HyperKähler Quotients & Lumps in SO(N) and USp(N) gauge theories

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Outline

Motivations

- The low energy effective theory on the Higgs branch in $\mathcal{N} = 2$ sQCD is hyperKähler

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  $\longrightarrow$ low energy effective action

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  $\longrightarrow$ topological excitations in the gauge theories

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  $\longrightarrow$ low energy effective action
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  $\longrightarrow$ topological excitations in the gauge theories
  $\longrightarrow$ non-perturbative effects

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- Topology, metric
  $\longrightarrow$ low energy effective action
- Topology
  $\longrightarrow$ topological excitations in the gauge theories
  $\longrightarrow$ non-perturbative effects
- In extension of the line of research of Nitta et.al. (BPS domain walls) and Eto et.al. (BPS domain walls and SO/USp vortices)

The model

Scalar field $\Phi = (\phi_1, \phi_2, \phi_3)$

target space : $S^2 \Rightarrow \Phi \cdot \Phi = 1$

$$\mathcal{L} = \frac{1}{4}\partial_\mu \Phi \cdot \partial^\mu \Phi + V(1 - |\Phi|^2)$$

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Lagrangian multiplier

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Lagrangian multiplier

Conformal FT

Non-linear σ-model

$$V = -\frac{1}{4}\Phi \cdot \Box \Phi$$

Non-linear σ-model

$$V = -\frac{1}{4}\Phi\circ\Box\Phi$$

$$\downarrow$$

non-linear σ-model

$$\Box\Phi - (\Phi\circ\Box\Phi)\Phi = 0$$

Energy

$$E = \frac{1}{4}\int_{\mathbb{C}} \partial_i \Phi \cdot \partial_i \Phi$$

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$$\downarrow \quad \Phi^{\infty} = (0,0,1) \in \text{const}$$

$$\downarrow \quad O(3) \xrightarrow[\text{ssb}]{} O(2)$$

Lumps : topological solitons

$$\Phi : S^2 \to S^2 \quad , \quad \pi_2(S^2) = \mathbb{Z}$$

$$N = \frac{1}{4\pi} \int_{\mathbb{C}} \Phi \cdot (\partial_1 \Phi \times \partial_2 \Phi) \in \mathbb{Z}$$

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- . . . size instability

[i.e. Leese, 1990]

Bogomol'nyi bound

$$E \geq 2\pi |N|$$

$$\partial_i \Phi \pm \varepsilon_{ij} \Phi \times \partial_j \Phi = 0$$

$\mathbb{C}P^1$-model

R : Riemann sphere coordinate on target space

$$R = \frac{\phi_1 + i\phi_2}{1+\phi_3} \qquad R = R(z, \bar{z})$$

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$$\mathcal{L} = \frac{\partial_\mu R\, \partial^\mu \bar{R}}{(1+|R|^2)^2} \quad : \quad \mathbb{C}P^1\text{-model}$$

[Belavin & Polyakov, 1975]

R coordinates : holomorphic functions

Bogomol'nyi Eq :

$\bar{\partial} R = 0$: Cauchy-Riemann Eq

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$\Downarrow$

$R = R(z)$: holomorphic !

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$\Downarrow$

$R = R(z)$: holomorphic!

