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Identification and Simulation of a Common Freeway Accident Mechanism: Collective Responsibility in Freeway Rear-end Collisions

Final Report

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16. Abstract (Limit: 200 words)

Determining whether or not an event was a cause of a road accident often involves determining the truth of a counterfactual conditional, where what happened is compared to what would have happened had the putative cause been absent. Using structural causal models, Pearl and his associates have recently developed a rigorous method for posing and answering causal questions, and this approach is especially well-suited to the reconstruction and analysis of road accidents. Here we apply these methods to freeway rear-end collisions. Starting with video recordings of accidents on I-94, trajectory information on a platoon of vehicles involved in a crash is extracted from the video record and these trajectories are then used to estimate each driver's initial speed, following distance, reaction time, and braking rate. Using Brill's model of rear-end accidents it is then possible to simulate what would have happened had, other things equal, certain driver reactions been other than they were. In each of three accidents we found evidence that:

(1) short following headways by the colliding drivers were probable causal factors for the collisions,

(2) for each collision at least one driver ahead of the colliding vehicles probably had a reaction time that was longer than his or her following headway, and

(3) had this driver's reaction time been equal to his or her following headway, the rear-end collision probably would not have happened.

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Introduction

Although traffic accidents on congested freeways do not usually result in fatal or even very severe injuries, they are responsible for a substantial fraction of the unpredictable delays many of us now regard as unavoidable aspects of urban life. Frequently, such accidents occur when a platoon of vehicles successively brake and the braking deceleration of at least one vehicle is not sufficient to prevent it from colliding with the vehicle ahead. Reducing the frequency of such collisions, for example by improving the competency of drivers or deploying in-vehicle collision-avoidance technology, could help reduce travel delays without resorting to expensive additions to highway capacity. Responsibility for preventing rear-ending collisions now rests mainly with drivers, and some may recall the older recommendation to maintain one car length of separation for each 10 mph of speed, or the more recent recommendation to maintain at least a 2.0 second gap with the vehicle ahead (e.g. DFT 1999). When a collision occurs, responsibility is almost always assigned to the drivers involved, and most often to the following driver.

In their classic car-following model, researchers at the General Motors Research Laboratories (Herman et al. 1959; see also Gazis 2002, pp. 21-33) modeled the behavior of drivers in a platoon of vehicles using a coupled system of differential equations, where each driver's acceleration or deceleration was, after a reaction time lag, assumed to be proportional to the difference between his or her speed and the speed of the vehicle ahead. One implication of this model was that, for certain combinations of reaction time and sensitivity to speed differences, the magnitude of a change in acceleration or deceleration by the leader of a platoon was amplified by each succeeding driver, so that if the platoon was long enough a collision became inevitable. Since all drivers in the platoon were assumed to have the same reaction times and sensitivities to speed differences, whether or not a driver was in a collision depended solely on his or her place in the platoon. Responsibility for the collision would then more appropriately be assigned to the platoon as a whole rather than to the colliding drivers. Herman et al (1959) also reported some limited empirical evidence supporting the notion that individuals tend to drive near the limit where this instability occurs.

Because the General Motors car-following model did not readily allow for individual differences, it was not possible to investigate situations where some drivers may have been more responsible than others. Brill (1972) described a relatively simple kinematic model of successive braking which supports these distinctions. Imagine a platoon of vehicles indexed in order from first to last by k=1,..,n, and let $v_1, v_2, \ldots v_n$ denote their speeds. At time t=0 the lead driver brakes to a stop, with deceleration a_1, and after a reaction time r_2 driver 2 also brakes to stop, with deceleration a_2, and so forth. A rear-end collision between vehicles k and k+1 will be avoided as long as the distance needed by driver k+1 to stop does not exceed the available stopping distance. That is,

$$x_{k+1} + \frac{v_k^2}{2a_k} \geq r_{k+1} v_{k+1} + \frac{v_{k+1}^2}{2a_{k+1}} \tag{1}$$

where x_{k+1} is the distance separating vehicle k's rear bumper from vehicle k+1's front bumper. Letting $x_{k+1}=v_{k+1}h_{k+1}$ express this distance in terms of driver k+1's speed and following headway, driver k+1 will stop before colliding if his or her deceleration satisfies

$$a_{k+1} \geq \frac{v_{k+1}^2}{\frac{v_k^2}{a_k} + 2v_{k+1}(h_{k+1} - r_{k+1})} \tag{2}$$

Inequality (2) has some interesting implications. Other things equal, the minimum deceleration required of driver k+1 increases as the deceleration used by driver k increases, since k+1's available stopping distance decreases as a_k increases. Also, other things equal, the minimum deceleration required by driver k+1 increases as the difference between k+1's following headway and reaction time ($h_{k+1}-r_{k+1}$) decreases. Together these features imply, as Brill pointed out, that if each driver in the platoon is little slow in reacting, so that his or her reaction time is longer than the following headway, the minimum required deceleration will tend to increase for each succeeding vehicle. If the platoon is long enough a collision again becomes inevitable, and as before it would appear reasonable to attribute the accident to the actions of each driver in the platoon, rather than to an egregious lapse by the last driver. But if the actions of drivers earlier in a platoon help set up the conditions for a collision then the traditional practice of penalizing only those drivers actually involved in the collision will leave the other drivers unaware of their contributions, and so be of limited effectiveness. But how can we assess the causal contributions, if any, of these other drivers?

Causal Concepts

Baker (1975) has noted that causal attributions in road safety take a number of forms, and are often invoked to achieve rhetorical, rather than scientific, objectives. He has also given an often-used definition of "causal factor" as a circumstance "contributing to a result without which the result could not have occurred." Implicit in this definition is first, that removal of a causal factor should be sufficient to prevent the result, and second that one determines whether or not a circumstance is a causal factor by carrying out a counterfactual test, where what happened is compared to what would have happened had the circumstance in question been absent. In practice, however, giving a rigorous yet general specification of such tests has proved somewhat daunting, the main challenge being to unambiguously specify what should count as the counterfactual condition. Since one can, with sufficient imagination, almost always describe a number of different scenarios where an accident is avoided, this test condition should involve a change that is in some sense minimal. Lewis (1973) has given a philosophical treatment of truth conditions for causal assertions using a comparison between what actually happened and what happens in a closest possible world where certain counterfactual assertions are true. What is meant by "closest possible world" is left deliberately vague, which improves the generality of Lewis's treatment but makes it difficult to apply to practical cases. Over the past 15 years or so however, there has been increased interest in causal inference as a component of artificial intelligence, and one especially useful approach is based on what Pearl (2000) calls a "causal model." This is "a mathematical object that assigns truth values to sentences involving causal relationships, actions, and counterfactuals." (Tian and Pearl 2000, p. 290) To construct a causal model one identifies a set of exogenous variables, a set of endogenous variables, and for each endogenous variable a structural equation describing how that variable changes in response to changes in the exogenous and/or other endogenous variables. Events are defined in terms of values taken on by the model's variables. The closest possible world where a set of variables takes on (counterfactual) values can be unambiguously defined as the outcome of a modified causal model, where the exogenous variables are set to the same values as in the actual condition, but where the structural equations associated with the counterfactual event are replaced by assignment statements. Arguably, this provides a rigorous specification of the insight underlying Baker's definition of causal factor.

