EXPLICIT EXACT SOLUTIONS FOR THE GENERALIZED NON-CONSERVATIVE ULTRASHORT PULSE PROPAGATION SYSTEM

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Explicit Exact Solutions for the Generalized non Conservative Ultrashort Pulse Propagation System.

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ABSTRACT

We consider the complex Ginzburg-Landau equation form which describes the femtosecond pulses propagation with higher-order time for narrow pulse widths and nonlinear dispersion. Using the separation method, we obtained under some specific constraint conditions the existence of dark soliton. In the particular case of degeneracy, a generalized method, which is generally called the projective Riccati equation method is used to construct new exact solutions of the above mentioned equation based on a system of Riccati equations. New exact travelling wave solutions are found which include bright soliton, dark soliton, new solitary wave, periodic and rational solutions.

1 INTRODUCTION

During the past decade, experimental achievements have increased the interest of the propagation of ultrashort pulses in optical fibers in the form of optical solitons. Several authors have discussed the new processes which present in the ultrashort pulse regime [1-3]. The possibility of compensating for the temporal broadening of a short pulse in the anomalous-dispersion regime of fibers by using nonlinearity (thus forming a so-called bright soliton) was pointed out by Hasegawa and Tappert in 1971 [4]. This prediction was subsequently confirmed by several experiments [5]. At the same time Hasegawa and Tappert proposed that in the normal dispersion regime of fiber, dark solitons might propagate in the form of dips embedded in a continuous-wave background [6]. It is well known that optical bright solitons can be used for long distance communication to drastically increase the bit rate of fiber transmission systems. Dark solitons are reflectiveness radiation modes of the wave guides, which also have a localized shape similar to bright solitons, but with complex envelope and non vanishing asymptotic.

In its original version, a soliton is a stable and localized nonlinear solution of an integrable, purely dispersive systems such as nonlinear Schrödinger (NLS) equation. However, in the past few years stable, localized solitary solutions have been found to occur in driven dissipative-dispersive systems such as the driven and damped NLS equation and the (quintic) complex Ginzburg-Landau (CGL) equation. Generic equation with both dissipation and dispersion and which describe systems near a subcritical bifurcations to travelling waves. These dissipative-dispersive localized (DDL) solutions share some properties with solitons, such as a fixed shape for the modulus and interaction behavior in which shape and size are preserved during collisions [7-12].

With the current interest of using solitons as pulse bits in long optical fibers for communication purposes, it is important for us to re-evaluate the practicality of using analytical techniques for predicting the behavior of such bits. Since such pulses are near a pure soliton solution, it becomes feasible to use analytical soliton techniques and the potentiality of obtaining useful analytical results becomes very high [13,14]. The convenience of using a single master equation for describing complicated phenomena such as ultrashort-pulse generation in laser systems is unquestioned [15]. This can be done in certain limits that are valid for variety of laser systems. For example, in order to increase the bit rates, it is necessary to decrease the pulse width. As pulse lengths become comparable to the wave lengths, however, the standard NLS equation becomes inadequate, additional terms have to be included and the resulting propagation is called as higher-order nonlinear Schrödinger (HNLS) equation [16]. This equation includes effects like third-order dispersion (TOD), self steepening (SS) and stimulated Raman scattering (SRS). It is well known that TOD effects splitting-up of higher order solitons. The inelastic Raman scattering is due to the delayed response of the medium which forces the pulse to undergo a frequency shift which is known as self-frequency shift (SFS). The effect of self-steepening is due to the intensity-dependent group velocity of optical pulse, which gives the pulse a very narrow width in the course of propagation. Many authors have analyzed the HNLS equation from different points of view (Painlevé analysis, Hirota direct method, inverse scattering transform, Bäcklung transformation and conservation laws). There has been many literatures giving bright soliton [17-24] solution and dark soliton [25-27] solution to HNLS equation.

In the case of a laser with a fast saturable absorber, the original equations can be reduced to a single CGL equation. It has been realized [28] that the simplest case of cubic CGL is not adequate for a realistic description of any actual system. The quintic nonlinearity is essential for ensuring the stability of soliton-like pulses [28]. Another restriction is that the spectral filtering in the CGL equation model is limited to a second-order term and can describe only a special response with a single maximum. It has been seen that the gain spectrum is usually wide and might have several maxima. It is clear that higher-order filtering terms are essential both for making the model more realistic, and for describing more involved pulse generation effects [29]. Thus a quintic nonlinearity term has been added to investigate the corrections caused by nonlinear amplification (absorption) and nonlinear refraction, resulting in new interesting solutions [30]. Deissler and Brand [31] have studied numerically the effect on nonlinear gradient terms on the DDL solutions of the quintic CGL equation. Yomba and Kofané have investigated analytically the quintic CGL equation with higher order terms [32]. Li et al [33] presented an exact analytical soliton-like solutions for femtosecond laser pulse including TOD, nonlinear dispersion and SFS and by employing numerical methods they proved the stability of the soliton-like solution under perturbations of amplitude, white noise and chirp.

In this paper we present new exact analytical solutions including dark soliton solution of the resulting equation for the propagation of femtosecond (e.g. 100 fs) pulses. More new solitary wave solutions are obtained in case of degeneracy, by expressing solutions of the equation as a polynomial in two elementary functions which satisfy a projective Riccati equation method.

For an optical system, including bandwidth-limited gain and higher order effects, when the period of the perturbation is small compared to amplifiers spacing, a so-called distributed model can be used. The related equation for the propagation of femtosecond pulses is governed by the generalized equation of the form:

$$E_z - i\sigma E_t - pE_{tt} - q|E|^2E - \delta E - \lambda E_{3t} - \mu(|E|^2E)_t - \nu E(|E|^2)_t = 0, \quad (1)$$

where E(t,z) is the slowly varying envelope of the electric field, z is the normalized propagation distance and t is the retarded time. The model parameters σ and δ are real constants; p, q, λ, μ, and ν can be complex. The corresponding effects of the model parameters can be found in [33].

This paper is organized as follows. In section II, we first follow the method of separation of E(z,t) into the amplitude $a(\xi)$ and nonlinear phase $\psi(\xi)$ to construct an exact solution that describes Eq.(1). In section III, the degeneracy of Eq.(1) is investigated and analytical solutions are presented. Conclusion is given in section IV.

2 Exact Analytical soliton-like solutions of femtosecond solitary laser pulse form system

The above generalized version of complex amplitude has been considered here for purpose of analyzing various new exact solutions from the point of view of separation method. Since, the chirped soliton solutions of Eq.(1) have already been reported in [33], we may only present here new solutions.

