Cosmic Ray Propagation in High β Plasmas

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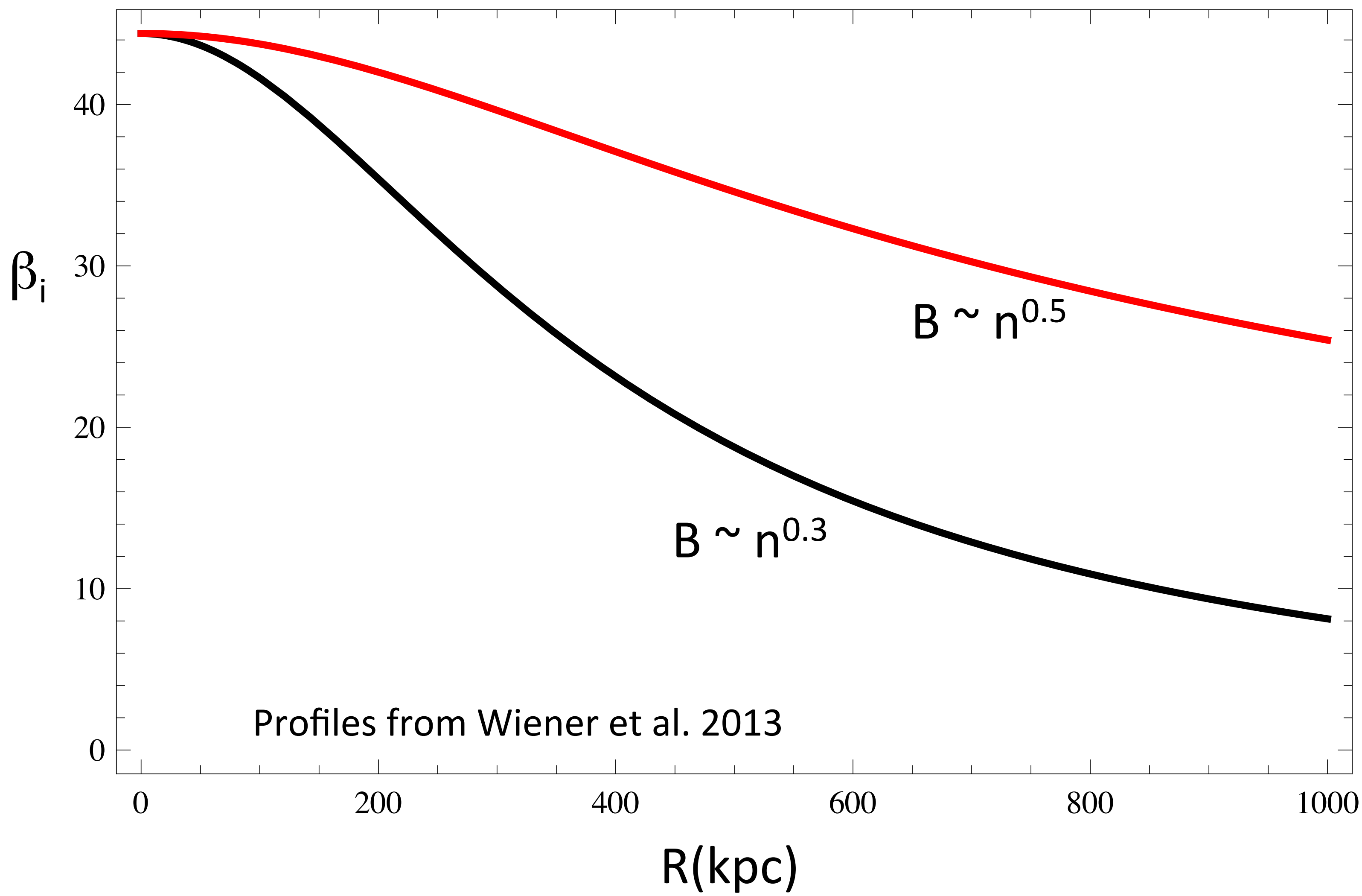
β in the Coma Cluster



High β Effects & Their Consequences

- What types of waves does cosmic ray streaming excite?
- How are these waves damped?
- What is the streaming speed or diffusion rate of cosmic rays relative to the background?
- How do cosmic rays transfer energy and momentum to the background medium?

- Is cosmic ray heating of ICM gas important?
- What is the origin of cosmic rays in clusters?
- What determines the cosmic ray pressure profile in clusters?

May be applicable to young galaxies & other environments.

