CONFINEMENT AND DUALITY IN QCD

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Large N parts in collaboration with L.G.Yaffe,

Finite N, with M.Shifman

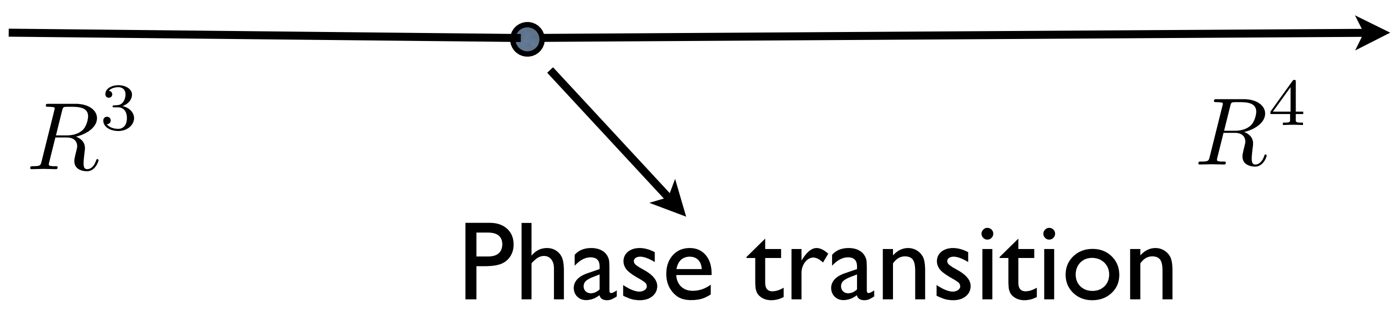
- Are there any QCD-like gauge theories which are analytically tractable in 4d?

- Meaning of the question? (like Polyakov model on R^3 or Seiberg-Witten)

- pure YM, Vector-like, chiral ? Obvious answer: No!

- Strong coupling gauge dynamics may also be relevant to (multi)-TeV scale physics.

Thermal vs nonthermal QCD on $\mathcal{R}^3 \times S^1$



- Thermal versus quantum fluctuations. (Forget thermal intuitions for the purpose of this talk.)
- Non-thermal compactifications: not all QCD-like theories alike.
- QCD(adj) with periodic spin connection. Center symmetry never breaks! (complex reps, not so.)

The dynamics of QCD(adj) on a small circle

- QCD(adj) is solvable on small circle, in the same sense as the Seiberg-Witten or Polyakov model. Since the theory is non-supersymmetric, it is more interesting.

- Exhibits (linear) confinement without continuous chiral symmetry breaking on small circle. Discrete chiral symmetry is always broken.

- There must exist a non-thermal chiral transition in the absence of any change in center symmetry realization!

- Massless fermions at small circle, massless Goldstone bosons at large!

SU(N) QCD(adj)

$$S = \int_{R^3 \times S^1} \frac{1}{g^2} \mathrm{tr} \left[\frac{1}{4} F_{MN}^2 + i \bar{\lambda}^I \bar{\sigma}^M D_M \lambda_I \right]$$

