THE EXTENDED F-EXPANSION METHOD AND ITS APPLICATION FOR SOLVING THE NONLINEAR WAVE, CKGZ, GDS, DS AND GZ EQUATIONS

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The extended F-expansion method and its application for solving the nonlinear wave, CKGZ, GDS, DS and GZ equations

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ABSTRACT

By using a simple transformation technique, we have shown that the nonlinear wave equation, the coupled Klein-Gordon-Zakharov (CKGZ) equations, the generalized Davey Stewartson (GDS) equations, the Davey Stewartson (DS) equations, the generalized Zakharov (GZ) equations can be reduced to the elliptic-like equation. Then, the extended F-expansion method is used to obtain a series of solutions including the single and the combined non-degenerative Jacobi elliptic function solutions and their degenerative solutions to the above mentioned class of NLPDEs.

1 INTRODUCTION

In the recent decade, the study of nonlinear partial differential equations (NLPDEs) modelling physical phenomena, has become an important tool. In this study, it appears that there are some basic relationships among many complicated nonlinear equations and some simple and solvable nonlinear ordinary differential equations (NODEs) such as Ricatti equation, sine-Gordon equation, sinh-Gordon, Weierstrass elliptic equation etc. In this attempt to use the solutions of NODEs, many powerful approaches have been presented. One of the most important approaches is the F-expansion method [1-8] which can be regarded as generalization of the Jacobi elliptic method [9-11]. In this method, elliptic equations have been applied as a mapping to obtain many kinds of travelling wave solutions to a large variety of NLPDEs whose odd- and even-order derivative terms do not coexist. The main advantages of the F-expansion approach are that: with the aid of elliptic equations one only needs to calculate the function which is solution of the elliptic equations, instead of calculating the Jacobi elliptic functions one by one; the coefficients of the elliptic equations can be selected so that the corresponding solution is a Jacobi elliptic function.

Very recently, the extended F-expansion method has been proposed to obtain not only the single non-degenerative Jacobi elliptic function solutions, but also the combined non-degenerative Jacobi elliptic solutions and their corresponding degenerative solutions [12,13].

In this letter, we will use the extended F-expansion method to construct exact solutions to some class of NLPDEs which can be reduced to a simple elliptic-like equation. By this technique, we have successfully found the solutions of the nonlinear wave equation [14], the coupled Klein-Gordon-Zakharov (CKGZ) equations [15], the generalized Davey-Stewartson (GDS) equations [16], the Davey-Stewartson (DS) equations [17], the generalized Zakharov (GZ) equations [18].

The letter is organized as follows: In section 2, first we briefly give the steps of the method and apply the method to solve the elliptic-like equation. In section 3, by using the results obtained in section 2, the corresponding solutions of the nonlinear wave equation, CKGZ equations, GDS equations, DS equations, GZ equations can be obtained.

2 Extended F-expansion method

Following the same line as in [12,13], the method simply proceeds as follows: Considering a given NLPDE with independent variables $x = (x_1, x_2, x_3, ..., x_l, t)$, and dependent variable u, the unknowns $u(x_1, x_2, x_3, ..., x_l, t)$ are solutions of the NODE obtained by the travelling wave reduction $(x_1, x_2, x_3, ..., x_l, t) \rightarrow u(\xi = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 + ... + \lambda_l x_l - \omega t)$. Thus, we get a NODE for $u(\xi)$ as follows

$$E(u, u', u'', ...) = 0. \qquad (\mathbf{1})$$

To determine $u(\xi)$ explicitly, we take the following four steps.

Step 1.

Supposing that $u(\xi)$ can be expanded as follows

$$u(\xi) = \sum_{j=0}^{n} \sum_{i=0}^{j} c_{ji} F^i(\xi) G^{j-i}(\xi), \quad c_{nn} \neq 0 \qquad (\mathbf{2})$$

or

$$u(\xi) = a_0 + \sum_{i=1}^{n} a_i F^i(\xi) + \sum_{j=1}^{n} b_j F^{-j}(\xi), \quad a_n \neq 0, \qquad (\mathbf{3})$$

where $c_{ji},\ a_0,\ a_i$, and b_j are constants to be determined, $F(\xi)$ and $G(\xi)$ satisfy the following relations

$$\begin{aligned} F'^2(\xi) &= P_1 F^4(\xi) + Q_1 F^2(\xi) + R_1, \\ G'^2(\xi) &= P_2 G^4(\xi) + Q_2 G^2(\xi) + R_2, \end{aligned}$$

$$G^2(\xi) = \mu F^2(\xi) + \nu, \qquad R_1 = \frac{(Q_1 - Q_2)^2 - 9P_2R_2}{9P_1}, \qquad (\mathbf{4})$$
$$\mu = \frac{P_1}{P_2}, \qquad \nu = \frac{Q_1 - Q_2}{3P_2} \qquad \nu \neq 0.$$

Integer n is the balance number determined by considering the homogeneous balance between the governing nonlinear terms and highest order derivatives of u in NODE (1).