To apply the separation method, we express E(z,t) as follows:

$$E(z,t) = a(\xi)e^{i[\psi(\xi)+Kt-\Omega z]}, \quad (2)$$

where ξ is the retarded time defined by $\xi = t - vz$. Substituting that ansatz into Eq.(1), one can separate the complex envelope equation into real and imaginary ordinary differential equations:

$$va_\xi + p_r(a_{\xi\xi} - (\psi_\xi + K)^2a) - p_i(\psi_{\xi\xi}a + 2(\psi_\xi + K)a_\xi) - \sigma(\psi_\xi + K)a + q_ra^3 + \delta a + \lambda_r[a_{3\xi} - 3(\psi_\xi + K)\psi_{\xi\xi}a - 3(\psi_\xi + K)^2a_\xi] - \lambda_i[\psi_{3\xi}a + 3\psi_{\xi\xi}a_\xi + 3(\psi_\xi + K)a_{\xi\xi} - (\psi_\xi + K)^3a] + 3\mu_ra_\xi a^2 - \mu_i(\psi_\xi + K)a^3 + 2\upsilon_ra_\xi a^2 = 0, \quad (3.a)$$

and

$$(v\psi_\xi + \Omega)a + p_i(a_{\xi\xi} - (\psi_\xi + K)^2a) + p_r(\psi_{\xi\xi}a + 2(\psi_\xi + K)a_\xi) + \sigma a_\xi + q_ia^3 + \lambda_i[a_{3\xi} - 3(\psi_\xi + K)\psi_{\xi\xi}a - 3(\psi_\xi + K)^2a_\xi] + \lambda_r[\psi_{3\xi}a + 3\psi_{\xi\xi}a_\xi + 3(\psi_\xi + K)a_{\xi\xi} - (\psi_\xi + K)^3a] + 3\mu_ia_\xi a^2 + \mu_r(\psi_\xi + K)a^3 + 2\upsilon_ia_\xi a^2 = 0, \quad (3.b)$$

Let

$$\psi_\xi = Ba + D, \tag{4}$$

taking into account (4) into Eqs.(3.a) and (3.b) respectively, then yields

$$\lambda_r a_{3\xi} - 4B\lambda_i a_{\xi\xi} a + [p_r - 3(D+K)\lambda_i]a_{\xi\xi} - 3B\lambda_i a_\xi^2 + [-6B^2\lambda_r + 3\mu_r + 2\upsilon_r]a_\xi a^2 + [-3Bp_i - 9B(D+K)\lambda_r]aa_\xi + [v - 2p_i(D+K) - 3\lambda_r(D+K)^2]a_\xi + [-\sigma(D+K) + \delta + \lambda_i(D+K)^3 - p_r(D+K)^2]a + B[-2(D+K)p_r - \sigma + 3(D+K)^2\lambda_i]a^2 + [-B^2p_r + q_r + 3B^2(D+K)\lambda_i - \mu_i(D+K)]a^3 + B[B^2\lambda_i - \mu_i]a^4 = 0, \tag{5.a}$$

and

$$\lambda_i a_{3\xi} + 4B\lambda_r a_{\xi\xi} a + [p_i + 3(D+K)\lambda_r]a_{\xi\xi} + 3B\lambda_r a_\xi^2 + [-6B^2\lambda_i + 3\mu_i + 2\upsilon_i]a_\xi a^2 + [3Bp_r - 9B(D+K)\lambda_i]aa_\xi + [\sigma + 2p_r(D+K) - 3\lambda_i(D+K)^2]a_\xi + [vD + \Omega - \lambda_r(D+K)^3 - p_i(D+K)^2]a + B[v - 2(D+K)p_i - 3(D+K)^2\lambda_r]a^2 + [-B^2p_i + q_i - 3B^2(D+K)\lambda_r + \mu_r(D+K)]a^3 - B[B^2\lambda_r - \mu_r]a^4 = 0, \tag{5.b}$$

by assuming that Eqs.(5.a) and (5.b) admit a solution of the form

$$a(\xi) = \eta \tanh k\xi. \tag{6}$$

Substituting Eq.(6) into Eqs.(5.a) and (5.b) and equating different powers of e, we obtain ten algebraic equations. We do not present them in detail here, but discuss their solutions. In general, these ten equations will determine ten parameters of problem. Seven of them ($K,\ \Omega,\ v,\ B,\ D,\ \eta$ and k) describe the characteristics of the dark soliton-like solution. The other three will give compatibility conditions for Eq.(1). By directly solving the ten equations, one can obtain the following relations

$$\eta = Bek; \quad D = \frac{(2-e^2B^4)p_r + 3eB^2p_i + B^2e^2q_r}{3[(2-e^2B^4)\lambda_i - eB^2\lambda_r] + e^2B^2\mu_i} - K; \quad v = 2p_i(D+K) + 3\lambda_r(D+K)^2 + (B^2\lambda_r - \mu_r)\eta^2; \quad \Omega = -vD + p_i(D+K)^2 + \lambda_r(D+K)^3 + [B^2p_i - q_i + 3B^2\lambda_r(D+K) - \mu_r(D+K)]\eta^2,$$
$$k^2 = \frac{(D+K)[\sigma - \lambda_i(D+K)^2 + p_r(D+K)] - \delta}{q_r - B^2p_r + (D+K)(3B^2\lambda_i - \mu_i)} > 0; \quad e = \frac{3|\lambda|^2}{\mu_i\lambda_r - \mu_r\lambda_i}, \tag{7}$$

where K is arbitrary constant and B verifies the following equation

$$\lambda_r[9|\lambda|^4B^4 + 9|\lambda|^2(\mu_r\lambda_r + \mu_i\lambda_i)B^2 + 2(\mu_i\lambda_r - \mu_r\lambda_i)^2] = 0. \tag{8}$$

The following constraint conditions should be verified

$$
\begin{aligned}
2\nu_r &= \tfrac{B^4e^3(B^2\lambda_i-\mu_i)-4\lambda_r-8e\lambda_iB^2}{B^2e^2}+5B^2\lambda_r-2\mu_r;\\
2\nu_i &= \tfrac{B^4e^3(-B^2\lambda_r+\mu_r)-4\lambda_i-8e\lambda_rB^2}{B^2e^2}+5B^2\lambda_i-2\mu_i; \qquad (9)\\
\sigma &= \tfrac{B^2e^2(B^2\lambda_i-\mu_i)[\lambda_i(D+K)^3-p_r(D+K)^2+\delta]-3(D+K)[q_r-B^2p_r+(D+K)(3B^2\lambda_i-\mu_i)][(D+K)\lambda_i-p_r]}{e^2B^2(D+K)(B^2\lambda_i-\mu_i)-[q_r-B^2p_r+(D+K)(3B^2\lambda_i-\mu_i)]};\\
q_i &= \tfrac{3eB^2p_r+(e^2B^4-2)p_i}{e^2B^2}+\tfrac{3[(e^2B^4-2)\lambda_r-3eB^2\lambda_i]-e^2B^2\mu_r}{3[(2-e^2B^4)\lambda_i-3eB^2\lambda_r]+e^2B^2\mu_i}\left[q_r+\tfrac{3eB^2p_i-(e^2B^4-2)p_r}{e^2B^2}\right].
\end{aligned}
$$