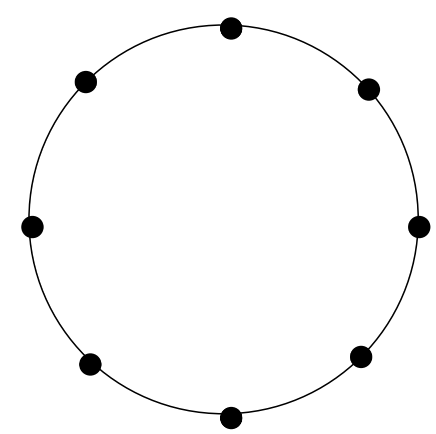
short distance

Center Z_{N_c}

Chiral $(SU(n_f) \times Z_{2N_c n_f})/Z_{n_f}$

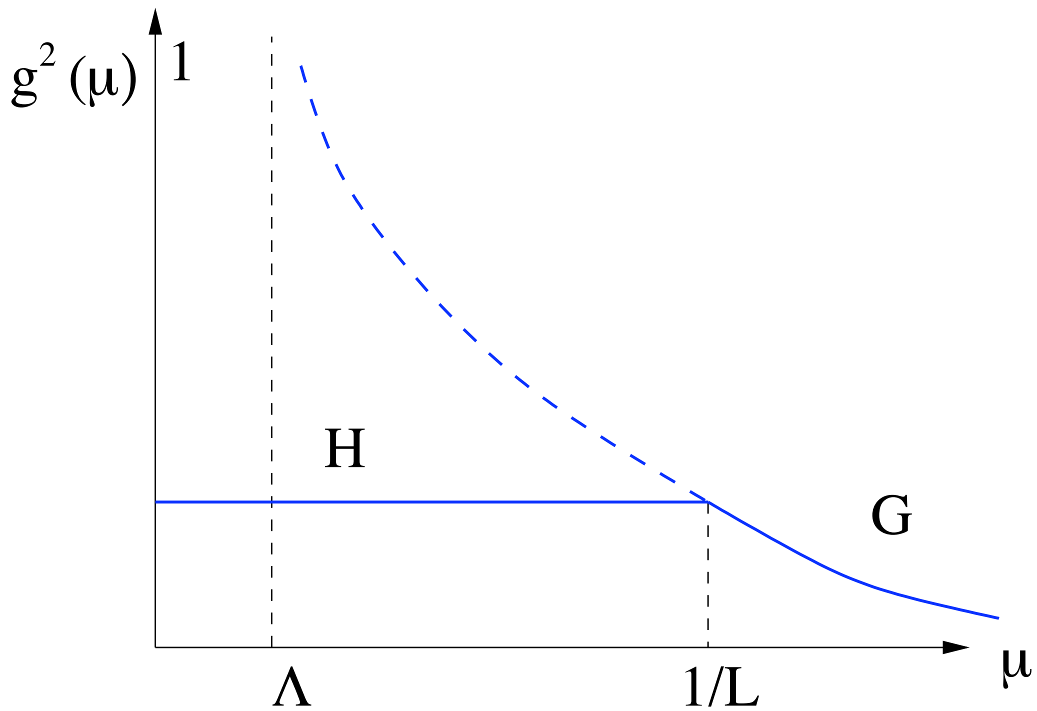
Evaluate the one loop effective potential for the Wilson line.

Eigenvalues repel. Minimum at



At weak coupling, the fluctuations are small, a "Higgs regime"

Perturbation theory



$$G = SU(2), \quad H = U(1)$$

IR in perturbation theory is a free theory of fermions and "photons". Is this perturbative fixed point destabilized non-perturbatively?

- Need to classify the effects of topological excitations in the theory. (Flux operators)

- The monopole operators will carry Jackiw-Rebbi zero modes. Hence, they can not induce mass gap in gauge sector. Need something new.

- Flux operators: Either magnetic flux or topological charge non-vanishing.

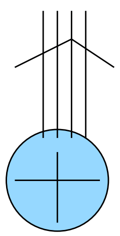
$$\left(\int_{S^2} F,\ \int_{R^3\times S^1} F\tilde{F}\right)$$

Magnetic Monopoles

Magnetic Bions

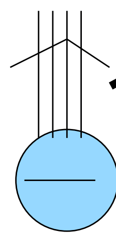
Also see, Bruckmann, Nogradi, van Baal 03

Rebbi-Jackiw fermionic zero modes



BPS

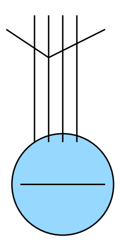
(1, 1/2)



KK

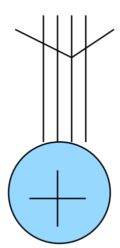
(–1, 1/2)

$$e^{-S_0} e^{i\sigma} \det_{I,J} \psi^I \psi^J,$$



$\overline{\text{BPS}}$

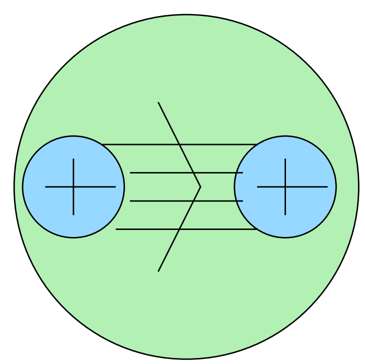
(–1, –1/2)



