Collective Flavor Transformation of Supernova Neutrinos

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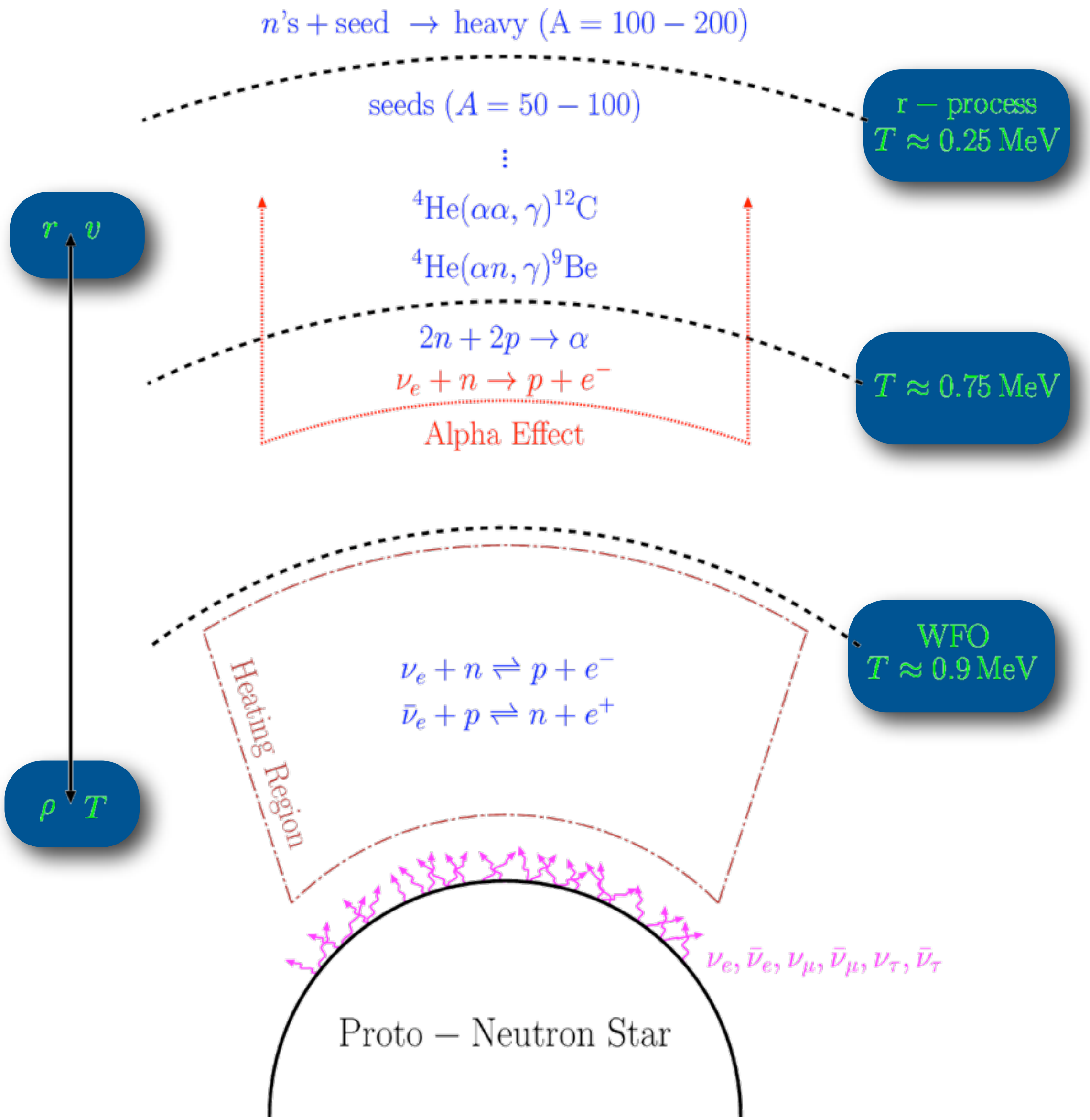
Neutrinos in SNe

- ★ Lots of neutrinos
- ★ Shock revival
- ★ Neutrino-driven wind
- ★ Nucleosynthesis
- ★ Infer neutrino mixing parameters
- ★ Probe conditions at the core
- ★ ......



Coherent limit

Coherent, 2x2: $$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}\begin{pmatrix} a_{\nu_e} \\ a_{\nu_{\tau'}} \end{pmatrix} = \mathsf{H}\begin{pmatrix} a_{\nu_e} \\ a_{\nu_{\tau'}} \end{pmatrix}$$

Vacuum mass differences:

$$\mathsf{H}_{\mathrm{vac}} \equiv \frac{\delta m^2}{4E_\nu}\begin{pmatrix} -\cos 2\theta_{\mathrm{v}} & \sin 2\theta_{\mathrm{v}} \\ \sin 2\theta_{\mathrm{v}} & \cos 2\theta_{\mathrm{v}} \end{pmatrix}$$

Charge current, forward scattering results in

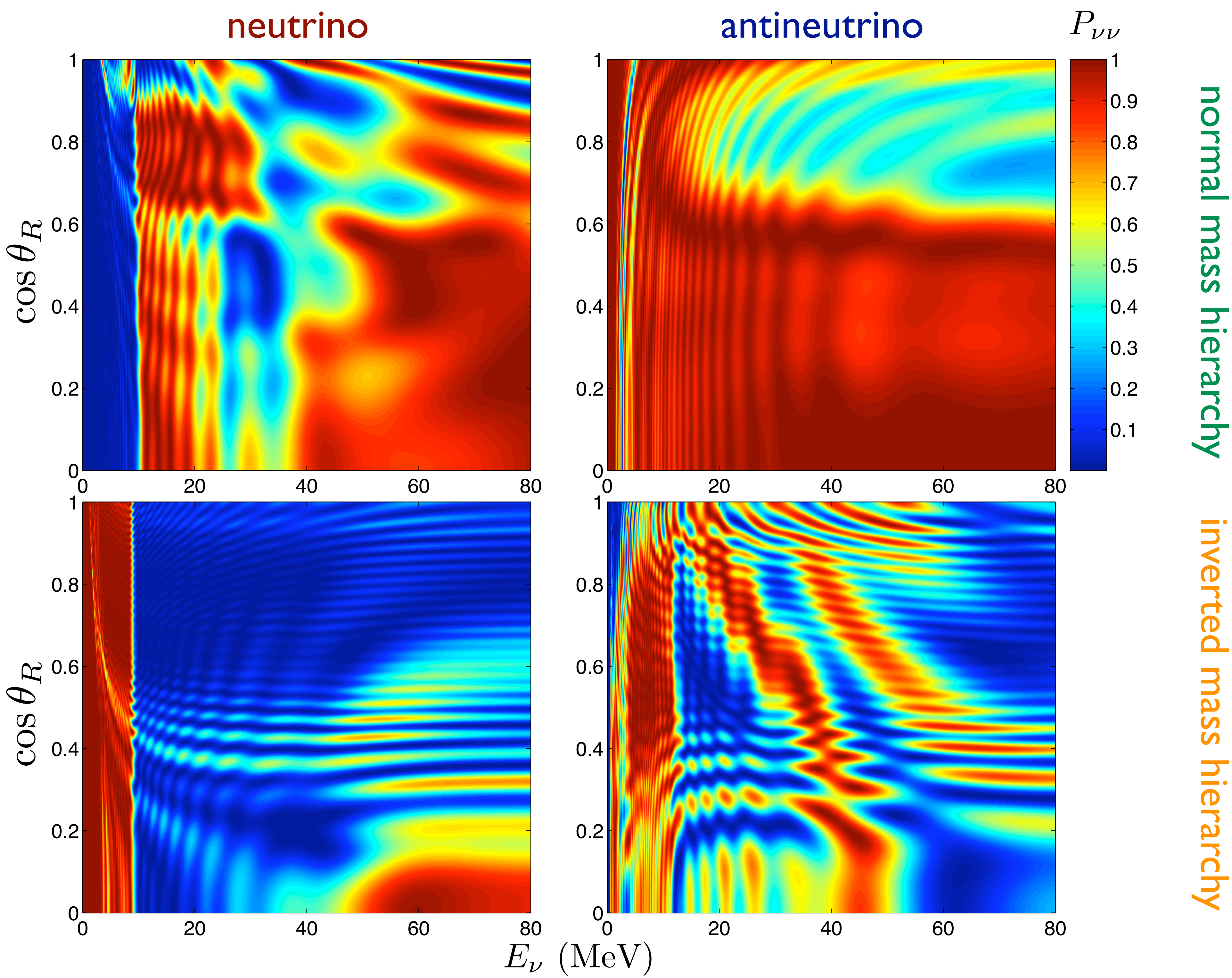
different refractive indices: $$\mathsf{H}_e \equiv \frac{G_{\mathrm{F}} n_e}{\sqrt{2}}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Neutrino-neutrino coupling

