Optimal Design for a Varying Environment

Serguei Lapin [*] Xuan Hien Nguyen [†] Jiyeon Oh [‡]
Daniel Vasiliu [§] Pei Yin [¶] Ningyi Zhang [‖]
Mentor: Dr. David Misemer [**]

June 3, 2002

1 Introduction

Lasers are currently used in many processes in which materials are manipulated, including abalation of polymers, cutting of both metals and nonmetals, and annealing of semiconductors. In many applications, the processes include not only changing the material properties but patterning them as well.

Computer-generated holograms are diffractive optical elements (DOE) that permit very general changes in phase and amplitude of an incoming wave. By adjusting the local phase function, one can create the desired target intensity in the image plane. The DOE responsible for the phase shifts is called a phase mask. Such optical elements can be used to shape a beam with Gaussian intensity profile into a uniform *top hat* shape (Fig. 1). Although it is now possible to create continuous masks to adjust the phase, the discrete ones are more common and inexpensive. They usually have 2 to 16 levels and alter the phase from $-\pi$ to π. Several additional requirements are imposed

[*] University of Houston
[†] University of Wisconsin, Madison
[‡] University of Cincinnati
[§] Michigan State University
[¶] University of Missouri - Columbia
[‖] University of Delaware
[**] 3M Company

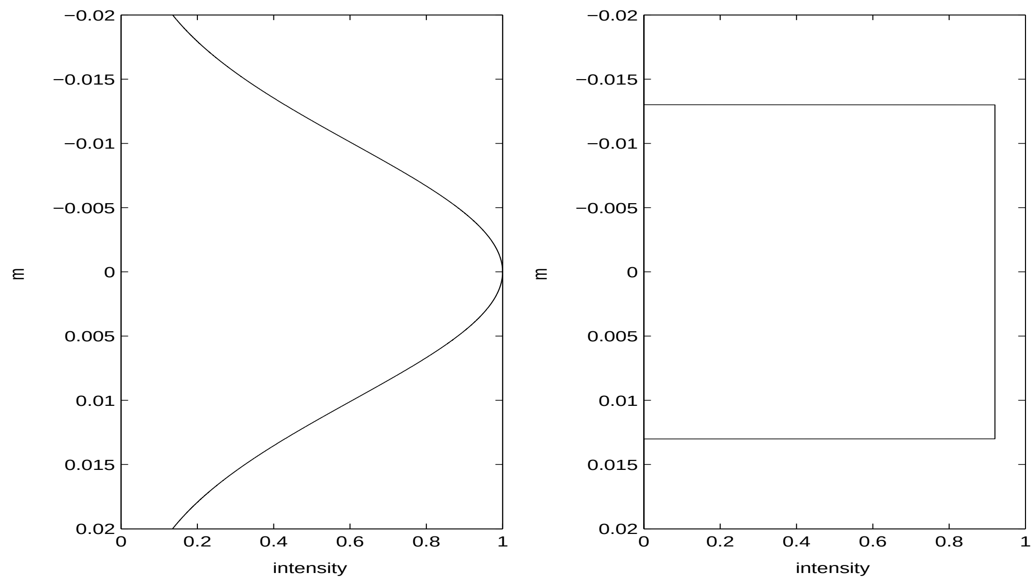

Figure 1: Input Gaussian intensity profile and desired output *top hat* intensity profile

on the design that severely restrict the design options. The most important of them is that the passive optical element has to maintain uniformity even as the beam profile shifts with time. In addition, the element must be very efficient; that is, a high percentage of the beam energy must be delivered to the work space. We also need to take into account the partial coherence of the laser beam. An extended light source consists of many atomic oscillators, which act as emitters light. The phase and amplitude of these emitters undergo random fluctuations between the two points are perfectly correlated then we say the emitters are coherent, and we fluctuations are independent, the difference between the phase decays gradually as the separation between the two emitters increases. We will demonstrate that this partial coherence has profound effects on the intensity generated by a DOE.