Canonical Lore

Gyroresonant Scattering is Fundamental Interaction

- Wave with frequency ω, parallel wavenumber k & amplitude δB:

$$\omega - kv\mu = +/- \, \omega_{ci}/\gamma$$

Scatter by

$$\mu = \mathbf{p.B}/pB$$

$$\delta\theta \sim \delta B/B;$$

$$\nu = \langle(\delta\theta)^2\rangle/\delta t \sim \omega_{ci}/\gamma \, (\delta B/B)^2$$

Gyroresonant Streaming Instability

$$\Gamma_{cr} = \frac{\pi^2 q^2}{2} \frac{v_A^2}{c^2} \sum_{\pm} \int \delta(\omega - kv\mu \pm \omega_c) v(1-\mu^2) \left[\frac{\partial f}{\partial p} + \left(\frac{kv}{\omega} - \mu \right) \frac{1}{p} \frac{\partial f}{\partial \mu} \right] p^2 dp d\mu,$$

resonance condition

damping

excitation by anisotropy

Simple approximation to the growth rate:

$$\Gamma_{cr} \sim C\omega_{ci} \frac{n_{cr}(> p_1)}{n_i} \left(\frac{v_D}{v_A} - 1 \right),$$

$v_D/v_A > 1$ in the presence of damping

$$p_1 \equiv \frac{m\omega_c}{k}$$

Minimum cosmic ray momentum that can resonate with a given k.

Historical Notes

- High β wave propagation considered by Foote & Kulsrud 1979.
- Holman et al. 1979 noted that thermal cyclotron damping of waves with $k_{ri} \sim 1$ would reduce pitch angle scattering for cosmic rays with $\mu \sim 0$.
- Felice & Kulsrud 2001 showed that such particles turn around through magnetic mirroring.

Energy Equation

Multiply F-P eqn. by particle energy ε & integrate over momentum space:

$$\frac{\partial U_c}{\partial t} + \nabla \cdot \tilde{W}_c = -\int d\omega dk 2\Gamma_{cr}(\omega, k) I(\omega, k)$$

Energy density Energy flux Energy transfer to waves

Full equation…not equation for pitch angle scattering alone.

Frequent Scattering Approximation

Relate anisotropy to spatial gradient:

$$D_{\mu\mu}\frac{\partial f_0}{\partial \mu} + D_{\mu p}\frac{\partial f_0}{\partial p} = -\frac{v(1-\mu^2)}{2}\frac{\partial f_0}{\partial z}$$

Lets us write Γ_{cr} in terms of density gradient.

Energy equation simplifies to:

$$\frac{\partial U_c}{\partial t} + \nabla \cdot \tilde{\boldsymbol{W}}_c = \boldsymbol{v}_A \cdot \nabla P_c.$$

"frictional heating" due to energy transfer to Alfven waves

At High β, Waves Are Dispersive

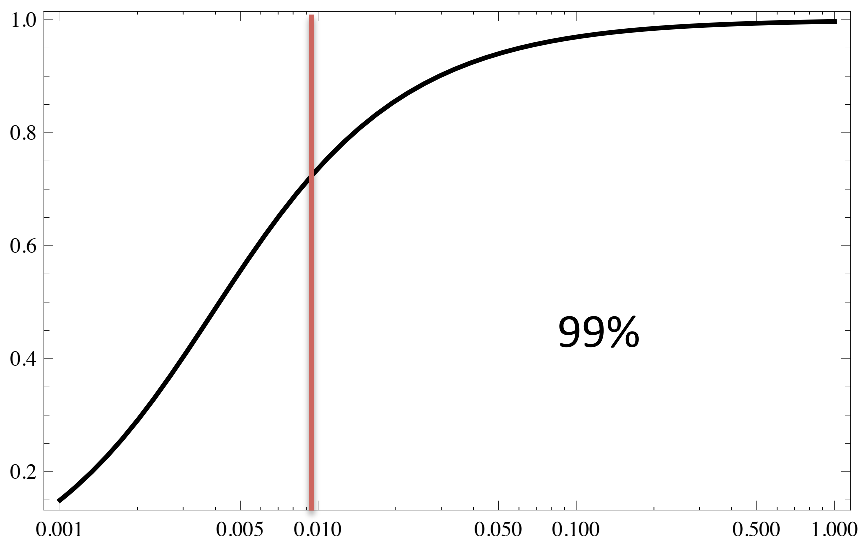
- Important for $kr_i\beta^{1/2} > 1$
  - $r_i = v_i/\omega_{ci}$ is the thermal ion gyroradius
  - Due to ions dropping out of the E x B drift
- Translate this to μ using the resonance condition

