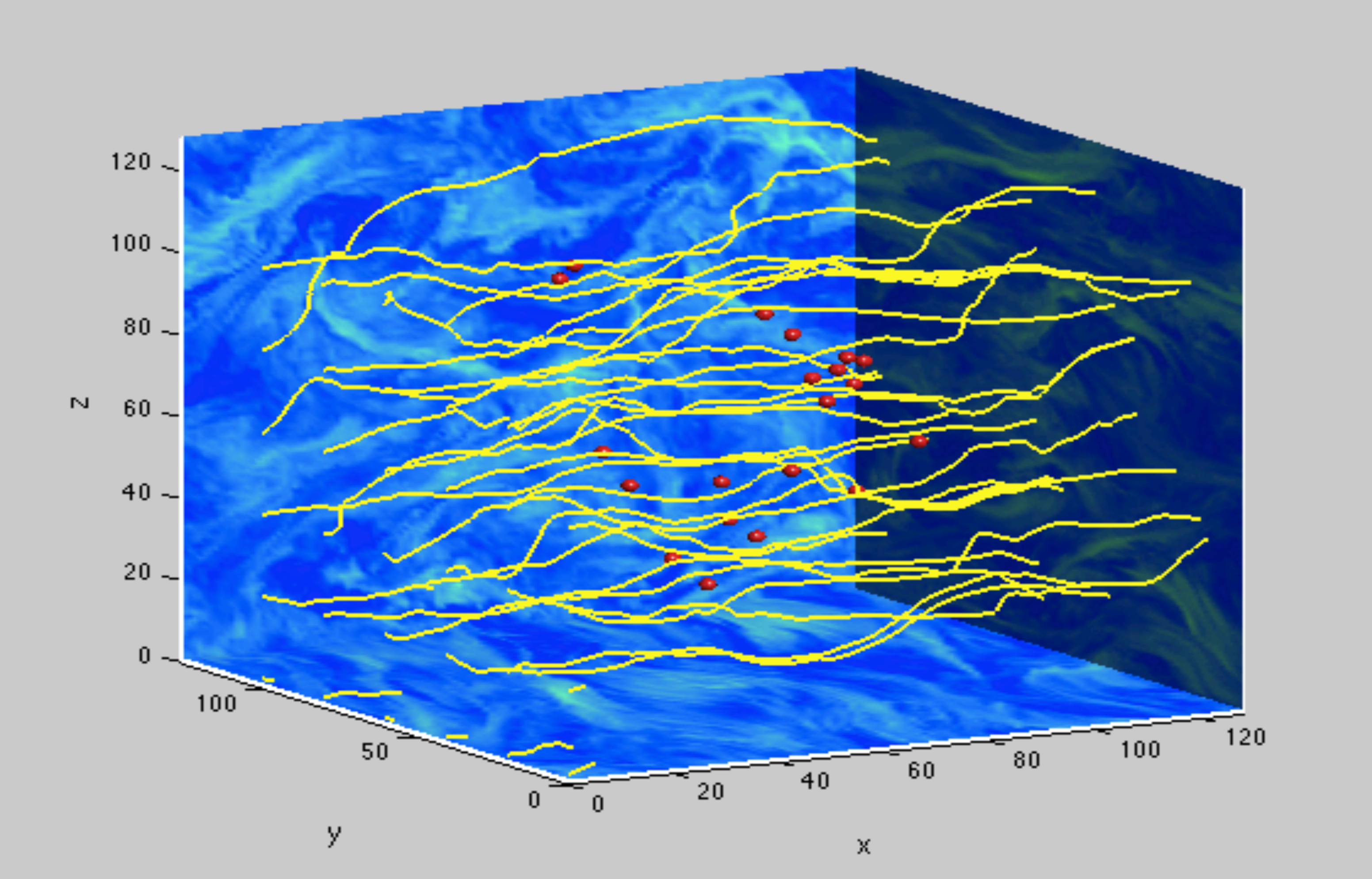
Superdiffusion and Streaming of Cosmic Rays



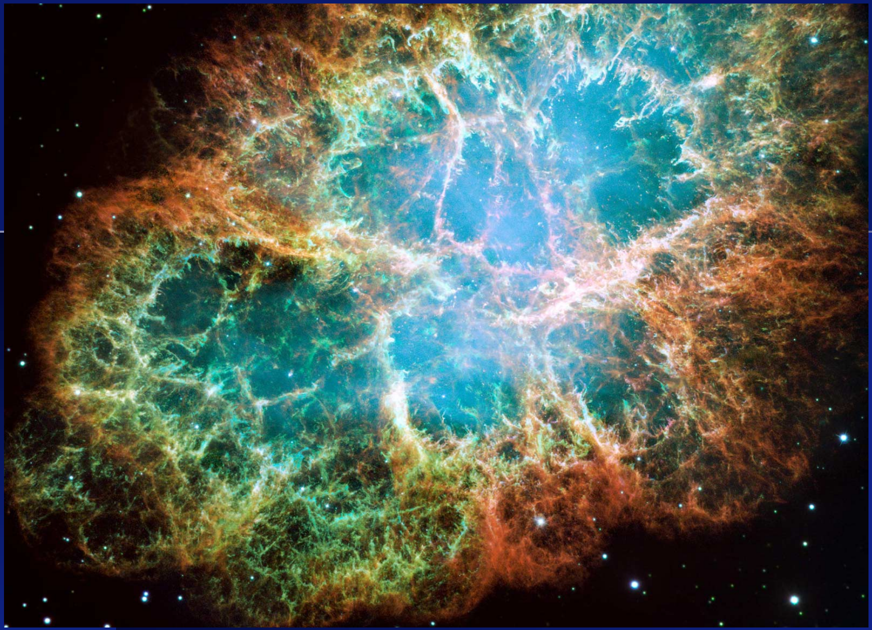
Alex Lazarian (Astronomy, Physics)

Special Thanks to G. Eyink, G. Kowal, E. Vishniac, H. Yan

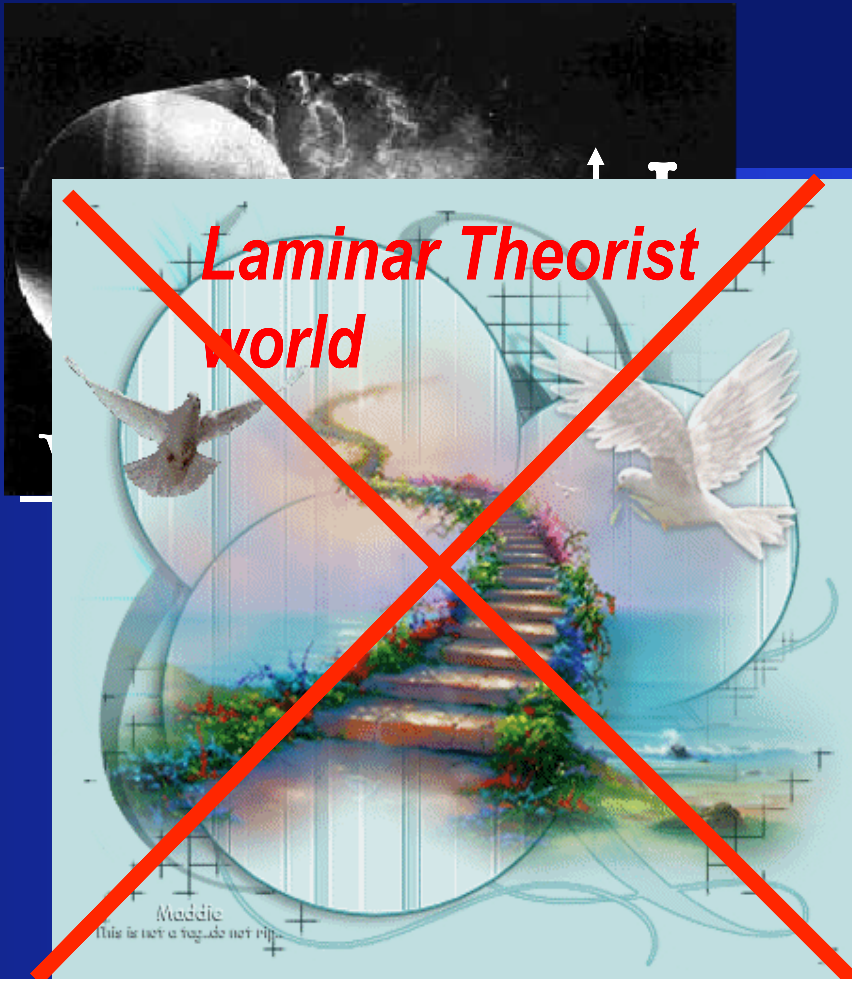
THE UNIVERSITY of WISCONSIN MADISON

Astrophysical fluids are generically turbulent

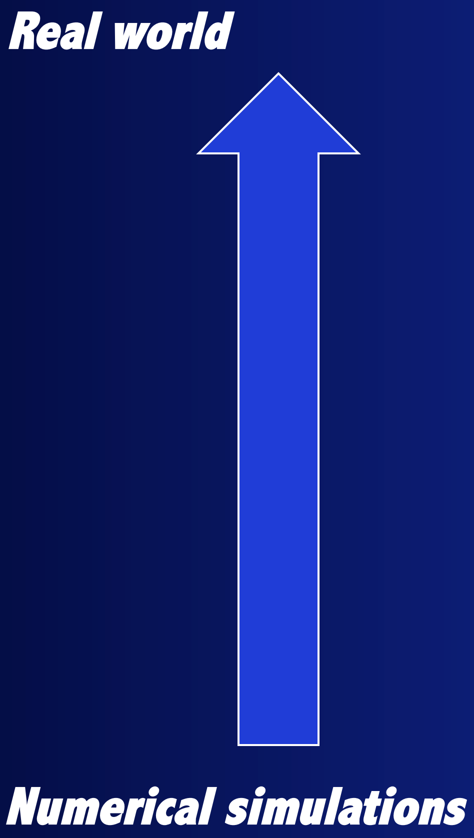
$$Re = LV/\nu = (L^2/\nu)/(L/V) = \tau_{diff}/\tau_{eddy}$$



Astrophysical flows have Re>10^{10}.

