Solvability of a nonlinear problem of Kirchhoff Shell

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Abstract

Conditions of existence and uniqueness of a weak solution are proven for a boundary problem for the Mushtari-Donnell-Vlasov system of equations

1 Statement of the problem

Let us consider the system of equations, corresponding to deformation of a sloping shell [1]

$$\frac{\partial N_1}{\partial x_1}+\frac{\partial T}{\partial x_2}+p_1=0, \quad \frac{\partial T}{\partial x_1}+\frac{\partial N_2}{\partial x_2}+p_2=0,$$

$$\frac{\partial^2 M_1}{\partial x_1^2}+2\frac{\partial^2 H}{\partial x_1 \partial x_2}+\frac{\partial^2 M_2}{\partial x_2^2}+k_1 N_1+k_2 N_2 \tag{1}$$

$$+\frac{\partial}{\partial x_1}\left(N_1\frac{\partial w}{\partial x_1}+T\frac{\partial w}{\partial x_2}\right)+\frac{\partial}{\partial x_2}\left(T\frac{\partial w}{\partial x_1}+N_2\frac{\partial w}{\partial x_2}\right)+q=0,$$

where

$$N_i=\frac{Eh}{1-\nu^2}\left\{\frac{\partial u_i}{\partial x_i}-k_i w+\frac{1}{2}\left(\frac{\partial w}{\partial x_i}\right)^2+\nu\left[\frac{\partial u_j}{\partial x_j}-k_j w+\frac{1}{2}\left(\frac{\partial w}{\partial x_j}\right)^2\right]\right\},$$

$$T=\frac{Eh}{2(1+\nu)}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}+\frac{\partial w}{\partial x_1}\frac{\partial w}{\partial x_2}\right), M_i=-\frac{Eh^3}{12(1-\nu^2)}\left(\frac{\partial^2 w}{\partial x_i^2}+\nu\frac{\partial^2 w}{\partial x_j^2}\right),$$

$$H=-\frac{Eh^3}{12(1+\nu)}\frac{\partial^2 w}{\partial x_1 \partial x_2}, \quad i,j=1,2, \quad i\neq j,$$

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u_1, u_2, w- displacement vector components, are unknown functions of the arguments x_1, x_2, $x = (x_1, x_2) \in \Omega$, Ω is a domain taken by shell in a plane view, $p_1 = p_1(x)$, $p_2 = p_2(x)$, $q = q(x)$- components of external loading on a shell, $k_1 = k_1(x)$, $k_2 = k_2(x)$ - curvatures of a shell, h-thickness, ν-Poisson's ratio, $0 < \nu < \frac{1}{2}$, E-modulus of elasticity.

Suppose that the shell is fixed on all boundaries

$$u_{1|\partial\Omega} = u_{2|\partial\Omega} = 0, \quad w|_{\partial\Omega} = 0, \quad \frac{\partial w}{\partial n}|_{\partial\Omega} = 0. \tag{2}$$

Here $\partial\Omega$ - boundary of Ω domain, n-external normal to $\partial\Omega$.

We say that the system of functions (u, w), where $u = (u_1, u_2) \in (\overset{\circ}{W}{}_2^1 (\Omega))^2$, $w \in \overset{\circ}{W}{}_2^2 (\Omega)$, is a weak solution of problem (1), (2) if it satisfies the equality

$$\begin{aligned}
&\int_\Omega \left[N_1 \left(\frac{\partial \eta_1}{\partial x_1} - k_1 \xi + \frac{\partial w}{\partial x_1} \frac{\partial \xi}{\partial x_1} \right) + N_2 \left(\frac{\partial \eta_2}{\partial x_2} - k_2 \xi + \frac{\partial w}{\partial x_2} \frac{\partial \xi}{\partial x_2} \right) \right. \\
&\left. + T \left(\frac{\partial \eta_1}{\partial x_2} + \frac{\partial \eta_2}{\partial x_1} + \frac{\partial w}{\partial x_1} \frac{\partial \xi}{\partial x_2} + \frac{\partial w}{\partial x_2} \frac{\partial \xi}{\partial x_1} \right) - M_1 \frac{\partial^2 \xi}{\partial x_1^2} - 2H \frac{\partial^2 \xi}{\partial x_1 \partial x_2} - M_2 \frac{\partial^2 \xi}{\partial x_2^2} \right] dx \\
&= \int_\Omega (p_1 \eta_1 + p_2 \eta_2 + q\xi) dx \qquad \forall \eta_1, \eta_2 \in \overset{\circ}{W}{}_2^1 (\Omega), \quad \xi \in \overset{\circ}{W}{}_2^2 (\Omega).
\end{aligned} \tag{3}$$

When $p_1 = p_2 = 0$ system (1) was studied by many authors (see bibliography in [2]). One of the first works are [3] and [4]. Note that the well-known von Karman system is the result of (1). For p_1 and p_2 different from zero the question of existence of solutions of system (1) is considered for the plate ($k_1 = k_2 = 0$) in [5] and for the shell in [6] and [7].