Our goal was to improve the existing algorithms, cut the time of computation down by using Fast Fourier Transform and find efficient algorithms to automate the search for an optimal phase mask. By examining different ways to discretize a function, we find that using uniform levels gives the result closest to the uniform function. To define the problem as an optimization problem, phase functions are generated by polynomials, we discretize them and minimize the error function.

The remaining part of the paper is organized as follows: we first describe the technical method to derive the output intensity for a given phase mask and incoming beam. In section 3, we determine the utility function that is the most suitable to our purpose. Some theoretical results and numerical experiments are then presented in section 4. We then explain the optimization algorithms used, along with numerical results in section 6.

2 Formulation of the problem

The problem of a beam homogenizer is divided into two parts, corresponding to the two following sections. First, one needs to represent the output intensity $I(P)$ in terms of the phase change $\phi(\xi)$ and the incoming wavefront $U_0(P')$ then build a transformation using this relation. The effects of partial coherence of the input intensity have to be taken into account in the transformation. Second, one needs to determine the phase function that creates the desired target intensity.

2.1 Calculation for full coherence

In this section, we follow the presentation of the paper by R. Rudnaya, D. Misemer and F. Santosa [4]. We denote the phase function by $\phi(\xi, \eta)$ and the distance between the phase mask and the image plane by z. Let $P(x, y, z)$ be a point in the image plane and let $P'(\xi, \eta, 0)$ be a point on the phase mask plane. A fully coherent incoming beam $U_0(P')$ will produce an electromagnetic field $U(P)$ on the image plane given by the Kirchoff's formula [2], p.41:

$$U(P) = \frac{z}{i\lambda} \iint U_0(P') \frac{e^{ikr}}{r^2} ds \tag{1}$$

where $r = |PP'|$ and $k = \frac{2\pi}{\lambda}$. When the distance between the lens and the image plane is much greater than the dimensions of the lens, we may use the Fresnel approximation

$$r \approx z \left[1 + \frac{1}{2} \left(\frac{(x-\xi)}{z} \right)^2 + \frac{1}{2} \left(\frac{(y-\eta)}{z} \right)^2 \right] \tag{2}$$

Combining (1) and (2) and letting $p(\xi,\eta) = U_0(P')e^{i\phi(\xi,\eta)}$ be the electromagnetic field exiting the phase mask, we obtain

$$U(x,y,z) = \frac{e^{ik\left(z+\frac{x^2+y^2}{2z}\right)}}{i\lambda z} \iint \left[p(\xi,\eta)e^{i\frac{k}{2z}(\xi^2+\eta^2)}\right] e^{-i\frac{k}{z}(x\xi+y\eta)} d\xi d\eta \tag{3}$$

The square modulus of the field gives the intensity on the image plane

$$I(P) = |U(P)|^2 \tag{4}$$

2.2 Calculation for partial coherence

A perfect ideal laser will emit a fully coherent light. For such a beam, one can predict the phase and amplitude at any point, given the phase and intensity at one point. The well manufactured lasers have a beam that is partially coherent, which means that the above prediction only works for points close to the given one. We will discover that an incoming partial coherent beam gives better results than the full coherent one.

Let $P'(\xi,\eta)$ and $P''(\xi',\eta')$ be points on the phase mask plane. For a partially coherent incoming beam, the output intensity is given by

$$I(P) = \frac{z^2}{\lambda^2} \iiiint U(P')e^{i\phi(\xi,\eta)} U^*(P'')e^{-i\phi(\xi,\eta)} \frac{e^{ik(r'-r'')}}{(r'r'')^2} \gamma(|P'P''|) ds' ds'' \tag{5}$$

where U^* denotes the complex conjugate of U, $r' = |PP'|$ and $r'' = |PP''|$. The coherence function $\gamma(r)$ is a symmetric nonnegative function of r such that $\gamma(0) = 1$ and $\gamma(r) \to 0$ when $r \to \infty$. $\gamma(r) \equiv 1$ corresponds to the full coherence model and (5) reduces to (3) and (4).