$\downarrow$

$$R(z) = \frac{p(z)}{q(z)}$$ rational maps

Kähler quotient in $SU(N)$ gauge theory

$\mathcal{N}=1$ $SU(N)$ SYM

N_F chiral superfields Q in $\square$-rep

V' vector multiplet

$$K = \mathrm{Tr}\left[QQ^\dagger e^{-V'}\right]$$

Complexified gauge invariance

Invariance : $SU(N)^{\mathbb{C}} = SL(N,\mathbb{C})$

Complexified gauge transformations

$$Q \to e^{i\Lambda'}$$

$$e^{V'} \to e^{i\Lambda'} e^{V'} e^{-i\Lambda'^{\dagger}}$$

$$e^{i\Lambda'} \in SL(N,\mathbb{C})$$

$1^{\rm st}$ road to a low energy effective theory

1st road

D-term potential in WZ-gauge

$$D^A = \mathrm{Tr}_F\left\{Q_{WZ}^\dagger T^A Q_{WZ}\right\} = 0$$

1st road to a low energy effective theory

1st road

D-term potential in WZ-gauge

$$D^A = \mathrm{Tr}_F\{Q^\dagger_{WZ}\, T^A\, Q_{WZ}\} = 0$$

$\rightarrow$ fixes $SU(N)^{\mathbb{C}}$ to $SU(N)$

Low energy effective theory

$g \to \infty$: $e^V \to \infty$ massive

$\to$ auxiliary field

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$\to$ non-linear σ-model

Low energy effective theory

$g \to \infty$: $e^{V} \to \infty$ massive

$\to$ auxiliary field

$\Rightarrow Q_{WZ} \in \{Q \mid D^A = 0\}$

$\longrightarrow$ non-linear σ-model

target space :

$$\mathcal{M} = \{ Q_{WZ} \mid Q_{WZ} Q_{WZ}^\dagger \propto \mathbb{1}_N , \ \text{rank}\, Q_{WZ} = N \} / SU(N)$$

$$\dim_{\mathbb{C}}(\mathcal{M}) = N(N_F - N) + 1$$

Full rank : full Higgs phase

$\operatorname{rank} Q_{wz} = N$

$\to$ full Higgs phase

Equivalent ways

1st road

Fix the gauge (WZ)

$g \to \infty$

mod out $SU(N)$

2nd road

$g \to \infty$

mod out $SU(N)^{\mathbb{C}}$

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well-defined iff

rank $Q = N$

$\Updownarrow$

full Higgs phase

Equivalent ways

1st road

Fix the gauge (WZ)

$g \to \infty$

mod out $SU(N)$

identical

2nd road

$g \to \infty$

mod out $SO(N)^{\mathbb{C}}$

well-defined iff

rank $Q = N$

$\Updownarrow$

full Higgs phase

An explicit map

$$Q_{wz} = e^{-\frac{V'}{2}} Q = \left[\det QQ^\dagger\right]^{\frac{1}{2N}} \frac{1}{\sqrt{QQ^\dagger}} Q$$

1st road $\leftrightarrow$ 2nd road

Yet another way to go

3rd road

target space $\longleftrightarrow$ classical vacuum manifold

Yet another way to go

3rd road

target space $\leftrightarrow$ classical vacuum manifold

$\updownarrow$

holomorphic invariants

see the talk by W. Vinci

Holomorphic invariants

$SU(N)^{\mathbb{C}}$ hol. :

$$B^{\langle A_1, A_2, \ldots, A_N \rangle} \equiv \det Q^{\langle A_1, \ldots, A_N \rangle}$$

minor $N \times N$'s

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minor $N \times N$'s

Plücker relation

$$B^{\langle A_1 \ldots A_{N-1}[B_1 \rangle} B^{\langle B_2 \ldots B_{N+1}] \rangle} = 0$$

$$\mathcal{M} = \left\{ B^{\langle A \rangle} \,\middle|\, \text{Plücker} \right\} - \left\{ B^{\langle A \rangle} = 0 \ \forall A \right\}$$

The Kähler quotient

$$K_{SU(N)} = N\left[\det QQ^\dagger\right]^{1/N} = N\left[\sum_{\langle A\rangle} |B^{\langle A\rangle}|^2\right]^{1/N}$$

Equivalent ways

1st road

Fix the gauge (WZ)

$g \to \infty$

mod out $SU(N)$

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Holomorphic inv's $\Big|_{\text{algebraic relations}}$

Coulomb phase

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$\det QQ^\dagger = 0$

point in target space

$\mathbb{Z}_N$ conifold singularity

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$\det QQ^\dagger = 0$

point in target space

↳ $\mathbb{Z}_N$ conifold singularity

Ex.: $N_F = N \rightarrow \mathbb{C}/\mathbb{Z}_N$

orbifold singularity

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$\mathbb{Z}_N$ conifold singularity