2 Statement of Theorem

Let us define and denote norms in spaces $\overset{\circ}{W}{}_2^m (\Omega)$, $\overset{\circ}{W}{}_4^1 (\Omega)$, $m = 1, 2$, in the following way

$$\|v\|_m = \left[\int_\Omega \sum_{|\ell|=m} (\mathcal{D}^\ell v)^2 dx \right]^{1/2}, \quad \|v\|_{1,4} = \left[\int_\Omega \sum_{|\ell|=1} (\mathcal{D}^\ell v)^4 dx \right]^{1/4},$$
$$\ell = (\ell_1, \ell_2).$$

By $\|\cdot\|, \|\cdot\|_c$ and $\|\cdot\|_{-m}$, we will mean norms in spaces $L_2(\Omega)$, $C(\Omega)$ and $W_2^{-m}(\Omega)$, $m = 1, 2$. For vector $v = (v_1, v_2)$ and integer m we denote $\|v\|_m = (\|v_1\|_m^2 + \|v_2\|_m^2)^{1/2}$.

We will need the inequalities [8]

$$\|v\|_{1,4} \le c_1 \|v\|_2, \tag{4.1}$$

$$\|v\| \le c_2 \|v\|_2, \tag{4.2}$$

which are true for any function v in the space $\overset{\circ}{W}{}_2^2(\Omega)$. In (4), c_1, c_2 are positive constants that do not depend on the function v.

Theorem. *Let the boundary of Ω domain be such that Sobolev's imbedding theorems are true and*

$$p = (p_1, p_2) \in (W_2^{-1}(\Omega))^2, \quad q \in W_2^{-2}(\Omega), \qquad k_1, k_2 \in C(\Omega).$$

If for $t = 0$

$$\|p\|_{-1} \le \frac{Eh}{c_1^2[32(3-\nu)(1-\nu)]^{1/2}} \left[\frac{h^2}{3(1+\nu)} - \frac{c_2^2 æ^2}{1 - t\alpha(1+\nu)} - \delta\right], \tag{5}$$

where

$$0 < \alpha < (1+\nu)^{-1}, \quad \delta > 0, \quad æ = \max_{i=1,2}(\|k_i\|_c),$$

then problem (1), (2) has at least one weak solution.

If for $t = 1$ inequality (5) is satisfied as well as the following inequality

$$\left[1 + \frac{1}{\alpha(1-\nu)}\right]\left\{\frac{1}{\delta}\left[4c_2 æ \left(\frac{1+\nu}{1-\nu}\right)^{1/2} \|p\|_{-1} \right.\right.$$
$$\left.\left. + \frac{1}{\sqrt{2}}\left(\frac{1-\nu}{1+\nu}\right)^{1/2} \|q\|_{-2}\right]^2 + 2\|p\|_{-1}^2\right\} < \left[\frac{Eh^3}{12c_1^2(1-\nu^2)}\right]^2, \tag{6}$$

then there exists a unique weak solution.

Remark. *For the case $t = 1$ the question of the best parameters α and δ are not discussed here.*

3 Proof of Theorem

3.1 Solvability

Put $\eta_1 = u_1, \eta_2 = u_2$, $\xi = 0$ in (3). Taking into account the boundary conditions (2), we get