As mentioned before, when the distance z between the lens and the image plane is much greater than the dimensions of the lens, we can use the Fresnel approximation for r to obtain

$$I(x,y,z) = \frac{1}{(\lambda z)^2} \iiiint \tilde{p}(\xi,\eta)\tilde{p}^*(\xi,\eta)e^{-i\frac{k}{z}(x(\xi-\xi')+y(\eta-\eta'))}\gamma(r) d\xi d\eta d\xi' d\eta'$$

where $\tilde{p}(\xi,\eta) = p(\xi,\eta)e^{i\frac{k}{2z}(\xi^2+\eta^2)}$ and $\tilde{p}^*$ is the complex conjugate of $\tilde{p}$.

The design problem can be greatly simplified if the amplitude of the incident beam $A(\xi, \eta)$, and γ are separable functions of ξ and η, and $I_T(x, y)$ is a separable function of x and y. In the rest of the paper, we will assume that these conditions are satisfied, so the two dimensional problem can be reduced to two independent one-dimensional cases, which are treated in the remaining analysis.

3 The choice of utility function

Ideally our goal is to achieve the output intensity as a rectangular shape (*top hat* function). Practically, it turns out that we can approximate this shape pretty well. We denote by $I_{Target}(x)$ the top hat function

$$I_{Target}(x) = \begin{cases} c > 0 & \text{if} \quad -T \leq x \leq T \\ 0 & \text{if} \quad otherwise \end{cases}$$

I_{Out} depends on the choice of the phase mask shape which is represented by a function ϕ. We have that $I_{Out}(\phi)$ is a bounded function and we make the remark that any bounded function is in the class of BMO (Bounded Mean Oscillation) function i.e.

$$\sup_I \inf_c \frac{1}{|I|} \int_I |f(x) - c| dt = ||f||_{BMO} \leq ||f||_\infty$$

Ideally we would like to be able to find a sequence ϕ_n such that

$$||I_{Out}(\phi_n) - I_{Target}||_\infty \to 0$$

Practically we can allow bigger oscillations on sets of very small measure and thus we look for a sequence ϕ_n such that

$$||I_{Out}(\phi_n) - I_{Target}||_{L^2} \to 0$$

It will follow also that

$$I_{Out}(\phi_n) \to I_{Target} \ a.e.$$

Thus we consider appropriate for our problem to minimize the following utility function:

$$J(\phi) = \int (I_{Out}(\phi)(x) - I_{Target}(x))^2 dx \tag{6}$$

4 A numerical experiment: comparison between number of lenslets

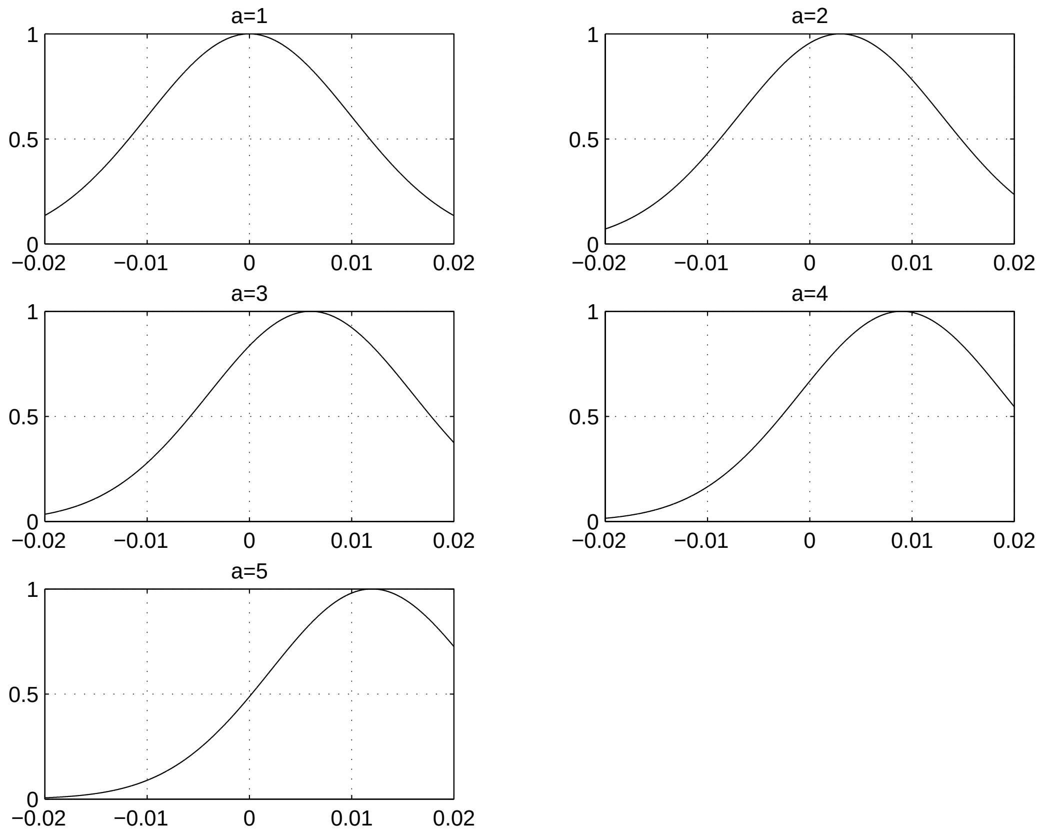

Figure 2: Input Gaussian intensity profiles

The diffractive optical element can be divided into different numbers of lenslets [4]. We have performed numerical experiments using the same phase mask as in [4] and have compared the difference in the shape of the output intensity and its closeness to the target intensity for optical elements consisting of three, six and ten lenslets.

For each given partition of the DOE we considered five different initial (Gausian) beam profiles (Fig. 2). Each profile is shifted from the origin by $s = 0.003(a-1)$ where a respectively takes the values $1, 2, 3, 4, 5$. In the case