$$kc\mu \sim \omega_{ci}/\gamma$$

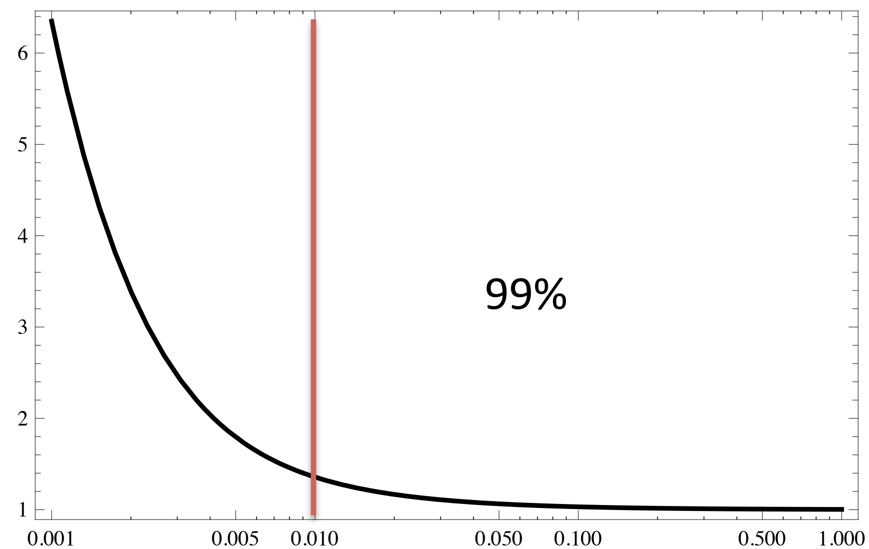
- Example for $\beta = 100$, $T = 10^8$, $\gamma = 3$

Wave Speeds V_ϕ *vs* μ

LCP



RCP



V_ϕ/V_A *vs* μ

Both waves destabilized by CR streaming; slower one sets more stringent limit. Small effect for most cosmic rays, unless $\beta >> 1$.

Wave Damping

- Waves propagating at an angle θ > 0 are thermally damped by the Landau resonance

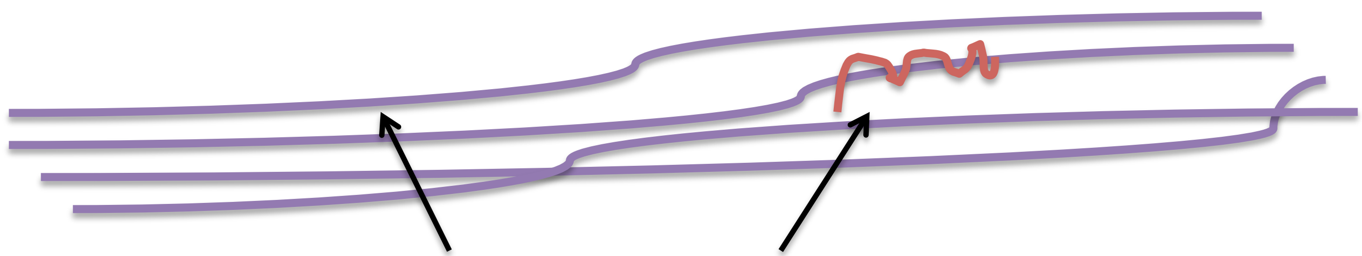
$$\omega = k_{||} v_{||}$$

- For small propagation angles & k not too large,

$$\Gamma_L \sim \frac{\sqrt{\pi}}{4} k v_A (\beta^{1/4}\theta)^2$$

What is θ ?

- Streaming instability growth rate Γ_{cr} maximized for $\theta = 0$.
- Background field line wandering sets a minimum θ, θ_{min} .



- Shearing apart of wave packets argued to be "turbulent damping" (Farmer & Goldreich, Yan & Lazarian).

Landau Damping Rate

$$\theta_{min} \sim \frac{\lambda_{\|}}{\lambda_{\perp}} \rightarrow \left(\frac{r_{cr}}{L}\right)^{1/4}$$