Step 2.

Substituting (2) or (3) into NODE (1) and using (4), we obtain a series in $F^p G^q$ ($p = 0, 1, 2, ...l;\ q = 0, 1$) (or F^p, $p = 0, 1, 2, ...l$). Equating each coefficient of $F^p G^q$ (or F^p) to zero yields a system of algebraic equations for c_{ji} ($j = 0, 1, 2, ...n;\ i = 0, ..., j$) and $\lambda_i,\ \omega$, (or $a_i, b_j,\ i = 1, 2, ...n;\ j = 1, 2, ...n; \lambda_i, \omega$).

Step 3.

Solving these equations, $c_{ij},\ \lambda_i$ and ω can be expressed in terms of $P_i,\ Q_i,\ R_i,\ \mu,\ \nu$ and the parameters of NODE (1), (or $a_i,\ b_j,\ \lambda_i$ and ω can be expressed in terms of $P_1,\ Q_1,\ R_1$, and the parameters of NODE (1)) . Substituting these results into (2) or (3) gives the general form of travelling wave solutions [see Appendix A].

Step 4.

With the aid of Appendices A and B and the relation (4) the appropriate

kinds of the Jacobi elliptic function solutions of (1) including the single functions and the combined function solutions can be chosen. As we know, when $m \to 1$, Jacobi elliptic functions degenerate as hyperbolic functions in the manner of Appendix C.
When $m \to 0$, Jacobi elliptic functions degenerate as trigonometric functions in the manner of Appendix C.
So we can get the corresponding hyperbolic function solutions and trigonometric function solutions. In the next section, we will first apply this method to the elliptic-like equation.
It is important to say that, we have preferred the expansion (2) the same as in [12] which is more general than the expansion (5) in [13]. One can verify that for $n \geq 2$ the expansion in [13] will miss some terms, for example at $n = 2$, $c_{20}G^2(\xi)$ can not be obtained by the authors of [13].

3 Solutions of the elliptic-like equation

Now let us choose the following elliptic-like equation

$$A\phi''(\xi) + B\phi(\xi) + D\phi^3(\xi) = 0, \tag{\textbf{5}}$$

where A, B and D are arbitrary constants.
Considering the homogeneous balance between $\phi''(\xi)$ and $\phi^3(\xi)$ we get $n = 1$, so the solution of the NODE (5) is the form

$$\phi(\xi) = a_0 + a_1 F(\xi) + b_1 G(\xi), \tag{\textbf{6}}$$

where $a_0,\ a_1$ and b_1 are constants to be determined, $F(\xi)$ and $G(\xi)$ satisfy ODE (4). Substituting (6) into (5) along with (4) and collecting coefficients of the $F^p(\xi)G^q(\xi)$ ($p = 0, 1, 2, 3;\ q = 0, 1$) we have the following algebraic equations

$$\begin{aligned}
&F^0: \quad a_0(B + Da_0^2 + 3D\nu b_1^2) = 0,\\
&F: \quad a_1(B + 3Da_0^2 + 3D\nu b_1^2 + AQ_1) = 0,\\
&G: \quad b_1(B + 3Da_0^2 + AQ_2 + D\nu b_1^2 + 2A\nu P_2) = 0,\\
&F^2: \quad 3Da_0(a_1^2 + \mu b_1^2) = 0,\\
&FG: \quad 6a_0a_1b_1 = 0,\\
&F^3: \quad a_1(Da_1^2 + 3\mu D b_1^2 + 2AP_1) = 0,\\
&F^2G: \quad b_1(BDa_1^2 + D\mu b_1^2 + 2A\mu P_2) = 0.
\end{aligned} \tag{\textbf{7}}$$

From which we get the following two solutions

$$a_0 = 0, \quad b_1 = 0, \quad a_1^2 = -\frac{2AP_1}{D}, \tag{\textbf{8.a}}$$

the following constraint among the coefficients A and B of the elliptic-like equation should be respected
$B+AQ_1 = 0.$ **(8.b)**

$a_0 = 0, \quad a_1^2 = -\frac{P_1 A}{2D}, \quad b_1^2 = -\frac{P_1 A}{2\mu D},$ **(9.a)**
for this case, the following constraint among the coefficients A and B of the elliptic-like equation has been obtained
$2B\mu + A(2\mu Q_1 - 3\nu P_1) = 0.$ **(9.b)**
We have used in the second solution, the identities $P_1 = \mu P_2$ and $\nu P_2 = (Q_1 - Q_2)/3$.
Substituting (8.a) into the relation (6) we obtain the following travelling wave solutions