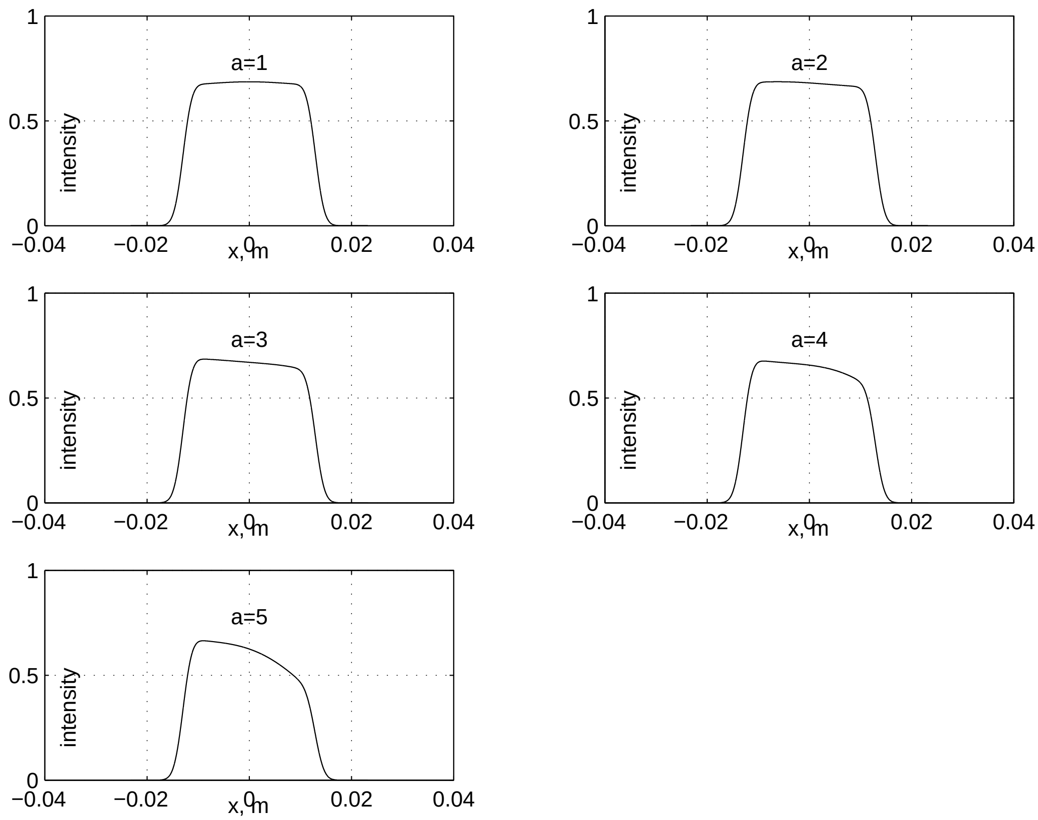

Figure 3: Output intensity profiles for 3 lenslets

of 3 lenslets, the shape of the output intensity for a shifted initial beam is not very satisfactory(Fig. 3). As for 6 lenslets (Fig. 4) and 10 lenslets (Fig. 5) the output intensity profiles are sufficiently close to each other, especially when the input Gaussian beam is symmetric with respect to the origin.

The error (utility) function (6) for the partially coherent output intensity slowly grows as we shift input intensity profile further away from the origin. The value of the utility function for all 3 different partitions of the DOE are relatively close to each other (Fig. 6). We can therefore conclude that 10 lenslet and 6 lenslet phase masks are comparably efficient in the sense of output intensity shape profiles and closeness to the desired solution.