To illustrate how these ideas might be applied to a freeway rear-end accident consider Figure 1 on page 15, which displays Brill's sequential braking model (in this case involving a three-vehicle platoon) as a directed acyclic graph. The nodes of the graph represent the model's variables while the arrows indicate the presence and direction of causal dependencies. Those nodes without arrows pointing toward them (such as v_1) represent exogenous variables, while the others (such as a_{20}) represent endogenous variables. To complete the model we need to specify, for each endogenous variable, a structural equation. The variables a_{20} and a_{30} are the minimal decelerations needed, for vehicles 2 and 3 respectively, to stop before colliding with the vehicle ahead. These are determined from the right-hand side of relation (2). The actual decelerations are then assumed to be determined as

$$a_k = \min(a_{k0} + u_k, \boldsymbol{a}) \tag{3}$$

where $\boldsymbol{a}$ is a maximum achievable deceleration, and u_k accounts for the difference between observed and minimum deceleration. Finally, the variable y is a collision indicator, and is assumed to be determined via

$$y = \begin{cases} 0, \text{ if } a_{30} \le \boldsymbol{a} \\ 1, \text{ if } a_{30} > \boldsymbol{a} \end{cases} \tag{4}$$

For example, suppose $v_1=v_2=v_3=40$ fps, that the maximum achievable deceleration is $a= 20$ fps^2, and that driver 1 brakes to a stop with $a_1=5$ fps^2. Suppose also that $h_2=2$ seconds but $r_2=4$ seconds, so that driver 2's minimum deceleration is $a_{20}=10$ fps^2, and that driver 2 then decelerates at 10.5 fps^2 (which means that $u_2=0.5$ fps^2). Further, suppose driver 3 is tailgating a bit, with $h_3=1.5$ seconds, and reacts after $r_3=2.5$ seconds. The minimum deceleration for driver 3 is then $a_{30}=22.1$ fps^2, which exceeds $a=20$ fps^2, and a rear-end collision between vehicle 2 and vehicle 3 occurs. Driver 3's tailgating can be considered a causal factor of this collision, since, if we counterfactually set $h_2=2.0$ seconds but fix v_2, a_2 and v_3 at their actual values, the minimum deceleration needed by driver 3 falls to $a_{30}=14.2$ fps^2, and so, other things equal, the collision is avoided. But driver 2's long reaction time could also be considered a causal factor, since setting $r_2=2.5$ seconds, but keeping $u_2=0.5$ fps^2, leads to $a_{30}=9.0$ fps^2.

In the above example we assumed exact knowledge of the values taken on by the collision model's variables, but in practice knowledge of these values will almost always be to some degree uncertain. Pearl defines a probabilistic causal model as a causal model augmented with a probability distribution over the values taken on by the model's exogenous variables, and this probability distribution can be used to determine the probabilities attached to the truth or falsity of counterfactual propositions. Balke and Pearl (1994) showed how, for models which admit a graphical representation such as that in Figure 1, the probabilities attached to counterfactual statements can be computed by augmenting the model with nodes reflecting the counterfactual situation and then using algorithms for performing Bayesian updating for graphical models. Existing algorithms for performing exact updating are not well-suited to accident modeling, but it is fairly straightforward to compute approximate updates using Markov Chain Monte Carlo computational methods (Davis 2003).

Application to Actual Collisions

As part of an ITS Institute study on "Identification of Accident Prone Conditions," video cameras were installed on high-rise buildings adjacent to Interstate 94 (I-94) south of downtown Minneapolis. This location has the distinction of producing a large number of accidents due to its heavy traffic conditions throughout the day and also to the frequency of weaving and merging maneuvers resulting from a rather complicated interchange geometry. The cameras were connected to a computer that recorded the weekday traffic movements from the early-morning rush hour to the early evening. Video records were saved in one-hour segments on the computer's hard drive. Accident reports filed with the State Patrol and incident reports recorded by the Minnesota Dept. of Transportation's Traffic Management Center were then used to determine which video segments contained accident footage. As of May 31, 2003 three collisions had occurred at locations where a camera's distance and angle made it possible to measure vehicle trajectory information from the video recordings.

The computer program VideoPoint was used to extract the (x,y) screen coordinates of vehicles from a frame of the recorded video by clicking on a discernable point on the object of interest. The program then advanced the movie one frame and the process was repeated, so by successively clicking on the same point of a vehicle's image it was possible to record the sequence of coordinates representing the vehicle's trajectory. Standard photogrammetry transformations (Bleyl 1976) were then used to convert the screen coordinates to the corresponding real-world coordinates. For example, Figure 2 shows the trajectories of a platoon of seven vehicles involved in sequential braking maneuvers, recorded during an afternoon peak period on westbound I-94, where the seventh vehicle was observed colliding with the sixth.

To assess the possible causal contributions of the drivers in a platoon, it was first necessary to determine values for the individual speeds, decelerations, reaction times and following headways. This was done by fitting trajectory models of the form,

$$x_k(t) = \begin{cases} v_k t, & t \le t_{0k} \\ v_k t - 0.5\, a_k(t-t_{0k})^2, & t_{0k} < t \le t_{0k}+v_k/a_k \\ v_k t_{0k} + v_k^2/2a_k, & t > t_{0k}+v_k/a_k \end{cases} \qquad (5)$$

where t_{0k} is the time at which driver k began braking. That is, prior to the initiation of braking at time t_{0k} the vehicle was assumed to be travelling at a constant speed v_k, and that after coming to a stop at time $t_{0k} + v_k/a_k$ the vehicle's location remains unchanged. Equation (5) can be viewed as a nonlinear model for predicting a vehicle's location, parameterized by the initial speed v_k, the braking initiation time t_{0k}, and the braking deceleration a_k. In principle one could then use nonlinear least-squares to identify those parameter values which gave the best fit of equation (5) to a vehicle's trajectory data. Alternatively, one could assume non-informative prior probability distributions for these parameters, and then treat the actual trajectory data as error-prone measurements from a process governed by equation (5). Bayes theorem could then be used to compute posterior probability distributions for the trajectory model parameters. The Markov Chain Monte Carlo (MCMC) program WinBUGS (Spiegelhalter et al 2000) was used to compute Bayes estimates of the trajectory model parameters for each of the vehicles in each of our three accidents. A listing of the WinBUGS model used for one of our accidents is given in the Appendix.

The term "space headway" is used to describe the distance between two successive vehicle front ends at the instant the leading vehicle begins braking. These values can be determined from the trajectories using the estimated braking times, as depicted in Figure 5. Space headways can then be converted to following distances by subtracting the length of the leading vehicle from the space headway, and these in turn converted into separation headways (h_k) by dividing by the speed of the following vehicle. Finally, reaction times were defined as the difference in time between when the leading vehicle began to brake and the time when the following vehicle began to brake, and Figure 6 illustrates how these can be determined from the vehicle trajectories.

Figure 2 on page 16 shows the observed vehicle trajectories for two vehicles involved in a rear-ending crash on December 30, 2002 at about 4:55 PM, along with trajectories for the five vehicles preceding these. Numbering the vehicles from left to right, it can be seen that the driver of vehicle 1 came to a stop first about 34.3 seconds from the beginning of the video clip, the driver for vehicle 2 came to a stop about 2 seconds later, and so forth. The collision was between the two left-most vehicles, 6 and 7, and inspection of the accident report revealed that the investigating officer cited contributing factors for driver 7, while the driver of vehicle 6 was considered to have done nothing improper.

Table 1 on page 20 displays summaries of the posterior distributions for the collision model variables, and the entries in Table 1 tell an interesting story. If we take the posterior means as the best point estimates of the variables' values, it appears that driver 1 was initially travelling at about 50 fps and at 28.2 seconds after the start of the video segment began braking to a stop with a deceleration of about 6.8 fps^2. About 1.9 seconds later driver 2 began braking with a deceleration of about 6.5 fps^2. Since driver 2's following headway and reaction time were approximately equal and driver 2's speed was probably less than that of driver 1, it was possible for driver 2 to decelerate at about the same rate as driver 1. Driver 3 on the other hand, although travelling slower than driver 2, probably needed over 4 seconds to react, and because 3's following headway was only around 2.0 seconds, the minimum deceleration for driver 3 jumped to about 11.4 fps^2, with an actual deceleration of about 12.6 fps^2. For drivers 4, 5 and 6 roughly comparable reaction times and small differences in speed appear to cancel each other, with the net effect that the minimum necessary deceleration increased somewhat for each. When we come to driver 7, whose reaction time was approximately 0.4 seconds longer than his/her headway, the minimum deceleration jumped to about 24.8 fps^2, which exceeds the 20.3 fps^2 observed to have been used by driver 7, and a collision resulted.

