Sharp Trace Regularity for the Solutions of the Equations of Dynamic Elasticity

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1 Introduction

Sharp trace regularity results have proven themselves to be of critical importance in the study of controllability and stabilizability of various systems, as well as being of great interest in their own right. Particular cases include the wave equation (see [9]) and both linear and nonlinear plate equations (see [5], [8]). In our study of the three-dimensional system of linear elasticity, we focus on results for the wave equation. This is due to the fact that, under appropriate assumptions, the system of elasticity can be decoupled into three wave equations. Thus, we would hope that results, analagous to those available for the wave equation, would hold in the fully coupled case.

Our motivation in developing these trace estimates arises from the desire to eliminate the strong geometric constraints assumed to hold in most results on boundary stabilization for the system of elasticity (see e.g. [7, 6]). In the case of the wave equation, stabilization results are numerous. However, until the works of Lasiecka and Triggiani [9] and Bardos, Lebeau and Rauch [2], most results were based on the assumption that the geometry of the domain satisfied strict constraints. A critical step in removing these constraints in [9] was a pseudodifferential analysis which permits certain boundary traces of the solution to the wave equation to be expressed in terms of other traces modulo lower-order interior terms.

Estimates of solutions near the boundary have a long history, dating back to such works as that of Agmon, Douglis and Nirenberg [1]. In what follows, we will focus on the proof of trace regularity, while the question of stabilization without geometric constraints will be addressed in a subsequent paper.

To formulate the system of elasticity, we begin with the following definitions. Let $u = (u_i)$, $1 \leq i \leq n$ be the displacement vector. Since we are considering a homogeneous, isotropic body, the strain tensor (ϵ_{ij}) is given by

$$\epsilon_{ij} \equiv \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \qquad 1 \leq i, j \leq n.$$

The stress-strain relation can be expressed as

$$\sigma_{ij} = \lambda \sum_{k=1}^{n} \epsilon_{kk}\delta_{ij} + 2\mu\epsilon_{ij} = \lambda(\mathrm{div} u)\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right),$$

where $\lambda, \mu > 0$ are Lame's coefficients and are constant. (In more general cases, λ and μ are assumed to be functions of position.) In the above equation, δ_{ij} is the Kronecker delta, i.e., $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$.

With the above definitions in mind, we now consider the system of linear elasticity defined in the open domain $\Omega \subset \mathbb{R}^n$ with smooth boundary $\partial\Omega$.

$$u_{tt} - \nabla \cdot \sigma(u) = f \quad \text{in } \Omega \times (0, T) \tag{1.1.a}$$

$$\sum_i \sigma_{ij}\nu_j|_{\partial\Omega} = g_j \quad \text{on } \partial\Omega \times (0, T) \tag{1.1.b}$$

$$u(x, 0) = \phi(x); \quad u_t(x, 0) = \psi(x) \qquad \text{in } \Omega, \tag{1.1.c}$$

and ν_i represents the components of the unit outward normal vector to $\partial\Omega$.

For this system, when $g \equiv 0$, the following well-posedness results can be shown to hold using standard linear semigroup theory (see, e.g. [10]). Alternatively, this can also be established via elliptic theory (see [11]).

Theorem 1.1 *(Wellposedness on $H^1(\Omega) \times L^2(\Omega)$.)*
Let $(\phi(x), \psi(x)) \in (H^1(\Omega))^n \times (L^2(\Omega))^n$. Then there exists a unique solution (in the sense of distributions),

$$(u(x, t), u_t(x, t)) \in C([0, T]; (H^1(\Omega))^n \times (L^2(\Omega))^n)$$

satisfying system (1.1).

We wish to address the question of trace regularity for the solution of this system. Our goal is to prove the following result that is in some sense better than what can be achieved by using standard trace theory.

Theorem 1.2 *(Trace regularity.)*
Let u be the solution to (1.1) and let $0 < \alpha < T/2$. Then u satisfies the following inequality:

$$\begin{aligned} &\|\nabla u \cdot \tau\|^2_{(L^2(\alpha, T-\alpha; \partial\Omega))^n} \\ &\quad \leq C\{\|u_t\|^2_{(L^2(0,T;\partial\Omega))^n} + \|\sigma(u) \cdot \nu\|^2_{(L^2(0,T;\partial\Omega))^n} \\ &\qquad + \|f\|^2_{(H^{-1/2}(0,T;\Omega))^n} + \|u\|^2_{(L^2(0,T;H^{1/2+\epsilon}(\Omega)))^n}\}, \end{aligned} \tag{1.2}$$

where ν and τ are, respectively, the unit normal and the unit tangent to the boundary.