The following particular case is obtained if $\lambda_r=0$

$$
\begin{aligned}
&\eta=-3\tfrac{\lambda_i}{\mu_r}Bk; \quad D=\tfrac{(9\lambda_i^2B^4-2\mu_r^2)p_r+9\lambda_iB^2(\mu_rp_i-\lambda_iq_r)}{3\lambda_i[3\lambda_iB^2(3\lambda_iB^2-\mu_i)-2\mu_r^2]}-K;\\
&v=2p_i(D+K)-\mu_r\eta^2; \quad \Omega=-vD+p_i(D+K)^2+[B^2p_i-q_i-\mu_r(D+K)]\eta^2;\\
&k^2=\tfrac{\mu_r^2}{9\lambda_i^2B^2}\tfrac{(D+K)[\sigma+p_r(D+K)-\lambda_i(D+K)^2]-\delta}{(D+K)(3\lambda_iB^2-\mu_i)+q_r-B^2p_r}>0, \qquad (10)
\end{aligned}
$$

where K is arbitrary constant and B verifies the following equation

$$\lambda_i[9\lambda_i^2B^4-9\lambda_i\mu_iB^2-2\mu_r^2]=0, \quad \lambda_i\neq 0. \qquad (11)$$

The following new constraint conditions should be satisfied

$$
\begin{aligned}
&\nu_r=\tfrac{4}{3}\mu_r+\tfrac{3}{2}\tfrac{\lambda_iB^2}{\mu_r}(\mu_i-B^2\lambda_i); \quad \nu_i=B^2\lambda_i-\mu_r-\tfrac{2\mu_i^2}{9\lambda_iB^2};\\
&\sigma=\tfrac{(B^2\lambda_i-\mu_i)[(D+K)^2(p_r-\lambda_i(D+K))-\delta]+(D+K)[q_r-B^2p_r+(D+K)(3B^2\lambda_i-\mu_i)][3(D+K)\lambda_i-2p_r]}{[q_r-B^2p_r+(D+K)(3B^2\lambda_i-\mu_i)]-(D+K)(B^2\lambda_i-\mu_i)};\\
&q_i=\tfrac{(9\lambda_i^2B^4-2\mu_r^2)p_i-9\lambda_i\mu_rp_rB^2}{9\lambda_i^2B^2}+\tfrac{2\mu_r}{3\lambda_i}\tfrac{(9\lambda_i^2B^4-2\mu_r^2)p_r+9\lambda_iB^2(\mu_rp_i-\lambda_iq_r)}{[3\lambda_iB^2(3\lambda_iB^2-\mu_i)-2\mu_r^2]}. \qquad (12)
\end{aligned}
$$

According to the results (7)-(12) along with (2) and (6), we can derive the dark soliton solutions for Eq.(1) as follows:

$$E(z,t)=\eta\tanh k(t-vz)\exp i[\psi(t-vz)+Kt-\Omega z], \qquad (13)$$

where $\psi(t-vz)$ is obtained after integrating (4) as

$$\psi(t-vz)=D(t-vz)+\frac{B\eta}{k}\ln(\cosh(k(t-vz))). \qquad (\mathbf{14})$$

3 Degeneracy of Eq.(1) and its solutions

The degeneracy of Eq.(1) is obtained by choosing the following assumption
$$\psi_\xi = 0 \Longrightarrow \qquad \psi = \psi_0. \tag{15}$$

Then ordinary equations (3.a) and (3.b) are reduced to

$$\lambda_r a_{3\xi} + (p_r - 3\lambda_i K)a_{\xi\xi} + (3\mu_r + 2\nu_r)a_\xi a^2 + (v - 2p_i K - 3\lambda_r K^2)a_\xi + (\delta - \sigma K - p_r K^2 + \lambda_i K^3)a + (q_r - \mu_i K)a^3 = 0, \tag{16.a}$$

$$\lambda_i a_{3\xi} + (p_i + 3\lambda_r K)a_{\xi\xi} + (3\mu_i + 2\nu_i)a_\xi a^2 + (\sigma + 2p_r K - 3\lambda_i K^2)a_\xi + (\Omega - p_i K^2 - \lambda_r K^3)a + (q_i + \mu_r K)a^3 = 0, \tag{16.b}$$

The particular solutions of the degeneracy of Eq.(1) may be obtained if we proceed in a similar way as presented in the previous section.

3.1 Dark soliton solutions

The dark soliton solutions read

$$\mathrm{E(z,t)}=\eta\ \tanh k(t - vz) \exp i(Kt - \Omega z + \psi_0), \tag{17}$$
where

$$\eta^2 = -\frac{6\lambda_r}{3\mu_r + 2\nu_r}k^2 > 0; \qquad K = \frac{p_r(3\mu_r+2\nu_r) - 3\lambda_r q_r}{6\lambda_r(\mu_i+\nu_i)}; \qquad v = 2\lambda_r k^2 + 2p_i K + 3\lambda_r K^2; \qquad \Omega = p_i K^2 + \lambda_r K^3 - (q_i + K\mu_r)\eta^2; \qquad k^2 = \frac{\sigma + 2p_r K - 3\lambda_i K^2}{2\lambda_i} > 0, \tag{18}$$
with the following constraint conditions

$$\begin{gathered}\lambda_r(3\mu_i + 2\nu_i) - \lambda_i(3\mu_r + 2\nu_r) = 0;\\ q_i = \frac{p_i}{3\lambda_i}(3\mu_r + 2\nu_r) + \frac{\mu_r+\nu_r}{\mu_i+\nu_i}\left[\frac{p_r}{3\lambda_i}(3\mu_r + 2\nu_r) - q_r\right];\\ \delta = \sigma K + p_r K^2 - \lambda_i K^3 - (q_r - K\mu_i)\eta^2.\end{gathered} \tag{19}$$

This degeneracy includes the following two important subcases

first subcase $p_r = q_r = \delta = \lambda_r = \mu_r = \nu_r = 0$ which leads to one form of NLS equation written as:

$$iE_z + \sigma E_t + p_i E_{tt} + q_i|E|^2E + \lambda_i E_{3t} + \mu_i(|E|^2E)_t + \nu_i E(|E|^2)_t = 0, \quad (20)$$

the solutions are obtained as follows

$$\eta^2 = -\frac{6\lambda_i}{\mu_i}k^2 > 0; \quad K^2 = 2k^2; \quad v = 2p_iK; \quad \Omega = p_iK^2 - q_i\eta^2; \quad k^2 = \frac{\sigma}{8\lambda_i}, \quad (21)$$

The constraint conditions are

$$\nu_i = -\mu_i; \qquad q_i = \frac{p_i}{3\lambda_i}\mu_i. \quad (22)$$

Second subcase $p_i = q_i = \sigma = \lambda_i = \mu_i = \nu_i = 0$ the following real equation is obtained as

$$E_z - p_r E_{tt} - q_r|E|^2E - \delta E - \lambda_r E_{3t} - \mu_r(|E|^2E)_t - \nu_r E(|E|^2)_t = 0. \quad (23)$$

Then, we have

$$K = 0; \quad \Omega = 0; \quad \eta^2 = -\frac{6\lambda_r}{3\mu_r+2\nu_r}k^2 > 0; \quad v = 2\lambda_r k^2;$$
$$k^2 = \frac{\delta}{2p_r} > 0 \quad (24)$$

The constraint condition is

$$q_r = \frac{p_r}{3\lambda_r}(3\mu_r+2\nu_r). \quad (25)$$

Now, if we return again to Eqs.(16.a) and (16.b) and choose the following relations

$$\lambda_i = 0, \qquad 3\mu_i+2\nu_i = 0, \quad 2p_iK+\sigma-3\lambda_iK^2 = 0, \quad (26.a)$$

and

$$p_r-3\lambda_iK = 0, \qquad q_r-\mu_iK = 0, \quad \delta-\sigma K-p_rK^2+\lambda_iK^3 = 0. \quad (26.b)$$

