Exotic superfluidity in cold atoms

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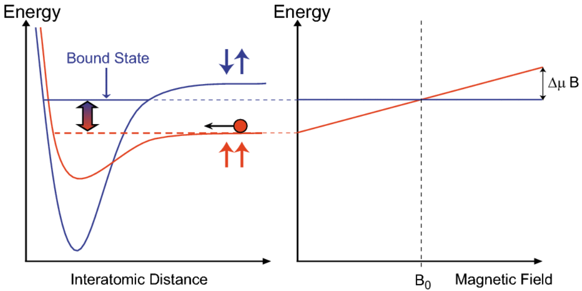
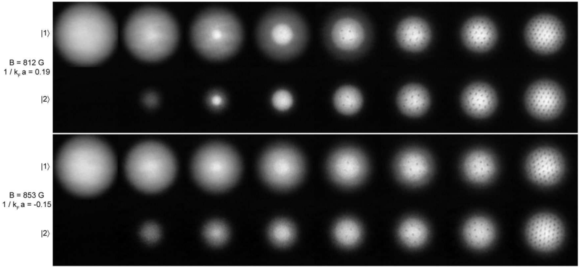
Experiments in cold atoms

Fig.1 (top): Zwierlein et al. 2008; Fig.2 (bottom): Ketterle et al. 2005





Theory of BCS-BEC crossover

Zero imbalance. $\mu_1 \approx \mu_2$
Nonzero polarization. Small $\delta\mu \leq \Delta/\sqrt{2}$. Large $\delta\mu \sim \Delta$.
Phase diagram at zero T. $\eta \sim \delta\mu$, $\kappa = -1/na^3$.

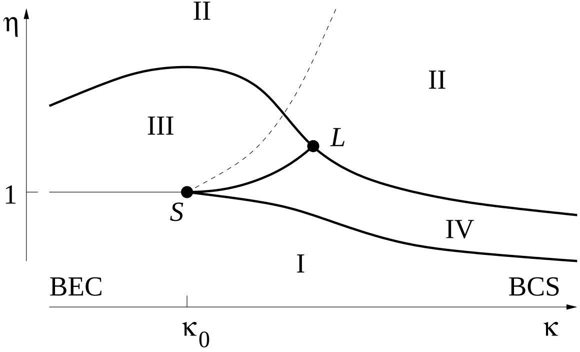


Figure: Son, Stephanov 2005.

Model and ansatz.

Two species of fermions ψ_1 and ψ_2 of equal mass m, $\mu_1 = \mu + \delta\mu$ and $\mu_2 = \mu - \delta\mu$.

$$g = \frac{1}{k_F a}$$

$n = k_F^3/(2\pi^2)$. BCS $g \to -\infty$, $a_s < 0$; BEC $g \to \infty$, $a_s > 0$.

$$\langle \psi_\alpha(x)\psi_\beta(x)\rangle = \frac{\Delta(x)}{\lambda}\varepsilon_{\alpha\beta}$$

$\Delta(x) = \Delta =$ const. Global $U(1)$ is broken.

$$\mathcal{L} = \Psi^\dagger \begin{pmatrix} i\partial_t - \xi(\mathbf{p}) + \delta\mu\sigma^3 & -\Delta(x)\varepsilon \\ \Delta^*(x)\varepsilon & i\partial_t + \xi(\mathbf{p}) - \delta\mu\sigma^3 \end{pmatrix} \Psi - \frac{|\Delta(x)|^2}{g}$$

$$\epsilon_+ = +\delta\mu + \sqrt{\xi(\mathbf{p})^2 + \Delta^2}\,, \qquad \epsilon_- = -\delta\mu + \sqrt{\xi(\mathbf{p})^2 + \Delta^2}$$

where $\xi = \mathbf{p}^2/2m - \mu$.