$$\phi^{sj}(\xi) = \pm\sqrt{\frac{-2AP_1}{D}} F(\xi), \tag{10}$$

where the superscript sj means single Jacobi, A and D should verify Eq.(8.b). These solutions are the single non-degenerative Jacobi elliptic function solutions. The relations between values of (R_1, Q_1, P_1) and the corresponding Jacobi elliptic function solution $F(\xi)$ of NODE (4) are given in Appendix A. Substituting the values of (R_1, Q_1, P_1) and the corresponding Jacobi $F(\xi)$ chosen from Appendix A into the general form of travelling wave solution (10), we can simultaneously obtain twelve periodic wave solutions to the elliptic-like equation (5).
For example, $R_1 = 1,\ Q_1 = -(1+m^2),\ P_1 = m^2$, then $F(\xi) = \mathrm{sn}(\xi)$, thus

$$\phi(\xi) = \pm m\sqrt{\frac{-2A}{D}}\mathrm{sn}(\xi), \tag{11}$$

where $AD < 0, \quad B = (1+m^2)A$.
Substituting (9.a) into (6) we obtain the combined non-degenerative Jacobi elliptic function solutions, solutions of the elliptic equation like (6), which are expressed by $F(\xi)$and $G(\xi)$

$$\phi^{cj}(\xi) = \pm\sqrt{\frac{-AP_1}{2\mu D}}[\sqrt{\mu}F(\xi) + \epsilon G(\xi)], \tag{12}$$

where the superscript cj means combined Jacobi, $\epsilon = 1$ or -1, $P_1 A/D < 0, \quad \mu > 0$, A and B should satisfied (9.b).
We will write down explicitly the solutions obtained for the case (12).

From Appendix B, if we select $\mathrm{dn}^2(\xi) = m^2\mathrm{cn}^2(\xi) + (1 - m^2)$, and set
$F(\xi) = \mathrm{cn}(\xi),\ \ G(\xi) = \mathrm{dn}(\xi),\ \ \mu = m^2,\ \ \nu = 1-m^2,$ (**13**)
and from Appendix A, we find that the respective coefficients of NODE for $\mathrm{cn}(\xi)$ and $\mathrm{dn}(\xi)$ are
$P_1 = -m^2, \quad Q_1 = 2m^2 - 1, \quad Q_2 = 2 - m^2.$ (**14**)
Inserting (13) and (14) into (12) we have

$$\phi_1^{cj}(\xi) = \pm\sqrt{\frac{-A}{2D}}[m\mathrm{cn}(\xi) + \epsilon\mathrm{dn}(\xi)], \tag{15}$$

where $\epsilon = 1$ or -1, $AD < 0$, $B = -(1 + m^2)A/2$.
From Appendix B, if we select $\mathrm{nd}^2(\xi) = m^2\mathrm{sd}^2(\xi) + 1$, and set

$F(\xi) = \mathrm{sd}(\xi),\ \ G(\xi) = \mathrm{nd}(\xi),\ \ \mu = m^2,\ \ \nu = 1,$ (**16**)
and from Appendix A, we find that the respective coefficients of NODE for $\mathrm{sd}(\xi)$ and $\mathrm{nd}(\xi)$ are
$P_1 = -m^2(1+m^2), \quad Q_1 = 2m^2-1, \quad Q_2 = 2-m^2.$ (**17**)
Inserting (16) and (17) into (12) we have

$$\phi_2^{cj}(\xi) = \pm\sqrt{\frac{(1 - m^2)A}{2D}}[m\mathrm{sd}(\xi) + \epsilon\mathrm{nd}(\xi)], \tag{18}$$

where $\epsilon = 1$ or -1, $AD > 0$, $B = -(1 + m^2)A/2$.
If we select $\mathrm{ns}^2(\xi) = \mathrm{cs}^2(\xi) + 1$ and set
$F(\xi) = \mathrm{cs}(\xi), \quad G(\xi) = \mathrm{ns}(\xi), \quad \mu = 1, \quad \nu = 1,$ (**19**)
$P_1 = 1, \quad Q_1 = 2 - m^2, \quad Q_2 = -(1 + m^2),$ (**20**)
then the solution is

$$\phi_3^{cj}(\xi) = \pm\sqrt{\frac{-A}{2D}}[\mathrm{cs}(\xi) + \epsilon\mathrm{ns}(\xi)], \tag{21}$$

where $\epsilon = 1$ or -1, $AD < 0$, $B = -(1 - 2m^2)A/2$.
In the same manner as mentioned above, we can obtain the following solutions

$$\phi_4^{cj}(\xi) = \pm\sqrt{\frac{-A}{2D}}[\mathrm{ds}(\xi) + \epsilon\mathrm{ns}(\xi)], \tag{22}$$

where $\epsilon = 1$ or -1, $AD < 0$, $B = -(m^2 - 2)A/2$.

$$\phi_5^{cj}(\xi) = \pm\sqrt{\frac{-A}{2D}}[\mathrm{cs}(\xi) + \epsilon\mathrm{ds}(\xi)], \tag{23}$$

where $\epsilon = 1$ or -1, $AD < 0$, $B = -(1+m^2)A/2$.