Taking into account (26.a) and (26.b) into (16.a) and (16.b), then yields to

$$\lambda_r a_{3\xi}+(3\mu_r+2\nu_r)a_\xi a^2+(v-2p_iK-3\lambda_rK^2)a_\xi = 0, \quad (27.a)$$

$$(p_i+3\lambda_rK)a_{\xi\xi}+(\Omega-p_iK^2-\lambda_rK^3)a+(q_i+\mu_rK)a^3 = 0, \quad (27.b)$$

It is easy to see that Eqs.(27.a) and (27.b) become

$$a_{\xi\xi}+\frac{-2p_iK-3\lambda_rK^2+v}{\lambda_r}a+\frac{3\mu_r+2\nu_r}{3\lambda_r}a^3=0, \tag{28}$$

under the constraint conditions

$$\lambda_i=0;\quad \nu_i=-\frac{3}{2}\mu_i;\qquad \delta=0;\quad \sigma=0;\quad p_r=0;\quad q_r=\frac{\mu_i}{3\lambda_r}(\lambda_r-p_i);\quad q_i=\mu_r+\frac{2}{3}\nu_r-\frac{\mu_r}{3\lambda_r}(\lambda_r-p_i);\quad K=\frac{1}{3\lambda_r}(\lambda_r-p_i);\quad \Omega=\frac{1}{27\lambda_r^2}(\lambda_r-p_i)[(\lambda_r-p_i)^2-18p_i\lambda_r-3(\lambda_r-p_i)(3\lambda_r-p_i)+v. \tag{29}$$

With this satisfied, Eq.(1) is now reduced to

$$E_z-ip_iE_{tt}-(q_r+iq_i)|E|^2E-\lambda_rE_{3t}-(\mu_r+i\mu_i)(|E|^2E)_t-(\nu_r+i\nu_i)E(|E|^2)_t=0, \tag{30}$$

In order to obtain new formal solutions for Eq.(30), under the constraint conditions (29). We will follow the method presented by Yan [34].

In effect, in [35], Conte et al presented a general ansatz to seek more new solitary wave solutions of some nonlinear partial differential equations (NLPDEs) that can be expressed as a polynomial in two elementary functions which satisfy a projective Riccati equations [36]. Recently, Yan developed Conte's method and presented the general projective Riccati equation method. Several authors used the Yan's technique to solve many NLPDEs [37,38]. The key idea of Yan's method is to extend the projective Riccati equation in a more general form:

$$\kappa'(\xi)=\epsilon\kappa(\xi)\tau(\xi),\ \ \tau'(\xi)=R+\epsilon\tau^2(\xi)-m\kappa(\xi),\ \ \epsilon=\pm1.\ m,\ R=\text{constant}. \tag{\textbf{31}}$$

We know from [34,37,38] that (31) admits the following solutions

Case 1 when $\epsilon=-1$, $R\neq0$

$$\kappa_1(\xi)=\frac{R\ sech(\sqrt{R}\xi)}{m\ sech(\sqrt{R}\xi)+1},\qquad \tau_1(\xi)=\frac{\sqrt{R}\ tanh(\sqrt{R}\xi)}{m\ sech(\sqrt{R}\xi)+1},$$
$$\kappa_2(\xi)=\frac{R\ csch(\sqrt{R}\xi)}{m\ csch(\sqrt{R}\xi)+1},\qquad \tau_2(\xi)=\frac{\sqrt{R}\ \coth(\sqrt{R}\xi)}{m\ csch(\sqrt{R}\xi)+1}, \tag{32}$$

Case 2 when $\epsilon=1$, $R\neq0$

$$\kappa_3(\xi)=\frac{R\ \sec(\sqrt{R}\xi)}{m\ \sec(\sqrt{R}\xi)+1},\qquad \tau_3(\xi)=\frac{\sqrt{R}\ \tan(\sqrt{R}\xi)}{m\ \sec(\sqrt{R}\xi)+1},$$
$$\kappa_4(\xi)=\frac{R\ \csc(\sqrt{R}\xi)}{m\ \csc(\sqrt{R}\xi)+1},\qquad \tau_4(\xi)=-\frac{\sqrt{R}\ \cot(\sqrt{R}\xi)}{m\ \csc(\sqrt{R}\xi)+1}. \tag{33}$$

The relations (32) and (33) satisfy

$$\tau^2(\xi) = -\epsilon\left[R - 2m\ \kappa(\xi) + \frac{m^2+r_i}{R}\kappa^2(\xi)\right], \tag{34}$$

with

$r_i = -1$ for $(\kappa_i,\ \tau_i)$ with i=1, 3, 4

$$r_i = 1 \quad \text{for } (\kappa_i,\ \tau_i) \text{ with i=2} \tag{35}$$

$Case\ 3$ when $m = 1,\ \ R = 0$

$$\kappa_5(\xi) = \frac{C}{\xi} = C\epsilon\tau_5(\xi), \qquad \tau_5(\xi) = \frac{1}{\epsilon\xi}. \tag{36}$$

C is a constant. Based upon the above analysis, we assume that Eq.(28) has the following two solutions

$Type\ I$ when $R \neq 0$

$$a(\xi) = \sum_{j=1}^{n} \kappa^{j-1}(\xi)[A_j\kappa(\xi)+B_j\tau(\xi)]+A_0, \tag{37}$$

where $\kappa,\ \ \tau$ satisfy (31) and (34).

$Type\ II$ when $R = m = 0$,

$$a(\xi) = \sum_{j=0}^{n} A_0\tau^j(\xi)], \tag{38}$$

where τ satisfies

$$\tau'(\xi) = \tau^2(\xi). \tag{39}$$

By balancing the highest-order linear term $a_{\xi\xi}$ and the nonlinear a^3 in (28), we obtain n=1. According to the above mentioned method, we assume that (28) has the following form of solutions with $R \neq 0$

$$a(\xi) = A_0+A_1\kappa(\xi)+B_1\tau(\xi), \tag{40}$$

where $A_0,\ \ A_1$ and $B_1$are constants to be determined later. $\kappa(\xi)$ and $\tau(\xi)$ satisfying (31) and (34).