Ex.: $N_F = N \to \mathbb{C}/\mathbb{Z}_N$

orbifold singularity

Non-linear σ-model breaks down

Removing the singularity

- Gauge overall $U(1)$: V_e vector multiplet
- FI parameter $\xi > 0$

$U(1) \times SU(N)$ Kähler quotient

$$G = U(1) \times SU(N)$$

$$\begin{aligned} K &= \mathrm{Tr}\left[Q Q^\dagger e^{-V_e} e^{-V'} \right] + \xi V_e \\ &= e^{-V_e} K_{SU(N)} + \xi V_e \\ &\simeq \xi \log K_{SU(N)} \\ &\simeq \frac{\xi}{N} \log \det Q Q^\dagger = \frac{\xi}{N} \log \left[\sum_{\langle A \rangle} |B^{\langle A \rangle}|^2 \right] \end{aligned}$$

Non-linear σ-model

$$\mathcal{L} = \int_{\mathbb{C}} g_{i\bar{i}}\, \partial_\mu Q^i \partial^\mu \bar{Q}^{\bar{i}}$$

$$g_{i\bar{i}} \equiv \frac{\partial^2 K}{\partial Q^i \partial \bar{Q}^{\bar{i}}}$$

$$K = K(Q^i, \bar{Q}^{\bar{i}}, \ldots)$$: Kähler potential

The $\mathbb{C}P^1$ model revisited

$$G = (G_1, G_2)$$

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$U(1)$ fixed $\rightarrow$ $G = (1, 6)$

The $\mathbb{C}P^1$ model revisited

$$G = (G_1, G_2)$$

$U(1)$ fixed $\to$ $G = (1, b)$

$$K = \xi \log(1+|b|^2)$$

$$\mathcal{L} = \xi \frac{\partial_\mu b \partial^\mu \bar{b}}{(1+|b|^2)^2}$$

Effective theory

$$Q(z) = (a, z - z_0)$$

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$\downarrow$ promotion of moduli parameters

$$Q(t,z) = \left(a(t), z - z_0(t)\right)$$

Effective theory for the $\mathbb{C}P^1$ model

$$\mathcal{L}^{\rm eff} = \xi \int_{\mathbb{C}} \partial_t \partial^{t\dagger} \log\left(|a(t)|^2 + |z - z_0(t)|^2 \right)$$

Effective theory for the $\mathbb{C}P^1$ model

$$\mathcal{L}^{\mathrm{eff}} = \xi \int_{\mathbb{C}} \delta_t \delta^{t\dagger} \log\left(|a(t)|^2 + |z - z_0(t)|^2\right)$$

$\longrightarrow$ divergent

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$\longrightarrow$ divergent $\Rightarrow$ $\partial_t a(t) = 0$

$$\mathcal{L}^{\mathrm{eff}} = \xi \pi |\dot{z}_0(t)|^2$$

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$\mathcal{N}=1$ $SO(N_c)$, $USp(2M_c=N_c)$ SYM

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$$K = \mathrm{Tr}\left[QQ^\dagger e^{-V'}\right]$$

$$V'^T J + J V' = 0$$

$$\Leftrightarrow e^{-V'^T} J e^{-V'} = J$$

$$J = \begin{pmatrix} 0 & \mathbb{1}_M \\ \epsilon \mathbb{1}_M & 0 \end{pmatrix}, \quad \epsilon = \begin{cases} + & SO \\ - & USp \end{cases}$$

D-flatness condition

D-flatness in WZ gauge

$$D^A = \mathrm{Tr}_F \{ Q_{WZ}^\dagger T^A Q_{WZ} \} = 0$$

$T^A \in so(N)$ or $usp(N)$

D-flatness condition

D-flatness in WZ gauge

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$T^A \in$ so(N) or usp(N)

