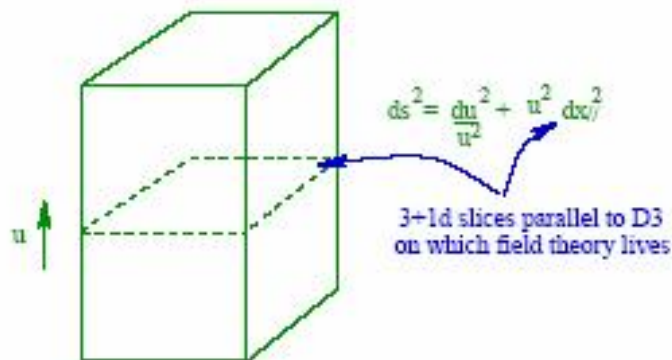# Holographic QCD

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- Dilaton flow QCD
  - Confinement
- Chiral symmetry breaking
  - High temp. phase

Minneapolis, May 2008

4d strongly coupled $\mathcal{N}=4$ SYM   =   IIB strings on $AdS_5 \times S^5$

(conformal)

Pretty well established by this point!

$$ds^2 = \frac{du^2}{u^2} + u^2 dx_{\parallel}^2$$

3+1d slices parallel to D3
on which field theory lives

u corresponds to energy (RG)
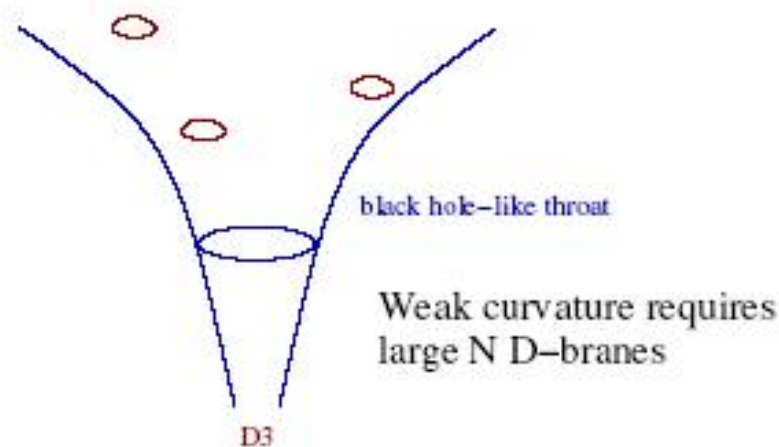scale in field theory

The SUGRA fields act as sources

$$\int d^4x \; \Phi_{SUGRA}(u_0)\lambda\lambda$$

eg asymptotic solution ($u \to \infty$) of scalar

$$\varphi \simeq \frac{m}{u} + \frac{\langle\lambda\lambda\rangle}{u^3}$$

Brane Construction

A large N stack of D3 branes generates curvature like a black hole:

black hole−like throat

Weak curvature requires
large N D−branes

D3

The Tension $\to \infty$ limit blows up the throat

$$ds^2 = u^2 dx_{//}^2 + \frac{du^2}{u^2} + d\Omega_5^2$$

This is $AdS_5 \times S^5$

In this limit higher dimension operators linking the gauge theory on brane and gravity fluctuations off are suppressed - the two descriptions decouple.

Towards Real QCD

4d strongly coupled $\mathcal{N}=4$ SYM $=$ IIB strings on $AdS_5 \times S^5$

Technology:

- Deform - break conformal invariance, running coupling
- Deform - break supersymmetry
- Add quarks - D7 probes

The simplest deformation is to add in SO(6) preserving scalar masses:

$$m^2(\varphi_1^2 + \varphi_2^2 + \varphi_3^2 + \varphi_4^2 + \varphi_5^2 + \varphi_6^2)$$

You can not decouple superpartners so this is as close as you can get to QCD!