With the aid of Mathematica, substituting (40) along with (31), (34) and (35) into (28) and collecting all terms with the same power in $\kappa^j(\xi)\tau^i(\xi),\ \ j = 0,\ 1,\ 2,\ 3,\ 4;\ \ i = 0,\ 1$. Setting the coefficients of these terms $\kappa^j(\xi)\tau^i(\xi)$ to zero yields a set of over-determined algebraic equations with respect to $A_0,\ A_1,\ B_1,\ \ R,\ v$ and m.

$$Const: \quad \frac{-2p_iK-3\lambda_rK^2+v}{\lambda_r}A_0+\frac{3\mu_r+2\nu_r}{3\lambda_r}[A_0^3-3\epsilon RA_0B_1^2] = 0, \tag{41}$$

$$\kappa(\xi): \quad \frac{-2p_iK-3\lambda_rK^2+v}{\lambda_r}A_1 + \frac{3\mu_r+2\nu_r}{3\lambda_r}[3A_0^2A_1 - 3\epsilon RA_1B_1^2 + 6\epsilon A_0B_1^2m] - \epsilon AR = 0, \tag{42}$$

$$\tau(\xi): \quad \frac{-2p_iK-3\lambda_rK^2+v}{\lambda_r}B_1+\frac{3\mu_r+2\nu_r}{3\lambda_r}[3A_0^2B_1-\epsilon RB_1^3] = 0, \tag{43}$$

$$\kappa(\xi)\tau(\xi): \quad \frac{3\mu_r+2\nu_r}{3\lambda_r}[6A_0A_1B_1+2\epsilon mB_1^3]+\epsilon mB_1 = 0, \tag{44}$$

$$\kappa^2(\xi): \quad \frac{3\mu_r+2\nu_r}{3\lambda_r}[3A_0^2A_1+6\epsilon mA_1B_1^2 - \frac{3\epsilon}{R}A_0B_1^2(m^2+r_i)]+3\epsilon mA_1 = 0, \tag{45}$$

$$\kappa^2(\xi)\tau(\xi): \frac{3\mu_r+2\nu_r}{3\lambda_r}[3A_1^2B_1 - \epsilon\frac{B_1^3}{R}(m^2+r_i)] - 2\epsilon\frac{m^2+r_i}{R}B_1 = 0, \tag{46}$$

$$\kappa^3(\xi): \frac{3\mu_r+2\nu_r}{3\lambda_r}[A_1^3 - 3\epsilon\frac{m^2+r_i}{R}A_1B_1^2] - 2\epsilon\frac{m^2+r_i}{R}A_1 = 0. \tag{47}$$

With the aid of Mathematica, solving Eqs.(41)-(47), we get the following results

$Case\ 1 \quad r_i = -1 \quad or \quad r_i = 1$

$$R = \frac{2p_iK+3\lambda_rK^2-v}{2\epsilon\lambda_r}, \quad A_0 = 0, \quad A_1 = 0, \quad m = 0, \quad B_1^2 = \frac{-6\lambda_r}{3\mu_r+2\nu_r}, \tag{48}$$

$Case\ 2 \quad r_i = -1$

$$R = \frac{2(2p_iK+3\lambda_rK^2-v)}{\epsilon\lambda_r}, \quad A_0 = 0, \quad A_1 = 0, \quad m = 1, \quad B_1^2 = \frac{-3\lambda_r}{2(3\mu_r+2\nu_r)}, \tag{49}$$

$Case\ 3 \quad r_i = -1$

$$R = \frac{2(2p_iK+3\lambda_rK^2-v)}{\epsilon\lambda_r}, \quad A_0 = 0, \quad A_1 = 0, \quad m = -1, \quad B_1^2 = \frac{-3\lambda_r}{2(3\mu_r+2\nu_r)}, \tag{50}$$

$Case\ 4 \quad r_i = -1$

$$R = \frac{v-2p_iK-3\lambda_rK^2}{\epsilon\lambda_r}, \ A_0 = 0, \ B_1 = 0, \ m = 0, \quad A_1^2 = \frac{-6\lambda_r^2}{(3\mu_r+2\nu_r)(v-2p_iK-3\lambda_rK^2)}, \tag{51}$$

$Case\ 5 \quad r_i = -1$

$$R = \frac{2(2p_iK+3\lambda_rK^2-v)}{\epsilon\lambda_r}, \ A_0 = 0, \ A_1^2 = \frac{-3(-1+m^2)\lambda_r^2}{4(3\mu_r+2\nu_r)(v-2p_iK-3\lambda_rK^2)}, \quad B_1^2 = \frac{-3\lambda_r}{2(3\mu_r+2\nu_r)}, \tag{52}$$

$Case\ 6 \quad r_i = 1$

$$R = \frac{v-2p_iK-3\lambda_rK^2}{\epsilon\lambda_r}, \ A_0 = 0, \ B_1 = 0, \ m = 0, \quad A_1^2 = \frac{6\lambda_r^2}{(3\mu_r+2\nu_r)(v-2p_iK-3\lambda_rK^2)}, \tag{53}$$

$Case\ 7 \quad r_i = 1$

$$R = \frac{2(2p_iK+3\lambda_rK^2-v)}{\epsilon\lambda_r}, \ A_0 = 0, \ A_1^2 = \frac{-3(1+m^2)\lambda_r^2}{4(3\mu_r+2\nu_r)(v-2p_iK-3\lambda_rK^2)}, \quad B_1^2 = \frac{-3\lambda_r}{2(3\mu_r+2\nu_r)}. \tag{54}$$

Thus from (32), (33), (40), (2) and (15) and results (48)-(54), we can derive many families of exact travelling wave solutions of Eq.(30)

Family 1. Dark soliton solutions

$$E_1(z,t) = \sqrt{\frac{-3(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}} \tanh\left[\sqrt{\frac{v-2p_iK-3\lambda_rK^2}{2\lambda_r}}(t-vz)\right] \exp i(\Omega z - Kt+\psi_0) \tag{55}$$

Family 2. Singular dark soliton solutions

$$E_2(z,t) = \sqrt{\frac{-3(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}} \coth\left[\sqrt{\frac{v-2p_iK-3\lambda_rK^2}{2\lambda_r}}(t-vz)\right] \exp i(\Omega z - Kt+\psi_0). \tag{56}$$

Family 3. Bright soliton solutions

$$E_3(z,t) = -\sqrt{\frac{-6(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}sech\left[\sqrt{-\frac{v-2p_iK-3\lambda_rK^2}{\lambda_r}}(t-vz)\right]\exp i(\Omega z-Kt+\psi_0) \tag{57}$$

Family 4. Singular soliton solutions

$$E_4(z,t) = -\sqrt{\frac{6(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}csch\left[\sqrt{-\frac{v-2p_iK-3\lambda_rK^2}{\lambda_r}}(t-vz)\right]\exp i(\Omega z-Kt+\psi_0) \tag{58}$$

Family 5. Periodic wave solutions

$$E_5(z,t) = \sqrt{\frac{3(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\tan\left[\sqrt{-\frac{v-2p_iK-3\lambda_rK^2}{2\lambda_r}}(t-vz)\right]\exp i(\Omega z-Kt+\psi_0) \tag{59}$$

Family 6. Periodic wave solutions

$$E_6(z,t) = -\sqrt{\frac{3(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\cot\left[\sqrt{-\frac{v-2p_iK-3\lambda_rK^2}{2\lambda_r}}(t-vz)\right]\exp i(\Omega z-Kt+\psi_0) \tag{60}$$

Family 7. Periodic wave solutions

$$E_7(z,t) = \sqrt{\frac{-6(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\sec\left[\sqrt{\frac{v-2p_iK-3\lambda_rK^2}{2\lambda_r}}(t-vz)\right]\exp i(\Omega z-Kt+\psi_0) \tag{61}$$

Family 8. Periodic wave solutions

$$E_8(z,t) = \sqrt{\frac{-6(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\csc\left[\sqrt{\frac{v-2p_iK-3\lambda_rK^2}{2\lambda_r}}(t-vz)\right]\exp i(\Omega z-Kt+\psi_0) \tag{62}$$

