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Constrained Regularization of Digital Terrain Elevation Data

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***Abstract*— A framework for geometric regularization of elevation maps is introduced in this paper. The framework takes into account errors in the data, which form part of standard elevation maps specifications, as well as possible additional user/application dependent constraints. The algorithm is based on adapting the theory of geometric active surfaces to the problem of regularizing elevation maps. We present the underlying concepts and numerical experiments showing the effectiveness and potential of this theory.**

Index terms: **Elevation data, geometric active surfaces, regularization, absolute errors, constraints.**

I. Introduction

Elevation data often comes with information about measurements uncertainty. In particular, the standard specification of the National-Geospatial Intelligence Agency (formerly NIMA) for digital terrain elevation data (DTED) includes information about absolute vertical and horizontal errors (see for example http://www.fas.org/irp/program/core/dted.htm and http://mountains.ece.umn.edu/∼guille/dtedspecification.pdf for a copy of the official DTED specification.). Any given post reading (e.g., with posts spaced approximately 30 meters apart for the DTED-2 data used for this paper) can actually "move" up/down or to the sides, following the absolute vertical and horizontal errors specified within the dataset. These errors/adjustments can be, for example, of 5-10 meters vertically and 10-15 meters horizontally.

Due to such intrinsic uncertainties in elevation data, its regularization is often needed, both for visualization and for processing (e.g., compression). In this paper we introduce a framework to perform this regularization, while maintaining the constraints given by the provided information about vertical and horizontal absolute errors. These errors are represented in our work by "uncertainty cylinders," centered at the given data points, see Figure 1. The cylinder's height represents the absolute vertical accuracy, while base diameter corresponds to horizontal accuracy.[1] We search for a smooth surface that deforms the original data while keeping the points inside their corresponding cylinders. Such an important constraint, that is dictated by the elevation dataset specifications, has not been exploited by previous state-of-the-art approaches such as those in [2], [13]. We thus examine techniques based on geometric partial differential equations [8], [9], [10], and in particular the geodesic active contours and minimal surfaces framework [1], [5]. In this note we propose a means to extend these works to the problem of regularization with constraints for elevation maps. We show that the framework is valid not only in order to respect the absolute vertical and horizontal errors provided by the DTED specification, but can also deal with other constraints, such as the preservation of geometric features.

The problem of regularization with constraints has been addressed in the literature. Kimeldorf and Wahba, [6], showed how to compute one dimensional splines with hard vertical-error constraints. While this elegant approach can easily be extended to higher dimensions, it doesn't include the horizontal freedom given by the horizontal absolute error. It is also not developed for the additional geometric constraints that are natural to add in our framework. The theory of *total least squares* [4] also addresses the "freedom of motion" of the given data, both in the vertical and horizontal position. In it's original form, although computationally very efficient, the framework does not provide hard constraints (that is, the error is not guaranteed to be bellow the allowed bounds), neither does include any kind of explicit regularization or geometric terms. In order to add these important constraints, the problem has to be transformed into a variational framework much of the flavor here introduced.

Before proceeding with the framework description, we should comment that we opt in this work to regularize the whole data set at once (as a function on the plane), and not just individual iso-height lines. Although we could use the same framework for regularizing lines, this is a non-consistent approach. That is, when regularizing "on demand" individual iso-height lines with the absolute errors constraints, terrain points can be assigned to multiple elevations. For example, considering a vertical error of 10 meters, a point measured at 95 meters could be assigned both to lines at 90 or 100 meters if the corresponding iso-level lines are treated independently.

II. Regularization with constraints via geometric active surfaces

Let us begin with the general description of the minimal surfaces approach [1], which we then adapt to the problem at hand. Let $\mathcal{S}$ be a two dimensional (2D) surface embedded in the Euclidean space R^3. Let also $I : R^3 \to R$ be some given

[1] If other noise models are considered, e.g., Gaussian distributions, the cylinders are to be replaced by other three dimensional shapes such as ellipsoids. We opt to work with cylinders since the standard DTED-2 specification does provide absolute errors.