Notice that to get this sharper bound on the trace, we must sacrifice part of the time interval. However, for some purposes, this bound is more useful than the bounds we would achieve using trace theory, bounds which would be in a higher norm than the natural energy space for this system, which is $(H^1(\Omega))^n \times (L^2(\Omega))^n$.

2 Preliminaries

In order to facilitate the proof of Theorem 1.2, we intend to take advantage of the techniques of pseudodifferential calculus. For further information on the inner workings of pseudodifferential operators, we refer the reader to Taylor [11]. In the remainder of the paper, we work in a half-space. Via a partition of unity, a smoothing of the boundary procedure, and a change of variable, system (1.1) can be shown to be equivalent to a more general problem which may be stated as follows. (See the books of Hörmander, e.g., [3], for further information on proving this equivalence of systems.)

Let $x > 0$ be a real-valued scalar variable and $y = (y_1, ..., y_{n-1}) \in \mathbb{R}^{n-1}$ be a $(n-1)$-dimensional vector with real components. In symbol notation, $x \in \mathbb{R}^1_{x+}$, $y \in \mathbb{R}^{n-1}_y$, $t \in \mathbb{R}^1_t$. The domain $\Omega \equiv \mathbb{R}^1_{x+} \times \mathbb{R}^{n-1}_y$ is the half-space with boundary $\partial\Omega \equiv \Omega|_{x=0} = \mathbb{R}^{n-1}_y$, where $n = \dim\Omega \geq 2$. In Ω, following second order system is considered:

$$\mathcal{P}u = -T(x,y)D_t^2 u + P(x,y,D_x,D_y)u = f \tag{2.1.a}$$

$$\mathcal{B}(0,y,D_x,D_y)u|_{\partial\Omega} = g, \tag{2.1.b}$$

with the differentials, D_t, D_x, D_{y_j}, defined by

$$D_t \equiv \frac{1}{\sqrt{-1}}\frac{\partial}{\partial t}, \quad D_x \equiv \frac{1}{\sqrt{-1}}\frac{\partial}{\partial x}, \quad D_{y_j} \equiv \frac{1}{\sqrt{-1}}\frac{\partial}{\partial y_j}. \tag{2.1.c}$$

$P(x,y,D_x,D_y)$ is an elliptic operator of order 2 in the variables x and y with symbol $p(x,y,\xi,\eta)$. Without loss of generality, the entries in the matrix $p(x,y,\xi,\eta)$ may be assumed to have the following form (modulo lower order terms):

$$p_{ij}(x,y,\xi,\eta) \equiv \begin{cases} \xi^2 + D_{ij}(\xi,\eta_k,\eta_k^2) & i = j, \ k = 1,2 \\ D_{ij}(\xi,\eta_k,\eta_k^2) & i \neq j, \ k = 1,2. \end{cases} \tag{2.2}$$

$T(x,y)$ is a positive definite diagonal matrix with entries $t_{ii}(x,y) > 0 \ \forall (x,y) \in \Omega$ and for $i = 1, ..., n$.

Throughout the following discussion, $\tau = \sigma - i\gamma$, $\gamma > 0$, $\sigma \in \mathbb{R}$, will denote the Laplace transform variable corresponding to t, i.e., $D_t \longrightarrow \tau$, while $\xi \in \mathbb{R}$ and $\eta \in \mathbb{R}^{n-1}$ are the Fourier transform variables corresponding to x and y, respectively, i.e., $D_x \longrightarrow \xi$, $D_{y_j} \longrightarrow \eta_j$.