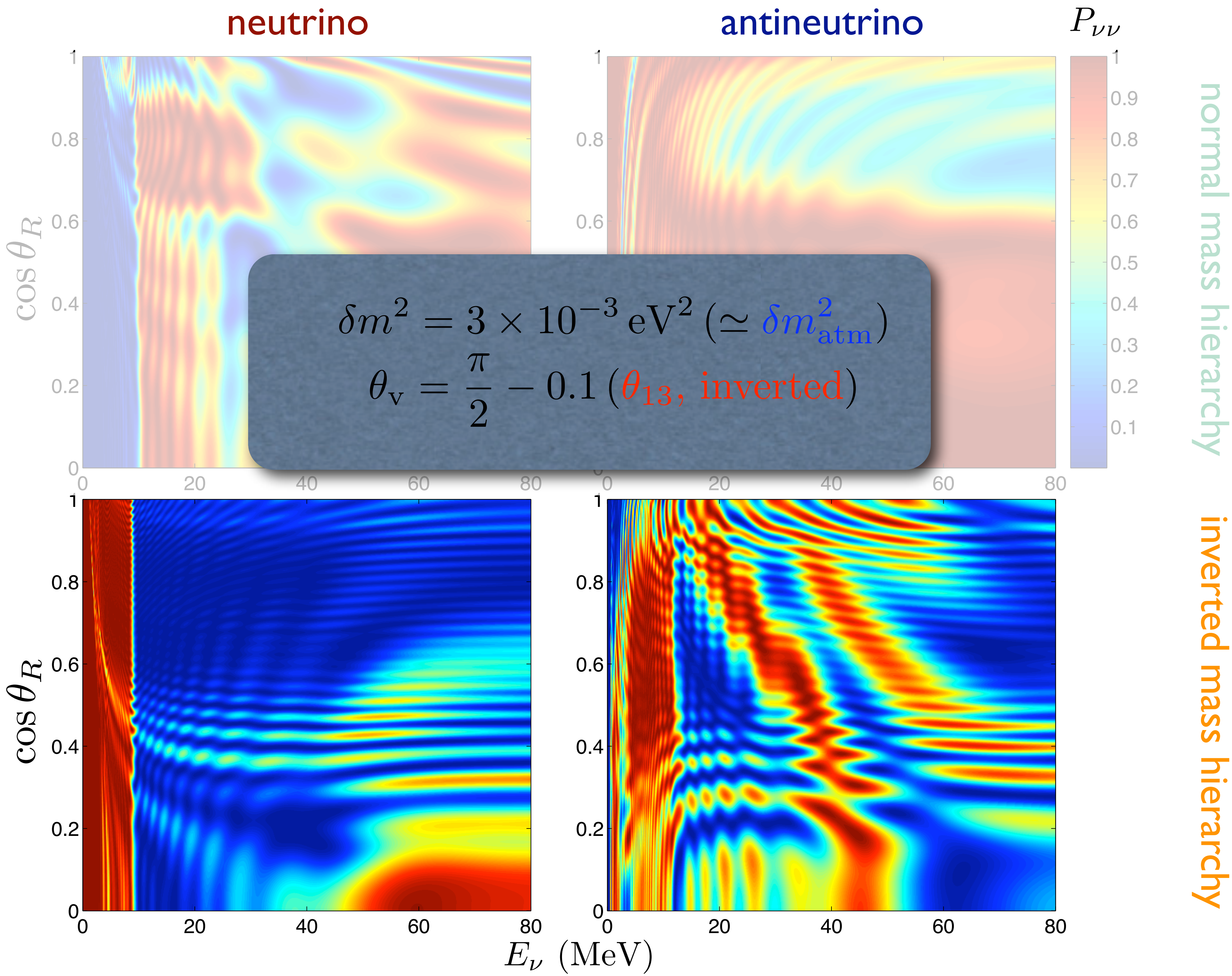
$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}\psi_{\nu,i} = (\mathsf{H}_{\mathrm{vac},i} + \mathsf{H}_e + \mathsf{H}_{\nu\nu,i})\psi_{\nu,i}$$

$$\mathsf{H}_{\nu\nu,i} \equiv \sqrt{2}G_{\mathrm{F}}\sum_j (1 - \hat{\mathbf{p}}_i \cdot \hat{\mathbf{p}}_j) n_{\nu,j}\psi_{\nu,j}\psi^\dagger_{\nu,j}$$
$$-\sqrt{2}G_{\mathrm{F}}\sum_{j'} (1 - \hat{\mathbf{p}}_i \cdot \hat{\mathbf{p}}_{j'}) n_{\bar{\nu},j'}(\psi_{\bar{\nu},j'}\psi^\dagger_{\bar{\nu},j'})^*$$