... difficult

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∘∘∘ difficult

∘∘∘ let's take another path

Enlarge the algebra

$$K = \mathrm{Tr}\left[QQ^{\dagger} e^{-V'} + \lambda \left(e^{-V'^{T}} J e^{-V'} - J \right) \right]$$

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$\cap$
$SU(N)$

Lagrangian multipliers

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$V' \in SU(N)$

λ: Lagrangian multipliers

$$\Rightarrow QQ^\dagger e^{-V'} + (\lambda + \epsilon\lambda^T)J = 0$$

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$\Cap$ SU(N)

Lagrangian multipliers

$$\Rightarrow QQ^\dagger e^{-V'} + (\lambda + \epsilon\lambda^T) J = 0$$

$$\Rightarrow QQ^\dagger e^{-V'} = e^{V'} J^\dagger (QQ^\dagger)^T J$$

Solution in terms of $Q, Q^\dagger$

$$X \equiv \sqrt{QQ^\dagger}\, e^{-V'} \sqrt{QQ^\dagger}$$

$$X^2 = \left(Q^T J \sqrt{QQ^\dagger}\right)^\dagger \left(Q^T J \sqrt{QQ^\dagger}\right)$$

Solution in terms of $Q, Q^\dagger$

$$X \equiv \sqrt{QQ^\dagger}\, e^{-V'} \sqrt{QQ^\dagger}$$

$$X^2 = \left(Q^T J \sqrt{QQ^\dagger}\right)^\dagger \left(Q^T J \sqrt{QQ^\dagger}\right)$$

$$K = \mathrm{Tr}\, X$$

Solution in terms of the holomorphic invariants

$$\mathrm{Tr}_C\sqrt{AA^\dagger} = \mathrm{Tr}_F\sqrt{A^\dagger A}$$

colour trace → flavour trace

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colour trace → flavour trace

$$\left\{ \begin{aligned} K_{SO,USp} &= \mathrm{Tr}_F\sqrt{MM^\dagger} \\ M &\equiv Q^T J Q \quad \text{meson} \end{aligned} \right\}$$

Target spaces

$$\mathcal{M}_{USp} = \left\{ M \;\middle|\; M \in \mathbb{C}^{N_F} \times \mathbb{C}^{N_F},\ M^T = -M,\ \mathrm{rank}\, M = 2M_c \right\}$$

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$\rightarrow$ no independent baryons

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$$\mathcal{M}_{SO} = \{ M, B^{\langle A \rangle} \mid M \in \mathbb{C}^{N_F} \times \mathbb{C}^{N_F},\ M^T = M,\ \det M^{\langle A \rangle \langle B \rangle} = (\det J)\, B^{\langle A \rangle} B^{\langle B \rangle},\ N_C - 1 \le \mathrm{rank}\, M \le N_C \}$$

Kähler quotients : $U(1) \times \{SO(N), USp(N)\}$ GTs

Gauging overall $U(1)$: $G = U(1) \times \begin{cases} SO(N) \\ USp(2M) \end{cases}$

FI $\xi > 0$, V_e vector multiplet

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... trick ...

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∘∘∘ trick ∘∘∘

$$K = \mathrm{Tr}\left[QQ^\dagger e^{-V'} e^{-V_e} + \lambda\left(e^{-V'^T} J e^{-V'} - J\right)\right] + \xi V_e$$