$$\phi_6^{cj}(\xi) = \pm\sqrt{\frac{-(1-m^2)A}{2D}}[\mathrm{sc}(\xi) + \epsilon\mathrm{nc}(\xi)], \qquad \mathbf{(24)}$$

where $\epsilon = 1$ or -1, $AD < 0$, $B = -(1+m^2)A/2$.

$$\phi_7^{cj}(\xi) = \pm\sqrt{\frac{-A}{2D}}[\sqrt{1-m^2}\mathrm{nc}(\xi) + \epsilon\mathrm{dc}(\xi)], \qquad \mathbf{(25)}$$

where $\epsilon = 1$ or -1, $AD < 0$, $B = -(m^2-2)A/2$.

$$\phi_8^{cj}(\xi) = \pm\sqrt{\frac{-A}{2D}}[\sqrt{1-m^2}\mathrm{sc}(\xi) + \epsilon\mathrm{dc}(\xi)], \qquad \mathbf{(26)}$$

where $\epsilon = 1$ or -1, $AD < 0$, $B = -(1-2m^2)A/2$.
Instead of expansion (6), if we assume that $\phi(\xi)$ can simply be expressed by

$$\phi(\xi) = a_0 + a_1F(\xi) + b_1F^{-1}(\xi), \qquad \mathbf{(27)}$$

where $a_0,\ a_1$ and b_1 are constants to be determined, $F(\xi)$ satisfies ODE (4). Substituting (27) into (5) along with (4) and equating the coefficients of the $F^p(\xi)$ $(p = 0, 1, ...6)$ to zero yields a system of algebraic equations for $a_0,\ a_1$ and b_1 as follows

$$\begin{aligned}
&F^0: \quad b_1(2AR_1 + Db_1^2) = 0,\\
&F: \quad 3Da_0b_1^2 = 0,\\
&F^2: \quad b_1(B + 3Da_0^2 + 3Da_1b_1 + AQ_1) = 0,\\
&F^3: \quad a_0(B + Da_0^2 + 6Da_1b_1) = 0,\\
&F^4: \quad a_1(B + 3Da_0^2 + 3Da_1b_1 + AQ_1) = 0,\\
&F^5: \quad a_0Da_1^2 = 0,\\
&F^6: \quad a_1(Da_1^2 + 2AP_1).
\end{aligned} \qquad \mathbf{(28)}$$

Solving system of algebraic equations (28) we obtain the following two cases
$a_0 = 0, \quad b_1 = 0, \quad a_1^2 = -\frac{2AP_1}{D},$ **(29.a)**
the following constraint among the coefficients A and B of the elliptic-like equation should be respected
$B + AQ_1 = 0.$ **(29.b)**
This case coincides with (8).
$a_0 = 0, \quad a_1^2 = -\frac{2P_1A}{D}, \quad b_1^2 = -\frac{2R_1A}{D},$ **(30.a)**
for this case, the following constraint among the coefficients A and B of the elliptic-like equation has been obtained

$B\mu+A(Q_1-6\sqrt{P_1R_1})=0.$ **(30.b)**

Substituting (30.a) into the relation (27) we obtain the following travelling wave solutions

$$\phi^{cj}(\xi)=\pm[\sqrt{\frac{-2AP_1}{D}}F(\xi)+\epsilon\sqrt{\frac{-2AR_1}{D}}F^{-1}(\xi)], \quad \textbf{(31)}$$

where $\epsilon=1$ or -1, A and B should verify Eq.(30.b).

With the aid of Appendix A, selecting

$F(\xi)=sn(\xi),\ F^{-1}(\xi)=ns(\xi),\ P_1=m^2,\ Q_1=-(1+m^2),\ R_1=1,$ **(32)**

and inserting (32) into (31) yields

$$\phi_9^{cj}(\xi)=\pm\sqrt{\frac{-2A}{D}}[m\ \mathrm{sn}(\xi)+\epsilon\mathrm{ns}(\xi)], \quad \textbf{(33.a)}$$

where $\epsilon=1$ or -1, $AD<0,\ B=(1+m^2+6m)A.$

Then if $sn(\xi)$ and $ns(\xi)$ are replaced by $cd(\xi)$ and $dc(\xi)$ respectively, we have

$$\phi_{10}^{cj}(\xi)=\pm\sqrt{\frac{-2A}{D}}[m\ \mathrm{cd}(\xi)+\epsilon\mathrm{dc}(\xi)], \quad \textbf{(33.b)}$$

where $\epsilon=1$ or -1, $AD<0,\ B=(1+m^2+6m)A.$

Inserting

$F(\xi)=\mathrm{sc}(\xi),\ \ F^{-1}(\xi)=\mathrm{cs}(\xi),\ \ P_1=1-m^2,\ \ Q_1=2-m^2,$

$$R_1=1, \quad \textbf{(34)}$$

into (31) yields

$$\phi_{11}^{cj}(\xi)=\pm\sqrt{\frac{-2A}{D}}[\sqrt{1-m^2}\ \mathrm{sc}(\xi)+\epsilon\ \mathrm{cs}(\xi)], \quad \textbf{(35)}$$