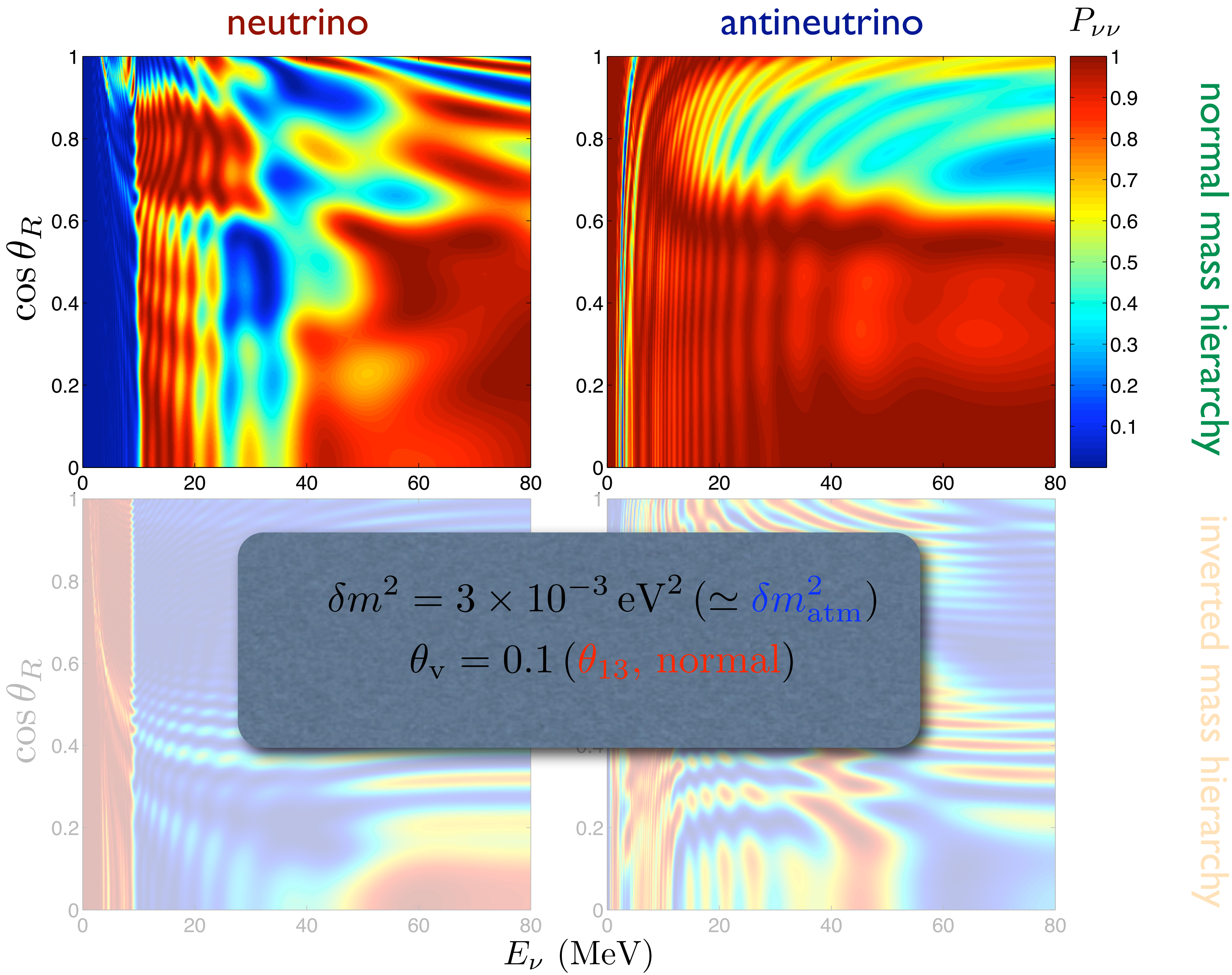
Neutrino oscillations in SNe



Neutrino oscillations in SNe



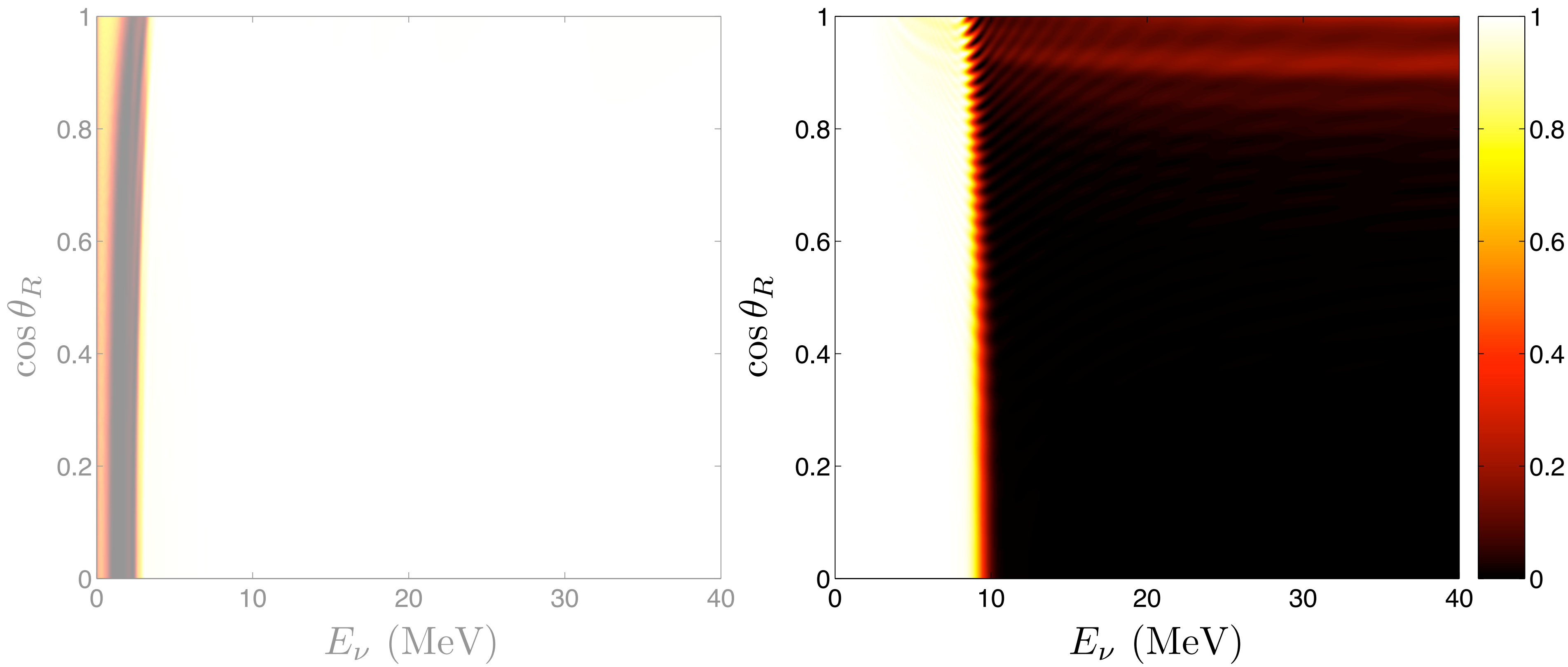
Neutrino oscillations in SNe



$$\delta m^2 = 3 \times 10^{-3}\,\mathrm{eV}^2 \; (\simeq \delta m^2_{\mathrm{atm}})$$

$$\theta_{\mathrm{v}} = 0.1 \; (\theta_{13},\ \mathrm{normal})$$

Sensitivity to θ_{13}



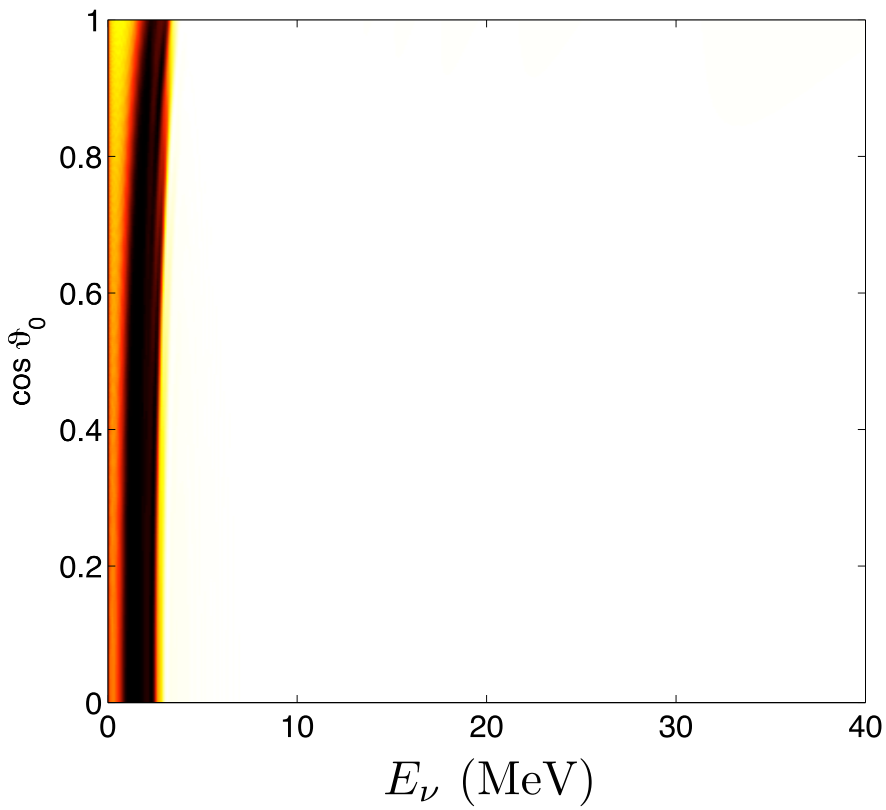
$$\delta m^2 = 3 \times 10^{-3}\,\mathrm{eV}^2\ (\simeq \delta m^2_{\mathrm{atm}})$$

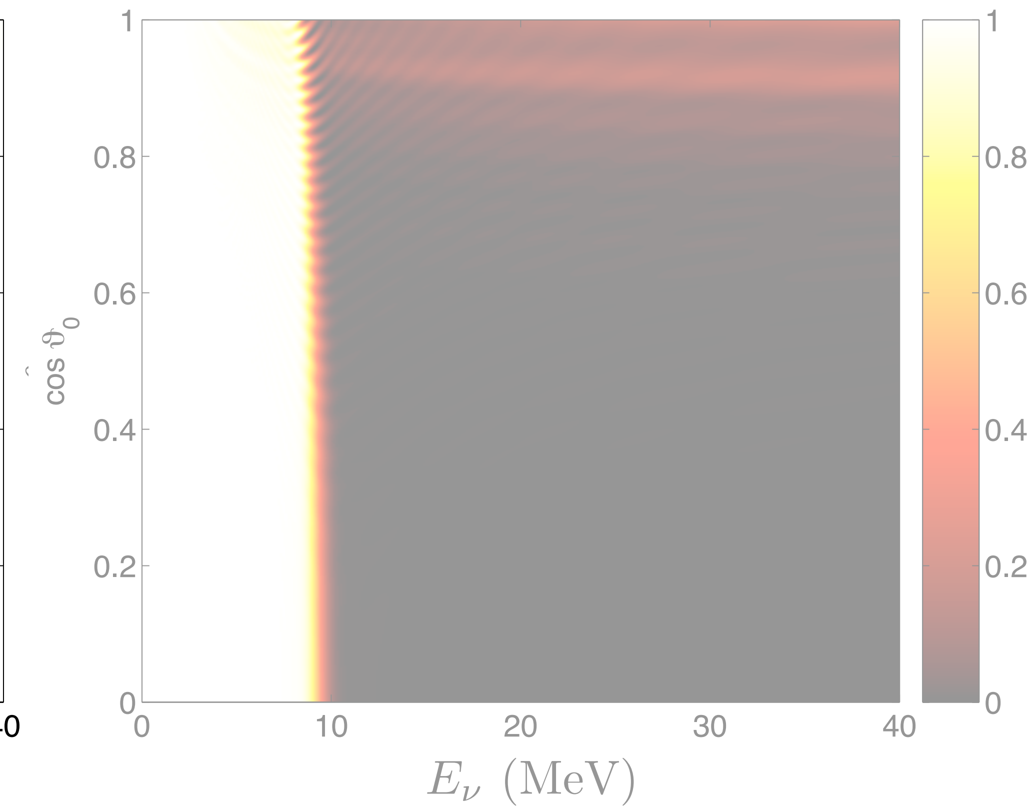
$$\theta_{\mathrm{v}} = 0.01\ (\theta_{13},\ \mathrm{normal})$$

$$\delta m^2 = 3 \times 10^{-3}\,\mathrm{eV}^2\ (\simeq \delta m^2_{\mathrm{atm}})$$

$$\theta_{\mathrm{v}} = \frac{\pi}{2} - 10^{-9}\ (\theta_{13},\ \mathrm{inverted})$$