So who, if anyone, was responsible for this accident? If we use the posterior means from Table 1 as the best estimates for the quantities appearing in relation (2), it is straightforward to verify that if driver 7 had had a following headway of 2.0 seconds, his or her minimum deceleration would decrease to about 13.0 fps^2, which is noticeably lower than driver 7's observed deceleration, and suggests that driver 7's failure to maintain a recommended following headway was a causal factor. In actuality we do not know any of the values taken on by the collision model's variables with certainty, and it may be that for plausible values, other than the posterior means, the collision still occurs. To assess this possibility, the Twin Network method of Balke and Pearl (1994) was used to compute the probability distribution for driver 7's minimum deceleration, on the (counterfactual) assumption that driver 7's following headway equaled 2.0 seconds. The posterior mean for driver 7's minimum deceleration under this counterfactual condition equaled about 13.0 fps^2, with a standard deviation of 0.5 fps^2, and the probability that the counterfactual minimum deceleration was less than driver 7's actual deceleration was 1.0. That is, in 15,000 iterations of the MCMC algorithm, an outcome where the actual deceleration was less than the counterfactual minimum never occurred. So we can conclude that quite probably driver 7's failure to maintain the recommended following headway was a causal factor for this accident. But now let's look at driver 3. His or her reaction time was clearly long compared to what other drivers in the platoon appeared capable of, and we can ask whether or not this long reaction time might also have been a causal factor. Counterfactually setting 3's reaction time to his/her following headway, leaving all observed speeds and headways, and all other observed reaction times alone, and then computing the actual decelerations for drivers 4, 5, and 6 by adding the observed differences u_k to the new computed minima, produced a posterior mean value for driver 7's minimum deceleration of about 12.2 fps^2, with a standard deviation of about 0.7 fps^2. The probability assigned to the set of situations where the counterfactual minimum deceleration for driver 7 is lower than his/her observed deceleration was 1.0. So it appears that had driver 3's reaction time been equal to his/her following headway the collision would, other things equal, also have been prevented and we can conclude that driver 3's long reaction time was also a causal factor. What is interesting here is that driver 3 was not actually involved in the collision, and may even have been unaware of his/her contribution to the accident.

Figure 3 shows trajectories for vehicles involved in a rear-ending collision on May 2, 2003, at about 10:46 AM. Again numbering the vehicles from left to right, vehicle 6 rear-ended vehicle 5 and shortly after that vehicle 7 rear-ended vehicle 6. No accident report was filed for this crash, as no injuries were reported, and so no "official" assignment of responsibility was available. Table 2 shows posterior summaries for the collision model variables. Here it appears that drivers 3, 4, and 7 probably had reaction times that were longer than their following headways, and for each of these drivers there was an increase in the minimum deceleration necessary to avoid collision. Driver 7, with a mean posterior reaction time of 3.2 seconds and a mean posterior following headway of 0.84 seconds, stands out prominently in this regard. A counterfactual test where driver 7's following headway was set to the recommended minimum of 2.0 seconds led to driver 7's minimum deceleration falling to about 12.6 fps^2, with a posterior standard deviation 0.4 fps^2. The probability that this minimum was less than driver 7's actual deceleration was again 1.0, and so one can say that driver 7's "inattention/distraction" was probably a causal factor for this collision.

Since drivers 5 and 6 both probably had reaction times that were less than their following headways it is difficult to attribute the collision between 5 and 6 to driver inattention/distraction but what stands out in Table 2 is that driver 6's initial speed was probably around 65 fps and so higher than that of the other drivers in the platoon. Counterfactually setting driver 6's initial speed to 60 fps, other things equal, reduced driver 6's minimum deceleration to about 10.2 fps^2, with a posterior standard deviation of about 0.6 fps^2, and the probability that this counterfactual minimum was less than driver 6's observed deceleration was again 1.0. That is, if driver 6 had been travelling at 60 fps and then decelerated at the same rate that he or she actually used then, other things equal, vehicle 6 would probably have stopped before colliding with vehicle 5, and we can conclude that driver 6's speed was probably a causal factor for the collision between 5 and 6. Finally, Table 2 indicates that driver 3's reaction time was around 2.5 seconds while his or her following headway was around 1.7 seconds, and we can ask whether or not this long reaction time was also a causal factor for the collision. Counterfactually setting driver 3's reaction time to the following headway produced posterior minimum decelerations of about 10.2 fps^2 (posterior standard deviation 0.3 fps^2) for driver 6 and 16.8 fps^2 (posterior standard deviation 1.2 fps^2) for driver 7. Since these are both below the observed decelerations for these drivers, we can conclude that driver 3's reaction time was also a probable causal factor.

Finally, Figure 4 shows trajectories for two vehicles involved in a rear-end collision on March 20, 2003 at about 7:49 AM, along with trajectories for the six vehicles preceding them, and Table 3 shows posterior estimation summaries for the fitted trajectory models. Numbering the vehicle trajectories in Figure 4 from left to right, it appears that driver 1 was initially travelling at about 33.7 fps, and about 18.2 seconds after the start of the video segment began, braking to stop with a deceleration of about 8.7 fps^2. Drivers 2 though 8 braked successively after this, and vehicle 8 rear-ended vehicle 7. Since this was a property damage only collision, no accident report was completed by a police officer. As with our other two collisions it is possible to identify drivers whose reaction times were probably longer than their following headways, in this case drivers 5, 6, and 8, and it is possible to carry out counterfactual tests in order to identify probable causal factors. Setting driver 8's following headway to 2.0 seconds reduces his or her minimum deceleration to about 10.0 fps^2 (posterior standard deviation 0.2 fps^2), and so it appears that driver 8's following too closely was probably a causal factor for this collision. Setting driver 5's reaction time to his or her following headway led to a reduction in driver 8's minimum deceleration to 18.6 fps^2 (posterior standard deviation 1.5 fps^2) and this was almost certainly less than driver 8's observed deceleration. So we can also conclude that driver 5's rather long reaction was also a causal factor.

Conclusion

Traditionally, responsibility for rear-ending collisions is usually assigned to the colliding driver, but in the 1950s researchers at the General Motors Research Laboratories showed how, in congested traffic, the actions of drivers not involved in the collision could help establish the conditions that made the collision inevitable. Brill (1972) presented a simple kinematic model of successive braking by vehicles in a platoon which revealed that a driver whose reaction time is longer than his or her following headway will, other things equal, have to brake with a higher deceleration than that used by the preceding driver in order to avoid colliding. By extracting vehicle trajectory measurement from video recordings of three rear-end collisions it was possible to compute Bayes estimates drivers' speeds, reaction times, following headways, and braking decelerations. The results indicated that of the 19 drivers for which it was possible to estimate reaction times and following headways, 10 probably had reaction times that were longer than their following headways. Counterfactual testing revealed that, for each of the three collisions, had the colliding driver maintained a following headway of 2.0 seconds the collision would probably have been prevented. More interesting though is that for each of the collisions it was also possible to identify actions by earlier drivers that also contributed to the occurrence of the collision, even though these drivers did not actually collide.

It has been observed that at night drivers sometimes "overdrive" their headlights, in that their speeds are such that the corresponding stopping distances exceed the distances they can see ahead. The above results suggest that in congested conditions some freeway drivers overdrive their reaction times, in that their reaction times tend to be longer than their following headways. For the three collisions we investigated, this overdriving appears to be locally benign, in that, based on what the vehicle ahead is doing and the expectation that the driver ahead will not decelerate too rapidly, sufficient space to slow or stop is available. More globally though this overdriving tends to set up conditions where vehicles farther behind in a platoon are more likely to collide. Preventing rear-end collisions on congested freeways would then appear to require that drivers base their actions on more than local information.