$$I \equiv \int_\Omega \left[\left(\frac{\partial u_1}{\partial x_1}\right)^2 + 2\nu \frac{\partial u_1}{\partial x_1}\frac{\partial u_2}{\partial x_2} + \left(\frac{\partial u_2}{\partial x_2}\right)^2 + \frac{1-\nu}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)^2 \right] dx$$
$$= -\frac{1}{2}\int_\Omega \left\{ \left(\frac{\partial w}{\partial x_1}\right)^2 \left(\frac{\partial u_1}{\partial x_1} + \nu\frac{\partial u_2}{\partial x_2}\right) + \left(\frac{\partial w}{\partial x_2}\right)^2 \left(\frac{\partial u_2}{\partial x_2} + \nu\frac{\partial u_1}{\partial x_1}\right) + (1-\nu)\frac{\partial w}{\partial x_1}\frac{\partial w}{\partial x_2}\left(\frac{\partial u_1}{\partial x_2}\right.\right.$$
$$\left.\left. + \frac{\partial u_2}{\partial x_1}\right) - 2w\left[(k_1+\nu k_2)\frac{\partial u_1}{\partial x_1} + (k_2+\nu k_1)\frac{\partial u_2}{\partial x_2}\right]\right\} dx + \frac{1-\nu^2}{Eh}\int_\Omega (p_1u_1 + p_2u_2)dx. \tag{7}$$

¿From (2)

$$\int_\Omega \frac{\partial u_1}{\partial x_2}\frac{\partial u_2}{\partial x_1}dx = \int_\Omega \frac{\partial u_1}{\partial x_1}\frac{\partial u_2}{\partial x_2}dx.$$

Therefore

$$I \geq \frac{1-\nu}{2}\|u\|_1^2.$$

Thus

$$\|u\|_1 \leq \left(\frac{2}{1-\nu}I\right)^{1/2}. \tag{8}$$

¿From (7), it follows

$$I \leq \frac{1}{2}\left\{\int_\Omega \left[\left(\frac{\partial u_1}{\partial x_1} + \nu\frac{\partial u_2}{\partial x_2}\right)^2 + \left(\frac{\partial u_2}{\partial x_2} + \nu\frac{\partial u_1}{\partial x_1}\right)^2 + (1-\nu)\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right)^2\right] dx\right\}^{1/2}$$
$$\times\left\{\left(1 + \frac{1-\nu}{2}\right)^{1/2}\|w\|_{1,4}^2\right\} + \sqrt{2}æ(1+\nu)\|u\|_1\|w\| + \frac{1-\nu^2}{Eh}\|p\|_{-1}\|u\|_1.$$

Using (4) and the inequality $1+\nu^2 < 2$, we find

$$I \leq \frac{c_1^2}{2}(3-\nu)^{1/2}I^{1/2}\|w\|_2^2 + \left[\sqrt{2}c_2æ(1+\nu)\|w\|_2 + \frac{1-\nu^2}{Eh}\|p\|_{-1}\right]\|u\|_1.$$

This together with (8) gives an estimate

$$\|u\|_1 \leq \frac{c_1^2}{\sqrt{2}}\left(\frac{3-\nu}{1-\nu}\right)^{1/2}\|w\|_2^2 + \frac{2\sqrt{2}c_2æ(1+\nu)}{1-\nu}\|w\|_2 + \frac{2(1+\nu)}{Eh}\|p\|_{-1}, \tag{9}$$

which will be needed later.

We now evaluate $\|w\|_2$. Combining (3) for $\eta_1 = 2u_1$, $\eta_2 = 2u_2$, $\xi = w$ and (2) yields

$$\frac{Eh^3}{12(1-\nu^2)}\|w\|_2^2 + \frac{2Eh}{1-\nu^2}\int_\Omega \left[\epsilon_1^2 + 2\nu\epsilon_1\epsilon_2 + \epsilon_2^2 + \frac{1}{2}k_1(\epsilon_1 + \nu\epsilon_2)w \right. \\ \left. + \frac{1}{2}k_2(\epsilon_2 + \nu\epsilon_1)w + \frac{1-\nu}{2}\gamma^2\right] dx - \int_\Omega (2p_1u_1 + 2p_2u_2 + qw)dx = 0, \tag{10}$$

where

$$\epsilon_i = \frac{\partial u_i}{\partial x_i} - k_i w + \frac{1}{2}\left(\frac{\partial w}{\partial x_i}\right)^2, \quad \gamma = \frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} + \frac{\partial w}{\partial x_1}\frac{\partial w}{\partial x_2}, \quad i = 1, 2.$$

¿From (10) we obtain that

$$\frac{Eh^3}{12(1-\nu^2)}\|w\|_2^2 + \frac{2Eh}{1-\nu^2}\int_\Omega \left\{[(1-\sigma(1+\nu^2))\epsilon_1^2 + 2\nu(1-2\sigma)\epsilon_1\epsilon_2 + (1-\sigma(1+\nu^2))\epsilon_2^2] \right. \\ \left. - \frac{1}{16\sigma}(k_1^2 + k_2^2)w^2 + \frac{1-\nu}{2}\gamma^2\right\} dx \le 2\|p\|_{-1}\|u\|_1 + \|q\|_{-2}\|w\|_2 \\ \forall \sigma > 0. \tag{11}$$