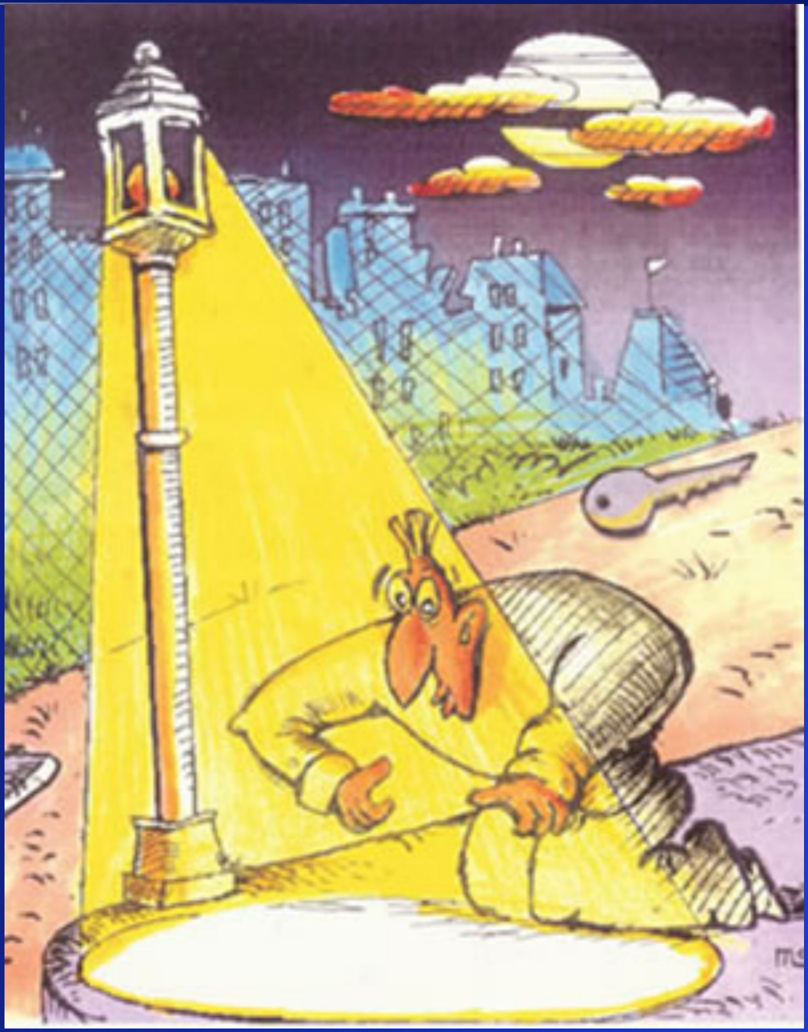


A lot of research is driven by what we can currently simulate, but simulating realistic turbulence is challenging/impossible

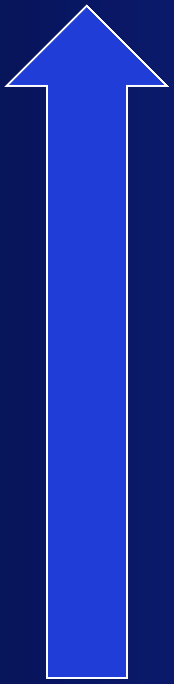


Real world

Numerical simulations



The studies extrapolate from low resolution numerical simulations to very different astrophysical regimes, while turbulence does require high resolution



Real world

Numerical simulations

Efforts scale as Re^4
Differences in Re can be more than 10^{10}

Main Points

Flux is not frozen in turbulent media
Turbulent magnetic fields present superdiffusion
MHD turbulence suppreses streaming instability, but does not create "streaming catastrophe"

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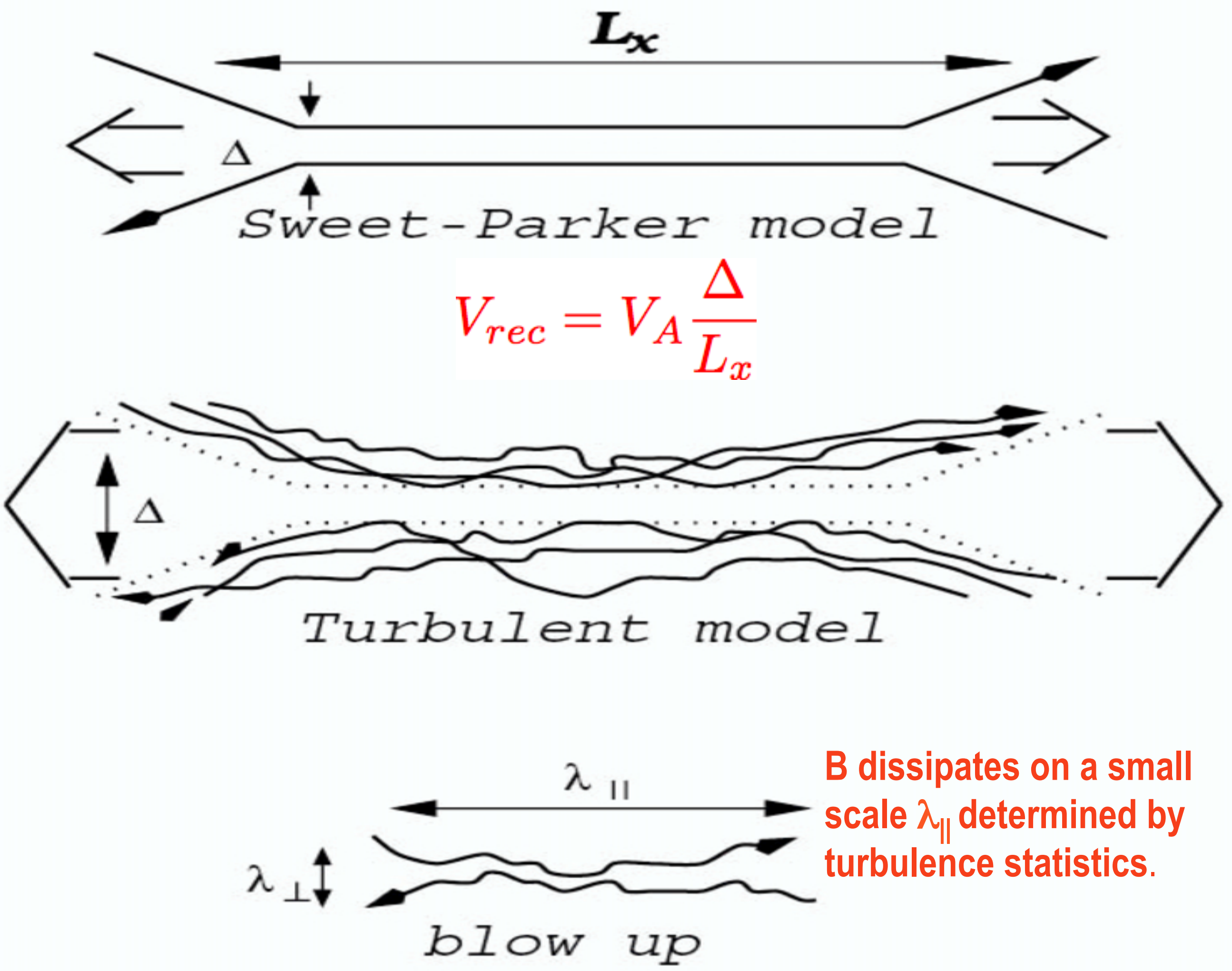
LV99 model extends Sweet-Parker model for turbulent astrophysical plasmas and makes reconnection fast

Turbulent reconnection:

Outflow is determined by field wandering.



$$V_{rec} = V_A \frac{\Delta}{L_x}$$

B dissipates on a small scale $\lambda_{\|}$ determined by turbulence statistics.