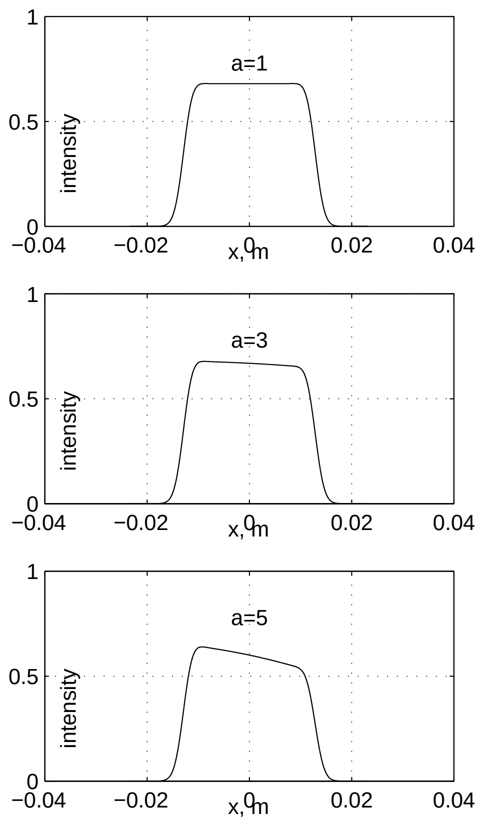

Figure 4: Output intensity profiles for 6 lenslets

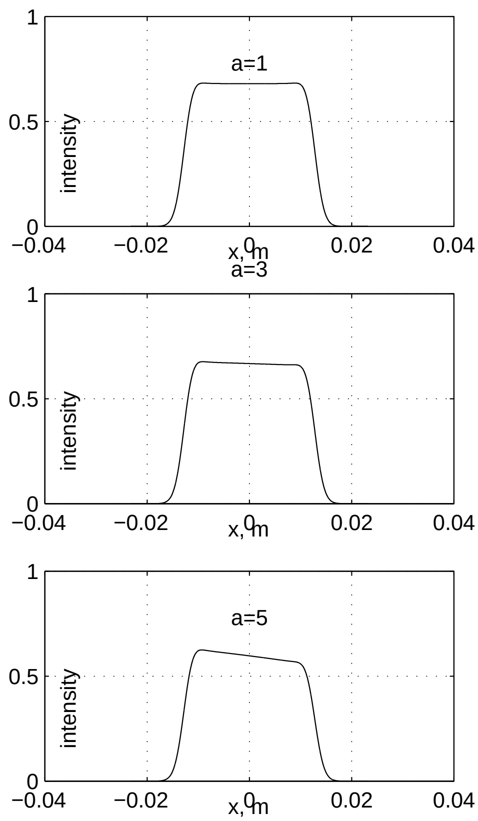

Figure 5: Output intensity profiles for 10 lenslets

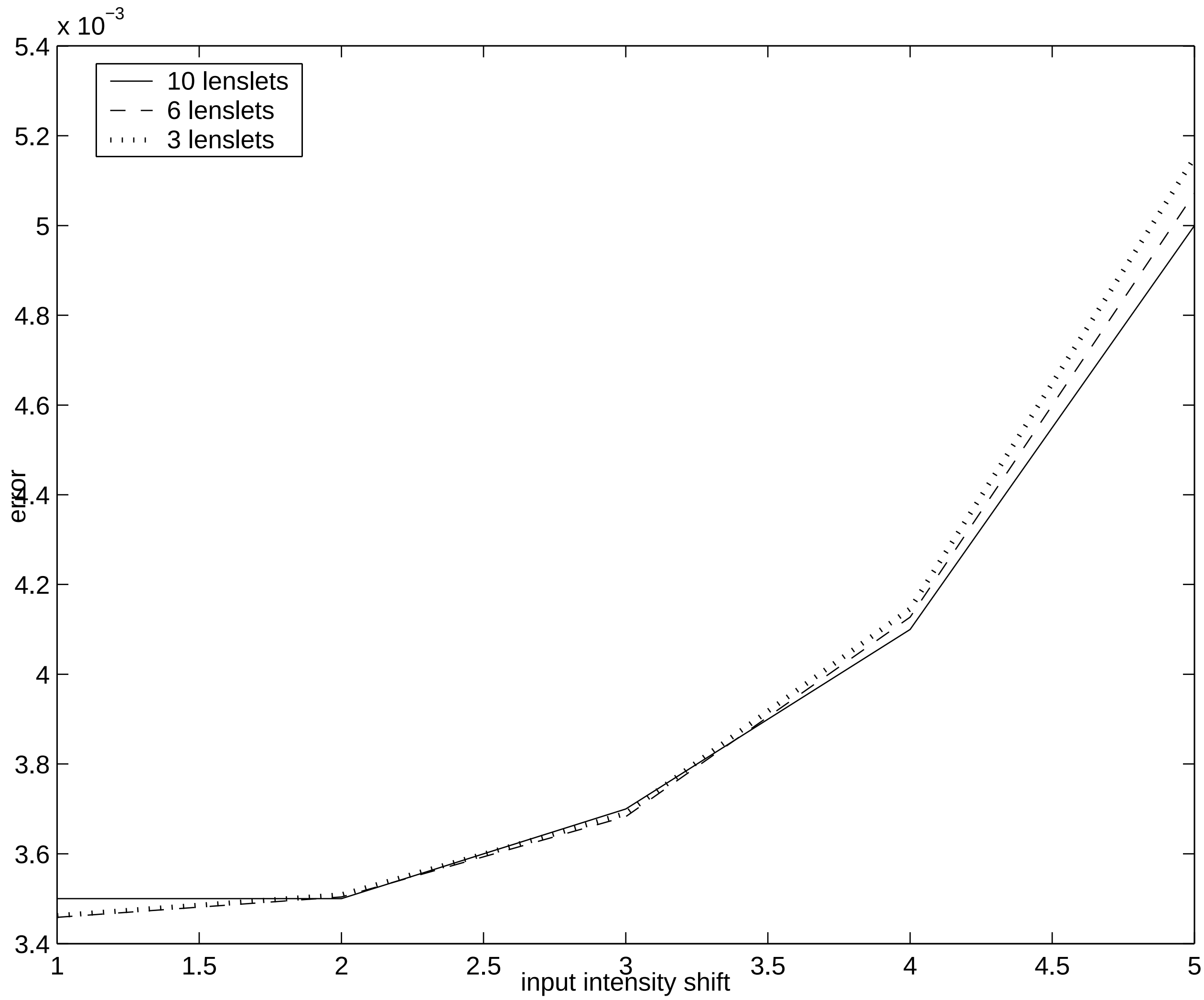

Figure 6: Utility function for 3, 6 and 10 lenslets

5 Discretization: uniform vs. nonuniform partition of the levels

We want to find the way to discretize a continuous function, which best approximates it. Only 4 level phase masks are considered here. We have tried some nonuniform partitions (different step size) and have calculated the error between the continuous and the discretized ones. Even though we did not test all possible step sizes, our results and experience let us conclude that the uniform partition is the one that produces the minimum error.

