

Soft wall dynamics in AdS/QCD *

Brian Batell

University of Minnesota

*with Tony Gherghetta
- [arXiv:0801.4383](https://arxiv.org/abs/0801.4383)

CAQCD08 5/17/08

Outline

- AdS/CFT
- AdS/QCD: Hard and soft walls
- A dynamical soft wall
- Scalar potential
- Match to QCD operators?

AdS/CFT duality

Maldacena '97

Gubser, Klebanov, Polyakov '97

Witten '97

String/gravity theory in higher dimensions **dual** \iff Strongly coupled gauge theory in 4D

Can we use it to understand real-world QCD?

Top-down AdS/CFT constructions:

- conformal
- nonconformal
- less or no SUSY
- confining
- flavor
- chiral symmetry breaking

but still no string/gravity dual to QCD

Work backwards: AdS/QCD

Erlich, Katz, Son, Stephanov '05
Da Rold, Pomarol '05

Basic idea:

Use facts about QCD to understand
properties of gravity dual

QCD ingredients:

- chiral symmetry \rightarrow bulk gauge symmetry
- QCD operators:
 - $(\bar{q}\gamma^\mu t^a q)_{L,R} \rightarrow A_{L,R}^{M a}(x, z)$
 - $\bar{q}q \rightarrow$ scalar $X(x, z)$ with $m_X^2 = -3$
- confinement?

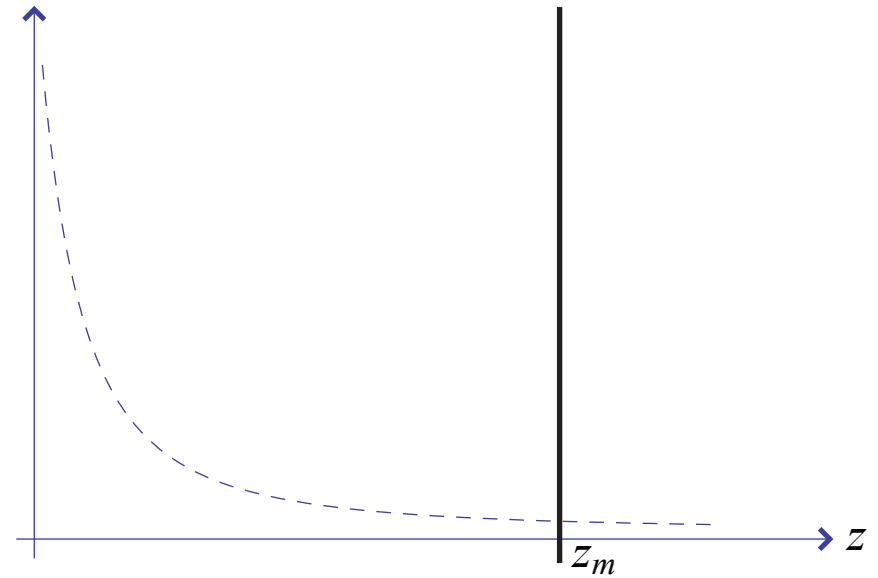
Hard wall

- Confinement implemented with a cutoff to AdS spacetime in the IR:

$$z_m \sim \Lambda_{QCD}^{-1}$$

- Predictions match QCD phenomenology with $O(10\%)$ error
- Highly excited states:

$$m_n^2 \sim n^2$$
$$m_S^2 \sim S^2$$



Hard wall model does not exhibit linear Regge behavior

Soft wall

Karch, Katz, Son, Stephanov '06

- Dilaton provides a smooth cutoff to spacetime

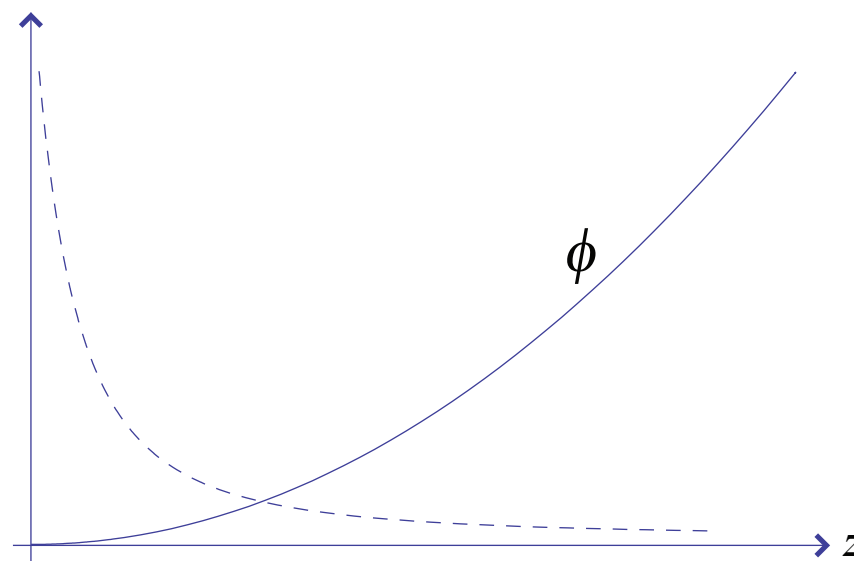
$$S = \int d^4x dz \sqrt{-g} e^{-\Phi} \mathcal{L}$$

- Simplest model:

$$g_{MN} = z^{-2} \eta_{MN}$$
$$\Phi(z) = z^2$$

- Regge trajectories:

$$m_{n,S}^2 \sim (n + S)$$



Soft wall dynamics

How could this background arise dynamically?