Key element:

$L/\lambda_{\|}$ reconnection simultaneous events

AL & Vishniac (1999)

henceforth referred to as LV99

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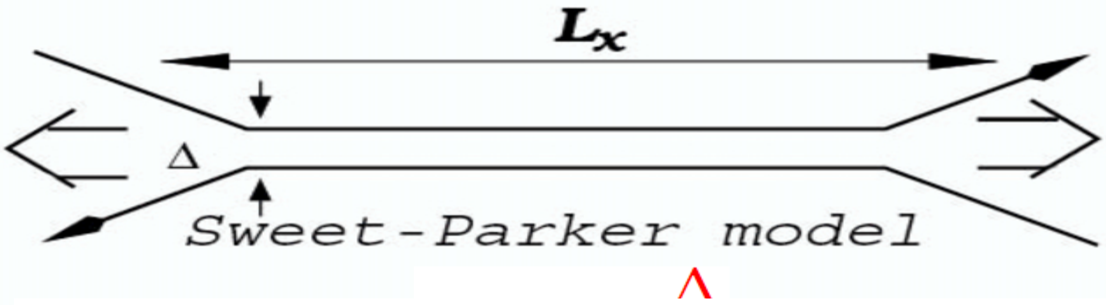

Without turbulence:

molecular diffusion coefficient D ~10^{-5} cm^2/sec

(← It's for small molecules in water.)

→ Mixing time ~ (size of the cup)^2/D ~ 10^7 sec ~ 0.3 year !

Key element:

$L/\lambda_{\|}$ reconnection simultaneous events

AL & Vishniac (1999)

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Example: Good correspondence between LV99 turbulent reconnection and solar wind data

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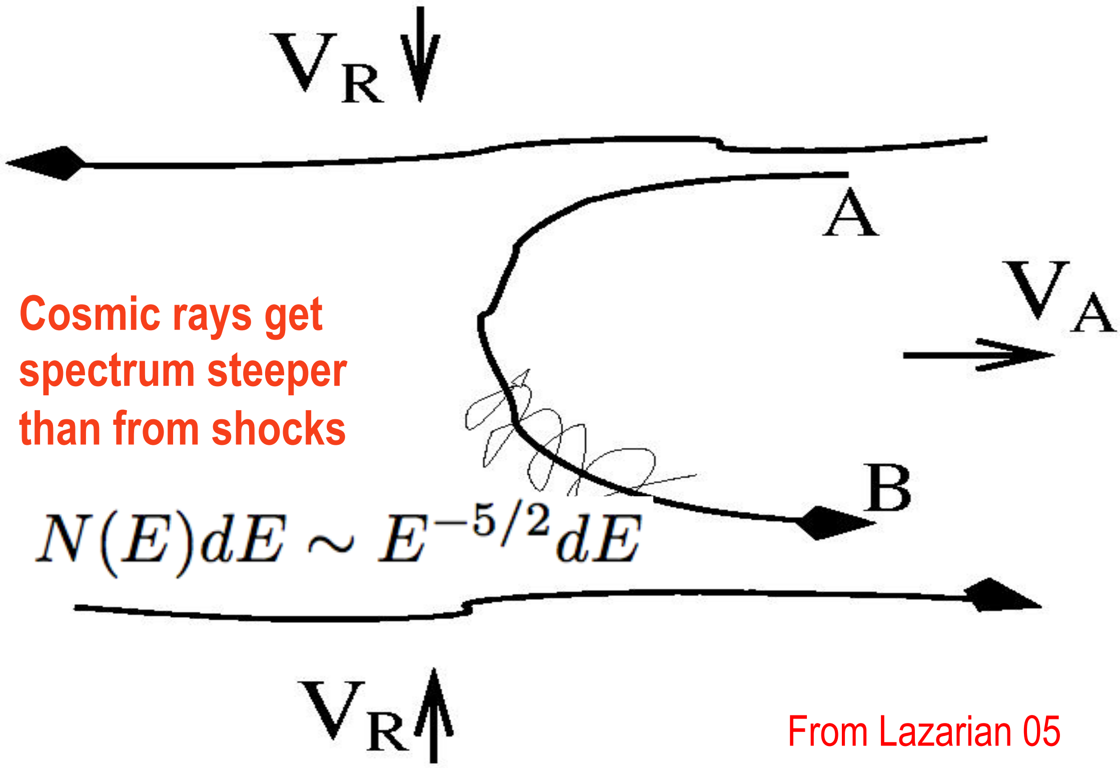
Lalescu et al. 2015

In LV99 model energetic particles get accelerated by First Order Fermi mechanism (as discussed by Gianfranco)



Cosmic rays get spectrum steeper than from shocks

$$N(E)dE \sim E^{-5/2}dE$$

From Lazarian 05

(cp. Drake 2006).

Published in De Gouveia Dal Pino & Lazarian 2003

Applications to pulsars, microquasars, solar flare acceleration (De Gouveia Dal Pino & Lazarian 00, 03, 05, Lazarian 05).

Big Implication: LV99 predicts that magnetic field in *turbulent fluids* is not frozen in

Hannes Alfven

Instead of flux freezing condition one should consider flux diffusion by turbulent flow. This has dramatic consequences for many areas of astrophysics including star formation!

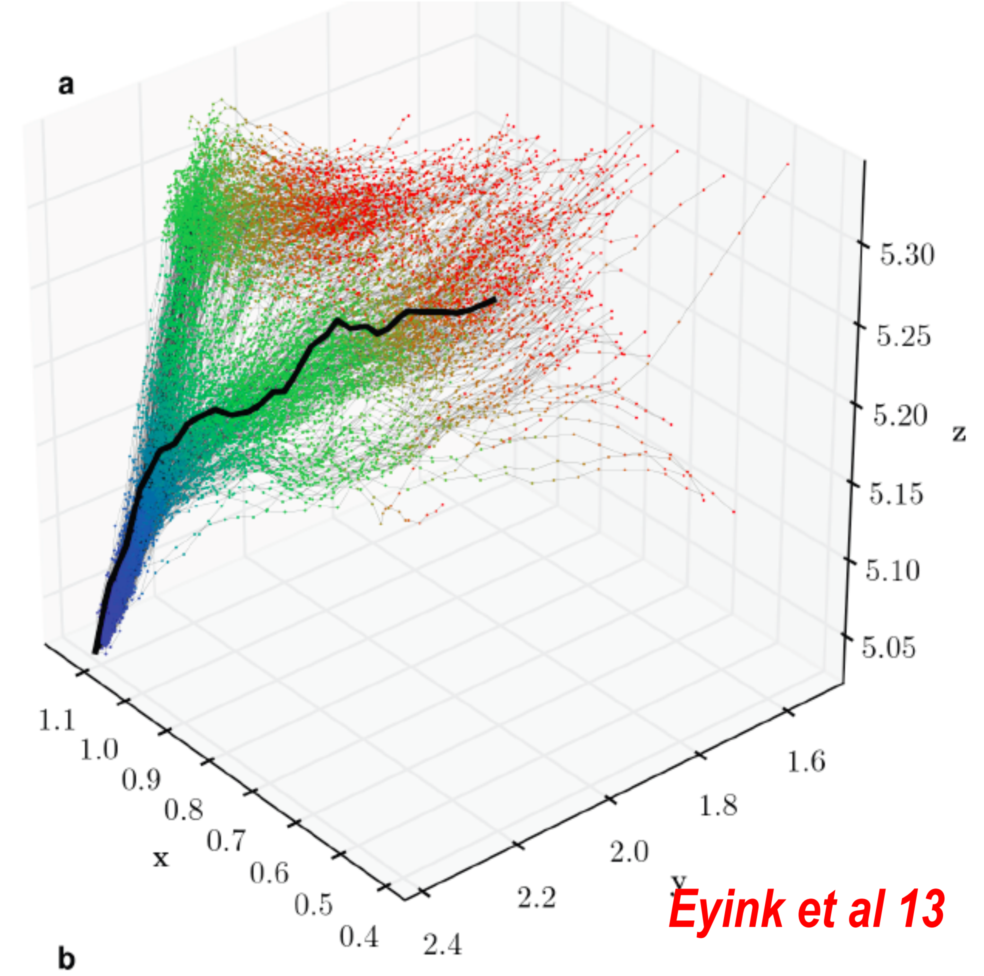
Violation of magnetic field frozen in condition in turbulent fluids proven in Eyink (2011). The equivalence of this and LV99 approach was demonstrated in Eyink, Lazarian & Vishniac 2011.