$$K_{U(1)\times SO, USp} = \mathrm{Tr}\sqrt{MM^\dagger}\, e^{-V_e} + \xi V_e$$

$$\simeq \xi \log \mathrm{Tr}\sqrt{MM^\dagger}$$

$U(1) \times SO(2)$ Lumps with $N_{\rm F} = 2$

$$\mathcal{M} = \mathbb{C}P^1 \times \mathbb{C}P^1$$

$$\pi_2(\mathcal{M}) = \mathbb{Z}_+ \oplus \mathbb{Z}_-$$

$$\Downarrow$$

$$(k_+, k_-)$$

$U(1) \times SO(2)$ Lumps with $N_{\mathrm{F}} = 2$

$$\mathcal{M} = \mathbb{C}P^1 \times \mathbb{C}P^1$$

$$\pi_2(\mathcal{M}) = \mathbb{Z}_+ \oplus \mathbb{Z}_- \ni (k_+, k_-)$$

$$Q(z) = \begin{pmatrix} Q_{+1} & Q_{+2} \\ Q_{-1} & Q_{-2} \end{pmatrix}$$

$$E = \int_{\mathbb{C}} 2\partial\bar{\partial}K = \pi\xi\overbrace{(k_+ + k_-)}^{\in\,\mathbb{Z}}$$

Minimal example : $k = 1$

$U(1) \times SO(2M_L)$:

$M = Q^T J Q = J z^k + \mathcal{O}(z^{k-1})$

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$U(1)\times SO(2M_L)$:

$$M = Q^T J Q = J z^k + \mathcal{O}(z^{k-1})$$

$k=1$ ex. :

$$Q = \begin{pmatrix} z\mathbb{1}_{M_L} - A & C \\ 0_{M_L} & \mathbb{1}_{M_L} \end{pmatrix}$$

$$A = \mathrm{diag}(z_1, \ldots, z_{M_L})$$

$$C = \mathrm{diag}(c_1, \ldots, c_{M_L})$$

Minimal example : $k = 1$

$SO(2)$ subgroup : $(MM^\dagger)_i^{SO(2)}$

$$K_i = \xi \log\left[2\sqrt{|z-z_i|^2+|c_i|^2}\right]$$

Minimal example : $k = 1$

$SO(2)$ subgroup : $\left(MM^\dagger\right)_i^{SO(2)}$

$$K_i = \xi \log\left[2\sqrt{|z-z_i|^2+|c_i|^2}\right]$$

$$K_{U(1)\times SO(2M_c)} = \xi \log\left[\sum_{i=1}^{M_c} 2\sqrt{|z-z_i|^2+|c_i|^2}\right]$$

$\mathcal{N}=2$, $SO(N_c)$, $USp(2M_c)$ hyperKähler quotients

$$\tilde{K} = \mathrm{Tr}\left[Q Q^\dagger e^{-V'} + \tilde{Q}^\dagger \tilde{Q} e^{V'} + \lambda \left(e^{-V'^T} J e^{-V'} - J \right) \right]$$

$$W = \mathrm{Tr}\left[Q \tilde{Q} \Sigma' + \chi \left(\Sigma'^T J + J \Sigma' \right) \right]$$

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$$\cap$$

$$SO(N_c), USp(2M_c)$$

$$\cup$$

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$\cap$

$SO(N_c), USp(2M_c)$

$\cup$

Lagrangian mult.

$$W = \mathrm{Tr}\left[Q \tilde{Q} \Sigma' + \chi \left(\Sigma'^T J + J \Sigma' \right) \right]$$

Obtaining the solution

$$(e^{V'})^T = J^T e^{-V'} J$$

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$$\Downarrow$$

$$\tilde{K} = \mathrm{Tr}\left[QQ^\dagger e^{-V'} + J^T e^{-V'} J (QQ^\dagger)^T \right]$$

Obtaining the solution

$$(e^{V'})^T = J^T e^{-V'} J$$

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$$\tilde{K} = \mathrm{Tr}\left[QQ^\dagger e^{-V'} + J^T e^{-V'} J (QQ^\dagger)^T \right]$$

$$= \mathrm{Tr}\left[qq^\dagger e^{-V'} \right]$$

$$q \equiv (Q, J\tilde{Q}^T)$$

Obtaining the solution

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$$= \mathrm{Tr}\left[qq^\dagger e^{-V'} \right]$$

$$q \equiv (Q, J\tilde{Q}^T)$$

$\Rightarrow \mathcal{N}=1$ sol applies...