Fluctuations

$\Delta(x) = \Delta + \eta(x)$, fluctuations $\eta(x) \ll \Delta$ and the mean field Δ.

$$Z = \int \mathcal{D}\eta^* \mathcal{D}\eta \mathcal{D}\Psi^\dagger \mathcal{D}\Psi e^{-\mathcal{S}[\Psi^\dagger, \Psi, \eta, \eta^*]}$$

$$\begin{aligned} S[\eta, \eta^*] &= \int d^4x \Big\{ \frac{1}{g} |\Delta + \eta(x)|^2 \Big\} \\ &- \Big\{ \frac{1}{2} \mathrm{Tr}\log \begin{pmatrix} -\partial_{x^4} - \xi(\mathbf{p}) + \delta\mu & -(\Delta + \eta(x)) \\ -(\Delta + \eta^*(x)) & -\partial_{x^4} + \xi(\mathbf{p}) + \delta\mu \end{pmatrix} \Big\} \end{aligned}$$

Expansion $\mathcal{S}[\eta, \eta^*] = \mathcal{S}^{(0)} + \mathcal{S}^{(1)} + \mathcal{S}^{(2)} + \dots$. Free energy $\mathcal{S}^{(0)}$, $\mathcal{S}^{(1)} = 0$,

$$\mathcal{S}^{(2)} = -\frac{1}{2} \frac{T}{V} \sum_k (\lambda(-k) \theta(-k)) \begin{pmatrix} M_{11}(k) & M_{12}(k) \\ M_{21}(k) & M_{22}(k) \end{pmatrix} \begin{pmatrix} \lambda(k) \\ \theta(k) \end{pmatrix}$$

with $\eta = (\lambda + i\theta)/\sqrt{2}$. $\mathcal{S}^{(2)}$ is UV finite.

Low energy effective Lagrangian

Approximations: small fluctuations, long wavelength

$$\mathcal{S}^{(2)} \to \begin{pmatrix} -C + Dk_0^2 - \frac{E}{3}k^2 & ik_0F \\ -ik_0F & Ak_0^2 - \frac{B}{3}k^2 \end{pmatrix}$$

Goldstone mode $\Delta \to \Delta e^{i\phi(x)}$, SS breaking total $n_1 + n_2$,

$$\mathcal{L}_\phi = \Delta^2 \left[A(\partial_t \phi(x))^2 - \frac{B}{3}(\partial_i \phi(x))^2 \right]$$

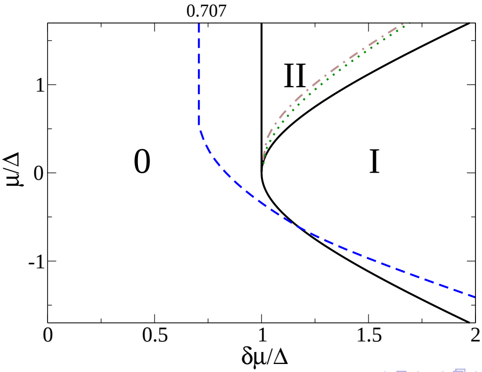
Speed of sound $\sqrt{B/3A}$; current instability $B < 0$. Higgs mode $\Delta \to \Delta + \lambda(x)/\sqrt{2}$,

$$\mathcal{L}_\lambda = -\frac{1}{2}C\lambda(x)^2 + \frac{1}{2}D(\partial_t \lambda(x))^2 - \frac{E}{6}(\partial_i \lambda(x))^2$$

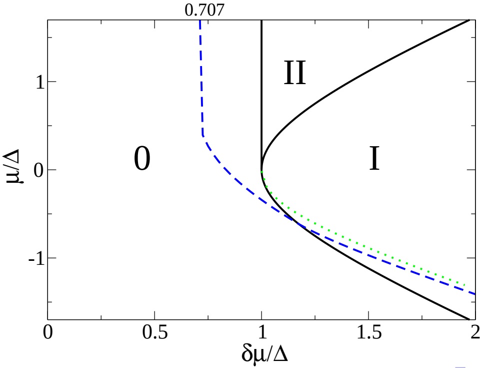
Higgs mass $m_H^2 = C/D$; $C < 0$ local max; $E < 0$ spatial modulation, no current. At weak coupling: $F = 0$, $B < 0$, $C < 0$, $E < 0$.

Analysis of stability at $T = 0$.

1. Real speed of sound for Goldstone and Higgs modes $(B, E > 0)$.
2. SF is a local minimum $(C > 0)$.
3. SF is a global minimum $(\Omega_s - \Omega_n < 0)$.