Sensitivity to θ13





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Neutrino flavor isospin

Neutrino flavor isospin

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}\psi_\nu = \mathsf{H}\psi_\nu$$

$$= -\vec{H}\cdot\frac{\vec{\sigma}}{2}\psi_\nu$$

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\vec{s} = \vec{s}\times\vec{H}$$

$$\begin{bmatrix} e^- \\ \nu_e \end{bmatrix} \quad \begin{bmatrix} \mu/\tau \\ \nu_\mu/\nu_\tau \end{bmatrix}$$

Weak Isospin

Flavor Isospin

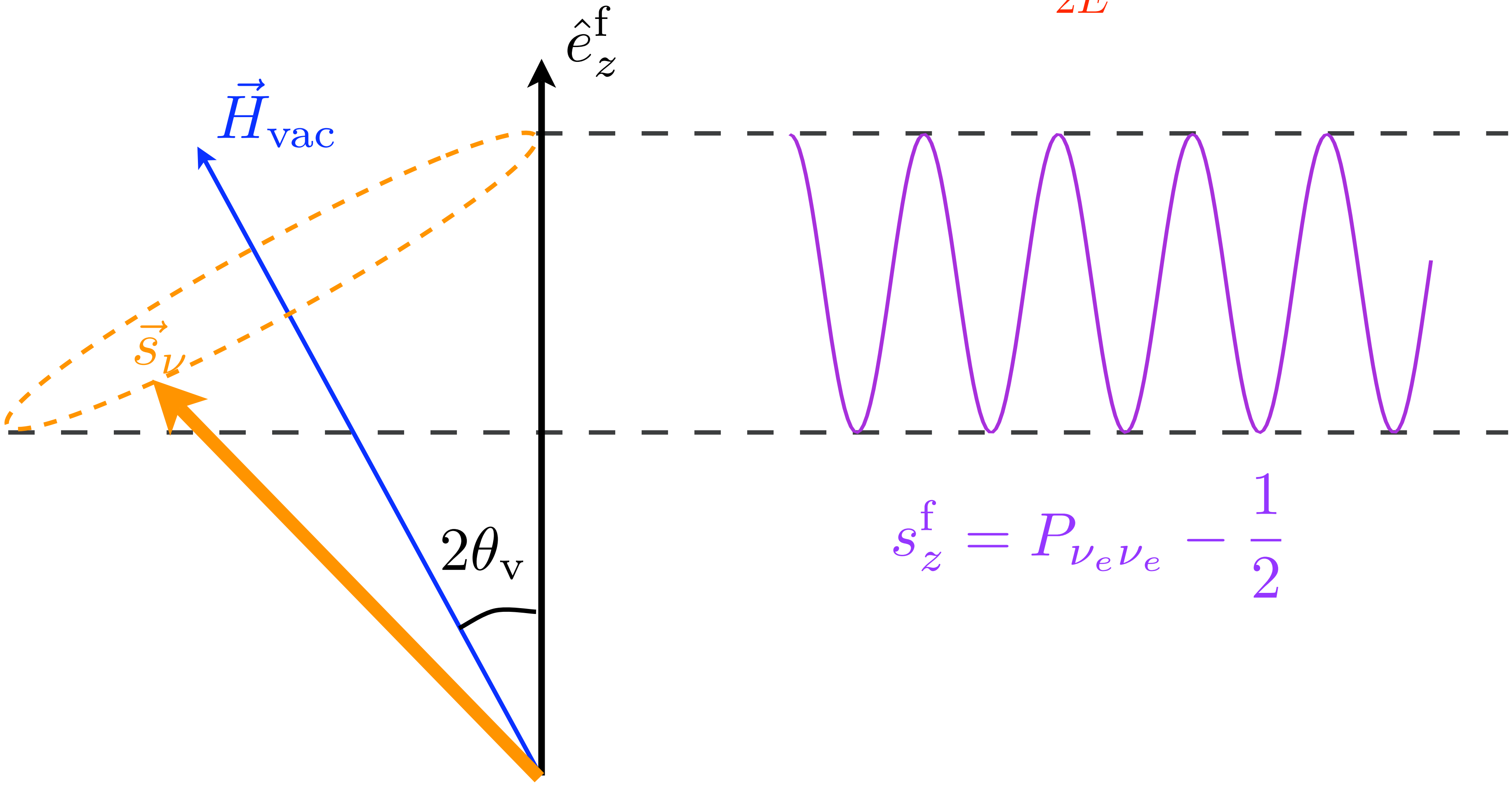
	e-flavor	τ'-flavor	maximally mixed
$\vec{s}_\nu \equiv \psi_\nu^\dagger \frac{\vec{\sigma}}{2}\psi_\nu$	$\uparrow$	$\downarrow$	$\rightarrow$
$\vec{s}_{\bar{\nu}} \equiv (\sigma_y\psi_{\bar{\nu}})^\dagger \frac{\vec{\sigma}}{2}(\sigma_y\psi_{\bar{\nu}})$	$\downarrow$	$\uparrow$	$\rightarrow$

Vacuum oscillations

$$\vec{H} = \omega \vec{H}_{\rm vac}$$

$$\vec{H}_{\rm vac} \equiv -\hat{e}^{\rm f}_x \sin 2\theta_{\rm v} + \hat{e}^{\rm f}_z \cos 2\theta_{\rm v}$$

$$\omega \equiv \pm \frac{\delta m^2}{2E}$$



$$s^{\rm f}_z = P_{\nu_e \nu_e} - \frac{1}{2}$$

Dense neutrino gases

Equation of motion: $$\frac{\mathrm{d}}{\mathrm{d}\lambda}\vec{s}_i = \vec{s}_i \times \vec{H}_i$$

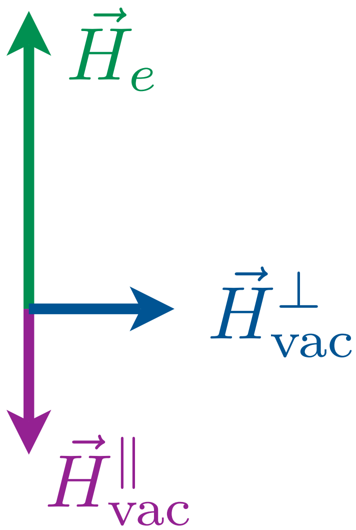
Effective field: $$\vec{H}_i = \omega_i \vec{H}_{\mathrm{vac}} + \vec{H}_e + \sum_j \mu_{ij} n_j \vec{s}_j$$

Matter field: $$\vec{H}_e \equiv -\hat{e}_z^{\mathrm{f}} \sqrt{2} G_{\mathrm{F}} n_e$$

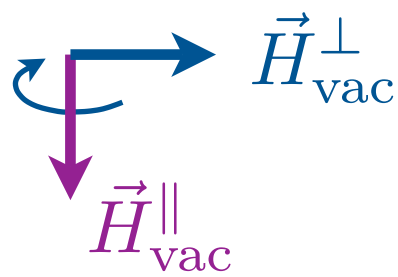
ν-ν coupling: $$\mu_{ij} \equiv -2\sqrt{2} G_{\mathrm{F}} (1 - \hat{\mathbf{p}}_i \cdot \hat{\mathbf{p}}_j)$$