where $\epsilon=1$ or -1, $AD<0,\ B=(m^2+6\sqrt{1-m^2}-2)A.$

Inserting

$F(\xi)=\mathrm{dn}(\xi),\ \ F^{-1}(\xi)=\mathrm{nd}(\xi),\ \ P_1=-1,\ \ Q_1=2-m^2,$

$$R_1=m^2-1, \quad \textbf{(36.a)}$$

into (31) yields

$$\phi_{12}^{cj}(\xi) = \pm\sqrt{\frac{2A}{D}}[\mathrm{dn}(\xi) + \epsilon\sqrt{1-m^2}\ \mathrm{nd}(\xi)], \qquad (\mathbf{36}.\mathbf{b})$$

where $\epsilon = 1$ or -1, $AD > 0$, $B = (m^2 + 6\sqrt{1-m^2} - 2)A$.

4 Exact solutions of some class of NLPDEs

in this section, by using the results obtained in the preceding section, we will construct the corresponding solutions of the nonlinear wave equation, CKGZ equations, GDS equations, DS equations, GZ equations .

4.1 nonlinear wave equation

Consider the nonlinear wave equation in Rev. [14]

$$u_{tt} + \alpha u_{xx} + \beta u + \gamma u^3 = 0, \qquad (\mathbf{37})$$

where α, β and γ are constants. Eq.(37) contains some particular important equations such as Duffing, Klein-Gordon and Landau-Ginzburg-Higgs equation. We assume that Eq.(37) has exact solution in the form
$u(x,t) = \phi(\xi), \quad \xi = px - \omega t.$ (**38**)
Substituting Eq.(38) into Eq.(37), we have

$$A\phi^{''}(\xi) + \phi(\xi) + D\phi^3(\xi) = 0. \qquad (\mathbf{39})$$

Eq.(39) coincides with Eq.(5) where A, B and D are defined by
$A = (\omega^2 + \alpha p^2), \quad B = \beta, \quad D = \gamma$ (**40**)
Then the solutions of (37) are

$$u = \phi(\xi). \qquad (\mathbf{41})$$

$\phi(\xi)$ is given by relations (10)-(26) and (31)-(36) and $\xi = px - \omega t$, A, B, D are defined by (40).

4.2 Coupled Klein-Gordon-Zakharov equations

The coupled nonlinear Klein-Gordon-Zakarov equations [15] read

$$u_{tt} - c_0^2\nabla^2 u + f_0^2 u + \delta uv = 0,$$
$$v_{tt} - c_0^2\nabla^2 v - \beta\nabla^2|u|^2 = 0. \qquad (\mathbf{42})$$

We seek its following wave packet solution

$u(x,y,z,t) = \phi(\xi)e^{i(kx+ly+nz-\Omega t)}, \quad v = v(\xi), \; \xi = px+qy+rz-\omega t \qquad (\mathbf{43})$

where both $\phi(\xi)$ and $v(\xi)$ are real functions. Substituting Eq.(43) into Eq.(42) yields

$(\omega^2 - c_0^2\,\mathbf{P^2})\,\phi^{''}(\xi) + 2i(\omega\Omega - c_0^2\,\mathbf{K}.\mathbf{P})\,\phi^{'}(\xi) - (\omega^2 - c_0^2\mathbf{K^2} - f_0^2)\phi(\xi) + \delta v\phi(\xi) = 0$

$(\omega^2 - c_0^2\,\mathbf{P^2})\,v^{''}(\xi) - \beta\,\mathbf{P^2}\,(\phi^2(\xi))^{''} = 0, \qquad (\mathbf{44})$

where

$\mathbf{K}=(k,l,n), \quad \mathbf{K^2}=k^2+l^2+n^2, \quad \mathbf{P}=(p,q,r),$

$\mathbf{P^2} = p^2+q^2+r^2, \quad \mathbf{K}.\mathbf{P}=kp+lq+nr. \qquad (\mathbf{45})$

If we take

$$\omega\Omega = c_0^2\mathbf{K}.\mathbf{P}, \qquad (\mathbf{46})$$

then (44) is reduced to

$$(\omega^2 - \mathbf{P^2}\,c_0^2)\phi^{''}(\xi) - (\omega^2 - \mathbf{K^2}\,c_0^2 - f_0^2)\phi(\xi) + \delta v\phi(\xi) = 0, \qquad (\mathbf{47.a})$$
$$(\omega^2 - \mathbf{P^2}\,c_0^2)v^{''}(\xi) - \beta\,\mathbf{P^2}\,(\phi^2(\xi))^{''} = 0. \qquad (\mathbf{47.b})$$