Eyink, AL & Vishniac 2011 related LV99 to the concept of Richardson diffusion

$$\langle|\mathbf{x}_1(t)-\mathbf{x}_2(t)|^2\rangle \sim t^3$$

Richardson's law

Eyink, AL & Vishniac 2011 related LV99 to the well-known concept of Richardson diffusion



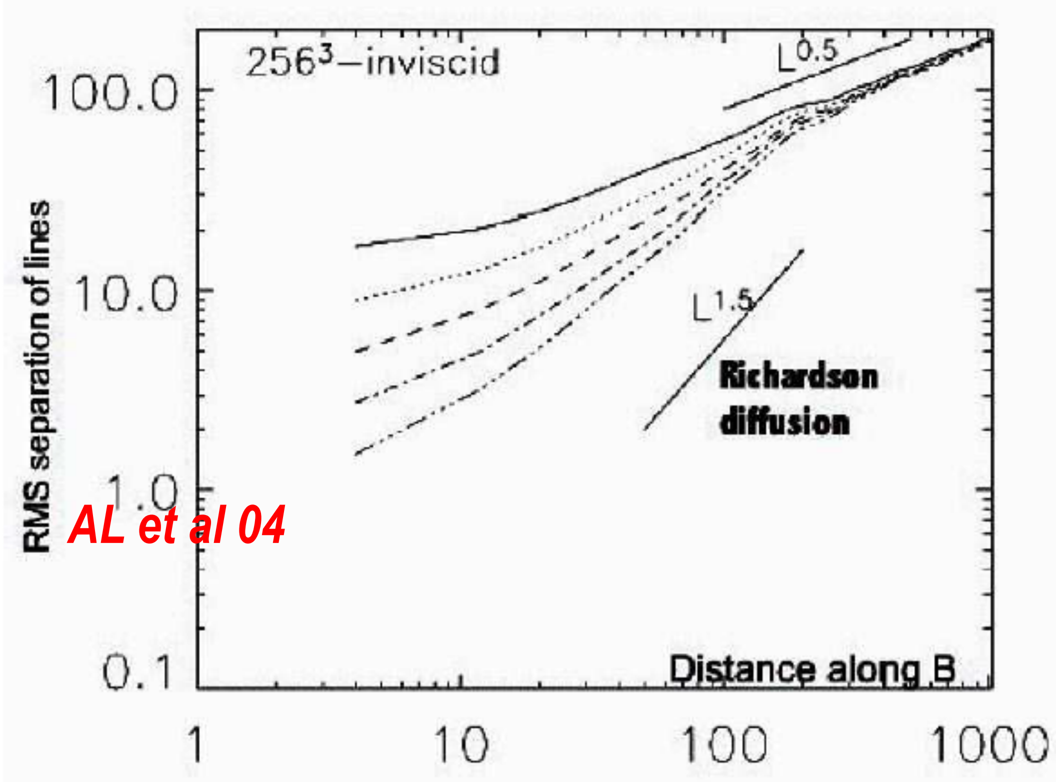
Eyink et al 13

$$\langle |\mathbf{x}_1(t) - \mathbf{x}_2(t)|^2 \rangle \sim t^3$$

Magnetic diffusion in time

If one traces magnetic field lines in the presence of Richardson diffusion than one gets the LV99 result for field wandering

Richardson diffusion measured in MHD



AL et al 04

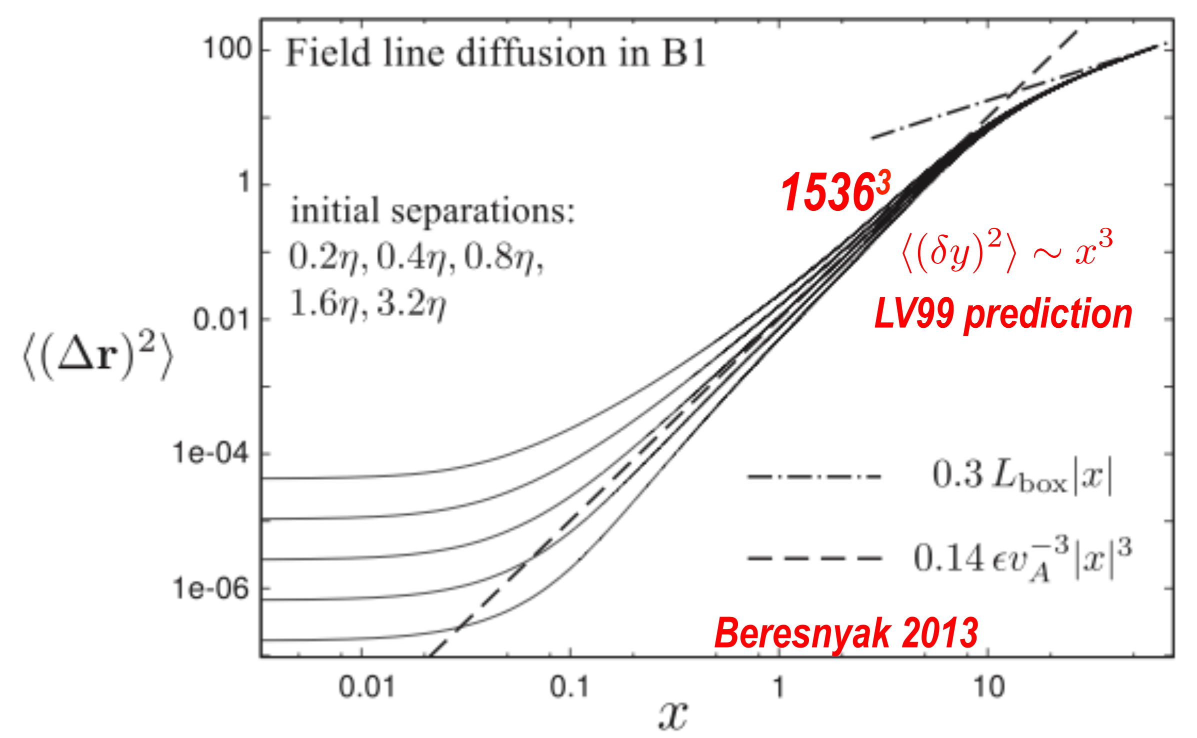
Magnetic diffusion in space: field wandering

$$\langle(\delta y)^2\rangle \sim x^3$$

We decided to keep the term Richardson diffusion

If one traces magnetic field lines in the presence of Richardson diffusion than one gets the LV99 result for field wandering

Richardson diffusion measured in MHD



1536^3

$\langle(\delta y)^2\rangle \sim x^3$

LV99 prediction

Beresnyak 2013

$$\langle(\delta y)^2\rangle \sim x^3$$

We decided to keep the term Richardson diffusion

Magnetic diffusion in space: field wandering

Main Points

Flux is not frozen in turbulent media

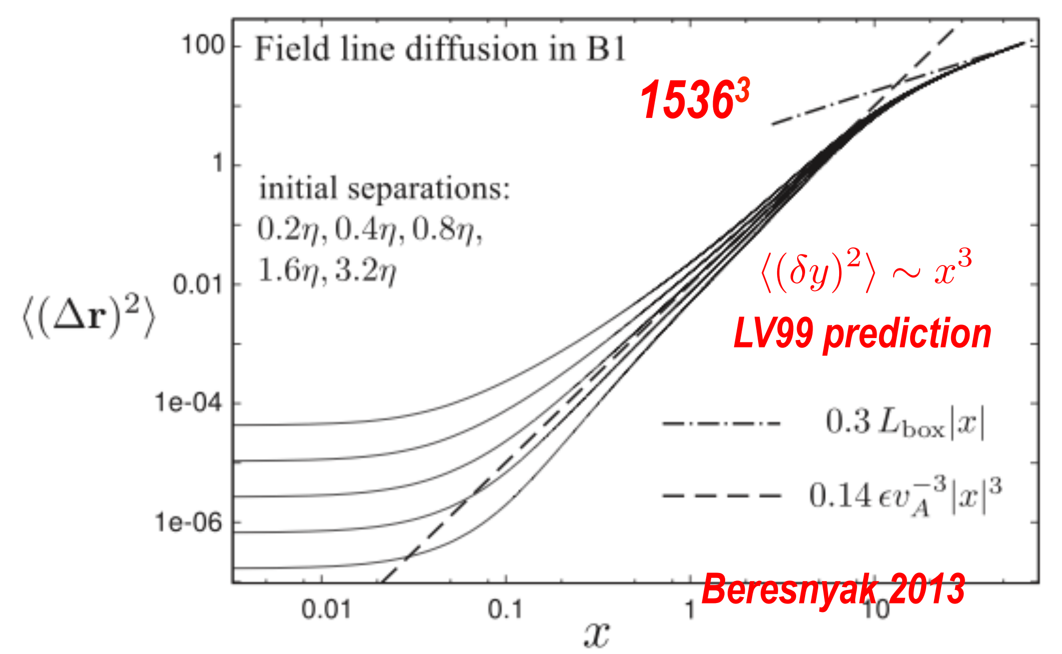
Turbulent magnetic fields present superdiffusion

MHD turbulence suppreses streaming instability, but does not create "streaming catastrophe"

Richardson diffusion in space means superdiffusion (superballistic behavior) for CRs following magnetic field

$$\langle (\delta y)^2 \rangle \sim x^3$$

Superdiffusion acts on scales x less than the injection scale of MHD turbulence



1536^3

$\langle (\delta y)^2 \rangle \sim x^3$

LV99 prediction

Beresnyak 2013

Injection scale of turbulence in the Galaxy is about 100 pc

Diffusion perpendicular to mean magnetic field direction is determined by magnetic field line wondering