Ordinary matter effects

static frame



corotating frame

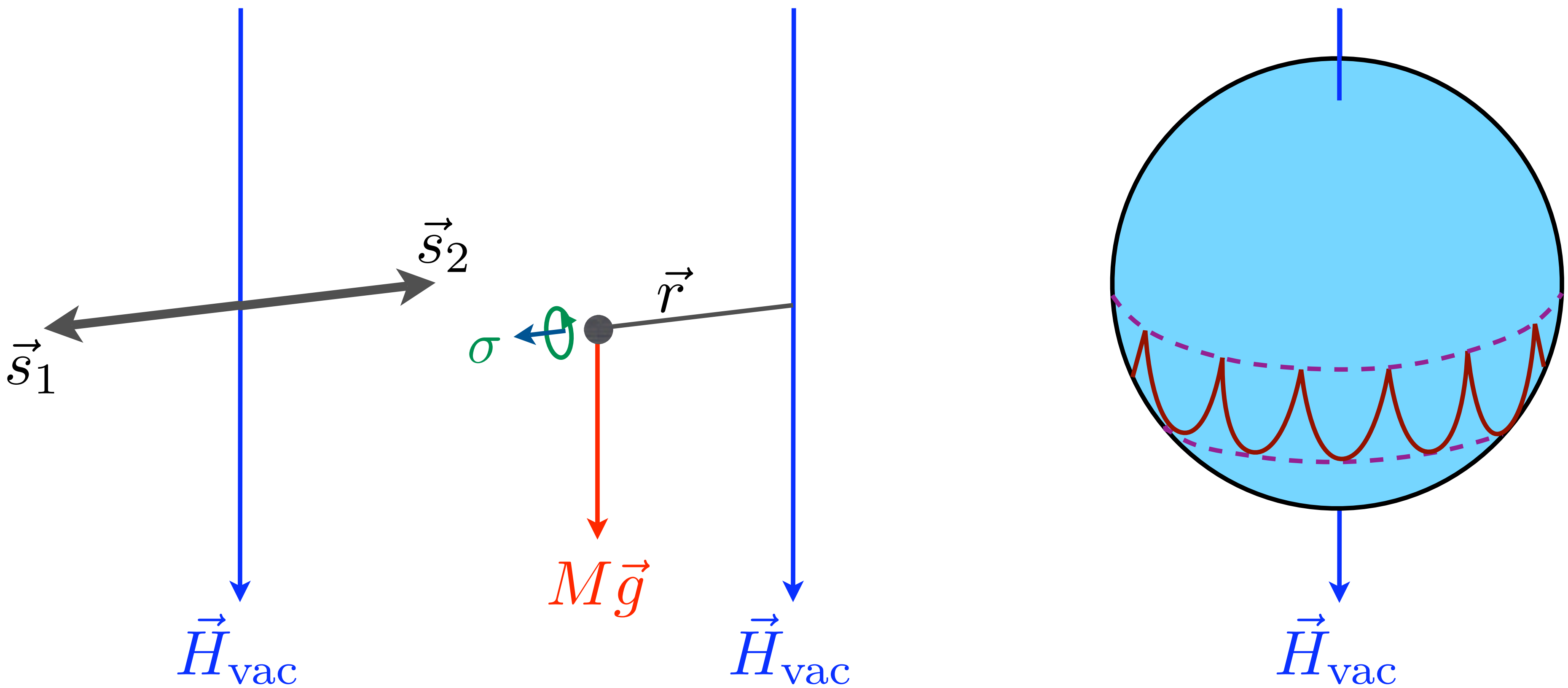


$\vec{H}_{\rm vac}^{\perp}$ can be ignored if $|\vec{H}_e + \vec{H}_{\rm vac}^{||}| \gg |\vec{H}_{\rm vac}^{\perp}|$

$$\vec{H}_{\rm vac} \longrightarrow \vec{H}_{\rm vac}^{||},\ \theta_{\rm v} \longrightarrow 0,\ \delta m^2 \longrightarrow \delta m^2 \cos 2\theta_{\rm v}$$

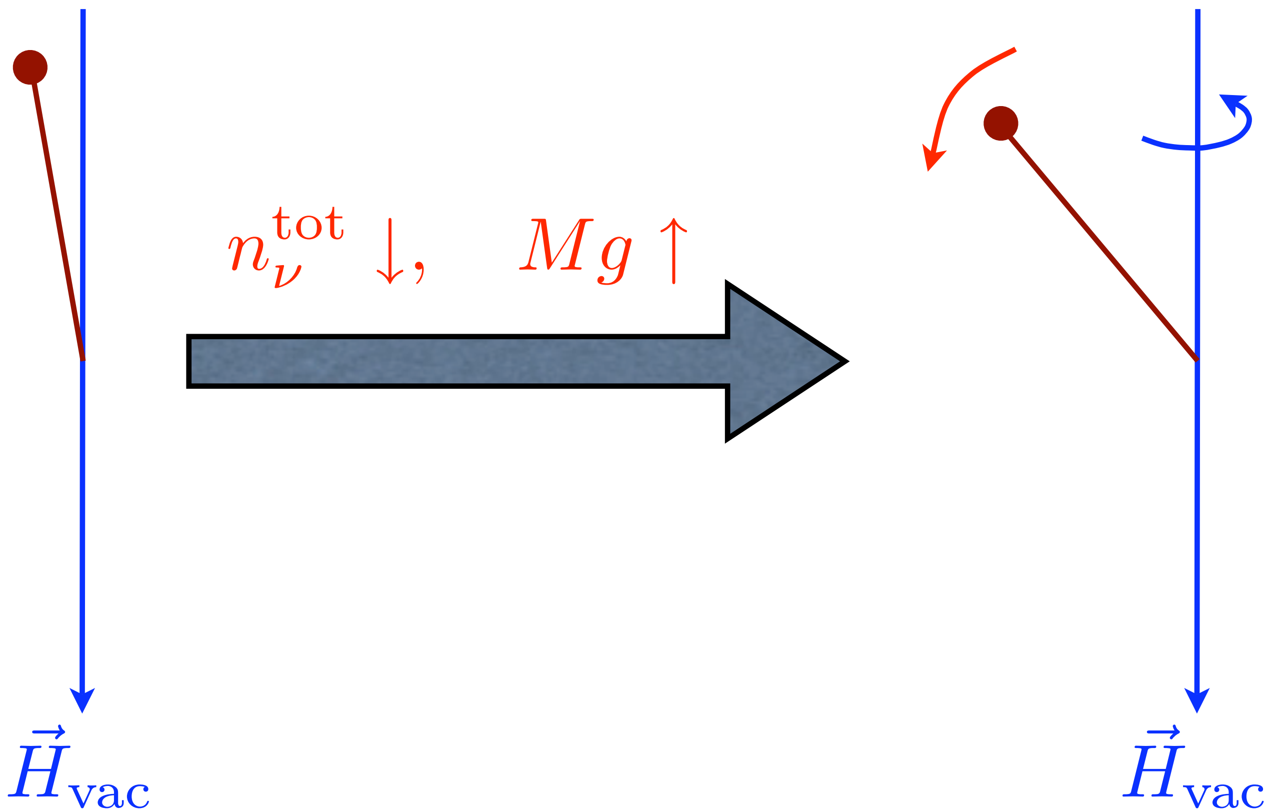
Flavor pendulum

Mono-energetic ν_e-$\bar{\nu}_e$ gas, $n_{\nu,1} = n_\nu$ and $n_{\nu,2} = \alpha n_\nu$.

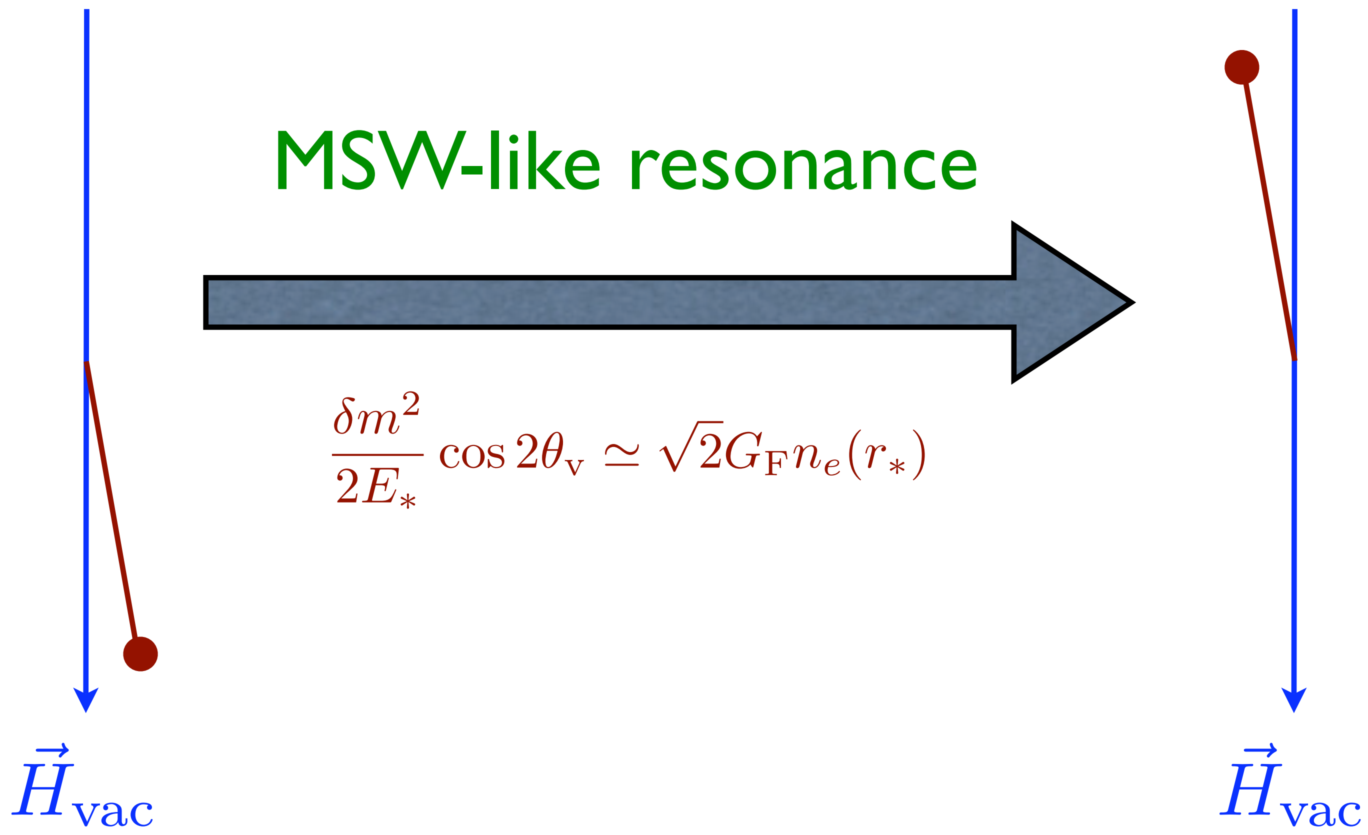


$$\sigma \sim \frac{n_{\nu_e} - n_{\bar{\nu}_e}}{n_{\nu_e} + n_{\bar{\nu}_e}} \qquad M\vec{g} \sim \frac{\vec{H}_{\rm vac}}{n_{\nu_e} + n_{\bar{\nu}_e}}$$