One interesting way to look at these results is in terms of inter-generational shared resource (ISGR) experiments, where each "generation decides how much of a resource to consume and how much to pass on to the succeeding generation (Sadrieh 2003). Inequality (1) reveals that the stopping distance available to a driver has two components, one provided by his or her following distance (x_{k+1}), and one provided by the braking distance of preceding driver ($v_k^2/2a_k$). We can consider the braking distance for the first vehicle in a platoon as a shared resource, and each succeeding driver can make more or less of this quantity available to followers. Posterior estimates of braking distances are easily computed in WinBUGS, and the columns labeled d_k in Tables 1-3 display estimation summaries for these quantities. Looking first at Table 1, we can see that the first driver "provided" a braking distance of about 185 feet, while the second driver "consumed" about six feet of this, leaving about 169 feet. Driver 3, with a long reaction time, consumed about 100 feet of the shared resource, while small reductions were made by drivers 4 and 5. Driver 6, with reaction time that was probably shorter than his or her following headway, may have managed to add a bit to the shared resource, but this was not sufficient to keep driver 7 from colliding. Looking at Tables 1-3, it can be seen that those drivers for which $P[r_k>h_k]$ exceeded 0.5 were also the ones who made reductions to the shared stopping-distance resource. What is interesting here is that the actual magnitude of a driver's following headway is less important than the relation of the following headway to the reaction time. For example, driver 6 in Table 1 had an estimated headway of about 1.17 seconds but an estimated reaction time of about 2.07 seconds and so was able to add a bit to the shared resource while driver 3 with an estimated headway of about 2.0 still made a substantial reduction to the shared resource. In Sadrieh's shared resource experiments subjects tended to fall into one of two groups, those who consumed the shared resource in order to maximize their own individual payoffs and those who exhibited altruistic behavior. Arguably the drivers in our study could be divided into two similar groups, although it is not clear that the drivers, unlike Sadrieh's subjects, were aware of the full consequences of their actions.

In conclusion, our finding that a substantial fraction of the drivers in our study showed reaction time longer than their following headways is consistent with the report by Herman et al. (1959) that in congested traffic drivers tend to create nearly unstable traffic conditions. Relatively small individual differences in following distances and reaction times, speeds and decelerations determine whether or not a stopping shock wave results in a collision. It has been pointed out that drivers often maintain relatively short following distances in order to discourage others from merging in front of them, and clearly short following headways translate into higher traffic flows, so one can argue that short headways help make effective use of limited freeway capacity. Unfortunately, our study also suggests that some of the costs associated with a driver's short headway are external in that they tend to fall disproportionately on the following drivers. This suggests that short headways in congested conditions will be "consumed" at levels exceeding what is socially optimal. As with other situations involving external costs, achieving socially optimal decisions would then require some form of coordination mechanism.

Figure 1. Directed Acyclic Graph Representation of Three-Vehicle Platoon Collision Model.

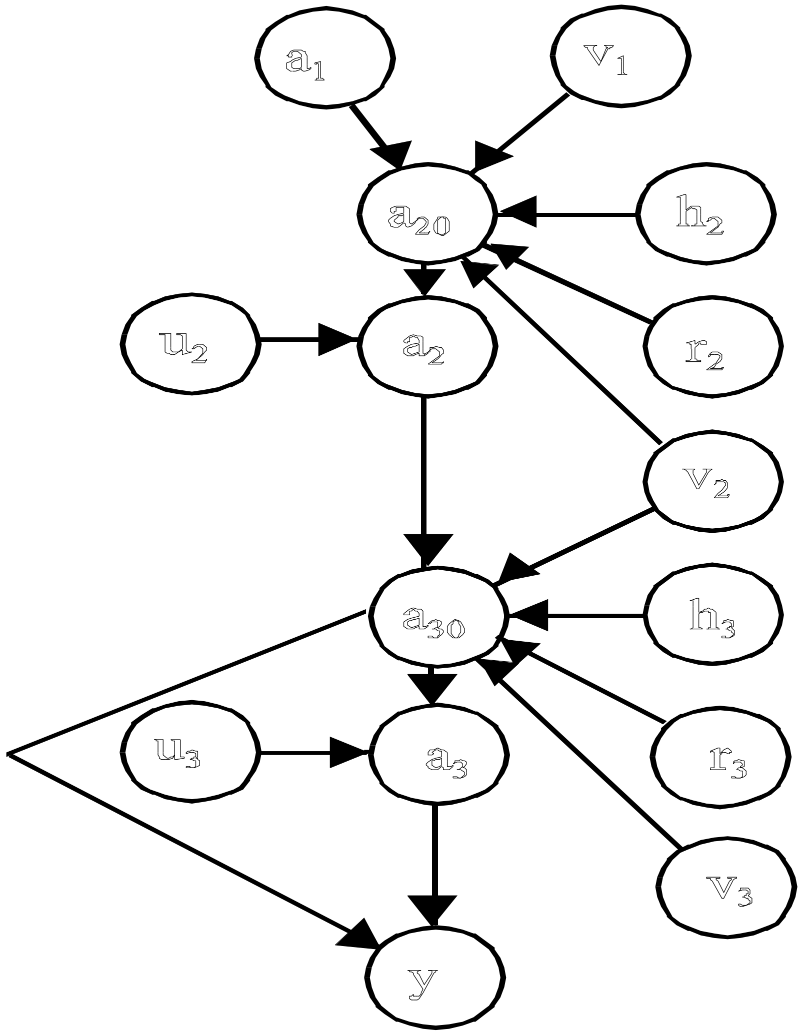

Figure 2. Trajectories of Vehicles Involved in Accident on December 30, 2002.

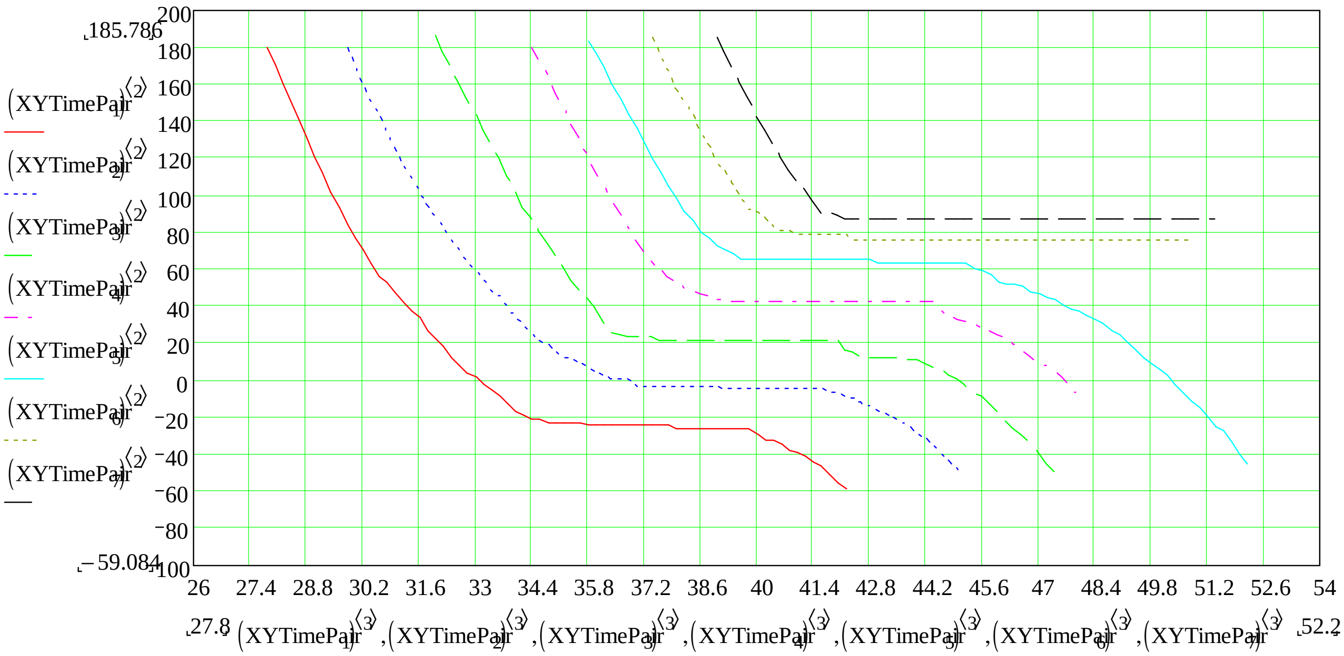

Figure 3. Trajectories of Vehicles Involved in Accident May 2, 2003.