6 Optimization of discrete phase mask based on polynomial of degree 6

6.1 Algorithm

The phase mask considered has $20,000$ pixels and each pixel can take four values. A program that tests all the different possibilities would run for an horribly long time. Therefore, we decided to use polynomials of the degree 6 to generate continuous phase masks and then discretize them. The number of lenslets has been reduced to 6, thanks to the previous discussion. The goal of the algorithm is to find the coefficients of the polynomial that minimize the utility funtion. In order to achieve this, we first need to write the error as a function of the coefficients and then use an algorithm that finds minima of functions of several variables. The first step is delicate and is described in detail in the following paragraph.

We first recall an equation of 2

$$U(x) = \frac{e^{ik\left(z+\frac{x^2}{2z}\right)}}{i\lambda z} \iint \left[p(\xi)e^{i\frac{k}{2z}(\xi^2)}\right] e^{-i\frac{k}{z}(x\xi)} d\xi$$

which is the one dimensional analogue of (3). In the paper [4], the authors defined and used a modified phase mask $\tilde{\phi}$ to compute the output intensity

$$\tilde{\phi}(\xi) = \phi(\xi) + \frac{k}{2z}(\xi^2) \tag{7}$$

where ϕ is the *real* phase mask(the actual profile of the DOE). The last term is part of the Green function used in the integration and should not be discretized. The remedy to the problem is to use real phase masks. For example, if the modified phase mask for the jth lenslet,

$$\tilde{\phi}(\xi) = \frac{\pi}{\lambda z}\left(\frac{T}{l}\right)(\xi - c(j))^2$$

where $c(j)$ is the center of the jth lenslet, l the length of a lenslet and the other relevant parameters are shown in the table 1.

Parameter	Description	Value
L	Aperture size	40 mm
T	Target size	26 mm
z	Target to image distance	1.4 m
λ	Wavelength	0.248 μm
d	Pixel size (on image plane)	4 μm
h	Coherence length	100 μm

Table 1: Parameters in the numerical experiment.

After computations, the corresponding real phase mask for the jth lenslet was found to be

$$\phi(\xi) = \frac{\pi}{\lambda z}\left(\frac{T}{l} - 1\right)(\xi - c(j))^2 - \frac{2\pi}{\lambda z}(\xi - c(j))c(j)$$

The last term shifts the output to the the desired target range and shows that a term of degree one is necessary in our polynomials. Thus, they are of the form

$$\phi(\xi) = a_1(\xi - c(j))^2 + a_2(\xi - c(j))^4 + a_3(\xi - c(j))^6 + a_4(\xi - c(j))c(j) \quad (8)$$

for the jth lenslet. The utility function can be now seen as a function of the four variables a_1, a_2, a_3 and a_4.

We now want to optimize discrete phase mask. For this purpose, we used *fminsearch* which is implemented in Matlab in order to minimize the error between the target intensity and partial coherence output intensity. As an utility function, we used least square function (6).

We choose the coefficient of the phase mask from [4] as our initial condition. The procedure of the optimization is that with the initial input coefficients we build a polynomial of the form (8), and then discretize it into 4 levels. We then compute the output intensity and plug it into utility function to obtain an error. After that, the error is minimized by *fminsearch*. The *fminsearch* produces a new set of coefficients and the procedure is repeated.

6.2 Numerical results

The table 3 shows our results.

Number of iterations	Error	Energy percentage
Initial	0.005146	65.5393
10	0.004954	65.5258
20	0.004945	65.5324
30	0.004942	65.5202
40	0.0049420	65.5196
50	0.0049420	65.5196

Table 2: Error in Non-Symmetric discrete phase masks (4 levels).