5d truncation ignoring the 5-sphere (SO(6) gauge symmetry)

$$S = \frac{1}{4\pi G_5} \int d^5 x \sqrt{-g} \left(\frac{1}{4} R - \frac{1}{8} g^{ab} \nabla_a \phi \nabla_b \phi + V \right)$$
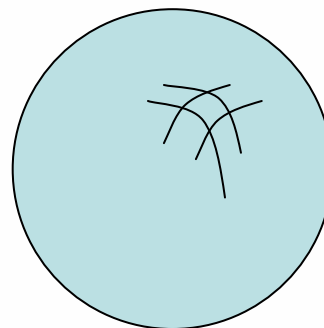
There are 42 scalars - a 20 of SO(6)

- a 10 and $\overline{10}$ of SO(6)

- scalar dilaton-axion, singlets of SO(6) – dual to $\frac{1}{g^2} F^{\mu\nu} F_{\mu\nu}$

$$m^2 Tr(\phi_1^2 + ... + \phi_6^2)$$

is invisible at supergravity level

These configs are dual to D3 5-balls – Gauss' law gives pure AdS background.

If we break susy though we expect all SO(6) singlets to switch on – there are non-trivial dilaton flows

$$\frac{1}{4} R_{ab} = \frac{1}{8} \partial_a \phi \partial_b \phi - g_{ab}$$

$$\nabla^2 \phi = 0$$

$$ds_5^2 = e^{2A} \left(-e^{2B} dt^2 + dx_3^2 \right) + dr^2$$

$$e^{4\bar{A}} = \frac{c_4^2 e^{8r} - c_3^2}{2c_4 e^{4r}}$$

$$\bar{B} = \frac{c_2}{4c_3} \ln \left(\frac{c_4 e^{4r} - c_3}{c_4 e^{4r} + c_3} \right) + B_0$$

$$\phi = \frac{c_1}{4c_3} \ln \left(\frac{c_4 e^{4r} + c_3}{c_4 e^{4r} - c_3} \right) + \phi_0$$

Analytic solutions that can be lifted to 10d supergravity

$$ds^2 = e^{\phi/2} \left(\frac{u^2}{L^2} \mathcal{A}^2(u) dx_4^2 + \frac{L^2}{u^2} du^2 + L^2 d\Omega_5^2 \right)$$
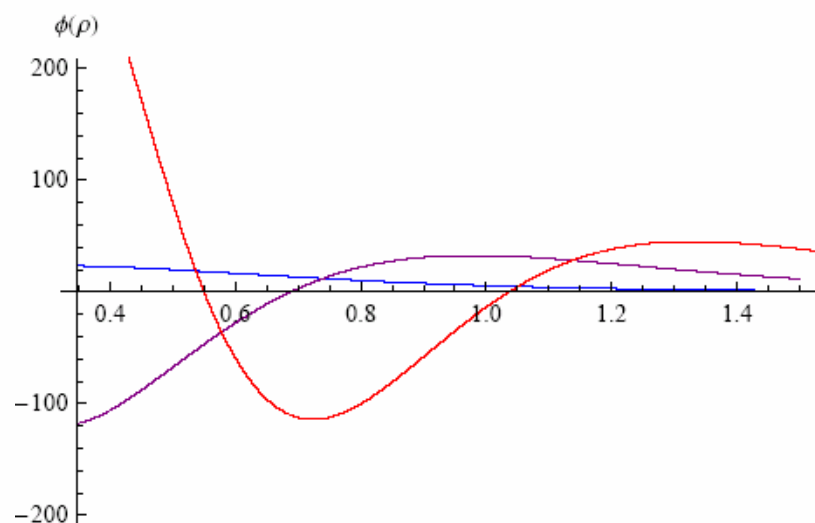
$$\mathcal{A}(u) = \left(1 - (\frac{u_0}{u})^8 \right)^{\frac{1}{4}}, \qquad e^{\phi} = \left(\frac{(u/u_0)^4 + 1}{(u/u_0)^4 - 1} \right)^{\sqrt{3/2}}$$