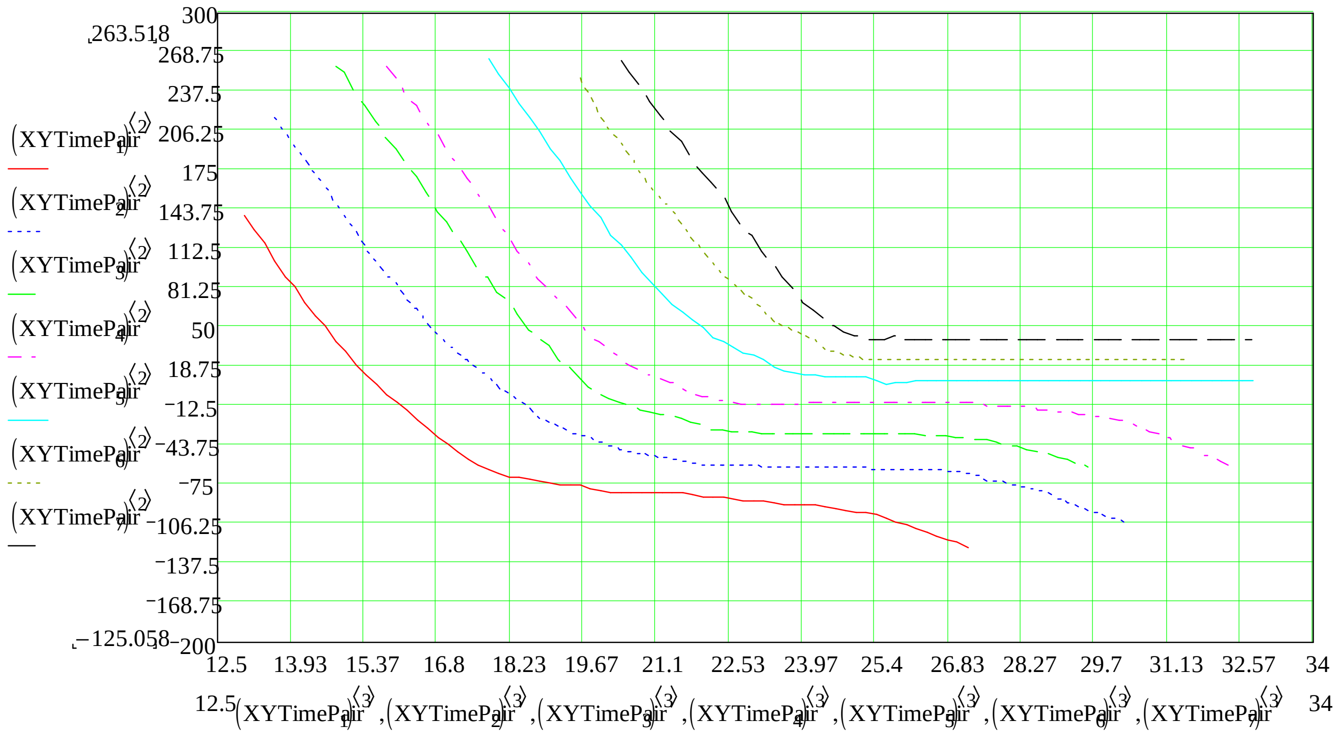

Figure 4. Trajectories for Vehicles Involved in Accident on March 20, 2003.

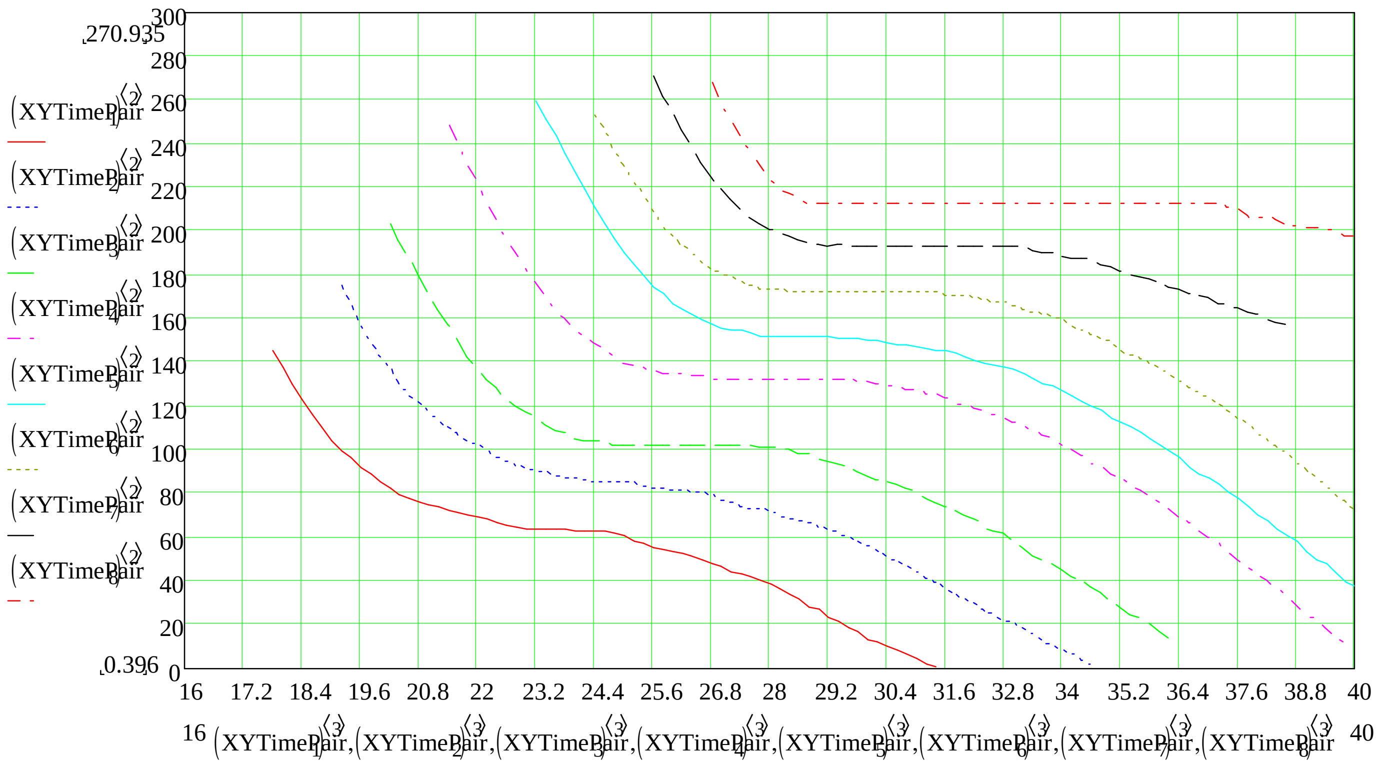

Figure 5. Example Computation of Space Headway.

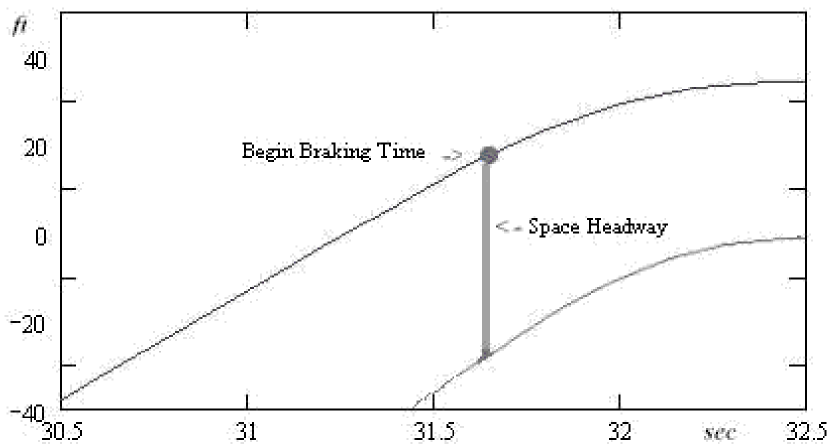

Figure 6. Example Computation of Reaction Time.