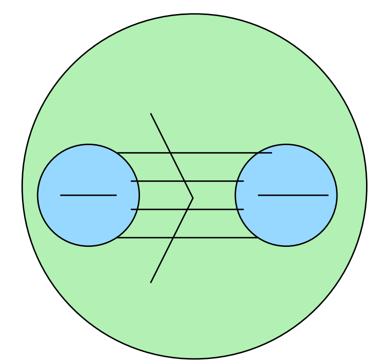
$\overline{\text{KK}}$

(1, –1/2)

$$e^{-S_0} e^{i\sigma} \det_{I,J} \bar{\psi}^I \bar{\psi}^J$$



(2,0)



(–2, 0)

$$e^{-2S_0}\left(e^{2i\sigma} + e^{-2i\sigma}\right)$$

Discrete shift symmetry : $\sigma \longrightarrow \sigma + \pi \qquad \psi^I \longrightarrow e^{i\frac{2\pi}{8}} \psi^I$

- In the absence of fermion zero modes, BPS and $\overline{\text{KK}}$ interact repulsively by Coulomb interactions.

- These two form magnetically charged stable pairs due a fermionic pairing mechanism. (magnetic bions)

- Atiyah-Singer index of an instanton = Sum of Callias indices of elementary monopoles.

- ‘t Hooft instanton vertex is product of monopole operators.

- Instanton: (0,1) = (+1, +1/2) + (-1, +1/2)

- Magnetic bions: (2,0)= (+1, +1/2) + (+1, -1/2)

Dual Formulation of QCD(adj)

$$L^{\rm dQCD} = \frac{1}{2}(\partial\sigma)^2 - b\, e^{-2S_0}\cos 2\sigma + i\bar{\psi}^I\gamma_\mu\partial_\mu\psi_I + c\, e^{-S_0}\cos\sigma(\det_{I,J}\psi^I\psi^J + {\rm c.c.})$$