Tr F^2 is non-zero

u_0 **is mass gap (position of D3 ball(?))**

We seek normalizable fluctuations in Tr F^2 / dilaton
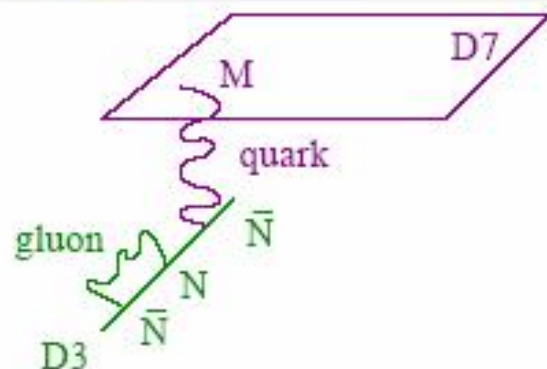
$$\delta\phi = f(r)e^{-ikx}, \qquad k^2 = -M^2$$

n	1	2	3	4	5
M_n	4.1	7.2	10.2	13.2	16.2

Table I: Lowest five glueball masses in the zero temperature dilaton flow geometry in units of the deformation scale u_0.
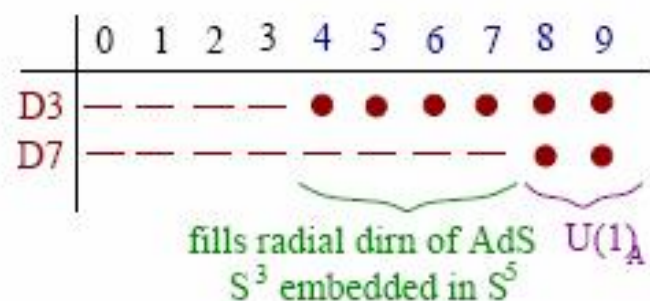
Quarks can be introduced via D7 branes in AdS

The brane set up is

	0	1	2	3	4	5	6	7	8	9
D3	—	—	—	—	●	●	●	●	●	●
D7	—	—	—	—	—	—	—	—	●	●

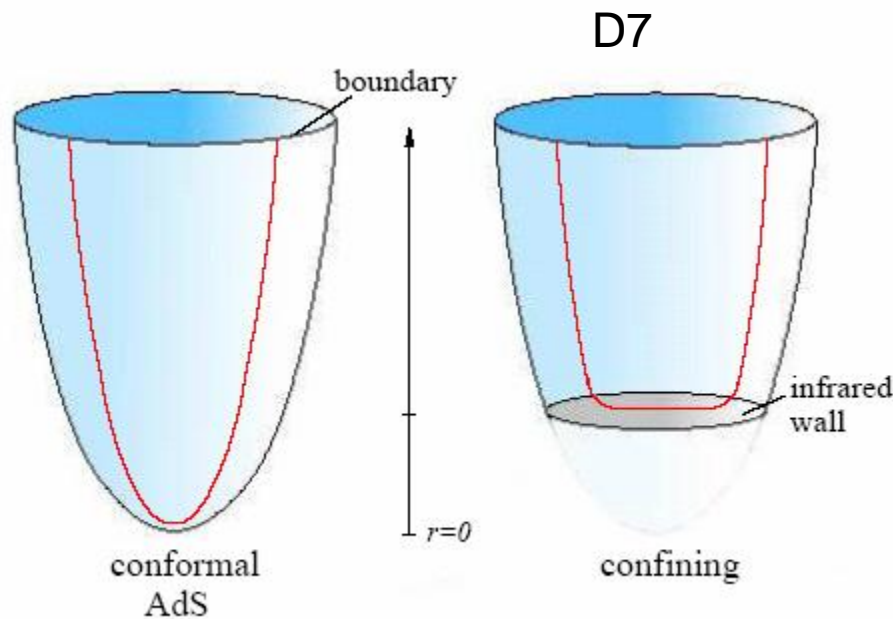fills radial dirn of AdS $U(1)_A$
S^3 embedded in S^5

We will treat D7 as a probe - quenching in the gauge theory.
Minimize D7 world volume with DBI action

$$S_{D7} = -T_7 \int d\xi^8 \sqrt{P[G_{ab}]}, \qquad P[G_{ab}] = G_{MN} \frac{dx^M}{d\xi^a} \frac{dx^N}{d\xi^b}$$