Family 9. combined formal soliton-like solutions

$$E_9(z,t) = \{\sqrt{\frac{-3(-1+m^2)(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\frac{sech\left[\sqrt{\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]}{m\ sech\left[\sqrt{\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]+1}+ \sqrt{\frac{-3(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\frac{\tanh\left[\sqrt{\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]}{m\ sech\left[\sqrt{\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]+1}\}\exp i(\Omega z-Kt+\psi_0) \tag{63}$$

Family 10. combined formal soliton-like solutions

$$E_{10}(z,t) = \{\sqrt{\frac{-3(1+m^2)(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\frac{csch\left[\sqrt{\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]}{m\ csch\left[\sqrt{\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]+1}+ \sqrt{\frac{-3(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\frac{\coth\left[\sqrt{\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]}{m\ csch\left[\sqrt{\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]+1}\}\exp i(\Omega z-Kt+\psi_0) \tag{64}$$

Family 11. combined formal periodic wave-like solutions

$$E_{11}(z,t) = \{-\sqrt{\frac{-3(-1+m^2)(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\frac{\sec\left[\sqrt{-\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]}{m\ \sec\left[\sqrt{-\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]+1}+$$

$$\sqrt{\frac{3(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\frac{\tan\left[\sqrt{-\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]}{m\ \sec\left[\sqrt{-\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]+1}\}\exp i(\Omega z-Kt+\psi_0) \quad (65)$$

Family 12. combined formal periodic wave-like solutions

$$E_{12}(z,t) = \{-\sqrt{\frac{3(-1+m^2)(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\frac{\csc\left[\sqrt{-\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]}{m\ \csc\left[\sqrt{-\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]+1}-$$

$$\sqrt{\frac{3(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\frac{\cot\left[\sqrt{-\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]}{m\ \csc\left[\sqrt{-\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]+1}\}\exp i(\Omega z-Kt+\psi_0) \quad (66)$$

When $m=\pm 1$ then we have the following solutions
Family 13. new soliton solutions

$$E_{13}(z,t) = \sqrt{\frac{-3(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\frac{\tanh\left[\sqrt{\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]}{\pm\ sech\left[\sqrt{\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]+1}\exp i(\Omega z-Kt+\psi_0) \quad (67)$$

Family 14. new periodic wave solutions

$$E_{14}(z,t) = \sqrt{\frac{3(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\frac{\tan\left[\sqrt{-\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]}{\pm\ \sec\left[\sqrt{-\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]+1}\exp i(\Omega z-Kt+\psi_0) \quad (68)$$

Family 15. new periodic wave solutions

$$E_{15}(z,t) = -\sqrt{\frac{3(v-2p_iK-3\lambda_rK^2)}{3\mu_r+2\nu_r}}\frac{\cot\left[\sqrt{-\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]}{\pm\ \csc\left[\sqrt{-\frac{2(v-2p_iK-3\lambda_rK^2)}{\lambda_r}}(t-vz)\right]+1}\exp i(\Omega z-Kt+\psi_0) \quad (69)$$

Family 16. Rational solutions: When setting the solutions of Eq.(28) in the form of (38), we obtain the following rational solutions for Eq.(30)

$$E_{16}(z,t) = \left[\pm\sqrt{\frac{-6\lambda_r}{3\mu_r+2\nu_r}}\frac{1}{t-(2p_iK+3\lambda_rK^2)z}\right]\exp i(\Omega z-Kt+\psi_0)\ , \quad (70)$$

where $v=2p_iK+3\lambda_rK^2$.

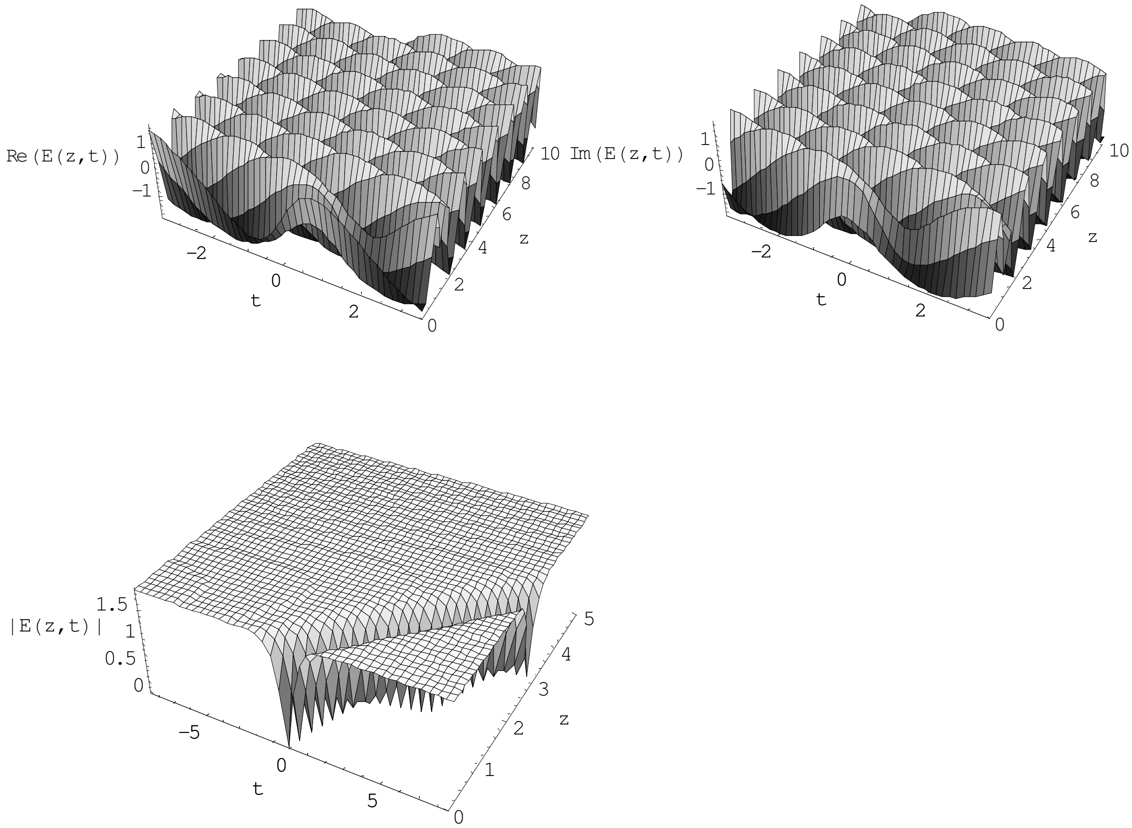
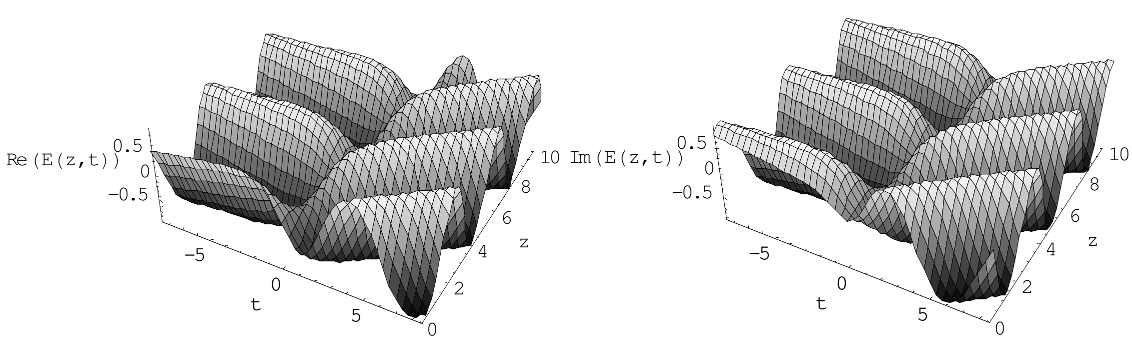