Let us choose the maximum σ for which the expression in square brackets in (11) is nonnegative, i.e. for which the following is satisfied:

$$1 - \sigma(1+\nu^2) \ge \nu|1-2\sigma|.$$

It is clear that $\sigma = (1+\nu)^{-1}$. Putting this value of σ in (11) and using (4.2), we get

$$\frac{Eh}{4(1-\nu^2)}\left[\frac{h^2}{3} - c_2^2æ^2(1+\nu)\right]\|w\|_2^2 + \frac{2Eh}{1-\nu^2}\int_\Omega \left[\frac{\nu(1-\nu)}{1+\nu}(\epsilon_1+\epsilon_2)^2\right.$$

$$\left. + \frac{1-\nu}{2}\gamma^2\right] dx \le 2\|p\|_{-1}\|u\|_1 + \|q\|_{-2}\|w\|_2.$$

This together with (9) gives

$$\left\{\frac{Eh}{4(1-\nu^2)}\left[\frac{h^2}{3} - c_2^2æ^2(1+\nu)\right] - \sqrt{2}c_1^2\left(\frac{3-\nu}{1-\nu}\right)^{\frac{1}{2}}\|p\|_{-1}\right\}\|w\|_2^2 \\ \le \left[\frac{4\sqrt{2}c_2æ(1+\nu)}{1-\nu}\|p\|_{-1} + \|q\|_{-2}\right]\|w\|_2 + \frac{4(1+\nu)}{Eh}\|p\|_{-1}^2. \tag{12}$$

Taking into account the validity of inequality (5) when $t=0$ the expression in fig brackets in (12) is not less than $\frac{1}{4}Eh\delta(1-\nu)^{-1}$. Therefore

$$\|w\|_2 \le c, \tag{13}$$

where positive constant c expressed by the norms $\|p\|_{-1}, \|q\|_{-2}$ and parameters æ, ν, δ, h, E, c_2.

Based on estimates (9), (13) and by using the Faedo-Galerkin method [4], [9], we get the solvability of problem (1), (2).

3.2 Existence of unique solution

Assume condition (5) holds when $t=1$. Then (5) will hold for $t=0$ as well. Thus, problem (1), (2) has a solution when $t=1$. We shall prove that if in addition (5), (6) holds, then the solution is unique.

Suppose the existence of two solutions $(u^{(\ell)}, w^{(\ell)})$, $\ell = 1,2$, $u^{(\ell)} = (u_1^{(\ell)}, u_2^{(\ell)})$. Denote ϵ_i, γ, $i=1,2$, corresponding to the solution by $\epsilon_i^{(\ell)}$, $\gamma^{(\ell)}$.

Let us write (3) for both solutions when $\eta_i = u_i^{(1)} - u_i^{(2)}$, $\xi = w^{(1)} - w^{(2)}$, $i=1,2$, and subtract the resulting equalities from each other. A simple calculation yields

$$\begin{aligned} &\frac{Eh^3}{12(1-\nu^2)}\|w^{(1)}-w^{(2)}\|_2^2 + \frac{Eh}{1-\nu^2}\int_\Omega \Big[(\epsilon_1^{(1)}-\epsilon_1^{(2)})^2 + 2\nu(\epsilon_1^{(1)}-\epsilon_1^{(2)})(\epsilon_2^{(1)}-\epsilon_2^{(2)}) \\ &+(\epsilon_2^{(1)}-\epsilon_2^{(2)})^2 + \frac{1-\nu}{2}(\gamma^{(1)}-\gamma^{(2)})^2\Big]\,dx + J = 0, \end{aligned} \tag{14}$$

where

$$\begin{aligned} J = \frac{Eh}{2(1-\nu^2)}\int_\Omega \Bigg\{ &\left[(\epsilon_1^{(1)}+\nu\epsilon_2^{(1)}) + (\epsilon_1^{(2)}+\nu\epsilon_2^{(2)})\right]\left[\frac{\partial(w^{(1)}-w^{(2)})}{\partial x_1}\right]^2 + \Big[(\epsilon_2^{(1)}+\nu\epsilon_1^{(1)}) \\ &+(\epsilon_2^{(2)}+\nu\epsilon_1^{(2)})\Big]\left[\frac{\partial(w^{(1)}-w^{(2)})}{\partial x_2}\right]^2 + (1-\nu)(\gamma^{(1)}+\gamma^{(2)})\frac{\partial(w^{(1)}-w^{(2)})}{\partial x_1}\frac{\partial(w^{(1)}-w^{(2)})}{\partial x_2}\Bigg\}\,dx. \end{aligned}$$