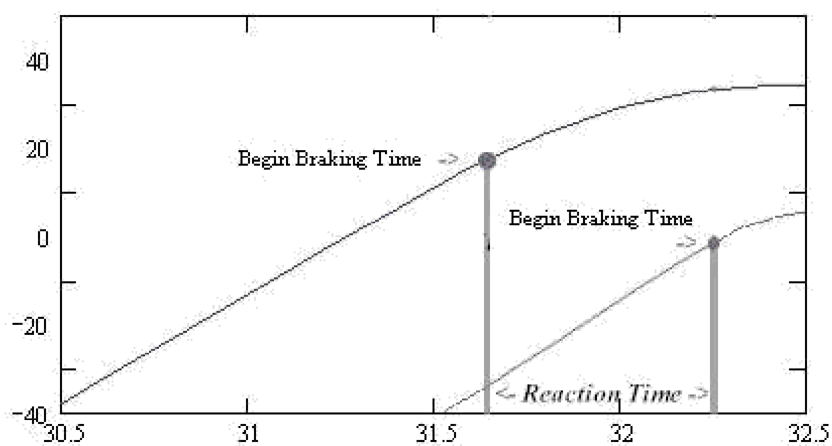

Table 1. Posterior Means and Standard Deviations for Vehicle and Driver Variables for Accident on December 30, 2002.

Vehicle	v_k (fps)	h_k (sec)	r_k (sec)	a_k (fps^2)	a_{k0} (fps^2)	t_{0k}	d_k (feet)	$P[r_k>h_k]$
1	50.0 (0.8)	--	--	6.8 (0.11)	--	28.2 (0.1)	185.3 (6.5)	--
2	46.7 (0.3)	1.69 (0.02)	1.91 (0.14)	6.5 (0.06)	6.2 (0.06)	30.1 (0.1)	168.7 (2.6)	0.90
3	41.8 (0.4)	2.00 (0.02)	4.21 (0.16)	12.6 (0.99)	11.4 (0.66)	34.3 (0.2)	69.6 (6.0)	1.0
4	42.3 (0.3)	1.87 (0.03)	1.86 (0.17)	14.2 (0.51)	12.8 (0.43)	36.1 (0.1)	62.9 (2.8)	0.47
5	39.3 (0.2)	1.21 (0.02)	1.44 (0.10)	16.0 (0.91)	14.4 (0.63)	37.6 (0.1)	48.5 (3.1)	0.99
6	42.3 (0.6)	1.17 (0.03)	1.07 (0.14)	17.3 (1.57)	17.1 (1.46)	38.7 (0.1)	52.1 (5.1)	0.19
7	41.7 (0.4)	1.24 (0.03)	1.65 (0.15)	20.3 (1.10)	24.8 (1.83)	40.3 (0.1)	42.9 (2.8)	0.99

Table 2. Posterior Means and Standard Deviations for Vehicle and Driver Variables for Accident on May 5, 2003.

Vehicle	v_k (fps)	h_k (sec)	r_k (sec)	a_k (fps^2)	a_{k0} (fps^2)	t_{0k}	d_k (feet)	$P[r_k>h_k]$
1	57.6 (0.7)	--	--	10.1 (0.2)	--	13.9 (0.1)	163.5 (6.3)	--
2	57.9 (0.5)	1.67 (0.03)	0.98 (0.16)	8.4 (0.1)	8.2 (0.1)	14.9 (0.1)	198.5 (6.3)	0
3	57.1 (0.4)	1.67 (0.02)	2.46 (0.16)	11.4 (0.4)	10.7 (0.3)	17.4 (0.1)	143.1 (6.2)	1.0
4	56.7 (0.2)	0.74 (0.02)	0.88 (0.14)	12.4 (0.3)	11.9 (0.2)	18.2 (0.1)	129.4 (3.6)	0.86
5	59.0 (0.3)	1.75 (0.02)	1.44 (0.09)	12.3 (0.2)	11.8 (0.2)	19.7 (0.1)	141.7 (3.3)	0
6	65.7 (1.5)	1.14 (0.03)	0.56 (0.20)	11.7 (0.3)	12.0 (0.3)	20.2 (0.2)	184.5 (11.9)	0
7	54.5 (0.2)	0.84 (0.03)	3.22 (0.21)	27.4 (1.8)	27.3 (1.7)	23.5 (0.1)	54.4 (3.7)	1.0

Table 3. Posterior Means and Standard Deviations for Vehicle and Driver Variables for Accident on March 20, 2003.

Vehicle	v_k (fps)	h_k (sec)	r_k (sec)	a_k (fps^2)	a_{k0} (fps^2)	t_{0k}	d_k (feet)	$P[r_k>h_k]$
1	33.7 (1.0)	--	--	8.7 (0.5)	--	18.2 (0.1)	65.2 (4.4)	--
2	37.6 (1.0)	1.76 (0.04)	1.35 (0.18)	9.4 (0.4)	8.8 (0.4)	19.5 (0.1)	75.0 (3.9)	0
3	42.2 (0.9)	1.27 (0.03)	1.07 (0.15)	10.8 (0.3)	10.7 (0.2)	20.6 (0.1)	83.1 (4.6)	0.05
4	46.0 (0.8)	1.81 (0.03)	1.23 (0.15)	11.3 (0.2)	9.7 (0.1)	21.8 (0.1)	93.6 (4.6)	0
5	41.0 (0.5)	1.74 (0.02)	2.19 (0.15)	11.5 (0.3)	11.2 (0.3)	24.0 (0.1)	72.8 (3.1)	0.99
6	39.6 (1.1)	0.68 (0.03)	0.82 (0.16)	12.6 (0.5)	11.6 (0.4)	24.8 (0.1)	62.5 (5.0)	0.81
7	43.4 (1.0)	1.22 (0.03)	1.07 (0.16)	14.8 (0.4)	13.7 (0.4)	25.9 (0.1)	64.1 (3.4)	0.14
8	44.6 (1.1)	0.71 (.02)	1.21 (0.11)	23.5 (0.9)	23.9 (0.9)	27.1 (0.1)	42.4 (3.1)	1.0

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Appendix A

WinBUGS Code Used to Analyze Accident on December 30, 2002.