- Factor $e^{-\Phi}$ in arises from open string/D-brane sector
- Start from string inspired action including closed string tachyon:

$$S = M^3 \int d^5x \sqrt{-g} e^{-2\Phi} \left(R + 4g^{MN} \partial_M \Phi \partial_N \Phi - \frac{1}{2} g^{MN} \partial_M T \partial_N T - V(\Phi, T) \right)$$

- What about the potential $V(\Phi, T)$? Don't know *a priori* so work backwards

Search for solution

1) Move to Einstein frame

- Conformal transformation $g_{MN} = e^{4\Phi/3} \tilde{g}_{MN}$; $\phi = \sqrt{8/3} \Phi$
- metric in Einstein frame: $\tilde{g}_{MN} = z^{-2} e^{-2az^2} \eta_{MN}$

2) Ansatz

$$ds^2 = e^{-2A(y)} dx^2 + dy^2,$$

$$\phi = \phi(y),$$

$$T = T(y).$$

3) “Superpotential method”

Skenderis, Townsend '99

DeWolfe, Freedman, Gubser, Karch '99

- converts second order system of equations into first order system
- introduce a “superpotential” $W(\phi, T)$

$$\begin{aligned}A' &= W, \\ \phi' &= 6 \frac{\partial W}{\partial \phi}, \\ T' &= 6 \frac{\partial W}{\partial T}.\end{aligned}$$

$$\tilde{V}(\phi, T) = 18 \left[\left(\frac{\partial W}{\partial \phi} \right)^2 + \left(\frac{\partial W}{\partial T} \right)^2 \right] - 12W^2.$$

4) Convert between z and y coordinates:

$$y = \int dz \frac{e^{-az^2}}{z} = \frac{1}{2} \text{Ei}(-az^2)$$

- Define the inverse function I through the relation

$$z^2 = -\frac{1}{a} \text{Ei}^{(-1)}(2y) \equiv -\frac{1}{a} I(2y)$$

- Inverse function I follows the differentiation rule:

$$\frac{dI}{dy} = 2Ie^{-I}$$

5) Turn the crank

$$A(y) = az^2 + \frac{1}{2} \log z^2 = -I + \frac{1}{2} \log \left(-\frac{I}{a} \right)$$

- Take derivatives

$$W' = A'' = -6Ie^{-2I} + 4I^2e^{-2I} = \frac{\phi'^2}{6} + \frac{T'^2}{6}$$

- Identify

$$\phi' = -2\sqrt{6}Ie^{-I}$$

$$T' = 6\sqrt{-I}e^{-I}$$

- Integrate

$$\phi = -\sqrt{6}I = \sqrt{6}az^2$$

$$T = 6\sqrt{-I} = 6\sqrt{az}$$

Solution

$$\Phi(z) = z^2$$

$$T(z) = \pm 2\sqrt{6}z$$

$$g_{MN} = \frac{\eta_{MN}}{z^2}$$

In the string frame, we have an AdS metric and quadratic dilaton!

Dilaton-tachyon potential

- Can invert solution to find superpotential $W(\phi, T)$
- One example:

$$W = -2 \left(1 - \frac{\phi}{\sqrt{6}} \right) e^{\phi/\sqrt{6}} + 3e^{T^2/36}$$

- leads to the potential

$$\tilde{V}(\phi, T) = \frac{T^2}{2} e^{T^2/18} + 2\phi^2 e^{2\phi/\sqrt{6}} - 12 \left[3e^{T^2/36} - 2 \left(1 - \frac{\phi}{\sqrt{6}} \right) e^{\phi/\sqrt{6}} \right]^2$$

- some terms in the potential are exotic; others are not
e.g. $e^{2\phi/\sqrt{6}}$ matches 5D noncritical string theory

Operator/field dictionary

- Expand potential:

$$\tilde{V}_{\pm}(\phi, T) \simeq -12 - 2\phi^2 - \frac{3}{2}T^2 + \dots$$

- Match bulk masses to operator dimensions:

$$\Delta_{\phi} = 2 + \sqrt{4 + m_{\phi}^2} = 2$$

$$\Delta_T = 2 + \sqrt{4 + m_T^2} = 3$$

Match to QCD operators?

$$\Delta_\phi = 2:$$

- No local, gauge invariant dimension 2 operator in QCD. So?
 - gluon mass?
 - SUSY QCD?
 - modified UV dynamics

$$\Delta_T = 2 + \sqrt{4 + m_T^2} = 3:$$

- Associate T with $\bar{q}q$ operator?
 - $T \sim z$: hard chiral symmetry breaking
 - constant mass splitting between chiral pairs
 - $\bar{q}q$ should originate from open string sector

Generalize solution: power-law dilaton and AdS_{d+1} metric

$$\begin{aligned}\Phi(z) &= z^\nu \\ T(z) &= \pm 4\sqrt{1 + 1/\nu} z^{\nu/2} \\ g_{MN} &= \frac{\eta_{MN}}{z^2}\end{aligned}$$

- 2D QCD - 't Hooft model : $d = \nu = 2$

- analytically soluble
- mesons follow linear Regge trajectories

$$\Delta_\phi = 2:$$

- Dimension 2 Gluon field strength squared: $G_{\mu\nu}^a G^{\mu\nu a}$

$$\Delta_T = 1:$$

- Dimension 1 $\bar{q}q$ operator

Conclusions

- 5D Gravity-dilaton-tachyon solution
- Dynamical model of linear confinement
- Exotic potential, but certain terms found in noncritical string theory
- Simple dimension 2 and 3 operators, but relation to QCD not clear
- 2D QCD: operator dimensions match $G_{\mu\nu}^a G^{\mu\nu a}$ and $\bar{q}q$
- More work: Scalar perturbations, stability, connect to string theory
...