For an anisotropic Goldreich-Sridhar cascade,

$$\lambda_{\|} \sim \lambda_{\perp}^{2/3} L^{1/3}.$$

Leading to

$$\Gamma_L \sim \frac{v_A \beta^{1/2}}{(r_{cr} L)^{1/2}}.$$

Compare to Turbulent Damping

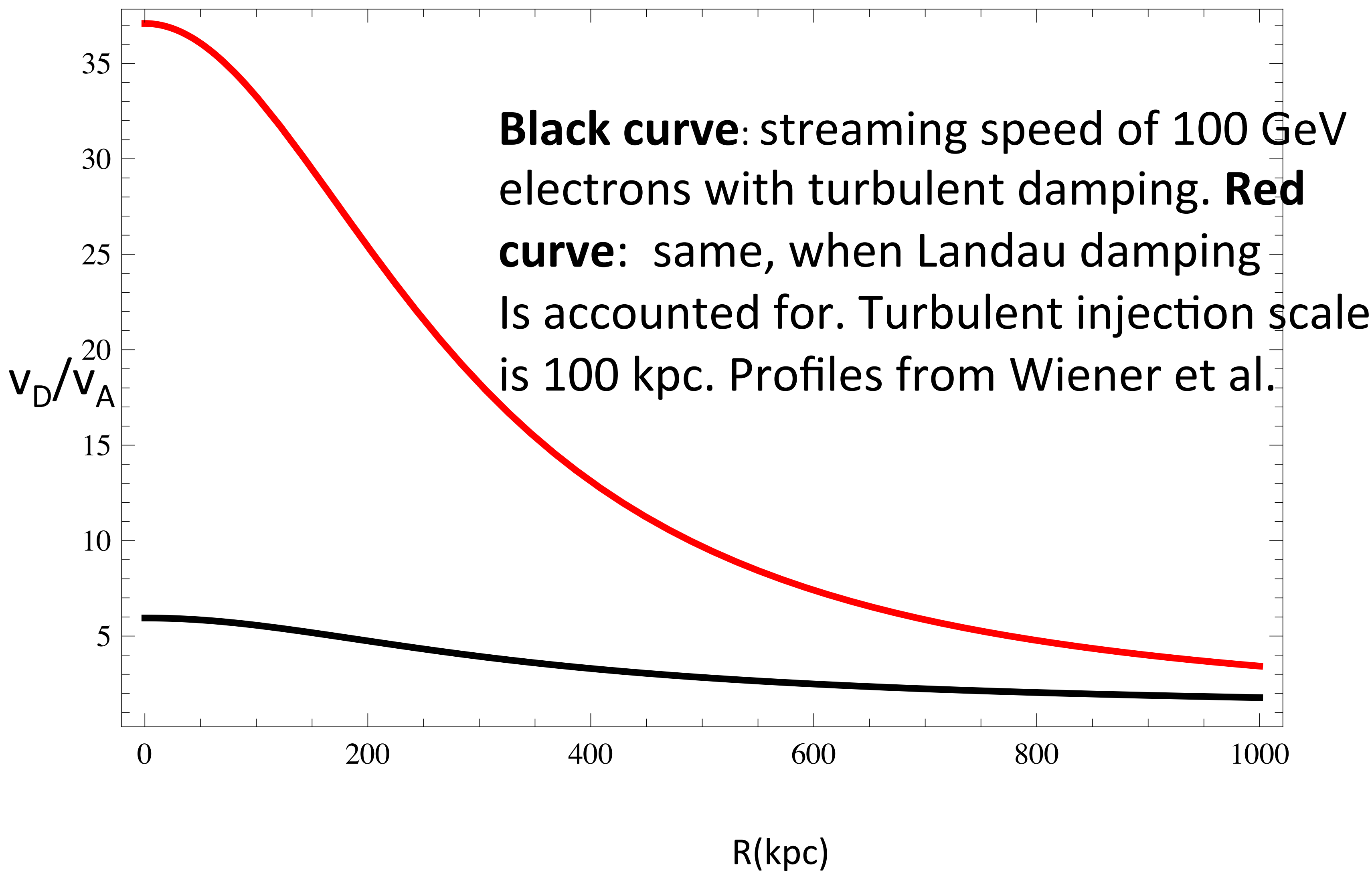
$$\Gamma_t \sim \frac{v_A}{\Lambda_{||}(\lambda_\perp)} \sim \frac{v_A}{(r_{cr}L)^{1/2}}.$$

So,

$$\frac{\Gamma_L}{\Gamma_t} \sim \beta^{1/2} > 1.$$

The implied streaming velocity is~ 5-10 x larger than predicted by Γ_t alone.

Landau + Turbulence in Coma



Conclusions

- At high β, Alfven waves split into 2 branches traveling slower & faster than v_A. Cosmic rays with $\gamma\mu < \beta^{1/2} v_i/c$ excite & scatter from these slower waves.
- Nonparallel propagation leads to rapid Landau damping which scales as $\theta\beta^{1/4}$. Background field line structure imposes $\theta > 0$, which exacerbates "turbulent damping" by $\beta^{1/2}$.