magnetic bions — magnetic monopoles

Same mechanism in N=1 SYM. Also see Hollowood, Khoze, ... 99

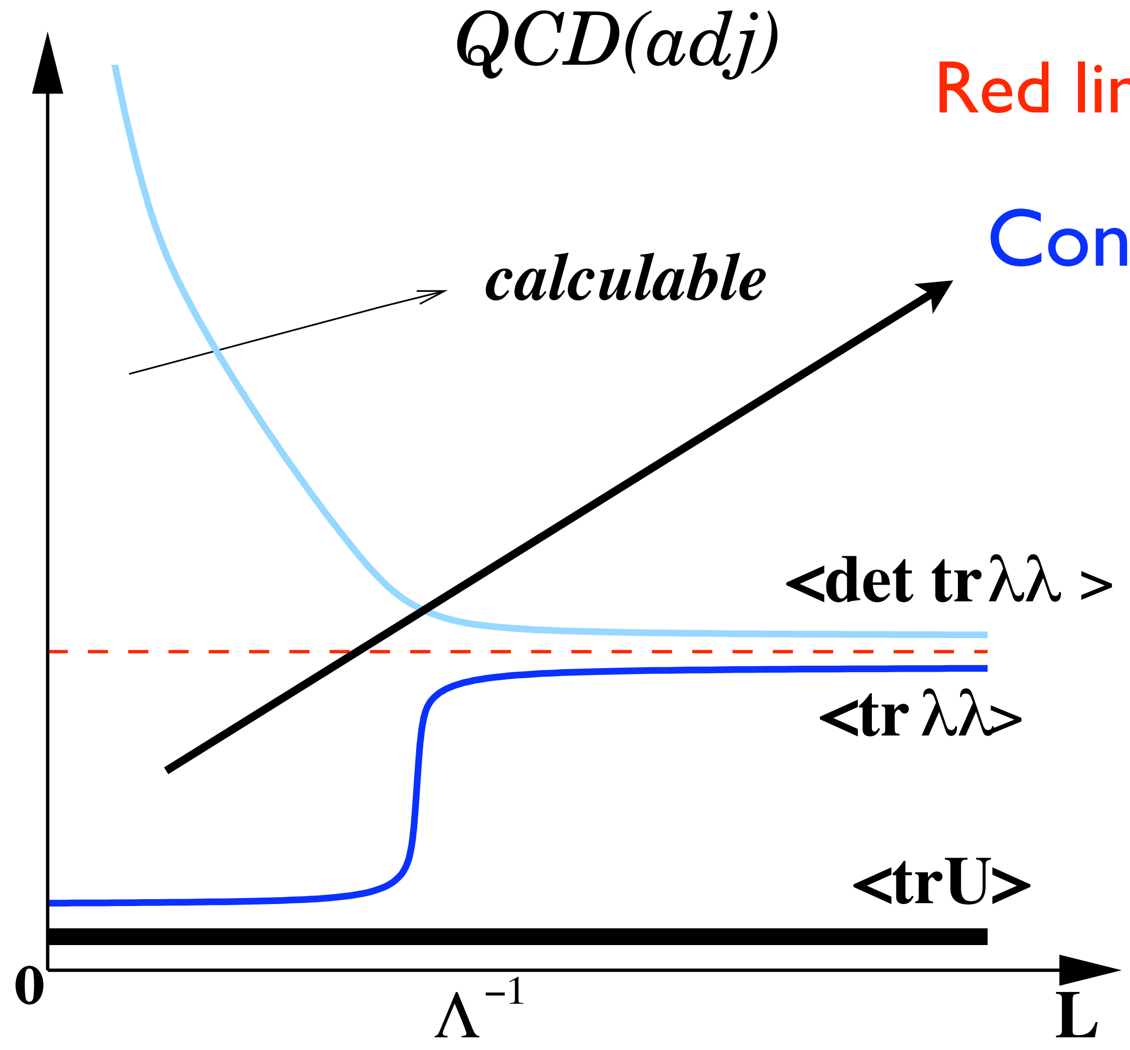
Important point in the solution of N=1 SYM is as well unbroken center symmetry, not supersymmetry.

In QCD(adj) and N=1 SYM, not the magnetic monopoles, the magnetic bions lead to mass gap, and linear confinement.

A) Mass gap in gauge sector due to magnetic bion mechanism, so is linear confinement, and stable flux tubes.

B) Discrete chiral symmetry is always broken.

C) Continuous chiral symmetry is unbroken at small radius.



Red line N=1SYM.

Confinement without chiral symmetry breaking

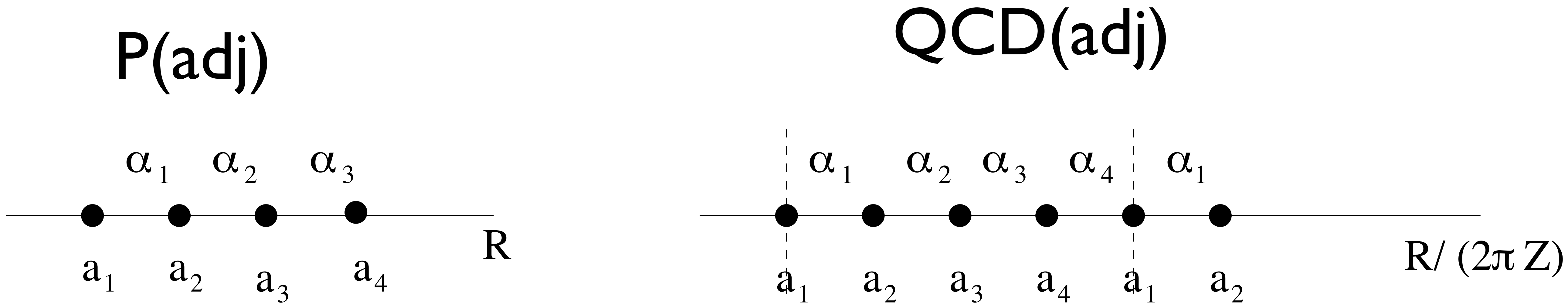
Is this Polyakov's model with adjoint fermions?