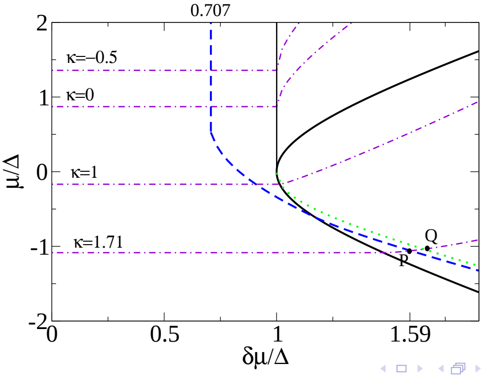


1. Real speed of sound equiv. to positive screening masses.
2. Local minimum is equiv. to positive number susceptibility.
3. Conclusion: there is gapless state with 1FS at BEC side.



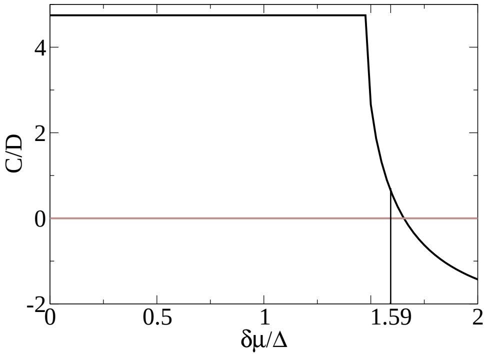
Parameters of the Higgs Lagrangian

Dimensionless $\kappa = \pi/2\sqrt{2m\Delta}a_s$. BCS at $\kappa \ll -1$, BEC at $\kappa \gg 1$.
P: $\delta\mu \approx 1.59$, Q: $\delta\mu \approx 1.66$.



Light Higgs in BEC

Higgs mass $m_H^2 = C/D$ along $\kappa = 1.71$; $g = \frac{1}{k_F a_s} \approx 1.31$.



Vortex configuration

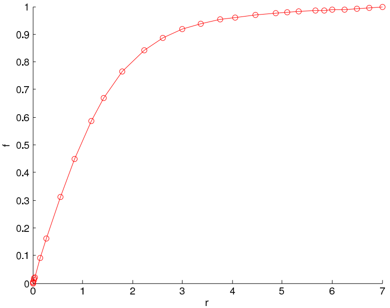
Correlation length $r_0 \sim 1/m_H$.
Condensate $\Delta(r) = f(r)e^{i\phi(\varphi)}$, $f(r) = \Delta + \rho(r)$

$$\Delta(r) - r_0^2 \nabla^2 \Delta(r) = const$$

$$f(r \to 0) = 0, \quad f(r \to \infty) = \Delta$$

$r_0 = \sqrt{E/3C}$ is outer core of a vortex configuration.

$f(r)/\Delta$



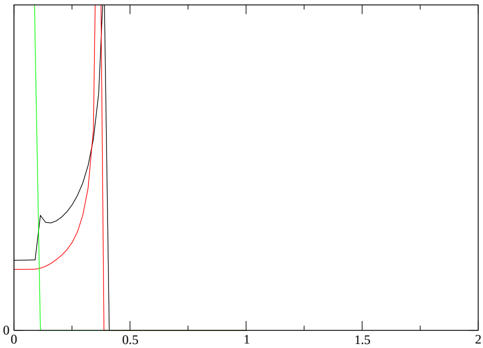
Vortex in a trap

Equate $E_k = E_{cond}$

$$E_k = n\frac{mv^2}{2},\ v = \frac{1}{2mr}e_\theta; \qquad E_{cond} = n\varepsilon_{cond}$$

Vortex radius $r_0 \sim 1/\sqrt{\varepsilon_{cond}}$.

$r_0^2(R)$



Conclusions

1. Effective theory for collective modes, Goldstone and Higgs. Interesting low energy content: gapless fermions, massless Goldstone, light Higgs.

2. Phase diagram. There is a gapless state with 1FS at BEC side. Instability towards LOFF only at weak coupling. The current instabilty (phase modulation) is less stringent than the Higgs instability (Higgs mass).

3. Possible experimental consequence: steep increase of a vortex size in a gapless phase. Tune n_1, n_2, a_s to a gapless BEC; flat traps.

4. Future work. Vortex core structure in asymmetric SF. Momentum/energy dependent interaction.