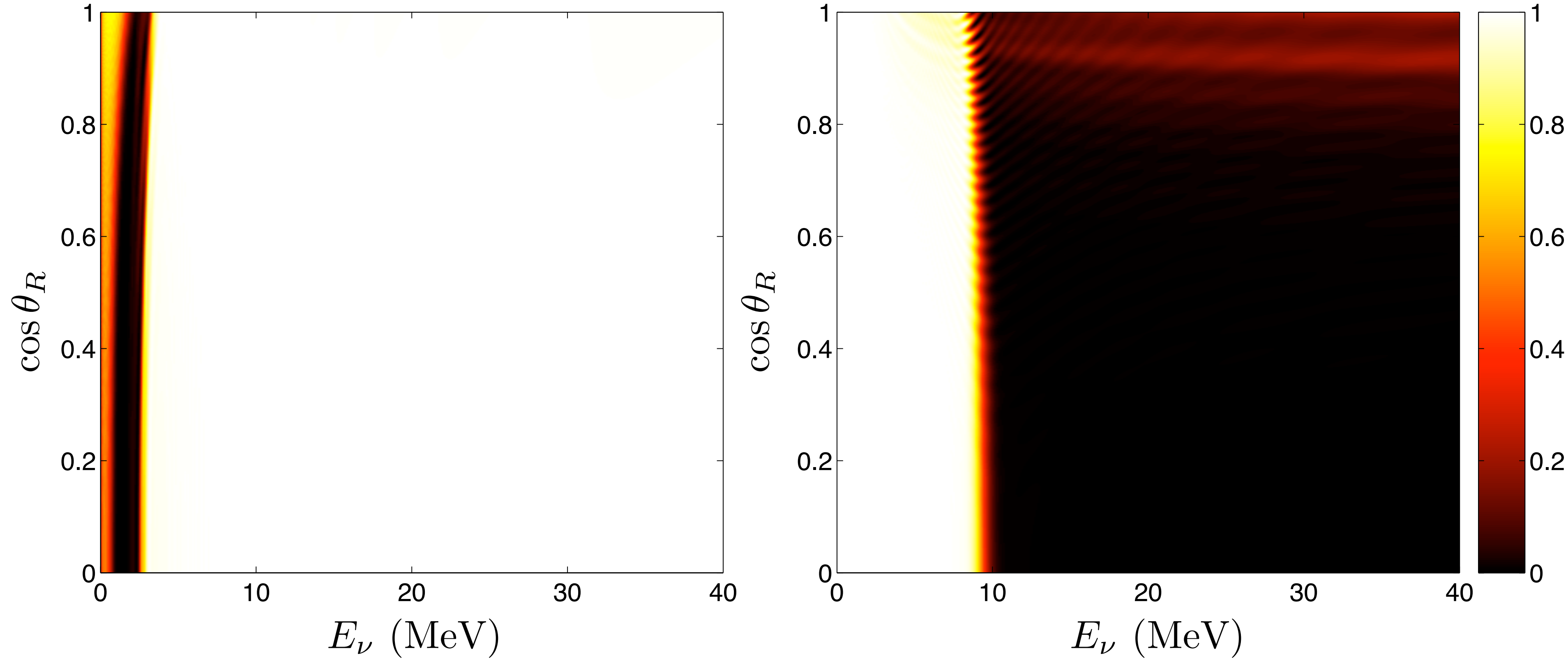
Inverted mass hierarchy
($\theta_V \approx \pi/2$)



Normal mass hierarchy ($\theta_V \approx 0$)



Sensitivity to θ13



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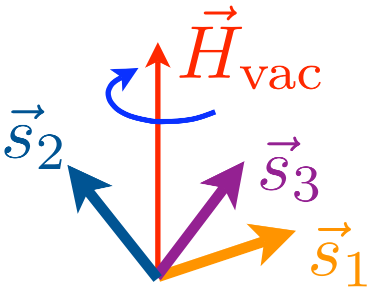
Symmetry in EOM

Assuming a static configuration:

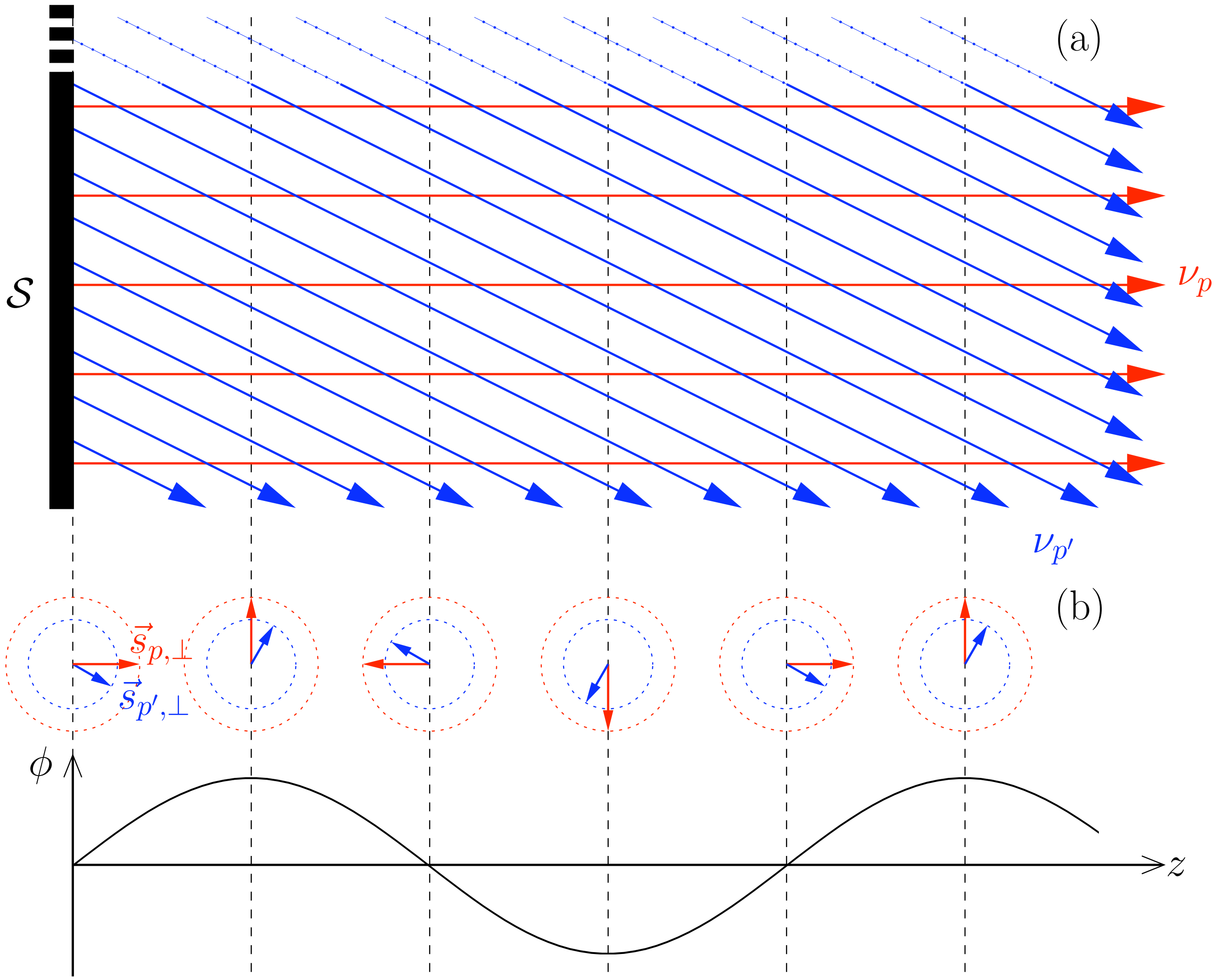
$$\hat{\mathbf{p}}_i \cdot \nabla \vec{s}_i(\mathbf{r}) = \vec{s}_i(\mathbf{r}) \times \left[\omega_i \vec{H}_{\rm vac} + \sum_j \mu_{ij} n_j(\mathbf{r}) \vec{s}_j(\mathbf{r})\right]$$