- Not same as Polyakov model with massless adjoint fermions which neither confines, nor has a mass gap. The masslessness of the photon is protected by U(1) shift symmetry. U(1) breaks spontaneously, and photon is the Goldstone boson. (Affleck, Harvey, Witten 82)

- Distinguishing notion between QCD(adj) and P(adj): discrete versus continuous topological symmetry.

- Topological symmetry is a sharp symmetry characterization for confinement! (Not explained in this talk.)

Is this Polyakov's model with adjoint fermions?



When the adjoint Higgs scalar is compactified, an extra topological excitation (on the same footing with the rest) moves in from infinity.

Region of validity of dual formulation

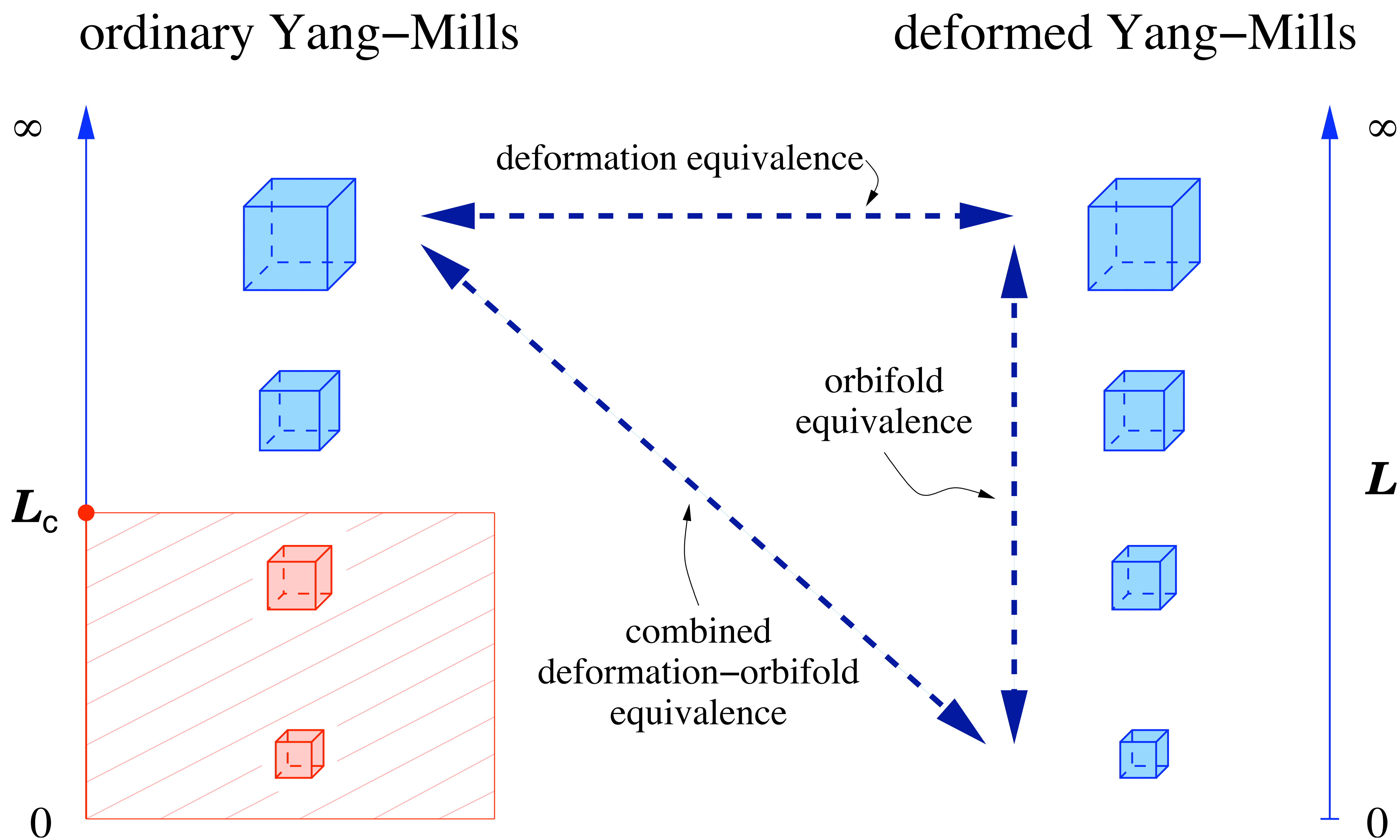
- $LN_c\Lambda \ll 1,$ **why not** $L\Lambda \ll 1$ **?**
- Separation of scales between W-bosons and dual photons.
- Deeper reason: Large N volume independence EK reduction
- The large N volume independence holds provided unbroken center symmetry. Thus, the chiral transition scale must move to arbitrarily small radius at large N.
- New dynamical scale in QCD: Λ^{-1}/N