On the boundary, $\partial\Omega$, the symbol of the operator $\mathcal{B}$ has the form (modulo lower order terms)

$$\tilde{b}_{ij}(y,\xi,\eta) = \begin{cases} \xi + b_{ij}(\eta_k) & i = j, \ k = 1, ..., n-1, \\ b_{ij}(\eta_k) & i \neq j, \ k = 1, ..., n-1. \end{cases} \tag{2.3}$$

In the above definitions, $D_{ij}(\xi, \eta_k, \eta_k^2)$ may be second order and contain second order combinations, e.g. $\xi\eta_k$, but does not contain ξ^2 terms. Additionally, $D_{ij}(\xi, \eta_k, \eta_k^2)$ may depend on the spatial variables x and y, but is independent of t. In the boundary operator, $b_{ij}(\eta_k)$ is assumed to be no worse than linear in all variables η_k, $k = 1, ..., n-1$, and mayΠ depend on y.
Remark: Note that the symbol associated with P (resp. $\mathcal{B}$), quadratic terms in ξ (resp. linear terms in ξ) appear only on the diagonal. Due to the form of the symbol of $\mathcal{B}$, this boundary operator may be thought of as a generalized normal derivative. These operators, P and $\mathcal{B}$ arise from the original problem, (1.1).

3 Proof of Theorem 1.2

Localization in (x, y, σ, η) Space

To establish the estimate of Theorem 1.2, the operator $\mathcal{P}$ is studied by considering subregions of the space-time place in which the symbol of $\mathcal{P}$ is defined. This idea is formulated more precisely in the following seven steps. Our goal is to take advantage of the behaviour of the symbol within these subregions and use the results to compare the norms of the time and tangential derivatives.

With the definition of $\mathcal{P}$ in (2.1.a), and the symbol $p(x, y; \xi, \eta)$ in (2.2) corresponding to P, the matrix $wp_{ij}(x, y; \tau, \xi, \eta)$ with entries

$$\wp_{ij}(x, y, \tau, \xi, \eta) = \begin{cases} -t_{ii}(x, y)\tau^2 + p_{ij}(x, y, \xi, \eta) & i = j \\ p_{ij}(x, y, \xi, \eta) & i \neq j \end{cases} \tag{3.1}$$

is the symbol corresponding to $\mathcal{P}$. Since $\tau = \sigma - i\gamma$, this can be rewritten as

$$\wp_{ij}(x, y, \tau, \xi, \eta) = \begin{cases} -t_{ii}(x, y)(\sigma^2 - \gamma^2) + 2i\sigma\gamma t_{ii} + p_{ij}(x, y, \xi, \eta) & i = j \\ p_{ij}(x, y, \xi, \eta) & i \neq j \end{cases} \tag{3.2}$$

Without loss of generality, we may assume $\gamma = 0$, as extension to the case $\gamma > 0$ follows similarly to the results for the wave equation considered in [9].

Because of the symmetry of $\wp_{ij}$ in σ and η at the highest order, we may restrict our focus to the region $\mathbb{R}^{2n}_{+} = \{(x, y; \sigma, \eta) : (x, y) \in \Omega, \quad \sigma, \eta_j > 0, \ j = 1, ..., n\}$.

Step 1: Cutoff in Time.

Although we only consider the quarter space $\mathbb{R}^{2n}_{+}$, the following arguments hold in any quarter and can then be combined to cover the entire space. To avoid difficulties near the origin, we begin by defining a cutoff solution $u_c(t)$. Let

$\zeta(t) \in C_0^\infty(\mathbb{R})$ be a cutoff function defined such that $0 \leq \zeta(t) \leq 1 \quad \forall t \in \mathbb{R}$ and

$$\zeta(t) = \begin{cases} 1 & t \in (\alpha, T-\alpha) \\ 0 & t \in (-\infty, 0) \cup (T, \infty). \end{cases} \tag{3.3}$$

Define a cutoff solution $u_c(t) \equiv \zeta(t)u(t)$, where u is the solution to (2.1). Then $u_c(t)$ satisfies

$$\begin{aligned} \mathcal{P}u_c &= [\mathcal{P}, \zeta]u + \zeta f \\ \mathcal{B}u_c &= \zeta g \end{aligned} \tag{3.4}$$

Note that this step forces the restriction of the time interval on the left-hand side of the estimate in Theorem 1.2 to $(\alpha, T-\alpha)$.

Step 2: Localization in σ/η Plane.

Consider the regions in Figure 1. Despite the two-dimensional drawing, keep in mind that $\eta \in \mathbb{R}^{n-1}$ and thus, the full $\sigma \times \eta$ region lies in $\mathbb{R}^n$. Let $\lambda(\sigma, \eta) \in C^\infty$ be a homogeneous symbol of order zero in both σ and η defined such that $0 \leq \lambda(\sigma, \eta) \leq 1 \quad \forall \sigma, \eta$ and

$$\lambda(\sigma, \eta) = \begin{cases} 1 & \text{in } \mathcal{R}_1 \\ 0 & \text{in } \mathcal{R}_2 \end{cases} \tag{3.5}$$

with $\text{supp}\lambda \subset \mathcal{R}_1 \cup \mathcal{R}_{tr}$.