Realized by Jokipii & Parker 69, Jokipii 73 but turbulence model was not right

In fact, this motivated my work in turbulent magnetic reconnection

The study with modern understanding of MHD turbulence is in AL& Vishniac 99

Strong subAlfvenic turbulence at scales $s<l_{trans}$ results in superdiffusion:

$$\frac{d}{ds}\ell_\perp^2 \sim D_\perp^B(\ell) \sim (\delta u_\ell/v_A)^2\ell_\parallel \sim L\left(\frac{\ell_\perp}{L}\right)^{4/3} M_A^{4/3},$$

$$\ell_\perp^2 \sim \frac{s^3}{27L}M_A^4,$$

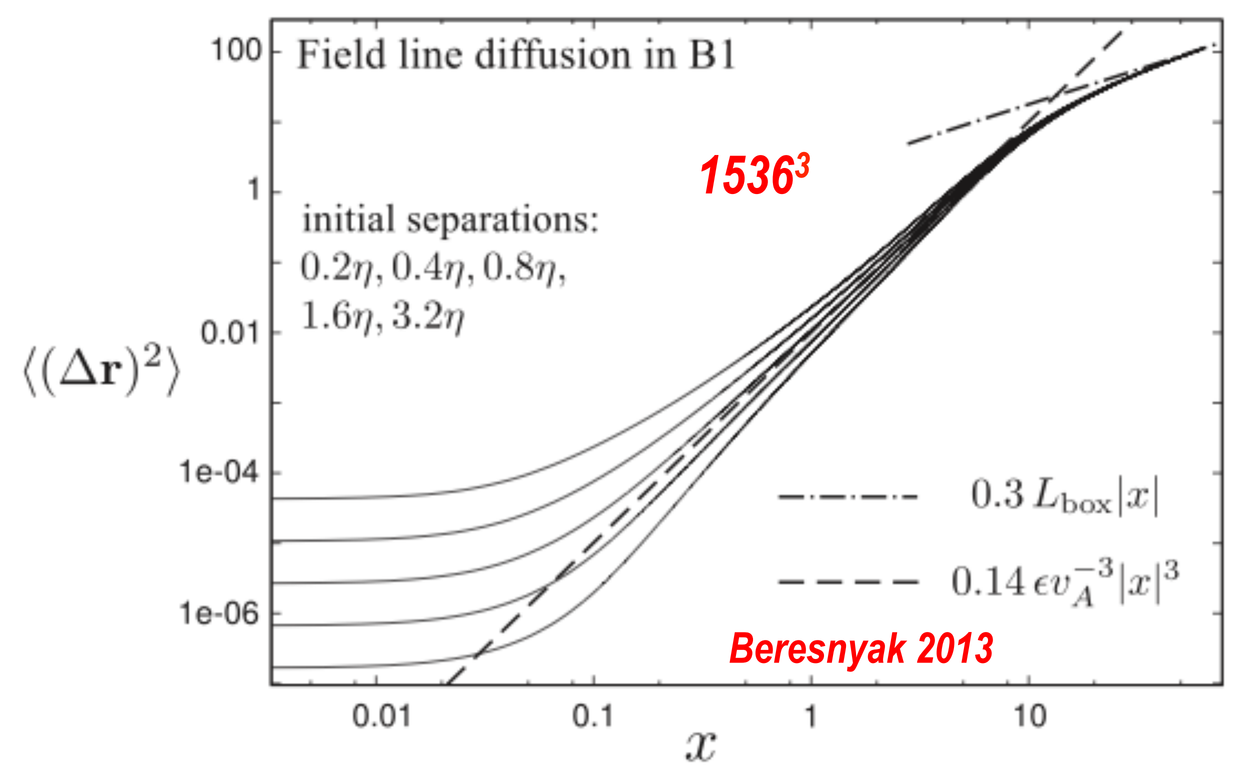
At scales $s>l_{trans}$ results in ordinary diffusion:

$$\ell_\perp^2 \sim sLM_A^4.$$

Diffusion is not applicable on scales less than the turbulence injection scale L

On scales < L and s< mfp, CRs trace magnetic field divergence

$$\ell_\perp^2 \sim \frac{s^3}{27L} M_A^4,$$



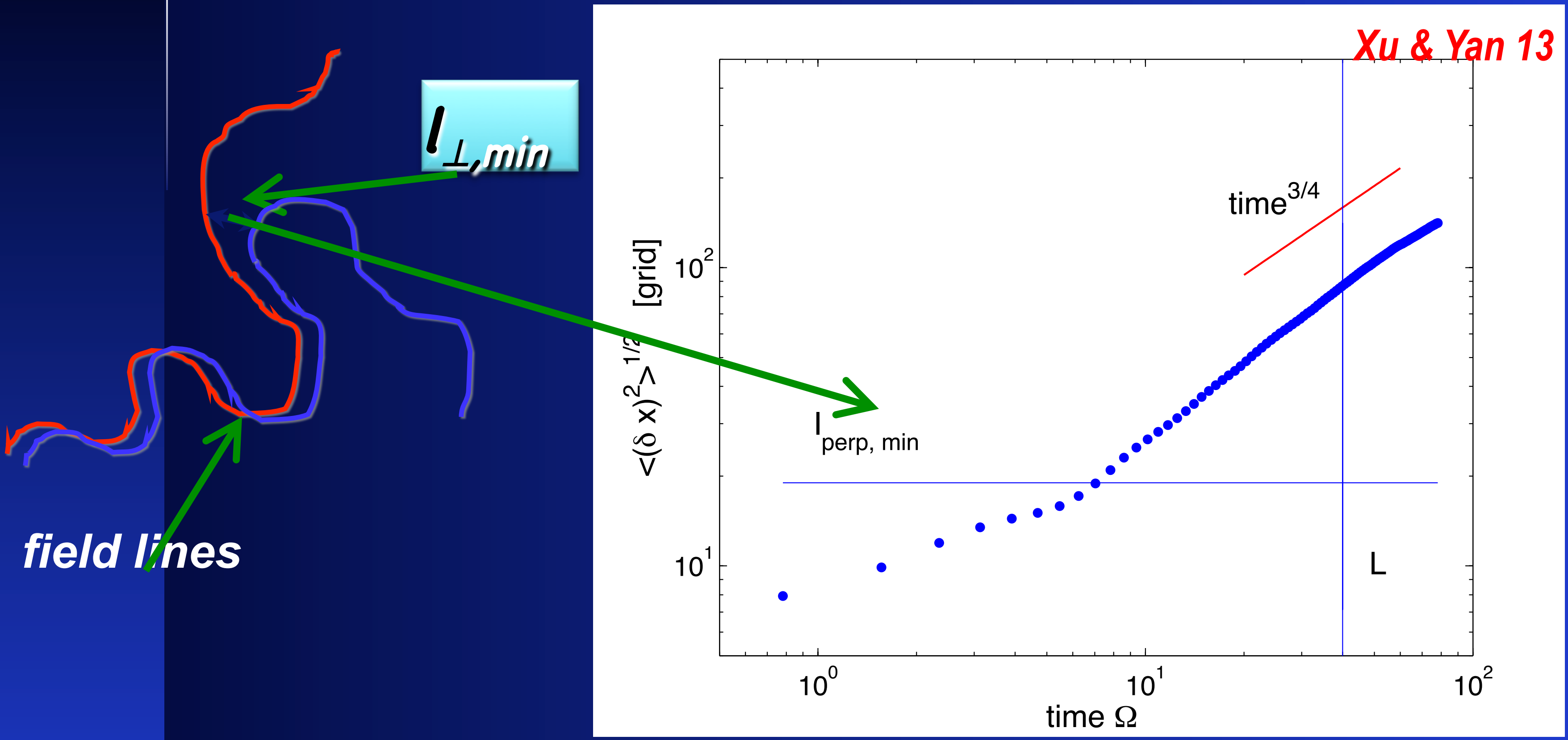
1536^3

Beresnyak 2013

For scales < L

Explosive separation of magnetic field lines is described analytically in AL & Vishniac 1999. Separtion ~ $X^{3/2}$

If CRs diffuse along the magnetic field lines the perpendicular motion Is still superdiffusive



To compare with

$$l_{\perp,\mathrm{CR}}^2 \sim \frac{(D_\parallel \delta t)^{3/2}}{27L} M_A^4, \quad M_A < 1,$$