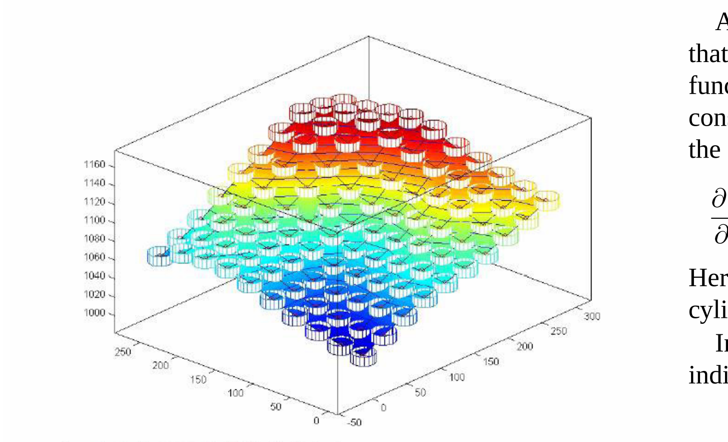

Fig. 1. Problem description (data in meters): The given surface is allowed to move inside the uncertainty cylinders, which are determined by the absolute horizontal and vertical accuracy of the measurements (as described in the NGA DTED Performance Specification Document and provided in the standard DTED file).

data, which constraints the surface (e.g., a gradient map if we are concerned with segmentation, the original motivation for the minimal surfaces model), and $g : R \to R$ a function that acts on this data. The basic idea is to find a surface $\mathcal{S}$ such that it minimizes the energy given by

$$\int_{\mathcal{S}} g(I(\mathcal{S}))da, \tag{1}$$

where da is the classical element of area on the surface $\mathcal{S}$. In words, the above energy is minimizing the measure g on the surface $\mathcal{S}$. In the case of image segmentation for example, g is often a decreasing function of the image gradient, and in such a way, $\mathcal{S}$ is constrained to be at regions of large gradient, thereby being attracted to object boundaries in simple models.

The above energy is often minimized via a gradient descent flow, leading to the evolution equation

$$\frac{\partial \mathcal{S}}{\partial t} = (\kappa g(I(\mathcal{S})) - \nabla g \cdot \vec{N})\vec{N}, \tag{2}$$

where κ is the surface mean curvature and $\vec{N}$ is the surface unit normal. The above equation is iteratively run to steady state starting from some initial guess $\mathcal{S}_0$. It has been shown in the literature, [1], that the flow is regularizing the surface (due to the curvature component) while being constraint to move it to areas of low g.

With respect to our surface elevation data, we select g to be a function of the distance to the uncertainty cylinders (see also [14] for the use of distance constraints in the problem of surface reconstruction from scattered data). That is, inside the cylinders we select $g = 0$ (or some very small $\epsilon > 0$ value for additional smoothness), and outside of the cylinders we select

$$g(x, y, z) = f(d(x, y, z)),$$

where $f(\cdot)$ is any increasing function (in our examples we select the unit constant function), and $d(x, y, z)$ is the Euclidean distance between the 3D point (x, y, z) and the set C_i of cylinders (see Figure 1).

Another important adaptation to our problem at hand is that the elevation data is not a generic surface but a height function $h(x, y) : R^2 \to R^+$ ($\mathcal{S} = (x, y, h(x, y))$). Thereby, considering the vector $\vec{z}$ a unit vector in the vertical direction, the above flow becomes a simple scalar motion:

$$\frac{\partial h}{\partial t} = (\kappa d(x, y, h(x, y)) - \nabla d(x, y, h(x, y)) \cdot \vec{N})\vec{N} \cdot \vec{z}. \tag{3}$$

Here, the generic g function has been replaced with the cylinder/distance-based version described above.

In addition, for functions, it is well known that (subscripts indicate derivatives)

$$\vec{N} = \frac{1}{\sqrt{h_x^2 + h_y^2 + 1}}(-h_x, -h_y, 1)$$

and

$$\kappa = -\frac{1}{2}\text{div}\left(\frac{1}{\sqrt{h_x^2 + h_y^2 + 1}}(-h_x, -h_y, 1)\right),$$

making the numerical implementation of the above mentioned flow quite straightforward.

A. Extensions

The framework introduced above is flexible and natural for extensions that are relevant for the regularization of elevation maps. For example, the following procedures may be implemented:

1) If, in-spite of the existence of the absolute vertical and horizontal errors, a certain feature in the original elevation data is not to be allowed to move, all that needs to be done is to place a zero height and zero width (or as small as desired) cylinder for that point. If for example a ridge is to be preserved, after it has been detected by a desired manual or automatic technique, the corresponding points in the DTED file are marked and their cylinders are reduced to the desired size (which can be null for absolute no motion). Note that this can be done for a group of points representing lines of interest as well as for entire regions. Of course, the algorithm works for cylinders of different sizes and also general shapes of "allowed motion." This is easily handled by the distance function and from it the penalty g function.
2) If other characteristics of the data are to be preserved, these can be incorporated in the form of hard or soft constraints to the variational formulation. For example, a slope preserving term can be easily added, leading to an energy of the form

$$\int_h g(h)da + \lambda \int \| \nabla h - \nabla h_0 \|^2 \, dxdy,$$

where h_0 is the original (noisy) data and λ is a standard Lagrange multiplier. Proximity constraints to desired coordinates can be added as well to force regions to be either preserved or moved towards a certain target.