WinBUGS Code Used to Analyze Accident on December 30, 2002.

```
model
# accident 02: all vehicles
# collision between 6&7 enforced; no collision between 5&6 enforced
# uncertainty for tc and vlen
# counterfactual conditions r3<- h3

{
t1[1] <- T1[1,2]
y1[1] <- T1[1,1]
y0[1] <- y1[1]-v[1]*t1[1]
for (k in 1:endstop1)
{nobrake1[k] <- step(t0[1]-T1[k,2])
stopped1[k] <- step(T1[k,2]-(t0[1]+v[1]/a[1]))
yhat1[k] <- nobrake1[k]*(y0[1]+v[1]*T1[k,2])+
stopped1[k]*(y0[1]+v[1]*t0[1]+(v[1]*v[1])/(2*a[1]))+(1-nobrake1[k])*(1-
stopped1[k])*(y0[1]+v[1]*T1[k,2]-(a[1]/2)*pow((T1[k,2]-t0[1]),2))
T1[k,1] ~ dnorm(yhat1[k],tau[1])}

t1[2] <- T2[1,2]
y1[2] <- T2[1,1]
y0[2] <- y1[2]-v[2]*t1[2]
for (k in 1:endstop2)
{nobrake2[k] <- step(t0[2]-T2[k,2])
stopped2[k] <- step(T2[k,2]-(t0[2]+v[2]/a[2]))
yhat2[k] <- nobrake2[k]*(y0[2]+v[2]*T2[k,2])+
stopped2[k]*(y0[2]+v[2]*t0[2]+(v[2]*v[2])/(2*a[2]))+(1-nobrake2[k])*(1-
stopped2[k])*(y0[2]+v[2]*T2[k,2]-(a[2]/2)*pow((T2[k,2]-t0[2]),2))
T2[k,1] ~ dnorm(yhat2[k],tau[2])}

t1[3] <- T3[1,2]
y1[3] <- T3[1,1]
y0[3] <- y1[3]-v[3]*t1[3]
for (k in 1:endstop3)
{nobrake3[k] <- step(t0[3]-T3[k,2])
stopped3[k] <- step(T3[k,2]-(t0[3]+v[3]/a[3]))
yhat3[k] <- nobrake3[k]*(y0[3]+v[3]*T3[k,2])+
stopped3[k]*(y0[3]+v[3]*t0[3]+(v[3]*v[3])/(2*a[3]))+(1-nobrake3[k])*(1-
stopped3[k])*(y0[3]+v[3]*T3[k,2]-(a[3]/2)*pow((T3[k,2]-t0[3]),2))
T3[k,1] ~ dnorm(yhat3[k],tau[3])}

t1[4] <- T4[1,2]
y1[4] <- T4[1,1]
y0[4] <- y1[4]-v[4]*t1[4]
for (k in 1:endstop4)
{nobrake4[k] <- step(t0[4]-T4[k,2])
stopped4[k] <- step(T4[k,2]-(t0[4]+v[4]/a[4]))
yhat4[k] <- nobrake4[k]*(y0[4]+v[4]*T4[k,2])+
stopped4[k]*(y0[4]+v[4]*t0[4]+(v[4]*v[4])/(2*a[4]))+(1-nobrake4[k])*(1-
stopped4[k])*(y0[4]+v[4]*T4[k,2]-(a[4]/2)*pow((T4[k,2]-t0[4]),2))
T4[k,1] ~ dnorm(yhat4[k],tau[4])}

t1[5] <- T5[1,2]
y1[5] <- T5[1,1]
y0[5] <- y1[5]-v[5]*t1[5]
for (k in 1:endstop5)
{nobrake5[k] <- step(t0[5]-T5[k,2])
```

```
stopped5[k] <- step(T5[k,2]-(t0[5]+v[5]/a[5]))
yhat5[k] <- nobrake5[k]*(y0[5]+v[5]*T5[k,2])+
stopped5[k]*(y0[5]+v[5]*t0[5]+(v[5]*v[5])/(2*a[5]))+(1-nobrake5[k])*(1-
stopped5[k])*(y0[5]+v[5]*T5[k,2]-(a[5]/2)*pow((T5[k,2]-t0[5]),2))
T5[k,1] ~ dnorm(yhat5[k],tau[5])}

t1[6] <- T6[1,2]
y1[6] <- T6[1,1]
y0[6] <- y1[6]-v[6]*t1[6]
for (k in 1:endstop6)
{nobrake6[k] <- step(t0[6]-T6[k,2])
stopped6[k] <- step(T6[k,2]-(t0[6]+v[6]/a[6]))
yhat6[k] <- nobrake6[k]*(y0[6]+v[6]*T6[k,2])+
stopped6[k]*(y0[6]+v[6]*t0[6]+(v[6]*v[6])/(2*a[6]))+(1-nobrake6[k])*(1-
stopped6[k])*(y0[6]+v[6]*T6[k,2]-(a[6]/2)*pow((T6[k,2]-t0[6]),2))
T6[k,1] ~ dnorm(yhat6[k],tau[6])}

t1[7] <- T7[1,2]
y1[7] <- T7[1,1]
y0[7] <- y1[7]-v[7]*t1[7]
for (k in 1:endstop7)
{nobrake7[k] <- step(t0[7]-T7[k,2])
stopped7[k] <- step(T7[k,2]-(t0[7]+v[7]/a[7]))
yhat7[k] <- nobrake7[k]*(y0[7]+v[7]*T7[k,2])+
stopped7[k]*(y0[7]+v[7]*t0[7]+(v[7]*v[7])/(2*a[7]))+(1-nobrake7[k])*(1-
stopped7[k])*(y0[7]+v[7]*T7[k,2]-(a[7]/2)*pow((T7[k,2]-t0[7]),2))
T7[k,1] ~ dnorm(yhat7[k],tau[7])}

for (j in 1:7) {
var[j] <- 1/tau[j]
a[j] ~ dnorm(0, 1.0E-06)I(,0)
v[j] ~ dnorm(0, 1.0E-06)I(,0)
t0[j] ~ dnorm(0, 1.0E-06)I(bound[j],)
tau[j] ~ dgamma(.001,.001) }
tc.hi <- tc.in+0.2
tc.lo <- tc.in-0.2
tc ~ dunif(tc.lo,tc.hi)

# enforce collision between 6 & 7 at time tc
nb6 <- step(t0[6]-tc)
st6 <- step(tc-(t0[6]+v[6]/a[6]))
yhit6 <- nb6*(y0[6]+v[6]*tc)+
st6*(y0[6]+v[6]*t0[6]+(v[6]*v[6])/(2*a[6]))+(1-nb6)*(1-
st6)*(y0[6]+v[6]*tc-(a[6]/2)*pow((tc-t0[6]),2))
nb7 <- step(t0[7]-tc)
st7 <- step(tc-(t0[7]+v[7]/a[7]))
yhit7 <- nb7*(y0[7]+v[7]*tc)+
st7*(y0[7]+v[7]*t0[7]+(v[7]*v[7])/(2*a[7]))+(1-nb7)*(1-
st7)*(y0[7]+v[7]*tc-(a[7]/2)*pow((tc-t0[7]),2))
phit7 <- .999*step(yhit6+vlen[6]-yhit7) + .001/2
hit7 ~ dbern(phit7)

for (j in 1 :7) {
ap[j] <- abs(a[j])
vp[j] <- abs(v[j])
bd[j] <- (v[j]*v[j])/(2*ap[j]) }

for (i in 1 : 6) {
vlen[i] ~ dunif(vlen.lo[i],vlen.hi[i])
s[i] <- (y0[i+1] + v[i+1]*t0[i])-(y0[i]+v[i]*t0[i])-vlen[i]
```

```
r[i] <- t0[i+1]-t0[i]
h[i] <- s[i]/vp[i+1]
slow[i] <- step(r[i]-h[i])
amin[i] <- (vp[i+1]*vp[i+1])/(((vp[i]*vp[i])/ap[i])+2*vp[i+1]*(h[i]-
r[i]))
u[i] <- ap[i+1]-amin[i]
}


# enforce no collision between 5 & 6
phit6 <- .999*step(amin[5]+a[6]) + .001/2
hit6 ~ dbern(phit6)


# counterfactual world
r.star[1] <- r[1]
r.star[2] <- h[2]
r.star[3] <- r[3]
r.star[4] <- r[4]
r.star[5] <- r[5]
r.star[6] <- r[6]
amin.star[1] <- ap[1]
a.star[1] <- ap[1]
for (i in 2:7){
amin.star[i] <- (vp[i]*vp[i])/(((vp[i-1]*vp[i-1])/a.star[i-
1])+2*vp[i]*(h[i-1]-r.star[i-1]))
a.star[i] <- amin.star[i]+u[i-1] }
nohit.star <- step(ap[7]-amin.star[7])

# tc=42.2  hit7=1, hit6=0,

}

Data list(tc.in=42.2  hit7=1,hit6=0,
vlen.lo=c(14,14,14,14,14,14),
vlen.hi=c(17,17,17,17,17,17),
endstop1=42,
endstop2=38,
endstop3=31,
endstop4=27,
endstop5=21,
endstop6=20,
endstop7=23,
bound=c(28.0,30.0,32.2,34.6,36.0,37.6,39.2),
T1=structure(.Data=
c(179.7915211695566,          27.8,
      170.4901441213861,      28,
    159.7323937256686,        28.2,
      150.59090005676214,     28.4,
      141.52181167374908,     28.6,
      130.9544131272984,      28.8,
      120.56126087331423,     29,
      111.72828295652243,     29.2,
      101.51001199660283,     29.4,
      92.82504015889617,      29.6,
      84.13456621201382,      29.8,
      76.93246667411574,      30,
      69.84849372098768,      30.2,
      62.73916454513461,      30.4,
      55.675889209595475,     30.6,
```

```
      52.821648072187,        30.8,
      47.20491529938064,      31,
      41.685721575034464,     31.2,
      37.49648593170208,      31.4,
      33.45897418618082,      31.6,
      26.650609190292688,     31.8,
      22.586031210853037,     32,
      18.602994667730755,     32.2,
      11.887052520579136,     32.4,
      7.942842867249993,      32.6,
      4.012820857921434,      32.8,
      1.2927747307599693,     33,
      -2.613175040702822,     33.2,
      -5.252117199175129,     33.4,
      -9.134570241863628,     33.6,
      -13.066577490519691,    33.8,
      -16.984096461399183,    34,
      -19.58776622804118,     34.2,
      -20.887207084737167,    34.4,
      -20.887207084737167,    34.6,
      -23.48131540200051,     34.8,
      -23.48131540200051,     35,
      -23.48131540200051,     35.2,
      -23.54376962319886,     35.4,
      -23.54376962319886,     35.6,
      -24.775988703892875,    35.8,
      -24.775988703892875,    36,
      -24.775988703892875,    36.2,
      -24.775988703892875,    36.4,
      -24.775988703892875,    36.6), .Dim=c(45,2)),
T2=structure(.Data=
c(179.87362460919954,   29.8,
170.5712968881511,      30,
159.7323937256686,      30.2,
150.5117748212538,      30.4,
143.0283347901886,      30.6,
133.94146581888043,     30.8,
125.08036801137727,     31,
117.60924311841933,     31.2,
108.79928050258279,     31.4,
101.51001199660283,     31.6,
94.2678575362593,       31.8,
88.50774991551404,      32,
82.77727335407333,      32.2,
75.6554991247228,       32.4,
69.99086217431929,      32.6,
62.95067842137142,      32.8,
58.748062807917265,     33,
53.16946147778543,      33.2,
47.61910347716653,      33.4,
44.854448783467845,     33.6,
37.973295201176896,     33.8,
33.86537433357719,      34,
29.772924365156722,     34.2,
25.69585806242112,      34.4,
21.634088846473333,     34.6,
18.934698622774906,     34.8,
15.030222205736694,     35,
12.347124571672316,     35.2,
11.008080273178189,     35.4,
```