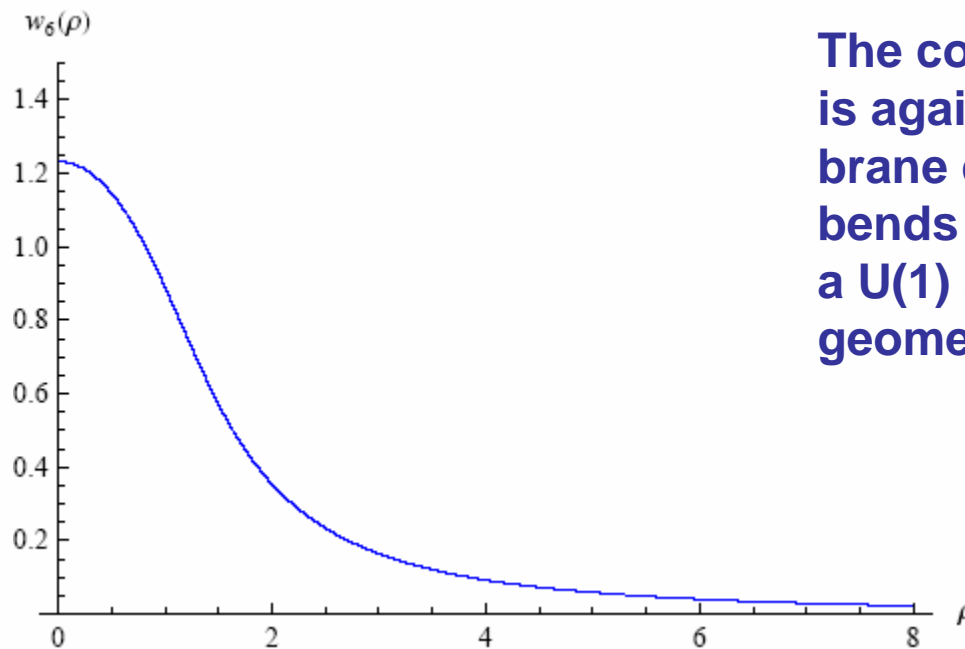**The interaction energy between a quark and an anti-quark is given by embedding a string in the space:**

D7

boundary

conformal
AdS

r=0

infrared
wall

confining

The singularity repels the string and it lies along the wall – a linear potential emerges
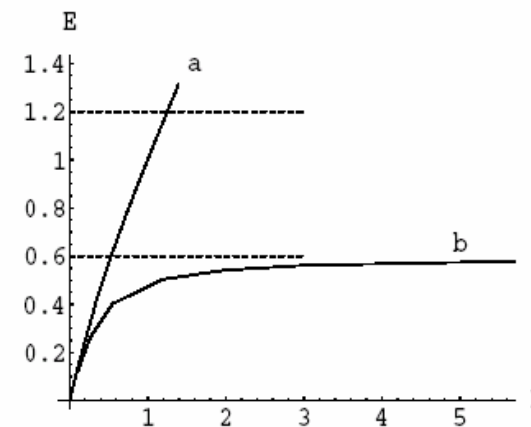
This theory has a U(1) symmetry that is dynamically broken by a quark condensate

$w_6(\rho)$

The core of the geometry is again repulsive - the D7 brane does not lie flat but bends explicitly breaking a U(1) symmetry of the geometry

$$\phi = m + \frac{c}{r^2} + \dots$$

$$\langle \bar{q}q \rangle = 1.51 u_0^3$$

We seek normalizable fluctuations of the D7

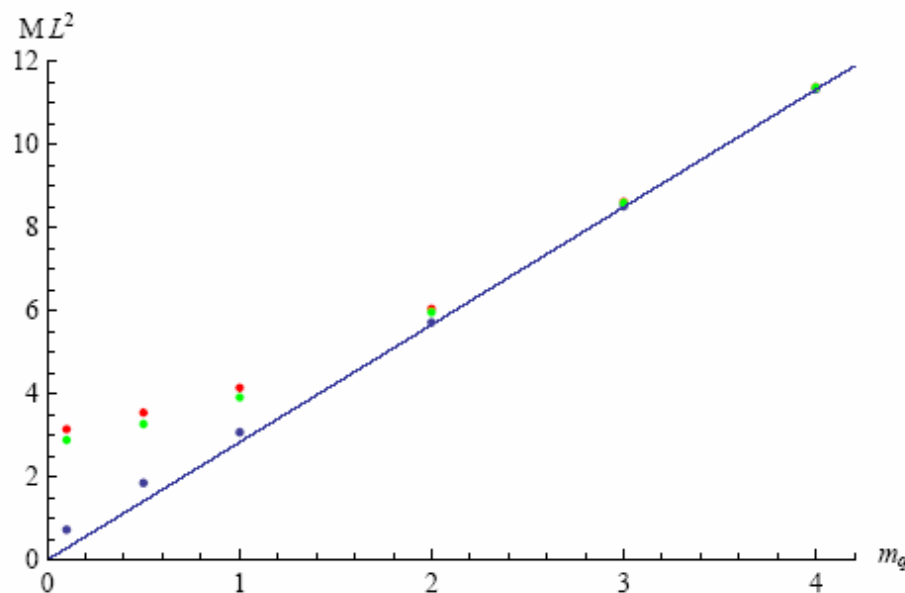$$\delta\phi = f(r)e^{-ikx}, \qquad k^2 = -M^2$$