For arbitrary $\alpha > 0$ we infer

$$\begin{aligned} |J| \le \frac{Eh}{2(1-\nu^2)}&\left\{\int_\Omega [\alpha(1-\nu)((\epsilon_1^{(1)}+\nu\epsilon_2^{(1)})^2 + (\epsilon_2^{(1)}+\nu\epsilon_1^{(1)})^2 + (\epsilon_1^{(2)}+\nu\epsilon_2^{(2)})^2\right. \\ &\left.+(\epsilon_2^{(2)}+\nu\epsilon_1^{(2)})^2) + \frac{(1-\nu)^2}{2}((\gamma^{(1)})^2+(\gamma^{(2)})^2]dx\right\}^{1/2} \\ &\times\left\{\left[2+\frac{2}{\alpha(1-\nu)}\right]\int_\Omega\left[\left(\frac{\partial(w^{(1)}-w^{(2)})}{\partial x_1}\right)^4 + \left(\frac{\partial(w^{(1)}-w^{(2)})}{\partial x_2}\right)^4\right]dx\right\}^{1/2}. \end{aligned}$$

By the use of (4.1), we obtain

$$|J| \leq \frac{Eh}{1-\nu^2} c_1^2 \left\{ \left[1 + \frac{1}{\alpha(1-\nu)} \right] (s^{(1)} + s^{(2)}) \right\}^{1/2} \|w^{(1)} - w^{(2)}\|_2^2, \qquad (15)$$

where

$$s^{(\ell)} = \int_\Omega \left\{ \frac{\alpha(1-\nu)}{2} [(1+\nu^2)(\epsilon_1^{(\ell)})^2 + 4\nu \epsilon_1^{(\ell)} \epsilon_2^{(\ell)} + (1+\nu^2)(\epsilon_2^{(\ell)})^2] + \right.$$
$$\left. + \left(\frac{1-\nu}{2} \right)^2 (\gamma^{(\ell)})^2 \right\} dx, \qquad \ell = 1, 2.$$

Let us determine estimate for $s^{(\ell)}$. From (10) and (4.2), it follows that

$$\begin{aligned} &\frac{Eh}{4(1-\nu^2)} \left(\frac{h^2}{3} - \frac{c_2^2 æ^2}{\sigma} \right) \|w^{(\ell)}\|_2^2 + \frac{2Eh}{1-\nu^2} \left\{ \frac{2}{1-\nu} s^{(\ell)} \right. \\ &+ \int_\Omega [(1-(\alpha+\sigma)(1+\nu^2))(\epsilon_1^{(\ell)})^2 + 2\nu(1-2\alpha-2\sigma)\epsilon_1^{(\ell)}\epsilon_2^{(\ell)} \\ &\left. +(1-(\alpha+\sigma)(1+\nu^2))(\epsilon_2^{(\ell)})^2] dx \right\} \leq 2\|p\|_{-1}\|u^{(\ell)}\|_1 + \|q\|_{-2}\|w^{\ell)}\|_2 \\ &\forall \sigma > 0. \end{aligned} \qquad (16)$$

In (16) choose the maximum value of σ for which under integral function is nonnegative, i.e. for which

$$1 - (\alpha+\sigma)(1+\nu^2) \geq \nu|1 - 2\alpha - 2\sigma|.$$

We obtain that such $\sigma = (1+\nu)^{-1} - \alpha$, $0 < \alpha < (1+\nu)^{-1}$. Substituting this expression in (16) and using (9), we get

$$\begin{aligned} &\left\{ \frac{Eh}{4(1-\nu)} \left[\frac{h^2}{3(1+\nu)} - \frac{c_2^2 æ^2}{1-\alpha(1+\nu)} - (\delta_1+\delta_2) \right] - \sqrt{2} c_1^2 \left(\frac{3-\nu}{1-\nu} \right)^{1/2} \|p\|_{-1} \right\} \|w^{(\ell)}\|_2^2 \\ &+ \frac{4Eh}{(1-\nu^2)(1-\nu)} s^{(\ell)} \leq \left[\frac{32 c_2^2 æ^2 (1+\nu)^2}{\delta_1 Eh(1-\nu)} + \frac{4(1+\nu)}{Eh} \right] \|p\|_{-1}^2 + \frac{1-\nu}{\delta_2 Eh} \|q\|_{-2}^2 \\ &\forall \delta_1, \delta_2 > 0. \end{aligned} \qquad (17)$$