III. Experimental Results

We experimented with DTED-2 data (30 meters post separation) with 10 meters of vertical absolute error and 13 meters of horizontal absolute error, as dictated by the data specification. To capture the size of these errors, we constructed a refined grid, 7 times larger in each horizontal dimension (that is, the original data is linearly interpolated to posts at 30/7 meters to produce h_0, the initial condition for Equation (3)). A finer refinement, which will increase the computational cost, was not found to be needed. This data was then regularized with the technique described above, using standard numerical techniques [10]. If desired, fast numerical implementations following [3], [7] could be used as well. Figures 2 and 3 present two examples showing original and regularized results. Figure 4 shows the histogram of the vertical motion (adjustment) introduced by the surface regularization algorithm. Note that while the data is being regularized, the motions are between the permitted margin dictated by the data.

IV. Concluding Remarks

In this paper we have exploited important components of DTED specifications that have been widely ignored when processing elevation maps. Extending work on geometric active surfaces we have shown how the maps can be regularized while maintaining the data in the intervals allowed by the pre-specified errors. Additional constraints can be easily incorporated to the framework as well. The framework can also be applied to non-grid data properly adapting the numerical implementation.

We are currently working with the framework described in [12] to provide an alternative to the approach here presented. We are also investigating the use of the results obtained here for compression of elevation maps, [11], and working on adding constraints to the regularization process. Results in these directions will be reported elsewhere.

V. Acknowledgments

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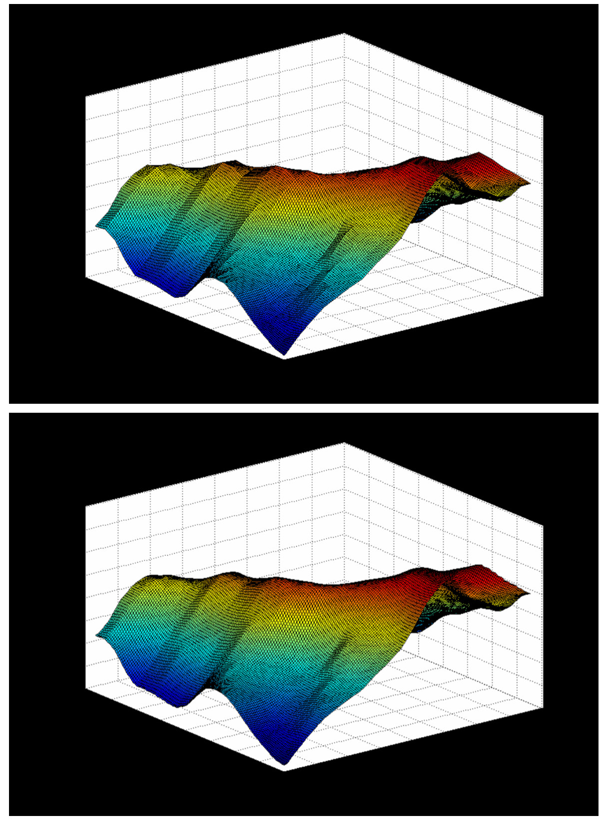
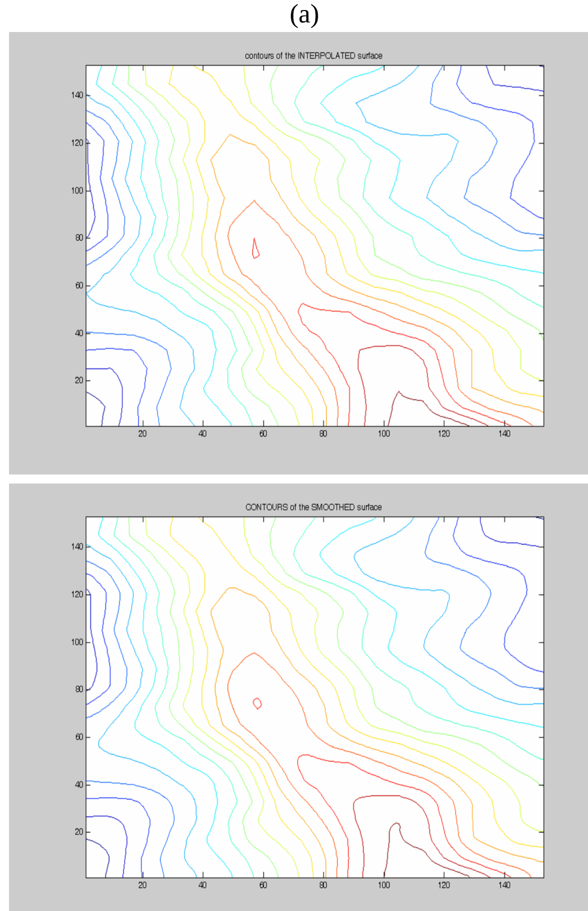