- Complex representation fermions, chiral theories, pure YM? Center is always broken at small radius regardless of spin connection of fermions.

- New idea: Add center stabilizing double trace deformations. Different theory? Not so fast.

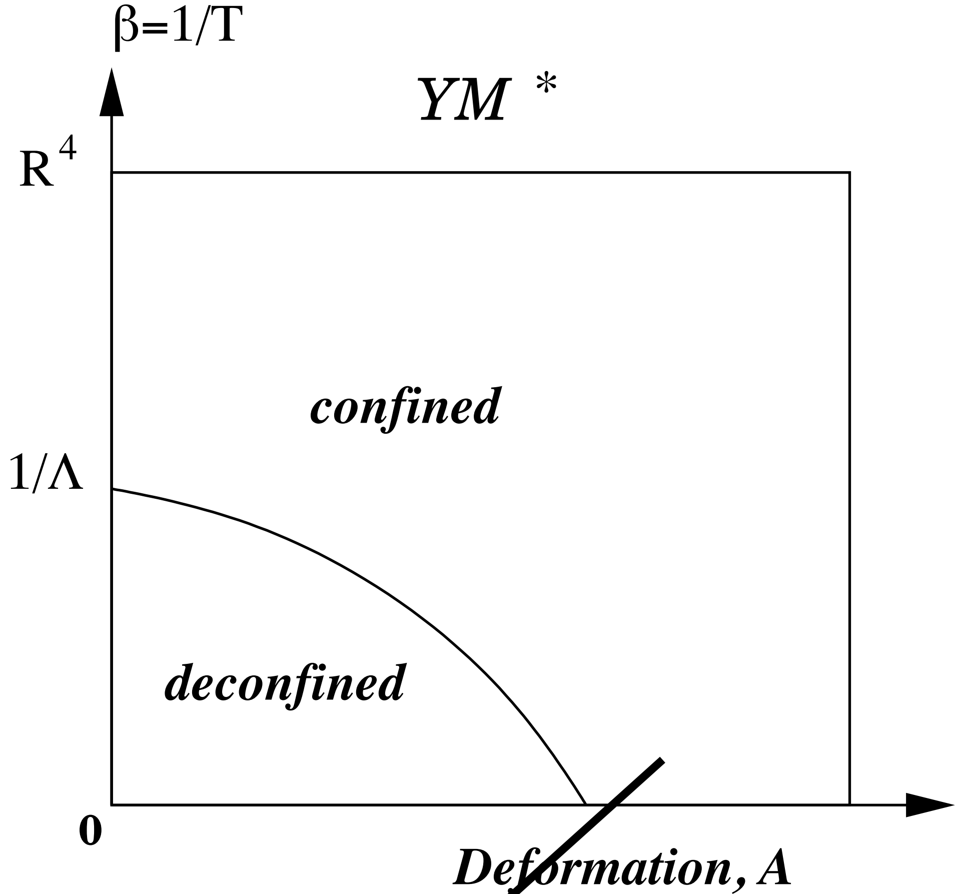
At finite N, (with Shifman): Conjecture: For the deformed theories without continuous global symmetries, the physics of the theory at large and small radius are smoothly connected. QCD* theories.

At large N, the deformation is a new cure to the old EK problem, and is a useful tool for QCD-like theories (with Yaffe).