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8.334985147138568,      35.6,
7.000928116346902,      35.8,
4.272771790521356,      36,
2.943796861350257,      36.2,
.2907842058291507,      36.4,
.2907842058291507,      36.6,
.2907842058291507,      36.8,
-3.6121617342252885,    37,
-3.6121617342252885,    37.2,
-3.6121617342252885,    37.4,
-3.6121617342252885,    37.6),  .Dim=c(40,2)),
T3=structure(.Data=
c(185.7860004878013,    32,
176.43746052650698,     32.2,
168.7042414847025,      32.4,
159.41226222764024,     32.6,
151.87162739401506,     32.8,
142.71493759695414,     33,
133.78656992254278,     33.2,
124.85067800163628,     33.4,
118.93234406532456,     33.6,
108.64936618468074,     33.8,
101.36152827919094,     34,
92.67825294315156,      34.2,
86.92669350866397,      34.4,
79.77879228244556,      34.6,
72.67664190222445,      34.8,
65.61980452004943,      35,
60.00666929680289,      35.2,
53.030309655183366,     35.4,
47.55005028632412,      35.6,
43.3374280433954,       35.8,
37.83702109737093,      36,
31.067862574976232,     36.2,
25.628869563383525,     36.4,
24.2733778291454,       36.6,
22.919582310618143,     36.8,
22.919582310618143,     37,
22.919582310618143,     37.2,
22.919582310618143,     37.4,
21.500879129514022,     37.6,
21.567479825913622,     37.8,
21.567479825913622,     38,
21.567479825913622,     38.2,
21.567479825913622,     38.4,
21.567479825913622,     38.6), .Dim=c(34,2)),
T4=structure(.Data=
c(179.95573906346272,   34.4,
172.1978206460925,      34.6,
164.49162671207685,     34.8,
153.7888719820317,      35,
146.28342542412534,     35.2,
137.16970677375505,     35.4,
128.20583600092334,     35.6,
120.78968904725888,     35.8,
111.9540927929076,      36,
104.56913281541345,     36.2,
97.38061562811492,      36.4,
88.72674114914166,      36.6,
81.56616534037225,      36.8,
```

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74.37984069449756,       37,
68.72136051769519,       37.2,
63.091732580774746,      37.4,
60.28766932954107,       37.6,
54.70090774050809,       37.8,
53.37825539615603,       38,
49.21166312179009,       38.2,
47.75723606555949,       38.4,
46.373772222459415,      38.6,
44.99205743617707,       38.8,
43.61208839191618,       39,
43.61208839191618,       39.2,
42.233861783251484,      39.4,
42.233861783251484,      39.6,
42.233861783251484,      39.8,
42.233861783251484,      40), .Dim=c(29,2)),
T5=structure(.Data=
c(182.74383561470776,    35.8,
176.51921281040853,      36,
168.78520459828545,      36.2,
159.57230669438167,      36.4,
151.9508847084348,       36.6,
142.94996997843447,      36.8,
135.43794155557006,      37,
127.8983410639413,       37.2,
118.93234406532456,      37.4,
111.65303260958271,      37.6,
104.34550812890879,      37.8,
98.45960637482004,       38,
91.2375789113328,        38.2,
85.49343198962045,       38.4,
79.77879228244556,       38.6,
76.93246667411574,       38.8,
72.60519932509949,       39,
69.84849372098768,       39.2,
68.43712970270256,       39.4,
65.54903975872517,       39.6,
65.54903975872517,       39.8,
65.54903975872517,       40,
65.54903975872517,       40.2,
65.54903975872517,       40.4), .Dim=c(24,2)),
 T6 = structure(.Data =
c(184.55294029594367,    37.4,
175.21311451951675,      37.6,
167.48708103893446,      37.8,
156.75688088847184,      38,
150.59090005676214,      38.2,
142.94996997843447,      38.4,
132.60159600423435,      38.6,
126.56524389686925,      38.8,
117.60924311841933,      39,
113.27105277529532,      39.2,
105.87789002877695,      39.4,
98.60752215642617,       39.6,
92.67825294315156,       39.8,
91.2375789113328,        40,
86.92669350866397,       40.2,
82.63243796275314,       40.4,
81.20469834984712,       40.6,
81.20469834984712,       40.8,
```

```
78.28272617876331,       41,
78.28272617876331,       41.2,
78.28272617876331,       41.4,
78.28272617876331,       41.6,
78.28272617876331,       41.8,
78.28272617876331,       42,
78.28272617876331,       42.2,
75.44032437009959,       42.4), .Dim = c(26,2)),
T7=structure(.Data=
c(184.4703567510986,     39,
176.60097604491705,      39.2,
167.32541447816502,      39.4,
159.65234488839573,      39.6,
152.03015252663425,      39.8,
141.44359794940897,      40,
133.94146581888043,      40.2,
126.48852302947321,      40.4,
119.00831076183903,      40.6,
111.65303260958271,      40.8,
107.1132097614176,       41,
102.51789180485875,      41.2,
96.56879303340622,       41.4,
90.50561260798713,       41.6,
90.35933257109365,       41.8,
88.7762322096289,        42,
87.34122490109584,       42.2,
87.34122490109584,       42.4,
87.34122490109584,       42.6,
87.34122490109584,       42.8,
87.34122490109584,       43,
87.26839685310837,       43.2,
87.26839685310837,       43.4,
87.26839685310837,       43.6,
87.26839685310837,       43.8,
87.26839685310837,       44), .Dim=c(26,2))
)


Inits
a[]         v[]         t0[]        tau[]
-6.8  -49.8 28.4        1
-6.5  -47.2 30.2        1
-12.5 -41.8 34.3        1
-14.0 -42.3 36.1        1
-15.8 -39.4 37.6        1
-17.3 -42.7 38.5        1
-20.3 -41.8 40.3        1
```