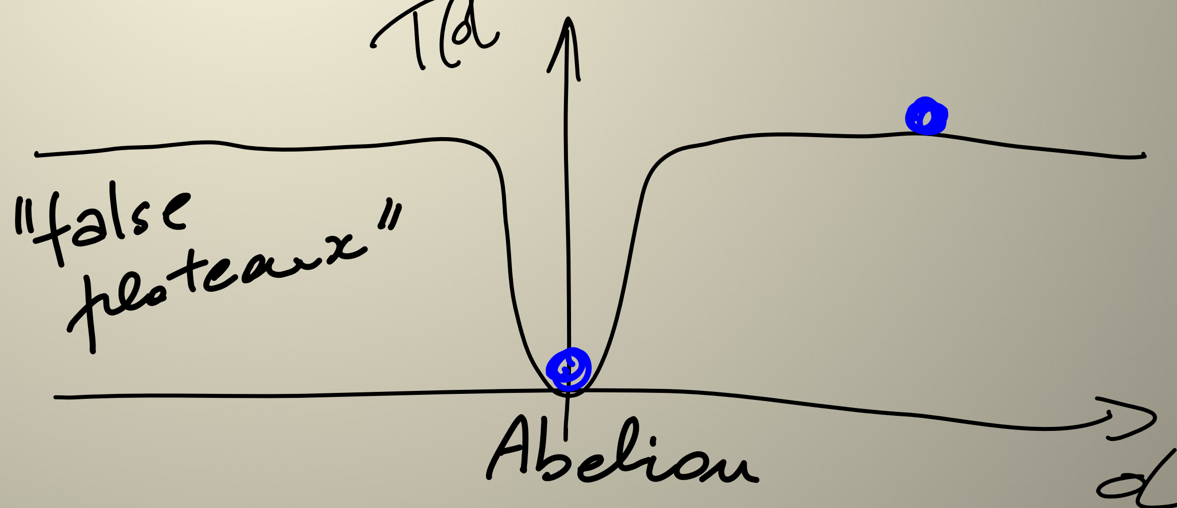




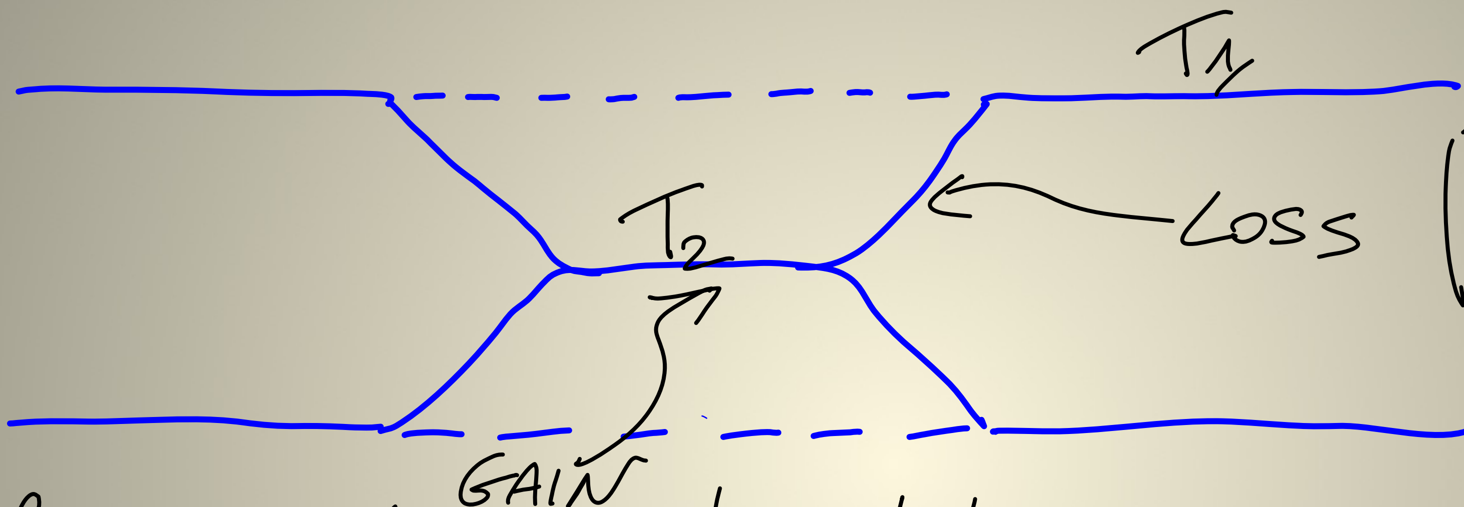


$$T_1 = |Z| + \delta$$

$$T_2 = 2|Z|$$



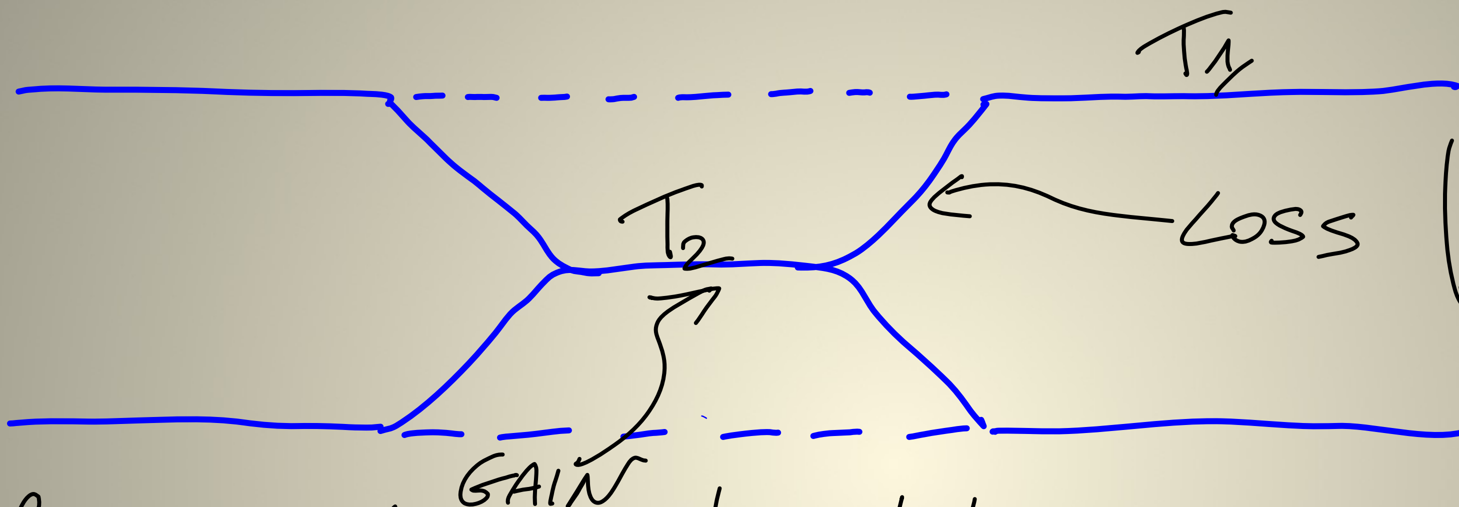
Tunneling



$$\boxed{T_2 < 2T_1}$$

Barrier due to kinetik energy

Tunneling

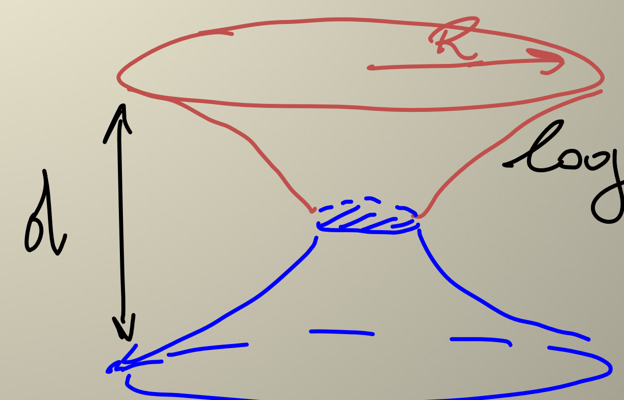


$$\boxed{T_2 < 2T_1}$$

Barrier due to kinetik energy

BOUNCE SOLUTION

$$S_B = \pi T_1 \frac{d^2}{2\log\left(R\delta/d^2\right)} + \dots$$



Index

	Q^1	Q^2	Φ	Q	Q	μ	ξ	$\Lambda^{2n_c-n_f}$
$U(1)_R$	1	1	2	0	0	-2	0	$2(2n_c-n_f)$
$U(1)_J$	1	-1	0	1	1	2	0	0
$U(1)_{J'}$	2	0	2	1	1	0	0	$2(2n_c-n_f)$