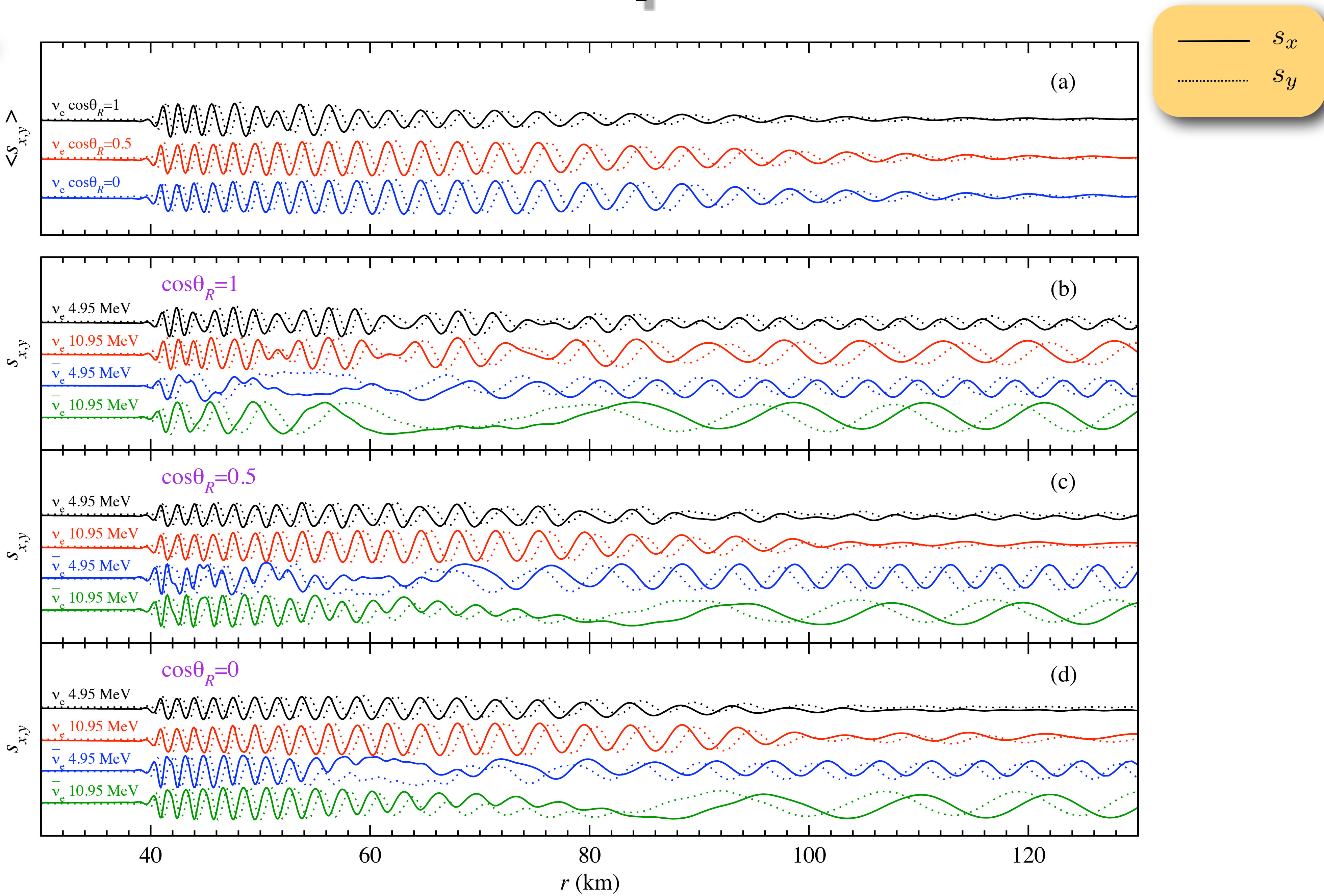
EOM is invariant under rotation around $\vec{H}_{\rm vac}$

Flavor spin wave

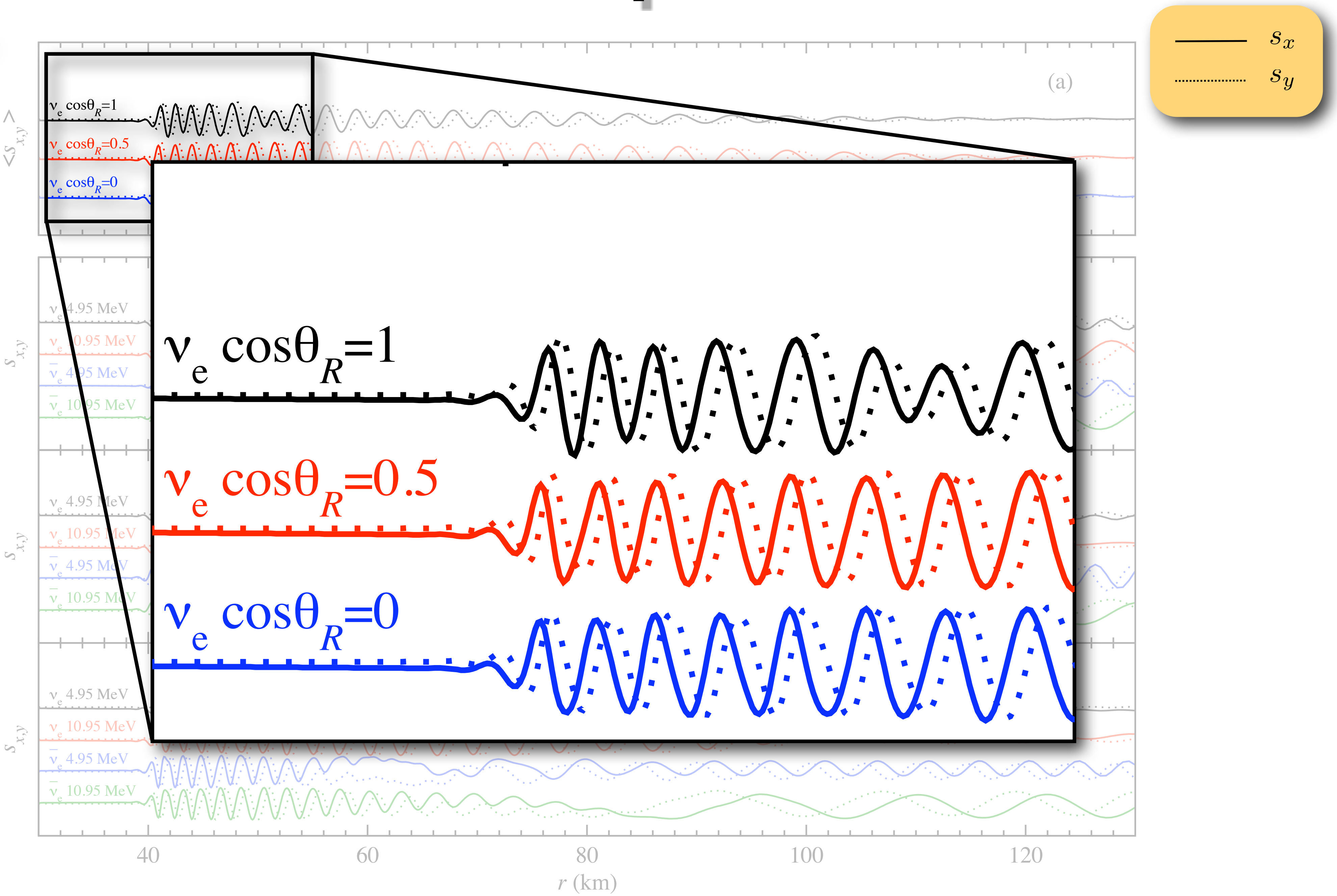
Static, homogeneous neutrino gas:



Flavor spin wave

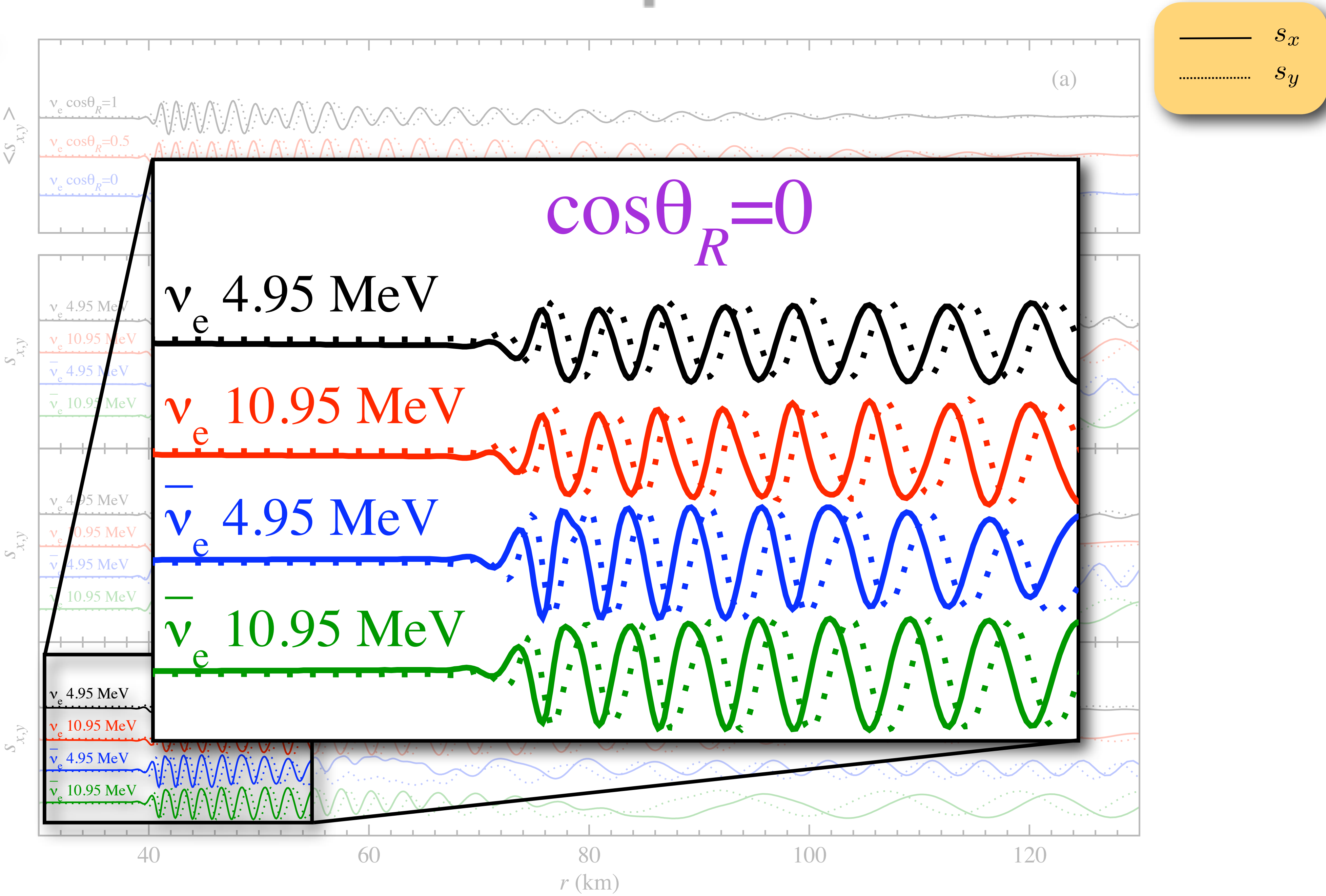


Flavor spin wave



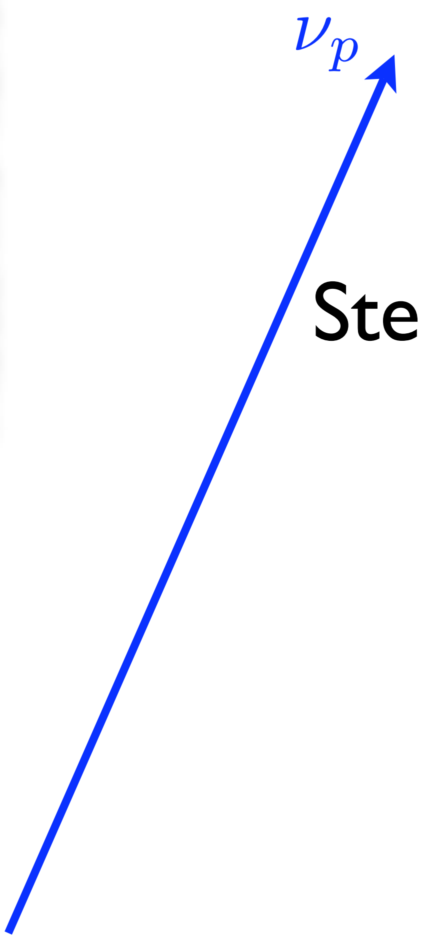
Neutrino flavor spin wave

Flavor spin wave



Flavor spin wave

Pure collective precession mode:



EOM: $$\frac{\mathrm{d}}{\mathrm{d}\lambda}\vec{s}_p = \vec{s}_p \times \vec{H}_p$$

Steady precession: $$\frac{\mathrm{d}}{\mathrm{d}\lambda}\vec{s}_p = \vec{s}_p \times (\mathbf{K}\cdot\hat{\mathbf{p}})\vec{H}_{\mathrm{vac}}$$

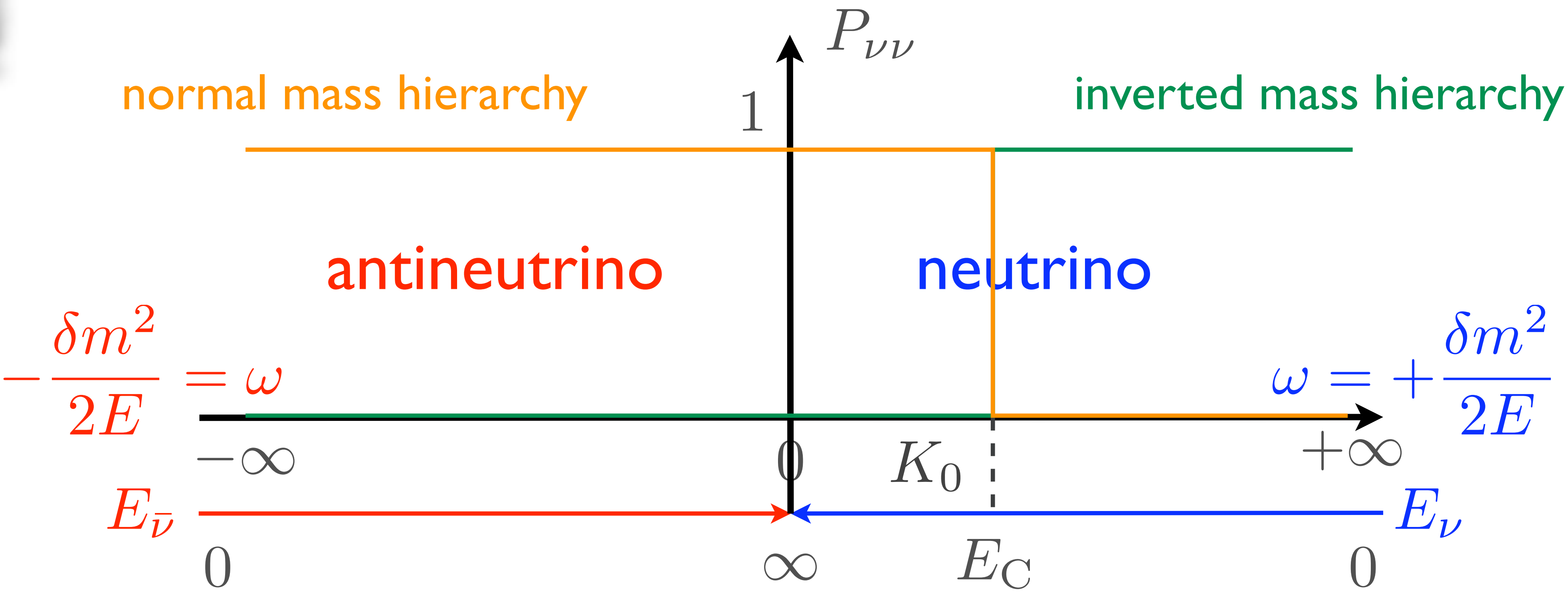
$$\vec{s}_p \parallel \frac{\epsilon_p}{2}[\vec{H}_p - (\mathbf{K}\cdot\hat{\mathbf{p}})\vec{H}_{\mathrm{vac}}]$$

$$\epsilon = \pm 1$$

Stepwise spectral swap

At large radius: $\hat{\mathbf{p}} \parallel \mathbf{K}, \quad \vec{H}_p \longrightarrow \omega \vec{H}_{\mathrm{vac}}$

$$\vec{s}_\omega = \frac{\epsilon_\omega \vec{H}_{\mathrm{vac}}}{2} \mathrm{sgn}(\omega - K_0)$$



Summary

- Collective neutrinos oscillations in supernovae because of large neutrino fluxes.
- Stepwise spectral swap can be used to probe the neutrino mass hierarchy even θ_{13} is tiny.
- A simple symmetry behind collective neutrino oscillations.