Fig. 1. represents the profile of a Dark, the given parameters in (7)-(9) and (13)-(14) are $q_r = 0.75;\ \lambda_r = 0.62;\ \lambda_i = -0.7;\ \mu_r = -0.81;\ \mu_i = 0.96;\ p_r = 0.52;\ p_i = 0.50;\ \delta = -0.58;\ K = 3.52.$ Then the solution (13)-(14) has the parameters $B = \pm 0.013120;\ D = -4.299140;\ k = 1.523960;$ $\eta = \pm 1.859950;\ v = 3.152490;\ \Omega = 9.369760.$ The constraint parameters are defined by $\sigma = -3.741100;\ q_i = 0.580936;\ \nu_r = 0.000212;\ \nu_i = -0.053564.$ The first, the second and the third plots give real part, imaginary part and modulus of the complex field $E(z,t)$.



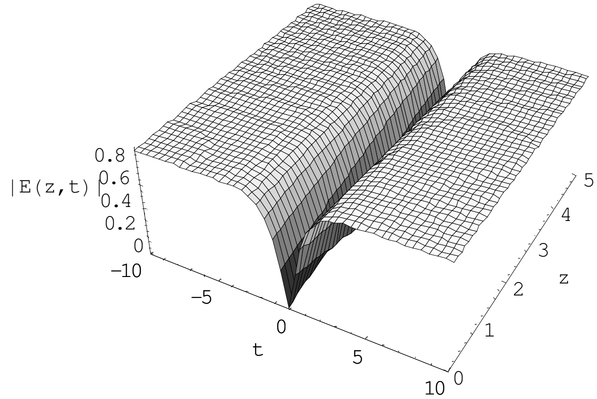
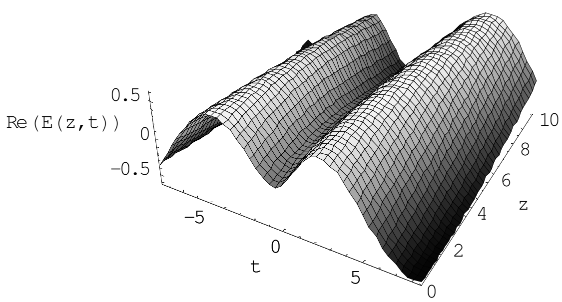
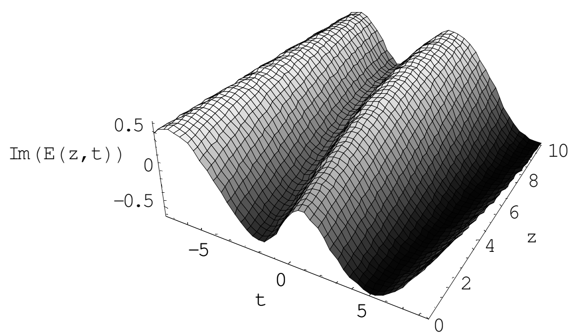


Fig. 2. gives the profile of another Dark solution, the given parameters in (10)-(14) are $q_r = 0.75$; $\lambda_r = 0$; $\lambda_i = -0.7$; $\mu_r = -0.81$; $\mu_i = 0.96$; $p_r = 0.52$; $p_i = 0.50$; $\delta = -0.58$; $K = 3.52$. Then the solution (13)-(14) has the parameters $B = \pm 0.436465$; $D = -3.980710$; $k = 0.782393$; $\eta = \mp 0.885338$; $v = 0.174182$; $\Omega = 1.00576$. The constraint parameters are defined by $\sigma = -0.823592$; $q_i = -0.541071$; $\nu_r = -0.810000$; $\nu_i = 2.212440$. The three plots represent the real part, the imaginary part and the modulus of the complex field $E(z,t)$.





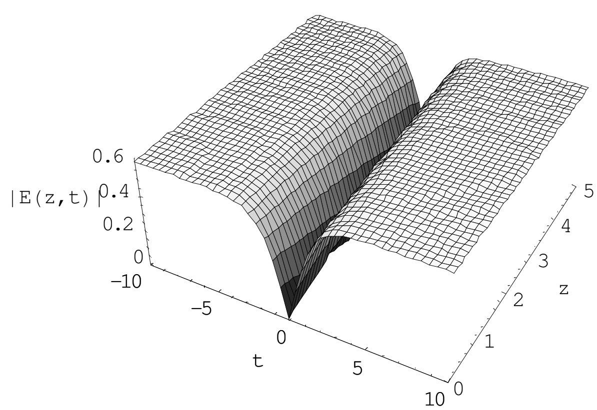


Fig. 3. is the profile of another Dark solution obtained for degeneracy of the system, the given parameters in (17)-(19) are $q_r = 0.75;\ \nu_r = 0.63;\ \nu_i = 0.93;\ \mu_r = -0.81;\ \mu_i = 0.96;\ p_r = 0.52;$ $\psi_0 = 1;\ \lambda_i = -0.70;\ p_i = 0.50;\ \sigma = -0.58.$ Then the solution (17) has the parameters $k = 0.635486;\quad \eta = 0.598193;\ K = -0.508919;\ v = -0.235111;\ \Omega = 0.152831.$ The constraint parameters are defined by $\delta = -0.105615;\ q_i = 0.322408;\ \lambda_r = 0.172785.$ In these three plots the real part, the imaginary part and the modulus of the complex field $E(z,t)$ are given respectively.