$\bullet$	$\times$	$\times$	$\bullet$
$\lvert v\rangle$	$Q^1\lvert v\rangle$	$Q^2\lvert v\rangle$	$Q^1 Q^2\lvert v\rangle$

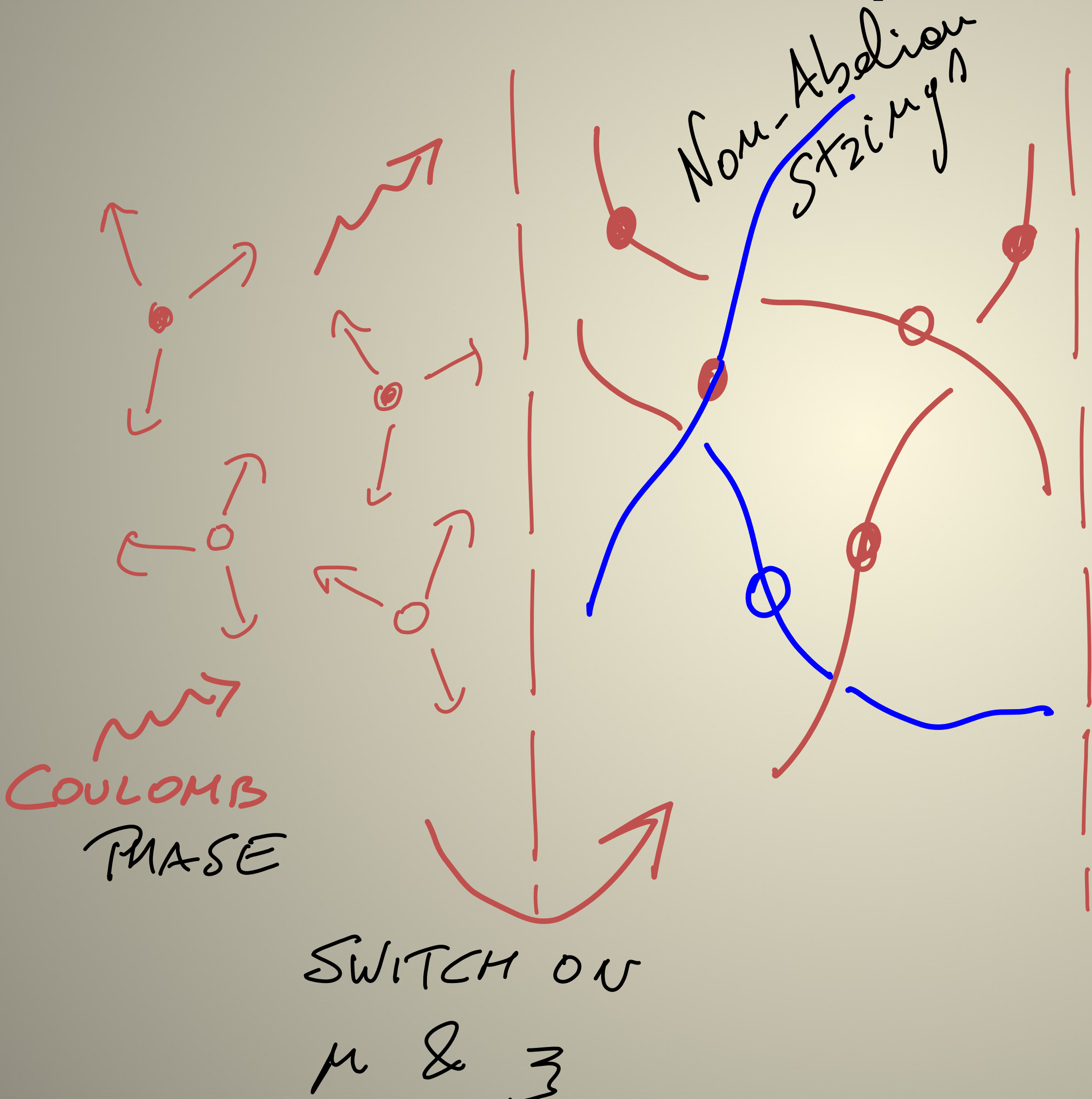
4 STATES

Short in $\mathcal{N}=2$

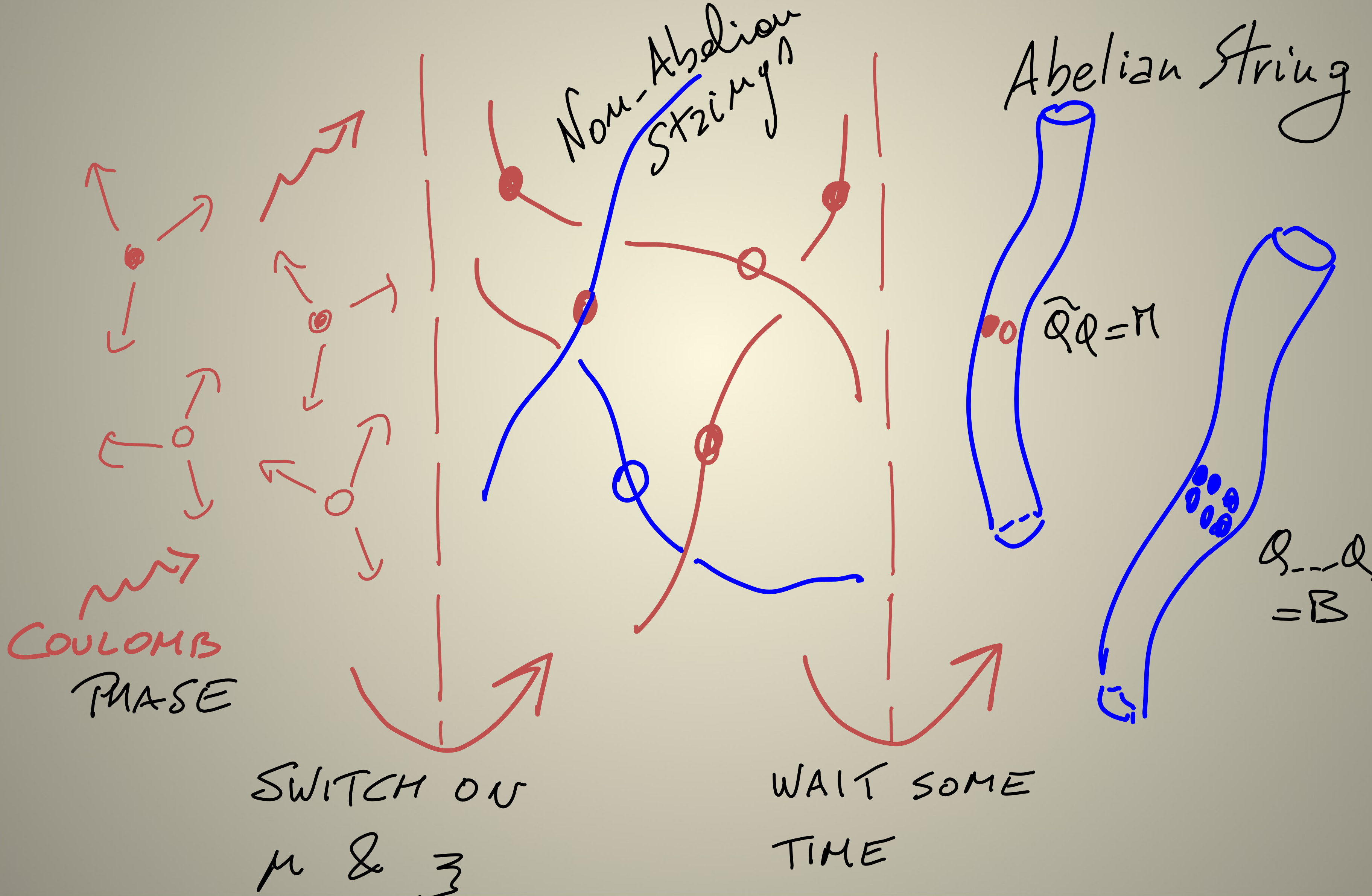
BUT Long in $\mathcal{N}=1$

$$U(1)_{J'} \longrightarrow \mathbb{Z}_{2n} \boxed{\longrightarrow \mathbb{Z}_2 \Longleftrightarrow \text{DSB}}$$

The final experiment …



The final experiment …



Conclusion

- Studied Heterotic Vortex

  $\mathcal{N}=1$, FI, $W'(m)=0$

- Explored (μ, ξ) parameter space

  DSB, n_fold deg. // New CFT Q.M.V.

- Abelian bound state

  remains BPS $\Longrightarrow$ Phase of SQCD + FI