Integrating (47.b) once with respect to ξ, we get

$$(\omega^2 - \mathbf{P^2}\,c_0^2)v^{'}(\xi) - \beta\,\mathbf{P^2}\,\phi^{2'}(\xi) = \tilde{c}. \qquad (\mathbf{48})$$

where $\tilde{c}$ is integration constant. Because we find the special form of exact solutions, we take $\tilde{c} = 0$ and integrating this formula once again, we have

$$v(\xi) = \frac{C}{\omega^2 - c_0^2\mathbf{P^2}} + \frac{\beta\mathbf{P^2}}{\omega^2 - c_0^2\mathbf{P^2}}\phi^2(\xi), \qquad (\mathbf{49})$$

where C is an integration constant. Substituting (49) into (47.a) yields

$$A\phi^{''}(\xi) + B\phi(\xi) + D\phi^3(\xi) = 0. \qquad (\mathbf{50})$$

It is easy to see that Eq.(50) coincides with Eq.(5) and A, B and D are defined by

$A = (\omega^2 - c_0^2\,\mathbf{P^2}\,)^2, \quad B = [(\omega^2 - c_0^2\,\mathbf{P^2}\,)(-\omega^2 + c_0^2\,\mathbf{K^2}\,c_0^2 + f_0^2) + \delta C],$

$$D = \delta\beta\mathbf{P^2}. \qquad (\mathbf{51})$$

Then solutions of Eq.(42) are defined as follows

$$u(x,y,z,t) = \phi(\xi)e^{i(kx+ly+nz-\Omega t)},$$

$$v(x,y,z,t) = \frac{C}{\omega^2 - c_0^2\mathbf{P^2}} + \frac{\beta\mathbf{P^2}}{\omega^2 - c_0^2\mathbf{P^2}}\phi^2(\xi), \tag{52}$$

$$\Omega = \frac{c_0^2\mathbf{K}.\mathbf{P}}{\omega}, \tag{53}$$

where $\phi(\xi)$ appearing in these solutions is given by relations (10)-(26) and (31)-(36) $A,\ B$ and D are defined by (51), $\xi = px + qy + rz - \omega t$

4.3 GDS, DS and GZ equations

We consider a class of NLPDEs with constant coefficients [16]

$$iu_t + \mu(u_{xx} + D_1u_{yy}) + E_1|u|^2u + C_1un = 0, \tag{54.a}$$

$$D_2n_{tt} + (n_{xx} - E_2u_{yy}) + C_2(|u|^2)_{xx} = 0, \tag{54.b}$$

where $\mu,\ D_i,\ E_i,\ C_i$ (i=1,2) are real constants and $\mu \neq 0,\ D_1 \neq 0,\ C_1 \neq 0,\ C_2 \neq 0$.
Eqs.(54.a), (54.b) are a class of physically important equations. In fact, if one takes
$\mu = \frac{1}{2}\kappa^2,\quad D_1 = 2\mu,\quad E_1 = \alpha,\quad C_1 = -1,$
$D_2 = 0,\quad E_2 = D_1,\quad C_2 = -2\alpha,\quad \kappa^2 = \pm 1,$ **(55)**
then Eqs.(54.a), (54.b) represent the Davey-Stewartson (DS) equations [17]

$$iu_t + \frac{1}{2}\kappa^2(u_{xx} + \kappa^2u_{yy}) + \alpha|u|^2u - un = 0, \tag{56.a}$$

$$n_{xx} - \kappa^2n_{yy} - 2\alpha(|u|^2)_{xx} = 0. \tag{56.b}$$

If one takes
$n = n(x,t)$ *i.e.* $n_y = 0,\ \mu = 1,\ D_1 = 0,\ E_1 = -2\lambda,$
$E_2 = -1,\quad C_2 = -1,\quad C_1 = 2,$ **(57)**
then Eqs.(54.a) and (54.b) become generalized Zakharov (GZ) equations [18]

$$iu_t + u_{xx} - 2\lambda|u|^2u + 2un = 0, \tag{58.a}$$

$$n_{tt} - n_{xx} + (|u|^2)_{xx} = 0. \tag{58.b}$$

Since u is a complex function, we assume that

$$u(x,y,t)=\phi(\xi)e^{i(kx+ly-\Omega t)},\quad v(x,y,t)=v(\xi),\ \ \xi=px+qy-\omega t \qquad (\mathbf{59})$$

where both $\phi(\xi)$ and $v(\xi)$ are real functions, k, l, p, q, Ω and ω are constants to be determined later. Substituting Eq.(59) into Eqs.(54.a) and (54.b), we have the following ODE for $\phi(\xi)$ and $v(\xi)$

$$\mu(p^2+D_1q^2)\phi^{''}(\xi)+[\Omega-\mu(k^2+D_1l^2)]\phi(\xi)+E_1\phi^3(\xi)+i[-\omega+2\mu(kp+D_1lq)]\phi^{'}(\xi)+C_1\phi(\xi)v(\xi)=0, \qquad (\mathbf{60}.\mathbf{a})$$

$$(D_2\omega^2+p^2-E_2q^2)v^{''}(\xi)+C_2p^2(\phi^2(\xi))^{''}=0. \qquad (\mathbf{60}.\mathbf{b})$$