(a)



(b)

Fig. 2. Example of constrained regularization of elevation maps. (a) The original surface is followed by the regularized one. (b) A corresponding set of original and regularized iso-height lines are shown, spaced at 9 meter intervals (from maximal contour elevation of 1159 meters to minimal contour elevation of 1025 meters).

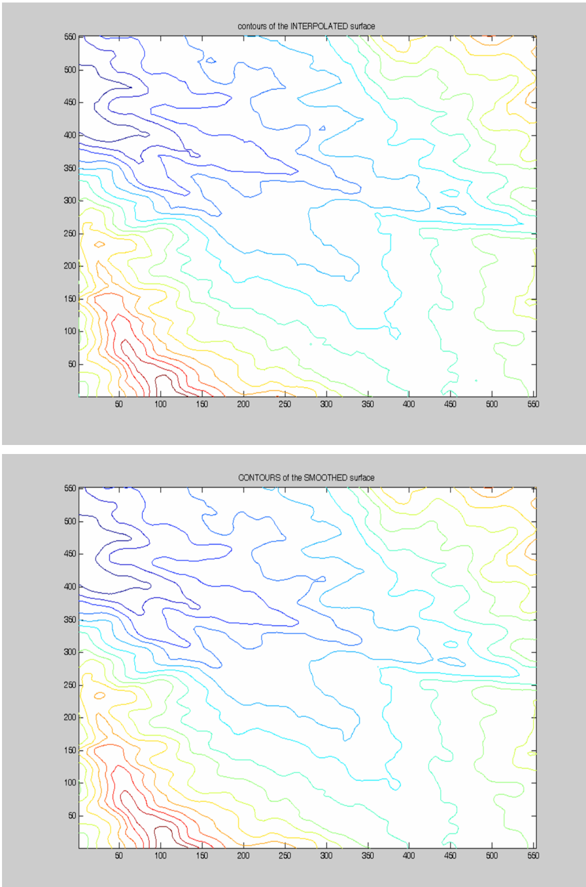


Fig. 3. Second example of constrained regularization of elevation maps. A set of original and regularized iso-height lines are shown, spaced at 16 meter intervals (from maximal contour elevation of 1159 meters to minimal contour elevation of 919 meters).

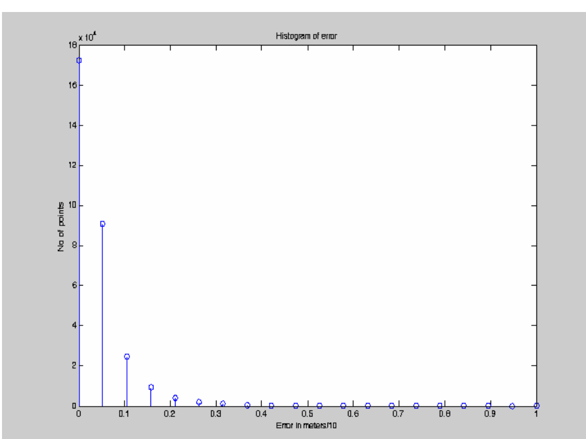


Fig. 4. Quantized vertical adjustments distribution. Note that following the imposed constraints, all points were adjusted less than 10 meters (the cylinders height), while the vast majority actually moved less than 2 meters. Other data sets might have a different distribution for the points vertical motion resulting from the regularization, being important to keep the cylinders of the size dictated by the DTED specification, as done in this work.

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