The dependence on forth power of Alfven Mach number is also confirmed

On scales s > L and s>> mfp the ordinary diffusion is present (AL06, Yan & AL08)

$$D_{\perp,\mathrm{global}} \approx D_{\parallel} M_A^4,$$

On scales < L and s< mfp, CRs trace magnetic field divergence

$$\ell_{\perp}^2 \sim \frac{s^3}{27L} M_A^4,$$

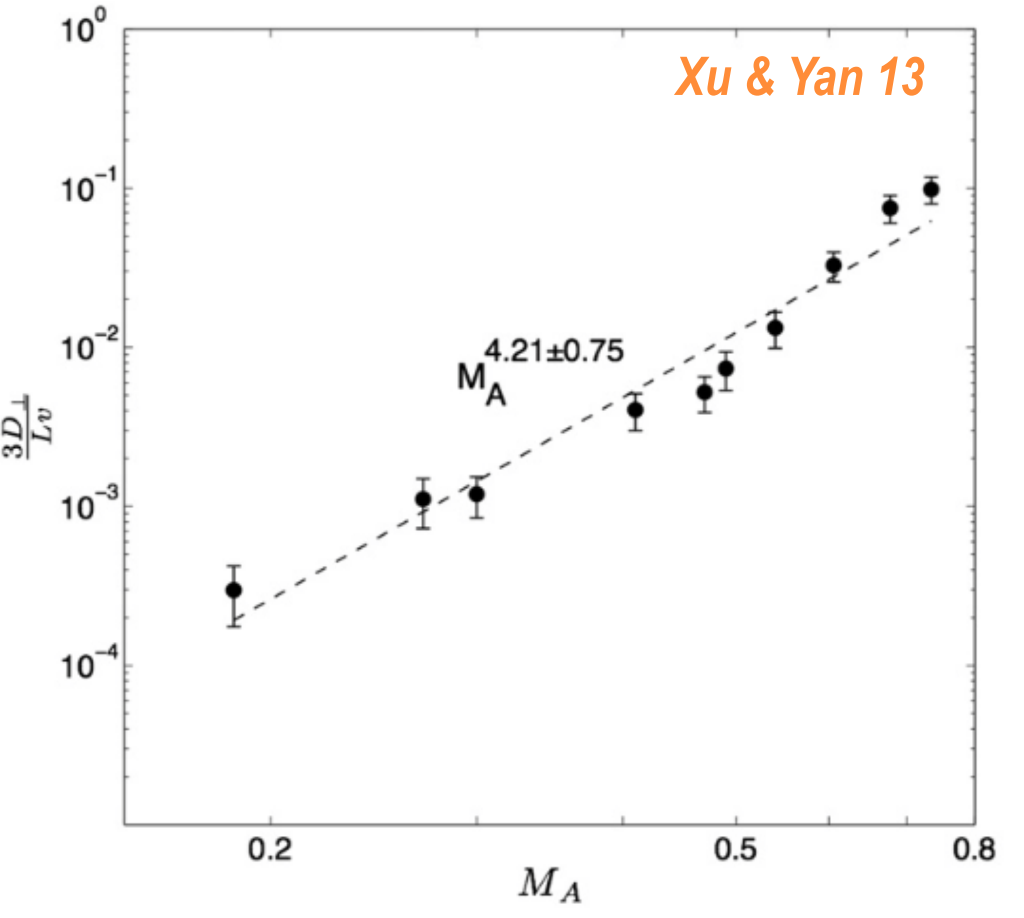
On scales < L and s> mfp, CRs trace magnetic field divergence, s is covered in diffusion process

$$l_{\perp,\mathrm{CR}}^2 \sim \frac{(D_{\parallel}\delta t)^{3/2}}{27L} M_A^4, \quad M_A < 1,$$

Differs from the textbook (see Jokipii & Parker 69) $M_A{}^2$ dependence

The dependence on forth power of Alfven Mach number is also confirmed

Diffusive regime

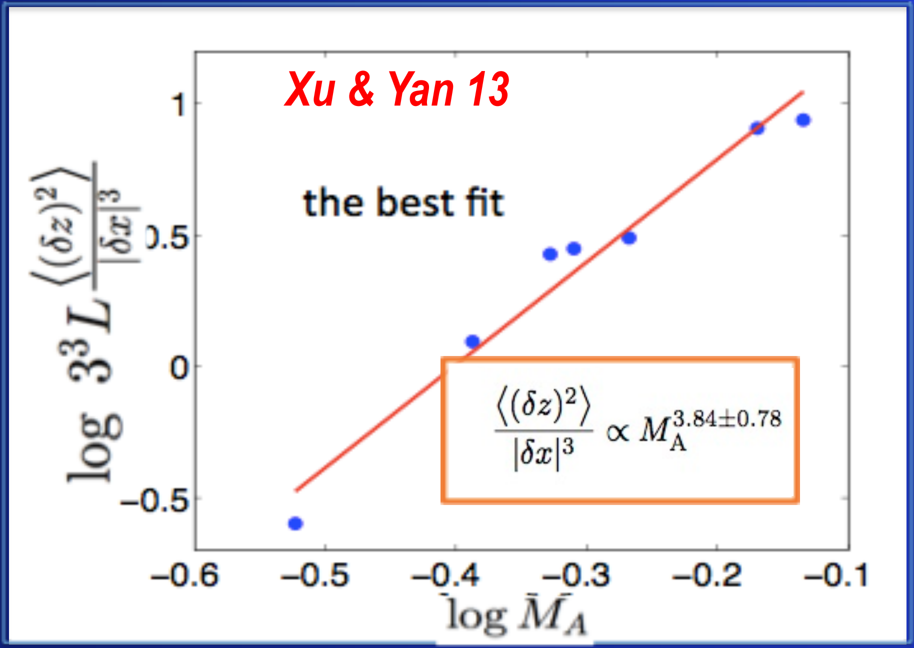


To compare with $D_{\perp,\mathrm{global}} \approx D_\parallel M_A^4,$ *in AL06, Yan & AL08*

The dependence on forth power of Alfven Mach number is also confirmed

Superdiffisive regime

$$< (\delta z)^2 >= \frac{|\delta x|^3}{3^3 L} M_A^4$$



To compare with

$$\ell_\perp^2 \sim \frac{s^3}{27L} M_A^4,$$

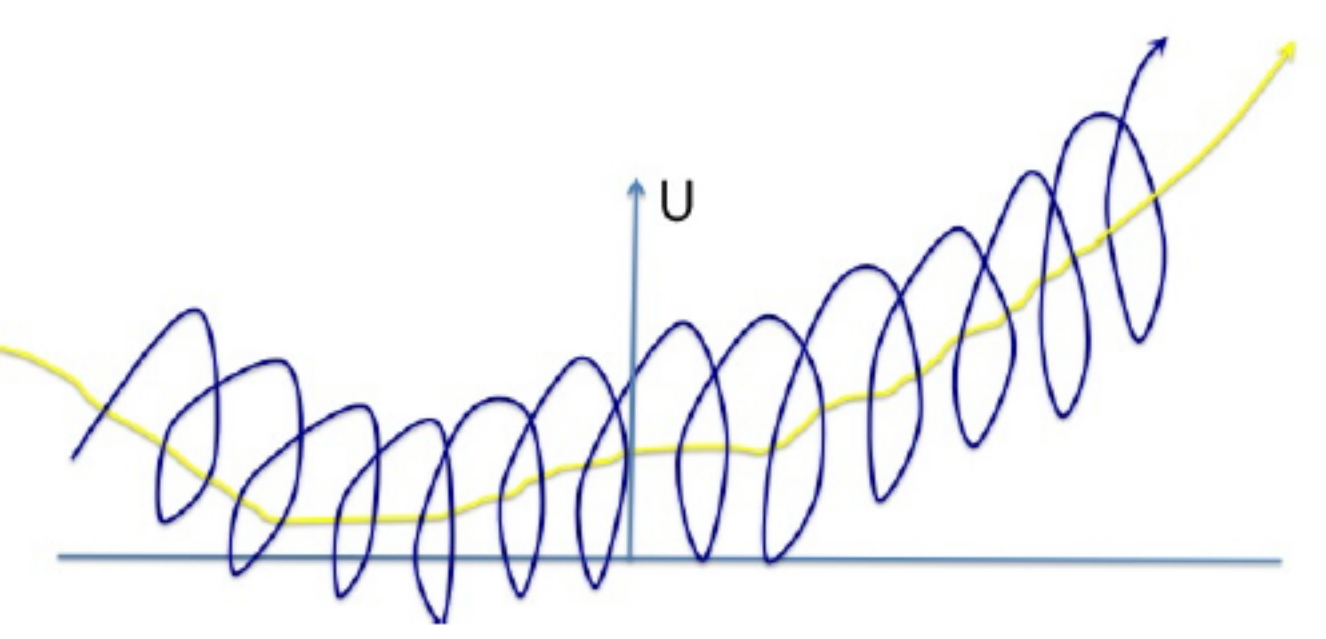
AL & Vishniac 1999; Yan & AL 2008

Superdiffusion changes the accepted formalism for parallel and perpendicular shock acceleration

$$\frac{\kappa_\perp}{\kappa_\|} = \frac{1}{1+(\lambda_{CR}/r_L)^2}$$

Accepted expression

In reality



Lazarian & Yan 2013

Main Points

Flux is not frozen in turbulent media

Turbulent magnetic fields present superdiffusion

MHD turbulence suppreses streaming instability, but does not create "streaming catastrophe"

Streaming instability damping by Alfvenic turbulence is suggested by Yan & AL 02 and quantified by Farmer & Goldreich 04

$$\Gamma_{cr} \approx \Omega_B \frac{n_{cr}(>\gamma)}{n_i} \left(\frac{v_{stream}}{V_A} - 1 \right),$$