$$\tilde{K} = \mathrm{Tr}\sqrt{MM^\dagger}, \quad M = q^T J q$$

Constraint coming from the superpotential

Superpotential →

$$q\tilde{J}q = 0\ , \quad \tilde{J} = \begin{pmatrix} 0_{N_F} & \mathbb{1}_{N_F} \\ -\epsilon\mathbb{1}_{N_F} & 0_{N_F} \end{pmatrix}$$

Constraint coming from the superpotential

Superpotential →

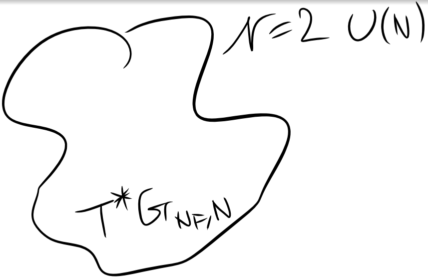
$$q\tilde{J}q = 0\ , \quad \tilde{J} = \begin{pmatrix} 0_{N_F} & \mathbb{1}_{N_F} \\ -\epsilon\mathbb{1}_{N_F} & 0_{N_F} \end{pmatrix}$$

↓ well-known result

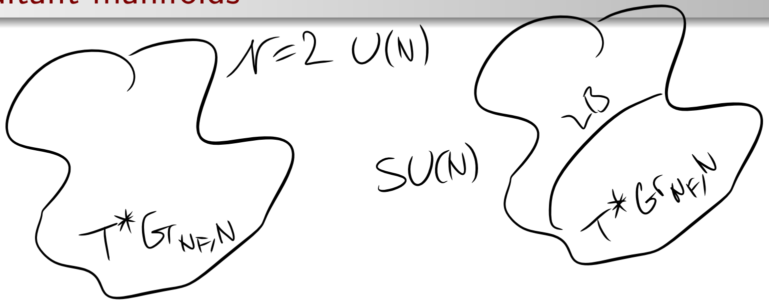
$SO(N_c)$: flavour symmetry $USp(2N_F)$

$USp(2M_c)$: flavour symmetry $SO(2N_F)$

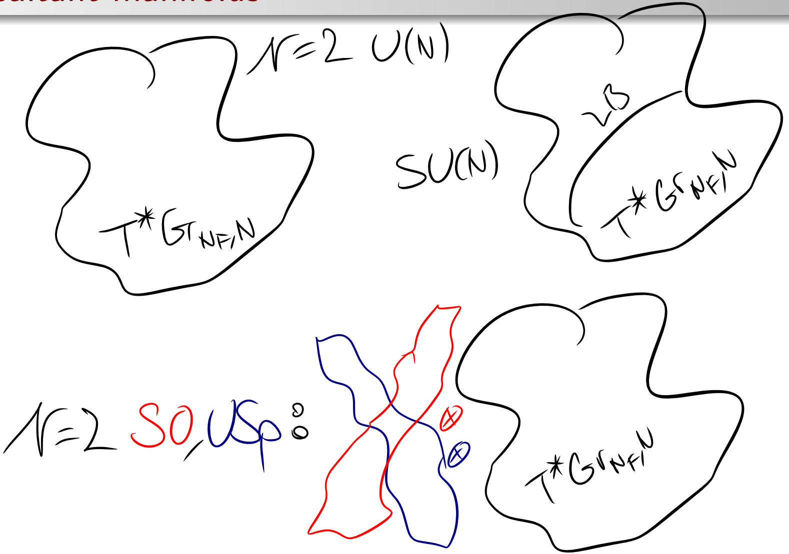
Resultant manifolds



Resultant manifolds



Resultant manifolds



Conclusion

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- Explicit construction of the up-to-now unknown hyperKähler metrics for SO/USp gauge groups.

Discussion

$SO(N), USp(2M)$ HK quotient – not $U(1) \times \{SO(N), USp(2M)\}$

Large N_c ...

Large dets

Expansions ...

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Questions?