If we set

$$\omega=2\mu(kp+D_1lq), \qquad (\mathbf{61})$$

then (60.a) and (60.b) reduce to

$$\mu(p^2+D_1q^2)\phi^{''}(\xi)+[\Omega-\mu(k^2+D_1l^2)]\phi(\xi)+E_1\phi^3(\xi)+C_1\phi(\xi)v(\xi)=0, \quad (\mathbf{62}.\mathbf{a})$$

$$(D_2\omega^2+p^2-E_2q^2)v^{''}(\xi)+C_2p^2(\phi^2(\xi))^{''}=0. \qquad (\mathbf{62}.\mathbf{b})$$

Integrating (62.b) once, we get

$$(D_2\omega^2+p^2-E_2q^2)v'(\xi)+C_2p^2(\phi^2(\xi))^{'}=\tilde{C}, \qquad (\mathbf{63})$$

where $\tilde{C}$ is integration constant, then we take $\tilde{C}=0$ and integrating the formula once again, we have

$$v(\xi)=\frac{C}{D_2\omega^2+p^2-E_2q^2}-\frac{C_2p^2}{D_2\omega^2+p^2-E_2q^2}\phi^2(\xi). \qquad (\mathbf{64})$$

Substituting (64) into (62.a) yields

$$A\phi^{''}(\xi)+B\phi(\xi)+D\phi^3(\xi)=0. \qquad (\mathbf{65})$$

Eq.(65) is identical to Eq.(5) and $A,\ B$ and D are defined by
$A=\mu(p^2+D_1q^2)(D_2\omega^2+p^2-E_2q^2),\ \ B=[C_1C-(D_2\omega^2+p^2-E_2q^2)(\Omega-\mu(k^2+D_1l^2))]$,

$$D=[E_1(D_2\omega^2+p^2-E_2q^2)-C_1C_2p^2]. \qquad (\mathbf{66})$$

Then the solutions of Eqs.(58) are

$$u(x,y,t)=\phi(\xi)e^{i(kx+ly-\Omega t)},$$

$$v(x,y,t)=\frac{C}{D_2\omega^2+p^2-E_2q^2}-\frac{C_2p^2}{D_2\omega^2+p^2-E_2q^2}\phi^2(\xi), \qquad \textbf{(67)}$$

$$\omega=2\mu(kp+D_1lq). \qquad \textbf{(68)}$$

The expression $\phi(\xi)$ appearing in these solutions are given by relations (10)-(26) and (31)-(36), $A,\ B$ and D are given by (66) and $\xi=px+qy-\omega t$
We may obtain from (56) that

$$\omega=\kappa^2(kp+\kappa^2lq), \qquad \textbf{(69)}$$

$$v(x,y,t)=\frac{C}{p^2-\kappa^2q^2}+\frac{2\alpha p^2}{p^2-\kappa^2q^2}\phi^2(\xi), \qquad \textbf{(70)}$$

$$u(x,y,t)=\phi(\xi)e^{i(kx+ly-\Omega t)},$$

$$A\phi^{''}(\xi)+B\phi(\xi)+D\phi^3(\xi)=0. \qquad \textbf{(71)}$$

This equation coincides also with Eq.(5), where A, B and D are defined as follows
$A=\kappa^2(p^2+\kappa^2q^2)(\kappa^2q^2-p^2),\quad B=2C+(p^2-\kappa^2q^2)(-2\Omega+\kappa^2(k^2+\kappa^2l^2))$

$$D=2\alpha\kappa^2(p^2+\kappa^2q^2). \qquad \textbf{(72)}$$

The expression $\phi(\xi)$ appearing in these solutions are given by relations (10)-(26) and (31)-(36), $A,\ B$ and D are given by (72) and $\xi=px+qy-\omega t$.
Then from (58) we have that

$$\omega=2kp, \qquad \textbf{(73)}$$

$$v(x,t)=\frac{C}{p^2-\omega^2}+\frac{p^2}{p^2-\omega^2}\phi^2(\xi), \qquad \textbf{(74)}$$

$$u(x,t)=\phi(\xi)e^{i(kx-\Omega t)},$$

$$A\phi^{''}(\xi)+B\phi(\xi)+D\phi^3(\xi)=0. \qquad \textbf{(75)}$$

Eq.(75) coincides also with Eq.(5), where $A,\ B$ and D are defined as follows
$A=p^2(p^2-\omega^2),\quad B=2C-(p^2-\omega^2)(\Omega-k^2))$

$$D=2(p^2-\lambda(p^2-\omega^2)). \qquad \textbf{(76)}$$

The expression $\phi(\xi)$ appearing in these solutions are given by relations (10)-(26) and (31)-(36), $A,\ B$ and D are defined by (72) and $\xi=px-\omega t$

5 Conclusion

In this paper, by using the extended F-expansion method, we have been able to obtain in a unified way, by the aid of symbolic computation system-Mathematica, a series of solutions including the single and the combined non-degenerative Jacobi elliptic function solutions and their degenerative solutions to a class of NLPDEs. This class of NLPDEs is characterized by the fact that it can be reduced through a simple transformation to the elliptic-like equation $A\phi^{''}(\xi) + B\phi(\xi) + D\phi(\xi)^3 = 0$. It is obvious that by using this simple transformation, the computation quantity involved in solving nonlinear equations is greatly reduced. Thus, the method has proved its efficiency to the nonlinear wave equation, the coupled Klein-Gordon-Zakharov equations, the generalized Davey Stewartson (GDS) equations, the Davey Stewartson (DS) equations, the generalized Zakharov (GZ) equations.