Streaming instability growth rate

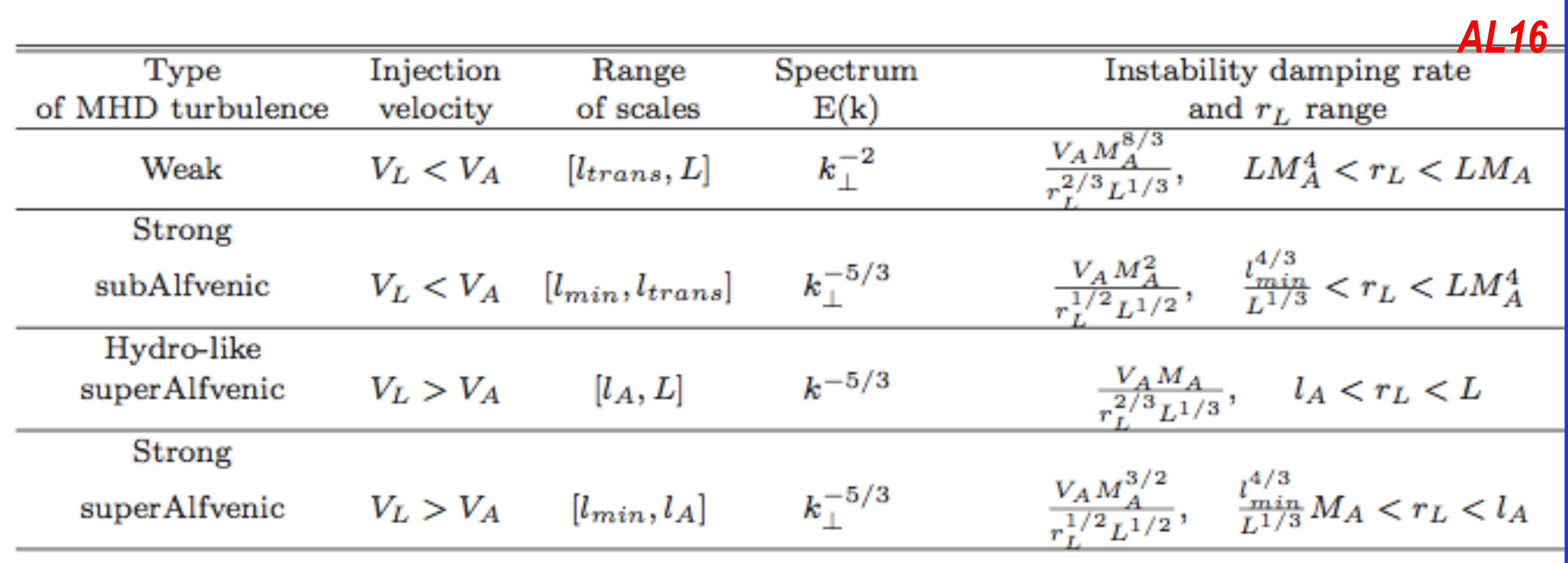
FG04 considered only damping by strong turbulence and assumed that the energy is injected with Alfven velocity. They also came to the conclusion that streaming instability should be suppressed in the Galaxy, which presented a problem with explaining cosmic ray isotropy measured.

The result of the pioneering FG04 study were overinterpreted in later works:

1. *That this is the model of damping of Alfven waves in turbulence in general sense.*
2. *That it covers both subAlfvenic and superAlfvenic damping of streaming.*
3. *That indeed "streaming catastrophe" exists for realistic Mikly Way model.*

Results in FG04, however, do not cover all important regimes of turbulence

$$M_A \equiv \frac{V_L}{V_A} = \frac{\delta B}{B}$$



AL16

Type of MHD turbulence	Injection velocity	Range of scales	Spectrum E(k)	Instability damping rate and r_L range
Weak	$V_L < V_A$	$[l_{trans}, L]$	$k_\perp^{-2}$	$\frac{V_A M_A^{8/3}}{r_L^{2/3} L^{1/3}}$, $LM_A^4 < r_L < LM_A$
Strong subAlfvenic	$V_L < V_A$	$[l_{min}, l_{trans}]$	$k_\perp^{-5/3}$	$\frac{V_A M_A^2}{r_L^{1/2} L^{1/2}}$, $\frac{l_{min}^{4/3}}{L^{1/3}} < r_L < LM_A^4$
Hydro-like superAlfvenic	$V_L > V_A$	$[l_A, L]$	$k^{-5/3}$	$\frac{V_A M_A}{r_L^{2/3} L^{1/3}}$, $l_A < r_L < L$
Strong superAlfvenic	$V_L > V_A$	$[l_{min}, l_A]$	$k_\perp^{-5/3}$	$\frac{V_A M_A^{3/2}}{r_L^{1/2} L^{1/2}}$, $\frac{l_{min}^{4/3}}{L^{1/3}} M_A < r_L < l_A$

For subAlfvenic turbulence the range for the strong turbulence damping is limited to $<LM_A^4$,
It extends from LM_A^4 to LM_A for weak turbulence

As for the suppression of streaming in the Galaxy FG04 uses a number of assumptions

1. The level of turbulent dissipation is estimated assuming that all the heating of halo gas is due to turbulent damping

This is not true as in the presence of streaming additional heating comes from CRs.

2. Streaming instability damping is induced by strong turbulence in the halo

Even for the turbulent dissipation rate that FG04 assumes (excessive) the damping of instability for high energy CRs is due not to strong turbulence, but to weak turbulence and therefore is reduced. Indeed $r_L<LM_A^4$ *and their assumption implies* $M_A=0.2$

3. A simple relation between the streaming velocity and degree anisotropy is assumed

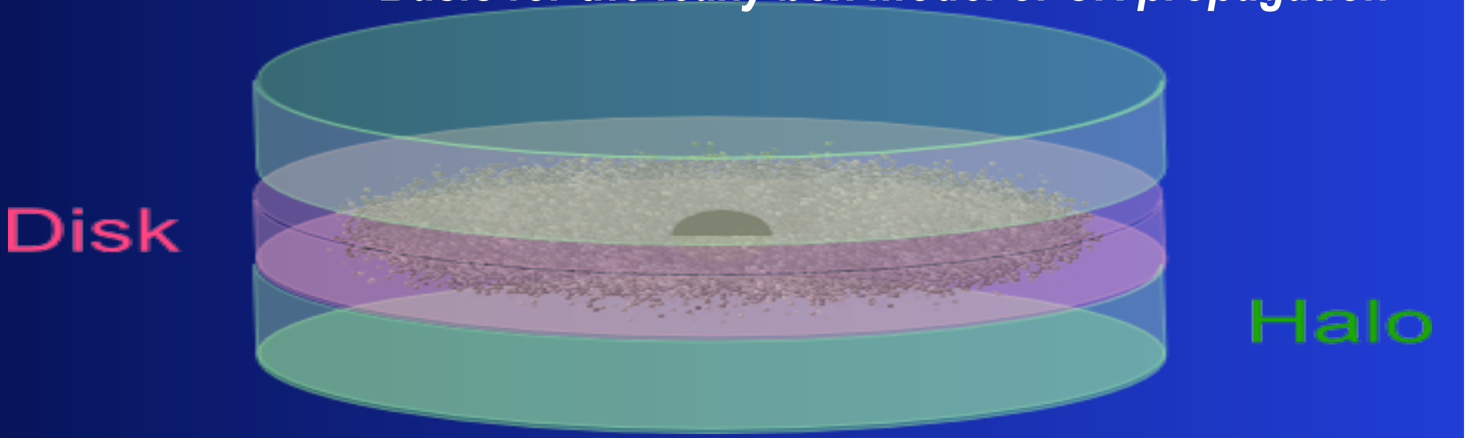
This ignores superdiffusion of magnetic field lines that we discussed.

Streaming instability damping by Alfvenic turbulence is suggested by Yan & AL 02 and quantified by Farmer & Goldreich 04

$$\Gamma_{cr} \approx \Omega_B \frac{n_{cr}(>\gamma)}{n_i} \left(\frac{v_{stream}}{V_A} - 1 \right),$$

Streaming instability

Basis for the leaky box model of CR propagation



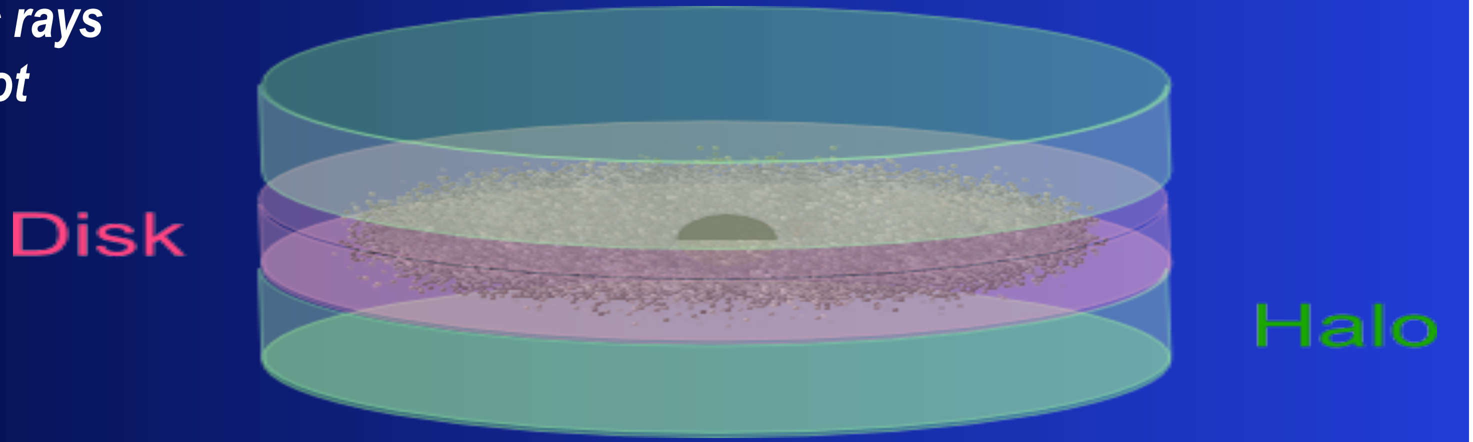
Quantified in Farmer & Goldreich 04 for strong transAlfvenic turbulence