FIG. 1: Pion (blue), sigma (red) and vector (green) masses as a function of quark mass - all in units of u_0. The line shows the large-m_q limit.

Massless Goldstone

Massive sigma and rho mesons

As in all AdS duals these states are tightly bound

$m \sim 1/$ lambda$^{1/2}$

The duals of high T gauge theories are black holes – their free energy scales like N^2 (Witten)

Analytically can show there is no black hole with dilaton hair – the only black hole in 5d supergravity is AdS-Schwarzchild

$$ds^2 = \frac{K(u)}{L^2} d\tau^2 + L^2 \frac{du^2}{K(u)} + \frac{u^2}{L^2} dx_4^2 + L^2 d\Omega_5^2 \qquad K(u) = u^2 - \frac{u_h^4}{u^2}$$

We conclude that Tr F^2 switches off at high T

Presumably the scalar mass is still there but invisible….

We can compare the free energies of the dilaton flow geometry (with compact Euclidean time) and the black hole

$$S_{DF} = \frac{1}{2G_5 L^2} \int_0^{\frac{\pi L^2}{u_h}\sqrt{1-\frac{u_h^4}{\Lambda^4}}} d\tau \int_{u_0}^{\Lambda} \sqrt{-g}\, dr$$

$$S_{BH} = \frac{1}{2G_5 u_h L^3} \int_{u_h}^{\Lambda} u^3 du = \frac{1}{8G_5 u_h L^3}\left(\Lambda^4 - u_h^4\right)$$

$\Lambda \to \infty$ limit

$$S_{BH} - S_{DF} = \frac{1}{16 G_5 u_h L^3}\left(4u_0^4 - u_h^4\right)$$

A first order transition

$$T_c = \frac{\sqrt{2}u_0}{\pi L^2}$$

$$T_c \sim 124 \text{ MeV}$$

(fixing the rho mass)

At the supergravity level this non-supersymmetric theory and the N=4 theory share the same supergravity dual

Regular glueball and meson solutions are replaced by in-falling quasi-normal modes… the bound states decay into the thermal bath

(Hoyos.., Myers…, Peeters…)

n	ω_n
1	$\pm\ 2.1988 - 1.7595\ i$
2	$\pm\ 4.2119 - 3.7749\ i$
3	$\pm\ 6.2155 - 5.7773\ i$
4	$\pm\ 8.2172 - 7.7781\ i$
5	$\pm\ 10.2181 - 9.7785\ i$

Table IV: the scalar mesonic quasinormal frequencies in the high T phase ($m_q = 0$) - in units of $\frac{u_h}{L^2}$.

$$\text{ratio of viscosity to entropy density} \qquad \frac{\eta}{s} = \frac{1}{4\pi}$$

(Son, Starinets,Policastro)

Summary

Dilaton Flow QCD is a holographic model with

- **Confinement**

- **Mass gap**

- **A quark condensate**

- **A massless pion & a massive rho**

- **A thermal deconfinement transition**

- **Meson melting at high T**

A review:

arXiv:0711.4467 [hep-th]

Also

arXiv:0805.0956 [hep-th]

It is derived from the AdS/CFT Correspondence so describes a real gauge theory and contains the DYNAMICS for all this physics (unlike AdS/QCD)

(Sakai-Sugimoto has in addition a non-abelian chiral symmetry but at the expense of being fundamentally five dimensional)