We claim that $\delta_1 + \delta_2 = \delta$. Then in power of condition (5) when $t = 1$ in (17) coefficient at $\|w^{(\ell)}\|_2^2$ will be nonnegative and therefore

$$s^{(\ell)} \leq \frac{(1-\nu^2)^2}{E^2 h^2} \left\{ \left[\frac{8 c_2^2 æ^2 (1+\nu)}{\delta_1 (1-\nu)} + 1 \right] \|p\|_{-1}^2 + \frac{(1-\nu)}{4(\delta-\delta_1)(1+\nu)} \|q\|_{-2}^2 \right\}. \qquad (18)$$

We optimize the estimate for $s^{(\ell)}$. By choosing $\delta_1 \in (0,\delta)$, let us get the minimum value of right hand side of (18). Consider that the minimum of the function $f(z) = az^{-1} + b(c-z)^{-1}$, $a, b > 0$, on interval $(0,c)$ is equal to $c^{-1}(\sqrt{a}+\sqrt{b})^2$. In result from (18) yields

$$s^{(\ell)} \le \frac{(1-\nu^2)^2}{E^2h^2}\left\{\frac{1}{\delta}[2\sqrt{2}c_2æ\left(\frac{1+\nu}{1-\nu}\right)^{1/2}\|p\|_{-1} + \frac{1}{2}\left(\frac{1-\nu}{1+\nu}\right)^{1/2}\|q\|_{-2}]^2 + \|p\|_{-1}^2\right\},$$
$\ell = 1, 2.$

Using this inequality in (15), we find

$$\begin{aligned} |J| \le c_1^2\left[2+\frac{2}{\alpha(1-\nu)}\right]^{1/2}&\left\{\frac{1}{\delta}\left[2\sqrt{2}c_2æ\left(\frac{1+\nu}{1-\nu}\right)^{1/2}\|p\|_{-1}\right.\right. \\ &\left.\left.+\frac{1}{2}\left(\frac{1-\nu}{1+\nu}\right)^{1/2}\|q\|_{-2}\right]^2 + \|p\|_{-1}^2\right\}^{1/2}\|w^{(1)}-w^{(2)}\|_2^2. \end{aligned} \tag{19}$$

Combining (14) and (19), gives us

$$\begin{aligned} &\left\{\frac{Eh^3}{12(1-\nu^2)} - c_1^2\left(2+\frac{2}{\alpha(1-\nu)}\right)^{1/2}\left[\frac{1}{\delta}\left(2\sqrt{2}c_2æ\left(\frac{1+\nu}{1-\nu}\right)^{1/2}\|p\|_{-1} + \frac{1}{2}\left(\frac{1-\nu}{1+\nu}\right)^{1/2}\|q\|_{-2}\right)^2\right.\right. \\ &\left.\left.+\|p\|_{-1}^2\right]^{1/2}\right\}\|w^{(1)}-w^{(2)}\|_2^2 + \frac{Eh}{1-\nu^2}\int_\Omega\left[(\epsilon_1^{(1)}-\epsilon_1^{(2)})^2 + 2\nu(\epsilon_1^{(1)}-\epsilon_1^{(2)})(\epsilon_2^{(1)}-\epsilon_2^{(2)})\right. \\ &\left.+(\epsilon_2^{(1)}-\epsilon_2^{(2)})^2 + \frac{1-\nu}{2}(\gamma^{(1)}-\gamma^{(2)})^2\right]dx \le 0. \end{aligned} \tag{20}$$

¿From the power of condition (6) the coefficient at $\|w^{(1)}-w^{(2)}\|_2^2$ in (20) has positive value. Therefore the following can be written

$$\|w^{(1)}-w^{(2)}\|_2 = 0, \quad \|\epsilon_i^{(1)}-\epsilon_i^{(2)}\| = 0,$$
$$\|\gamma^{(1)}-\gamma^{(2)}\| = 0, \qquad i = 1,2.$$

Now by using easy judgement we come to the relationships $u^{(1)} = u^{(2)}$, $w^{(1)} = w^{(2)}$.

Theorem is proven.

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