Number of iterations	a_1	a_2	a_3	a_4
Initial	$2.6240 * 10^7$	0	0	$-1.8097 * 10^7$
10	$2.4928 * 10^7$	$2.19 * 10^{-4}$	$2.19 * 10^{-4}$	$-1.7531 * 10^7$
20	$2.4780 * 10^7$	$2.38 * 10^{-4}$	$-1.63 * 10^{-4}$	$-1.7672 * 10^7$
30	$2.4780 * 10^7$	$2.39 * 10^{-4}$	$-1.6 * 10^{-4}$	$-1.7673 * 10^7$
40	$2.4780 * 10^7$	$2.39 * 10^{-4}$	$-1.6 * 10^{-4}$	$-1.7673 * 10^7$
50	$2.4780 * 10^7$	$-2.39 * 10^{-4}$	$-1.6 * 10^{-4}$	$-1.7673 * 10^7$

Table 3: Coefficients of polynomial in Non-Symmetric discrete phase masks (4 levels).

We noticed that we lose energy as we minimize the error.

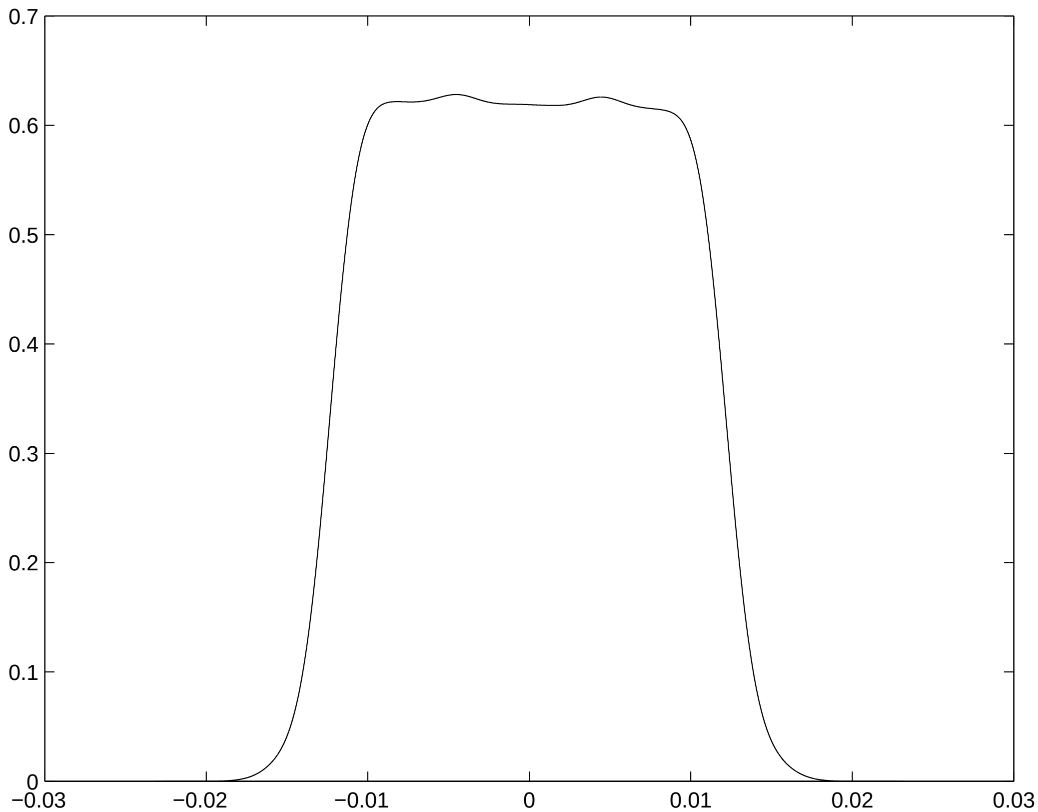

Figure 7: Intensity profile produced by optimized phase mask

The shape of the intensity using the phase mask with the coefficients obtained from 'fminsearch' after 50 iterations is shown in the figure (7).

7 Conclusion

We have investigated the design of a diffractive homogenizer for a laser beam with Gaussian intensity profile. An explanation about the choice of the most suitable utility (error) function has been given. We have showed numerically that 6 lenslet phase masks give us almost as good results as 10 lenslet phase masks. A new formulation of the optimal design problem has been defined, which allows us to automate the search for the best phase mask. We have obtained and compared results with continuous and discrete phase masks.

In the optimization problem we have restricted the class of admissible phase functions to the class of the polynomials of degree 6, for future work we can try to encompass a larger class of functions. More efficient optimization methods can be used to avoid local minima.

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