Appendix A

Relation between values of (P,Q,R) and corresponding $F(\xi)$in NODE
$F^{'2} = PF^4 + QF^2 + R$

P	Q	R	$F^{'2} = PF^4 + QF^2 + R$	F
m^2	$-(1-m^2)$	1	$F^{'2} = (1-F^2)(1-m^2F^2)$	sn ξ, cd $\xi = \frac{\text{cn } \xi}{\text{dn } \xi}$
$-m^2$	$2m^2-1$	$1-m^2$	$F^{'2} = (1-F^2)(m^2F^2+1-m^2)$	cn ξ
-1	$2-m^2$	m^2-1	$F^{'2} = (1-F^2)(F^2+m^2-1)$	dn ξ
1	$-(1+m^2)$	m^2	$F^{'2} = (1-F^2)(m^2-F^2)$	ns $\xi = (\text{sn } \xi)^{-1}$, dc $\xi = \frac{\text{dn } \xi}{\text{cn } \xi}$
$1-m^2$	$2m^2-1$	$-m^2$	$F^{'2} = (1-F^2)[(m^2-1)F^2-m^2]$	nc $\xi) = (\text{cn } \xi)^{-1}$
m^2-1	$2-m^2$	-1	$F^{'2} = (1-F^2)[(1-m^2)F^2-1]$	nd $\xi) = (\text{dn } \xi)^{-1}$
$1-m^2$	$2-m^2$	1	$F^{'2} = (1+F^2)[(1-m^2)F^2+1]$	sc $\xi = \frac{\text{sn } \xi}{\text{cn } \xi}$
$-m^2(1-m^2)$	$2m^2-1$	1	$F^{'2} = (1+m^2F^2)[1+(m^2-1)F^2]$	sd $\xi = \frac{\text{sn } \xi}{\text{dn } \xi}$
1	$2-m^2$	$1-m^2$	$F^{'2} = (1+F^2)[1-m^2+F^2]$	cs $\xi = \frac{\text{cn } \xi}{\text{sn } \xi}$
1	$2m^2-1$	$-m^2(1-m^2)$	$F^{'2} = (m^2+F^2)[m^2-1+F^2]$	ds $\xi = \frac{\text{dn } \xi}{\text{sn } \xi}$

Appendix B

Jacobi elliptic functions with modulus $m(O \le m \le 1)$ have the identity relations in the form of $G^2 = \mu F^2 + \nu$

$\text{cn}^2(\xi) = -\text{sn}^2(\xi) + 1,\ \text{dn}^2(\xi) = -m^2\text{sn}^2(\xi) + 1,\ \text{cd}^2(\xi) = \frac{m^2-1}{m^2}\text{nd}^2(\xi) + \frac{1}{m^2},$
$\text{cd}^2(\xi) = (m^2-1)\text{sd}^2(\xi)+1,\ \text{dn}^2(\xi) = m^2\text{cn}^2(\xi)+(1-m^2),\ \text{nd}^2(\xi) = m^2\text{sd}^2(\xi)+1,$
$\text{ns}^2(\xi) = \text{cs}^2(\xi) + 1,\ \text{ns}^2(\xi) = \text{ds}^2(\xi) + m^2,\ \text{ds}^2(\xi) = \text{cs}^2(\xi) + (1-m^2),$
$\text{nc}^2(\xi) = \text{sc}^2(\xi) + 1,\ \text{dc}^2(\xi) = (1-m^2)\text{nc}^2(\xi) + m^2,\ \text{dc}^2(\xi) = (1-m^2)\text{sc}^2(\xi) + 1,$

Appendix C

Jacobi elliptic functions degenerate as hyperbolic functions when $m \to 1$

sn ξ tanh ξ	cn ξ $sech\ \xi$	dn ξ $sech\ \xi$	sc ξ sinh ξ	sd ξ sinh ξ	cd ξ 1
ns ξ coth ξ	nc ξ cosh ξ	nd ξ cosh ξ	cs ξ $csch\ \xi$	ds ξ $csch\ \xi$	dc ξ 1

Jacobi elliptic functions degenerate as trigonometric functions when $m \to 0$

sn ξ sin ξ	cn ξ cos ξ	dn ξ 1	sc ξ tan ξ	sd ξ sin ξ	cd ξ cos ξ
ns ξ csc ξ	nc ξ sec ξ	nd ξ 1	cs ξ cot ξ	ds ξ csc ξ	dc ξ sec

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