Farmer & Goldrech 2004 challenged the "leaky box" for CR confinement and isotropization claiming that streaming instability cannot exist in the presence of turbulence

The model of damping by strong turbulence is used

The turbulence level was estimated using $\epsilon_{turb.dissipation} = radiation \;\; cooling$

Problem: cosmic rays stream and do not isotropize

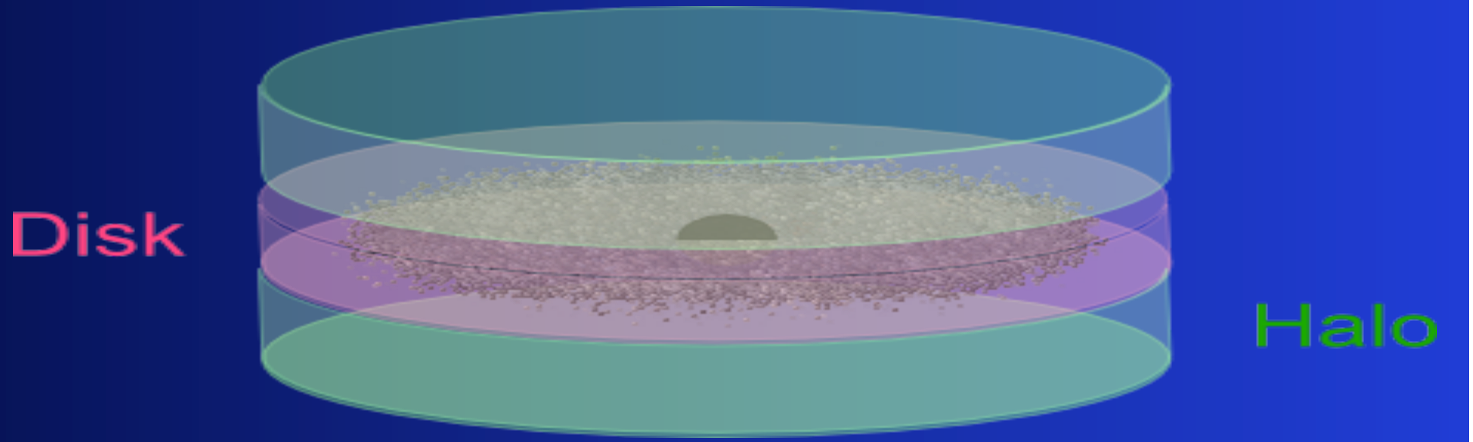


The work meant the crisis of the existing models of CR isotropization!

Detailed calculations in AL16 show that "leaky box" model is valid if it is accounted that scattering is by weak turbulence and the level of turbulence in Halo is small

The model of damping by weak turbulence is used

The turbulence level was estimated using $\epsilon_{turb.dissipation} < radiation \quad cooling$



CRs stream in the disk where turbulence is transAlfvenic and randomize by streaming instability in the halo. Streaming CR and not turbulence dissipation is the source of halo healing.

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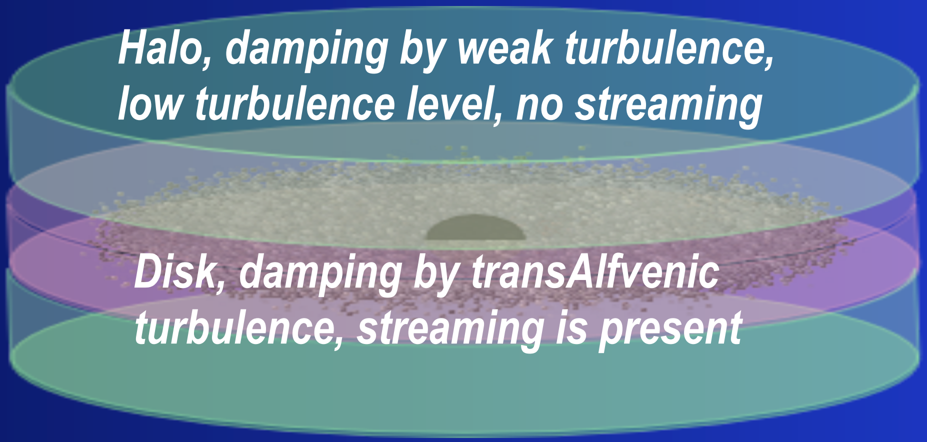
The model of damping by weak turbulence is used

The turbulence level was estimated using $\epsilon_{turb.dissipation} < radiation \quad cooling$

New understanding:

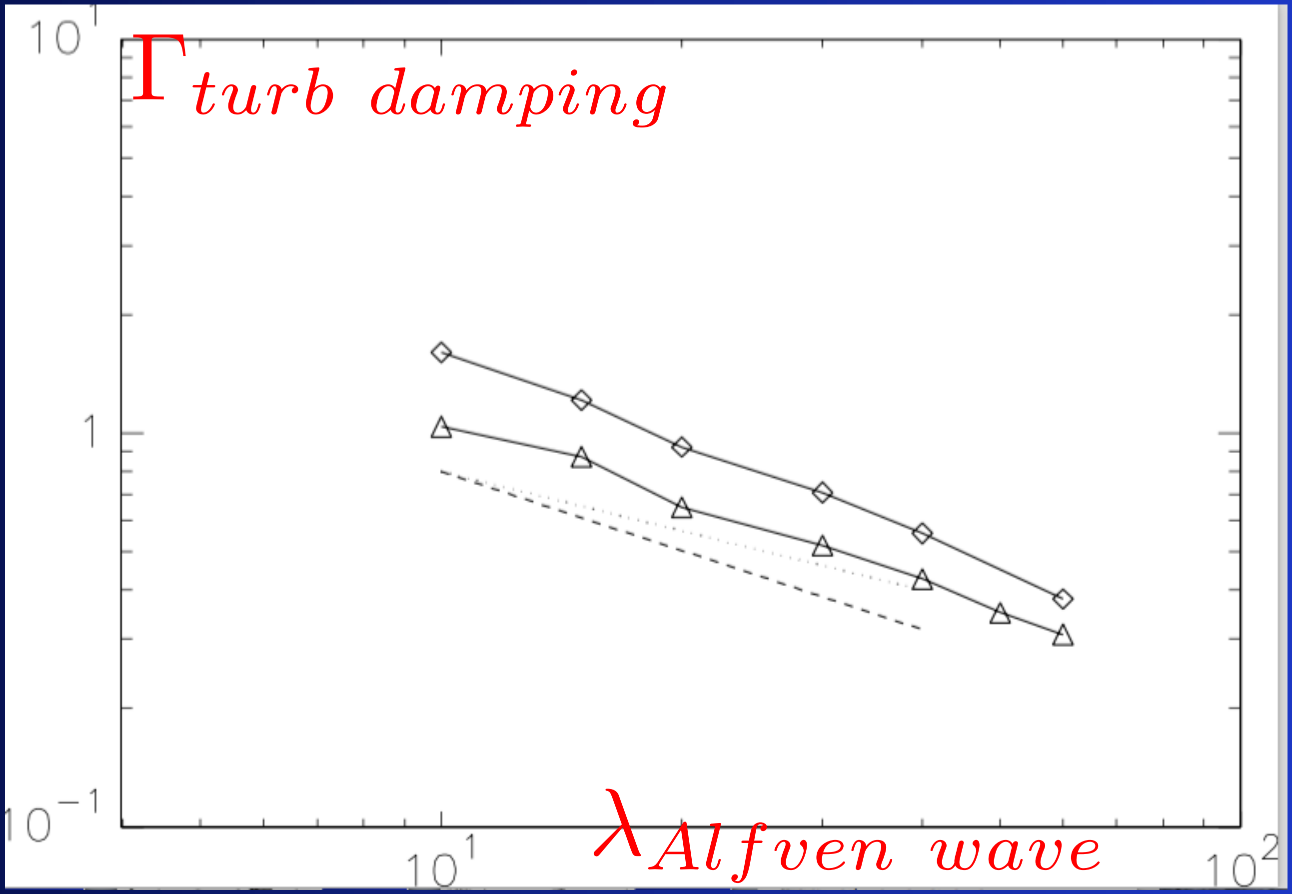


Halo, damping by weak turbulence, low turbulence level, no streaming

Disk, damping by transAlfvenic turbulence, streaming is present

CRs stream in the disk where turbulence is transAlfvenic and randomize by streaming instability in the halo. Streaming CR and not turbulence dissipation is the source of halo healing.

Numerical simulations confirm the predicted scaling of the Alfven wave damping



Cho & AL 16

Turbulence induces superdiffusion and damping of Alfven waves

Turbulence

superdiffusion

Suppression of streaming