Volume independence via center stabilizing deformations.

YM* theory at finite N



$$S^{\mathrm{YM}^*} = S^{\mathrm{YM}} + \int_{R^3 \times S^1} P[U(\mathbf{x})]$$

$$P[U] = A \frac{2}{\pi^2 \beta^4} \sum_{n=1}^{\lfloor N/2 \rfloor} \frac{1}{n^4} \left| \mathrm{tr}\,(U^n) \right|^2$$

$$S^{\mathrm{dual}} = \int_{R^3} \left[\frac{1}{2L} \left(\frac{g}{2\pi} \right)^2 (\partial \sigma)^2 - \zeta \sum_{i=1}^{N} \cos(\alpha_i \cdot \sigma) \right].$$

Monopole mechanism is valid in YM*, compare with QCD(adj).

Uses of deformations

- Deformed theories, 1-flavor QCD* (F/BF/AS/S) and YM* are analytically tractable at small radius. Confinement, mass gap, discrete chiral symmetry breaking can be shown analytically.

- There are difficulties for complex rep fermions, since one has both electrically and magnetically charged relevant excitations in the IR. However, this is surmountable.

- Double trace deformations are also useful for a large class of strongly coupled chiral gauge theories. It is fair to say that our current knowledge of such theories is next to nothing. (in progress, with M. Shifman.)

- The dynamics of chiral theories are really bizarre (or unique), and we have some promising analytical results.

- Progress in non-perturbative QCD by continuum methods is possible. I am optimistic that our understanding of QCD-like theories may exceed the level of supersymmetric ones (if we pursue QCD vigorously.)

- Long-distance theories may be derived starting with microscopic physics (rather than being modeled.)

Some of the questions during/after the talk and answers

- Vainstein (on page 3): You are not writing a thermal partition function, right?

- Correct, for QCD(adj), I am interested in a twisted partition function. The reason is opposite behaviors of thermal and quantum fluctuations in this theory. With thermal boundary conditions, temporal center is broken at small circle, and for spatial compactification, the spatial center symmetry is unbroken. The latter provides a calculable window in non-supersymmetric QCD-like gauge theories

Twisted partition function: $$\widetilde{Z} = \mathrm{tr}\left[e^{-\beta H}(-1)^F\right]$$

- Schaefer (page 9): Are these related to the zero modes studied van Baal et. al? What provides a region of analytical control in your theory, what is the separation of scales which makes this analysis reliable?

- Balitsky(page 9): Are these related to calorons?

- Indeed, Bruckmann, van Baal and Norgadi did very nice theoretical and lattice work for fundamental representation fermions in the background of non-trivial holonomy Wilson line showing to which constituent monopole the zero modes localize into. This work tries to understand strongly coupled regime by using non-trivial holonomy. In our case, the non-trivial holonomy is the minimum of the effective potential in a weakly coupled regime and the dynamics is completely under control.

- Calorons are there, but they appear in order exp(-NS_0) in semiclassical expansion in SU(N) theory. The monopoles are exp(-S_0) effects and magnetic bions (which lead to mass gap) are exp(-2S_0) effect.

- Konishi(page 9): There are topological objects with magnetic charge +2 in certain extended supersymmetric theories.

- Thanks for bringing this to my attention. My goal here (and more generally) is to understand dynamical non-perturbative effects in QCD-like gauge theories, which do not have any elementary scalars.

- Poppitz(page 11): The superpotential in N=1 SYM is correct, right?

- Certainly. The supersymmetric affine Toda description is correct and has the same region of validity with QCD(adj). My point is, incomplete deliberation is given into the origin of the bosonic potential in N=1 SYM. The origin of confinement in literature is asserted as monopoles per se which is incorrect.

- Even in QCD(adj), there exist a prepotential which serves an identical role with superpotential in N=1 SYM. The mechanism in these class of theories is same. Unfortunately, in N=1 SYM discussions in literature, no one bothered about the region of validity of the supersymmetric affine Toda description. It is sometimes incorrectly extended into decompactification limit where it is not valid at all. (except for few chiral ring operators.)