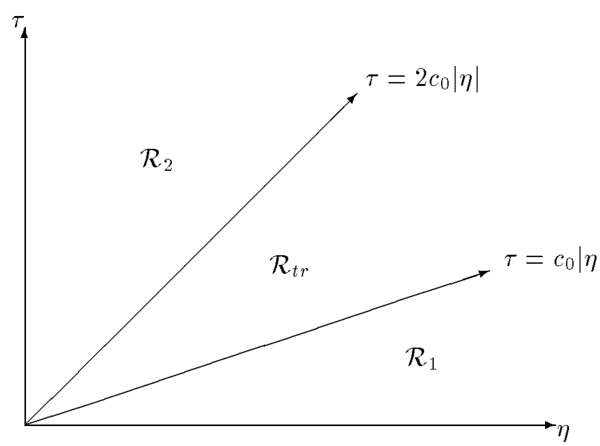

Figure 1: Elliptic and Non-elliptic Regions

Let $\Lambda \in OPS^0(\mathbb{R}^{n+1}_{tyx})$ denote the pseudodifferential operator corresponding to λ. Then, with reference to (3.4), Λu_c satisfies the following system.

$$\begin{aligned} \mathcal{P}(\Lambda u_c) &= [\mathcal{P}, \Lambda]u_c + \Lambda[\mathcal{P}, \zeta]u + \Lambda\zeta f \\ \mathcal{B}(\Lambda u_c) &= [\mathcal{B}, \Lambda]u_c + \Lambda\zeta g \end{aligned} \tag{3.6}$$

Step 3: Estimates in $\mathcal{E}_1 \equiv \mathcal{R}_1 \cup \mathcal{R}_{tr}$.

In the region $\mathcal{E}_1$, $\tau \leq 2c|\eta|$. Within the region $\mathcal{E}_1$, $\sigma \leq 2c_0|\eta|$. Since P is an elliptic operator of order two, there exists a constant $\beta > 0$ such that the symbol, $p(x, y; \xi, \eta)$, corresponding to P satisfies the inequality,

$$p(x, y, \xi, \eta)u \cdot u \geq \beta(|\xi|^2 + |\eta|^2)|u|^2.$$

Therefore, recalling the definition of $\wp(x, y; \sigma, \xi, \eta) = diag(-t_i\sigma^2) + p(x, y; \xi, \eta)$, the symbol corresponding to $\mathcal{P}$ satisfies

$$\begin{aligned} \wp(x, y; \sigma, \xi, \eta)u \cdot u \; &= -\textstyle\sum_1^n t_i \sigma^2 u_i^2 + p(x, y; \xi, \eta)u \cdot u \\ &\geq -(\min_{i=1,\ldots,n} t_i)\sigma^2|u|^2 + \beta(|\xi|^2 + |\eta|^2)|u|^2 \\ &\geq -4c_0(\min_{i=1,\ldots,n} t_i)|\eta|^2|u|^2 + \beta(|\xi|^2 + |\eta|^2)|u|^2 \qquad \text{in } \mathcal{E}_1. \end{aligned}$$

Hence, choosing c_0 to be sufficiently small and setting $\tilde{\beta} \equiv -4c_0 + \beta > 0$,

$$\wp(x, y, \xi, \eta)u \cdot u \geq \tilde{\beta}(|\xi|^2 + |\eta|^2)|u|^2.$$

Thus, $\mathcal{P}$ is elliptic of order two within the region $\mathcal{E}_1$ and system (3.6) satisfies elliptic estimates in all variables. In particular,

$$\begin{aligned} &||\Lambda u_c||_{(H^{3/2}(-\infty,\infty;\Omega))^n} + ||\Lambda u_c||_{(H^1(-\infty,\infty;\partial\Omega))^n} \\ &\quad \leq \{||\mathcal{B}(\Lambda u_c)||_{(L^2(-\infty,\infty;\partial\Omega))^n} + ||[\mathcal{P}, \Lambda]u_c||_{(H^{-1/2+\epsilon}(-\infty,\infty;\Omega))^n} \\ &\qquad + ||\Lambda[\mathcal{P}, \zeta]u||_{(H^{-1/2+\epsilon}(-\infty,\infty;\Omega))^n} + ||\Lambda\zeta f||_{(H^{-1/2}(-\infty,\infty;\Omega))^n}\}. \end{aligned} \tag{3.7}$$

Step 4: Estimates in $\mathcal{E}_2 \equiv \mathcal{R}_2 \cup \mathcal{R}_{tr}$.