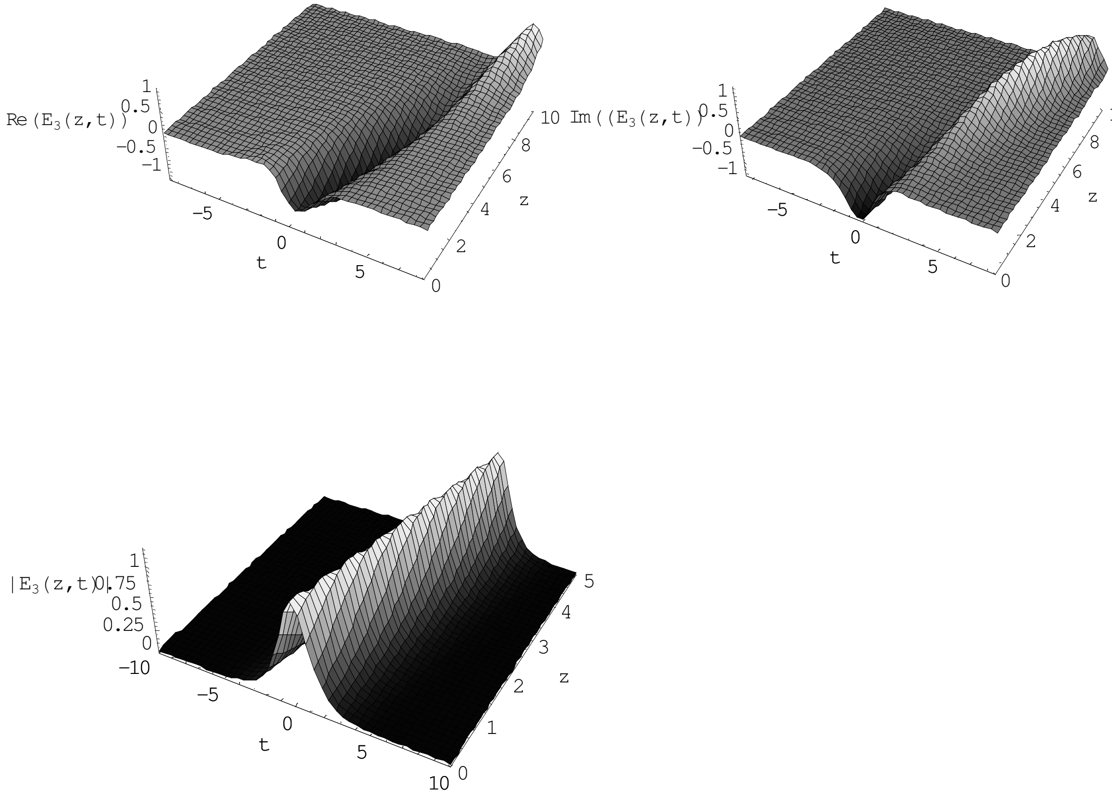


Fig. 4. shows the profile of the bright solution $E_3(z,t)$ obtained for degeneracy of the system, the given parameters are $p_i = 0.50,\ \mu_r = -0.81;\ \mu_i = 0.96;\ \lambda_r = -0.7;\ \nu_r = -0.63;$ $v = 0.75;\ \psi_0 = 1.$ Then the solution (57) has the parameters $K = 0.571429;\ \Omega = 0.078912.$ The constraint parameters are expressed by $\nu_i = -1.440000;\ q_r = 0.548571;\ q_i = -0.767143.$ The three plots represent real part, imaginary part and modulus of the complex field $E_3(z,t)$ a respectively.

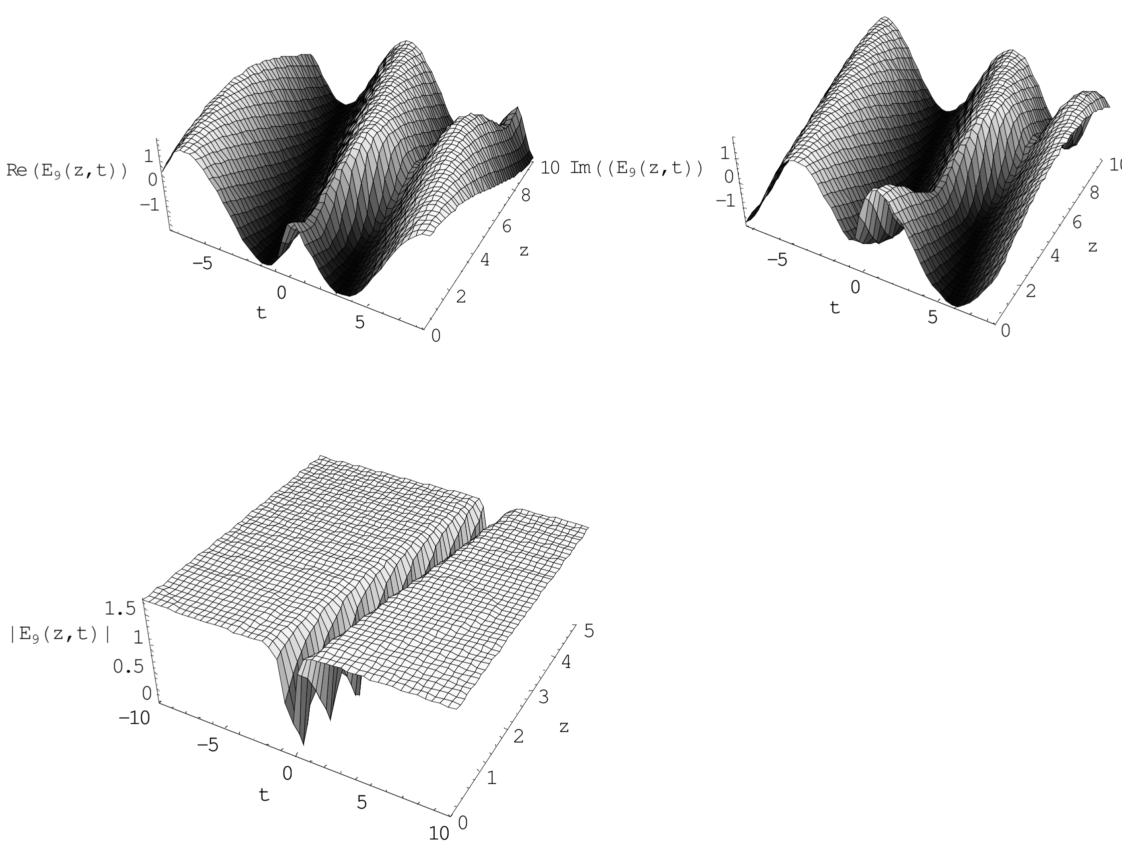


Fig. 5. Another family of dark solution $E_9(z,t)$ is shown for the following dimensionless parameters $\nu_r = 0.63;\ \lambda_i = -0.7;\ \mu_r = -0.81;\ \mu_i = 0.96;\ \lambda_r = 0.173;\ v = 0.75;\ \psi_0 = 1;\ m = 1.20.$ Then the solution (63) has the parameters $K = -0.631258;\ \Omega = 0.276027.$ The constraint parameters are $\nu_i = -1.440000;\ q_r = -0.606007;\ q_i = -0.901319.$

4. Conclusion

We have provided analytical evidence of existence of a distinct class of wave solutions than that reported in the literature [33], namely: dark soliton solutions for a generalized non conservative system which models ultrashort pulse propagation. Through separation of E(z,t) into the amplitude a(ξ) and nonlinear phase ψ(ξ), a new choice of parameters were given for the propagation of dark. As the dark solutions are preferred to the bright solutions because of their inherent stability resistance to the influence of noise and fiber loss, we believe that the study of dark solutions in higher-order system like what this paper addresses, will be useful for further application.

On the other hand, we have found the degeneracy of Eq.(1) as well as its solutions. An important special case of this degeneracy leads to Eq.(30). We have obtained many families of exact travelling wave solutions for this equation, based upon the improved projective Riccati equation method due to Yan [34]. The results indicate that the non conservative amplitude Eq.(30) has rich structures of the solutions. This equation possesses physically interesting types of travelling waves: bright soliton, dark soliton, periodic wave, new bright soliton, new dark soliton, combined bright-dark soliton, combined periodic wave, and rational solutions.

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