Within the region $\mathcal{E}_2$, $\sigma \geq c_0|\eta|$. Recall the definition of the symbol of an operator, the tangential derivative may be estimated directly.

$$\begin{aligned} &||\nabla(1-\lambda)u_c \cdot \tau||^2_{(L^2(-\infty,\infty;\partial\Omega))^n} \\ &\quad \leq \int_{-\infty}^{\infty}\int_{\mathbb{R}^{n-1}} |\eta|^2(1-\lambda)\hat{u}_c(\sigma, x, \eta)|_{x=0}|^2 d\eta d\sigma \\ &\quad \leq \frac{1}{c_0^2}\int_{-\infty}^{\infty}\int_{\mathbb{R}^{n-1}} |\sigma(1-\lambda)\hat{u}_c(\sigma, x, \eta)|_{x=0}|^2 d\eta d\sigma \\ &\quad = C||((1-\zeta)u_c)_t||^2_{(L^2(-\infty,\infty;\partial\Omega))^n} \\ &\quad \leq C||(u_c)_t||^2_{(L^2(-\infty,\infty;\partial\Omega))^n}, \end{aligned} \tag{3.8}$$

where $\hat{u}_c$ is a Fourier-Laplace transform of u_c, i.e., the Fourier transform in the tangential direction and the Laplace transform in time.

Step 5: Estimates in the Entire Region

To summarize what we have achieved by estimating the norm of the solution in localized regions, we combine the results of the previous two steps. By combining

(3.7) and (3.8), we arrive at

$$
\begin{aligned}
&\|\nabla u_c \cdot \tau\|_{(L^2(-\infty,\,\infty;\,\partial\Omega))^n} \\
&\quad \le C\{\|\nabla(1-\zeta)u_c \cdot \tau\|_{(L^2(-\infty,\,\infty;\,\partial\Omega))^n} + \|\mathcal{B}(\Lambda u_c)\|_{(L^2(-\infty,\,\infty;\,\partial\Omega))^n} \\
&\qquad + \|[\mathcal{P},\Lambda]u_c\|_{(H^{-1/2+\epsilon}(-\infty,\,\infty;\,\Omega))^n} + \|\Lambda[\mathcal{P},\zeta]u\|_{(H^{-1/2+\epsilon}(-\infty,\,\infty;\,\Omega))^n} \\
&\qquad + \|\Lambda\zeta f\|_{(H^{-1/2}(-\infty,\,\infty;\,\Omega))^n}\} \\
&\quad \le C\{\|u_t\|_{(L^2(0,\,T;\,\partial\Omega))^n} + \|\mathcal{B}(\Lambda u_c)\|_{(L^2(-\infty,\,\infty;\,\partial\Omega))^n} \\
&\qquad + \|[\mathcal{P},\Lambda]u_c\|_{(H^{-1/2+\epsilon}(-\infty,\,\infty;\,\Omega))^n} + \|\Lambda[\mathcal{P},\zeta]u\|_{(H^{-1/2+\epsilon}(-\infty,\,\infty;\,\Omega))^n} \\
&\qquad + \|\Lambda\zeta f\|_{(H^{-1/2}(-\infty,\,\infty;\,\Omega))^n}\}.
\end{aligned} \tag{3.9}
$$

Step 6: Commutator Estimates.

In the previous steps, terms arising from the principal part of the operator in system (3.6) have been estimated. However, in defining cutoff solutions, a number of lower order tems have arisen due to the commutators of the operators. To achieve the finial estimate, these terms must be removed as well.

By using formulas for an asymptotic expansion of the symbols corresponding to the appropriate commutators (see [4], page 70) and recalling that $supp\lambda \subset \mathcal{E}_1$, we obtain

$$
\begin{cases} symb\{[\mathcal{P},\Lambda]\} = \mathcal{O}(|\xi| + |\eta|) & \text{in } \mathcal{E}_1 \\ supp\ symb\{[\mathcal{P},\Lambda]\} \subset \mathcal{E}_1 \end{cases} \tag{3.10}
$$

$$
\begin{cases} symb\{\Lambda[\mathcal{P},\zeta]\} = \mathcal{O}(|\xi| + |\eta|) & \text{in } \mathcal{E}_1 \\ supp\ symb\{\Lambda[\mathcal{P},\zeta]\} \subset \mathcal{E}_1 \end{cases} \tag{3.11}
$$

$$
\begin{cases} symb\{[\mathcal{B},\Lambda]\} = \mathcal{O}(1) & \text{in } \mathcal{E}_1 \\ supp\ symb\{[\mathcal{B},\Lambda]\} \subset \mathcal{E}_1 \end{cases} \tag{3.12}
$$

Hence, in particular for any $0 < \epsilon < \frac{1}{2}$,

$$
[\mathcal{P},\Lambda] \in \mathcal{L}((L^2(-\infty,\infty;H^{1/2+\epsilon}(\Omega))^n \to (H^{-1/2+\epsilon}(-\infty,\infty;\Omega))^n) \tag{3.13}
$$

$$
\Lambda[\mathcal{P},\zeta] \in \mathcal{L}((L^2(-\infty,\infty;H^{1/2+\epsilon}(\Omega))^n \to (H^{-1/2+\epsilon}(-\infty,\infty;\Omega))^n) \tag{3.14}
$$

$$
[\mathcal{B},\Lambda] \in \mathcal{L}((L^2(-\infty,\infty;\partial\Omega))^n \to (L^2(-\infty,\infty;\partial\Omega))^n) \tag{3.15}
$$

From (3.13) and (3.14),

$$
\|[\mathcal{P},\Lambda]u_c\|_{(H^{-1/2+\epsilon}(-\infty,\,\infty;\,\Omega))^n} \le C\|u\|_{(L^2(0,\,T;\,H^{1/2+\epsilon}(\Omega)))^n}, \tag{3.16}
$$

$$
\|\Lambda[\mathcal{P},\zeta]u_c\|_{(H^{-1/2+\epsilon}(-\infty,\,\infty;\,\Omega))^n} \le C\|u\|_{(L^2(0,\,T;\,H^{1/2+\epsilon}(\Omega)))^n}, \tag{3.17}
$$

hence, the last three terms in (3.9) are bounded by

$$
C\{\|u\|_{(L^2(0,\,T;\,H^{1/2+\epsilon}(\Omega)))^n} + \|f\|_{(H^{-1/2+\epsilon}(0,\,T;\,\Omega))^n}\}. \tag{3.18}
$$

The second term on the right-hand side of (3.9) is estimated by

$$\begin{aligned}
&\|\mathcal{B}(\Lambda u_c)\|_{(L^2(-\infty,\,\infty;\,\partial\Omega))^n} \\
&\quad\le \|[\mathcal{B},\Lambda]u_c\|_{(L^2(-\infty,\,\infty;\,\partial\Omega))^n} + \|\Lambda\zeta g\|_{(L^2(-\infty,\,\infty;\,\partial\Omega))^n} \\
&\quad\le C\{\|u_c\|_{(L^2(-\infty,\,\infty;\,\partial\Omega))^n} + \|g\|_{(L^2(0,\,T;\,\partial\Omega))^n}\} \\
&\quad\le C\{\|u\|_{(L^2(0,\,T;\,\partial\Omega))^n} + \|g\|_{(L^2(0,\,T;\,\partial\Omega))^n}\}.
\end{aligned} \tag{3.19}$$

Step 7: Final estimate.

Combining (3.18) and (3.19) with (3.9), we arrive at our desired estimate,

$$\begin{aligned}
\|\nabla u\cdot\tau\|_{(L^2(\alpha,\,T\,-\,\alpha;\,\partial\Omega))^n} &\le \|\nabla u_c\cdot\tau\|_{(L^2(-\infty,\,\infty;\,\partial\Omega))^n} \\
&\le C\{\|u_t\|_{(L^2(0,\,T;\,\partial\Omega))^n} + \|u\|_{(L^2(0,\,T;\,H^{1/2+\epsilon}(\Omega)))^n} \\
&\qquad + \|f\|_{(H^{-1/2}(0,\,T;\,\Omega))^n} + \|g\|_{(L^2(0,\,T;\,\partial\Omega))^n}\}.
\end{aligned} \tag{3.20}$$

Application of this result to system (1.1) gives us the estimate of Theorem 1.2 once we note that $g = \sigma(u)\cdot\nu$.

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