

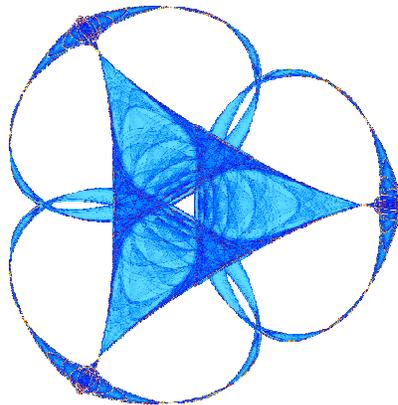
VALIDATION OF SERVICE CONCEPTS FOR OIL DRILLING BY SIMULATION

By

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Validation of Service Concepts for Oil Drilling by Simulation

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Abstract

We wish to validate various service concepts for oil drilling. A Monte Carlo Simulation is used to generate failure data. We calculate functions for the cost of hiring technicians, purchasing spare parts, fuel, and automobiles for each year of the service period. We also calculate a function for the availability of the wells. We optimize the number of technicians to hire each year in order to maximize the availability while minimizing cost and considering different strategies with respect to spare parts.

1 Introduction

Significant uncertainty and risk is associated with operation and maintenance costs for oil drilling. The requirement for service concepts is to guarantee high availability and productivity at low service costs during the service period. Failures of the oil rigs, the number of service technicians and the availability of spare parts are the relevant parameters influencing the service costs. In addition various strategies combining scheduled and unscheduled maintenance and different contract issues for technicians can be considered. To support this process, simulation methods can be used to establish and optimize operation and maintenance strategies.

We develop a model for the service of oil wells and determine service concepts for oil drilling that are optimal with respect to high availability and low costs. We analyze the problem to service 100 identical oil wells that are at a distance of 10 km apart. These are arranged in a 10×10 grid with the home base located 10km south of the grid (see Figure 1). The costs for the labor, spare parts, and transportation are given. The service company hires technicians to carry out scheduled and corrective maintenance on the oil wells.

Input values for our problem are listed in Appendix A. We consider two different types of failures with different failure rates and delivery times for spare parts.

For the specific problem we consider we make the following assumptions.

Assumptions on Technicians:

1. There is only one level of technician.
2. Technicians work five out of seven days each week in their contract (meaning weekends may be split; weekends do not have to be two consecutive days).

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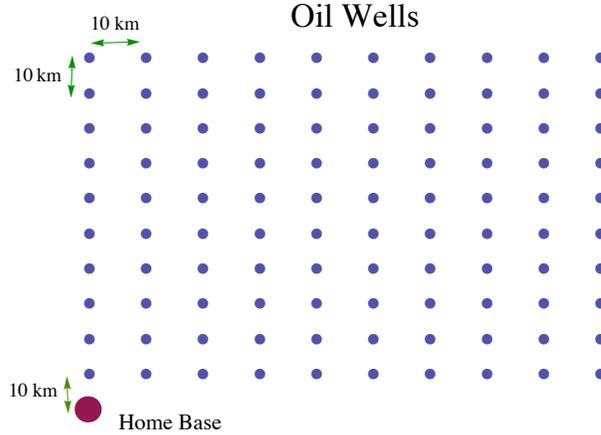


Figure 1: Grid showing location of Oil Wells

3. Technicians work eight hours a day and no more than ten hours per day including overtime.
4. Technicians work in teams of two, because two technicians are needed to perform each type of failure maintenance.

Assumptions on Maintenance:

1. Maintenance shuts down a well for the entire day.
2. No team performs both scheduled and corrective maintenance on the same day.
3. Corrective maintenance is always prioritized over schedule maintenance.
4. A team only fixes one type of failure per day.
5. Scheduled maintenance must be completed in five weeks.

Assumptions on Travel:

1. Travel time between wells is negligible so that one team can perform at most two corrective maintenance tasks in one day or four scheduled maintenance tasks.

In order to solve this problem, we need data about the actual number of failures that will occur in a given time period, because the repair costs and the availability function are dependent on this. Since we cannot predict the future exactly, we look at given failure rates for similar oil wells and use Monte Carlo simulations to predict oil well failure for our problem.

After obtaining data about the number of failures in the service period, we model a cost function and an availability function. These functions will be optimized to determine the number of technicians to hire within the service period. Finally, This model is refined to include spare parts storage. We perform an optimization in order to determine the optimal size of storage.

2 Modeling the Oil Well Failures

We use the Monte Carlo simulation method to predict the number of oil well failures. Monte Carlo simulation is a type of stochastic simulation that relies on repeated random sampling which can be used to determine the properties of some phenomenon [3].

We generate a large sample of random numbers that are uniformly distributed on the interval $[0, 1]$ from Matlab. Then we assign a 1 to any value less than the failure probability and a 0 to any value greater than the failure probability.

Given the daily failure rates, we compute the probabilities in order to use the Monte Carlo method. Let $\lambda(t)$ be the failure rate where t stands for time. Here,

$$\lambda(t) = \frac{f(t)}{R(t)}$$

where $f(t)$ is the failure density function and $R(t)$ is the probability of no failure before time t . Note that

$$R(t) = 1 - F(t)$$

where $F(t)$ is the probability of failure.[1]

$$F(t) = 1 - e^{-\int_0^t \lambda(x) dx}$$

Since our failure rate is constant, the probability of failure in the time period reduces to the formula

$$F(t) = 1 - e^{-\lambda(t)t}$$

We are interested in observing how many failures we have per year. Also, we need to know the maximal number of failures we have in any one day, the longest run of consecutive days with at least one failure per day, and the number of failures per month. We code a Monte Carlo simulation in Matlab to find this information. (see Appendix 8.3)

2.1 Interpretation of Simulation Results

Our simulation for Type I and Type II failures yields the results shown in Figure 2

Years	0.5		1		1.5		2		2.5		3		3.5		4		4.5		5	
1000 Simulations	T1	T2	T1	T2																
Total # of Failures	20	5	14	5	8	5	5	6	6	4	8	4	7	3	11	3	11	4	10	4
# of Days with 2 Failures	3	1	1	0	1	1	1	1	1	1	1	0	1	0	2	1	2	1	1	1
# of Days with 3 Failures	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
# of Days with 4 Failures	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
# of Failures in Month 1	7	2			3	2			3	2			3	2			3	2		
# of Failures in Month 2	7	2			5	2			4	2			3	2			4	2		
# of Failures in Month 3	7	2			3	3			3	2			3	2			4	2		
# of Failures in Month 4	6	2			4	2			3	2			3	2			3	1		
# of Failures in Month 5	7	2			4	3			3	2			4	2			4	2		
# of Failures in Month 6	8	3			4	2			2	2			3	2			3	2		
# of Failures in Month 7			6	2			3	2			3	2			4	2			4	2
# of Failures in Month 8			5	2			3	3			3	2			4	2			4	3
# of Failures in Month 9			4	2			2	2			2	2			3	2			4	2
# of Failures in Month 10			5	2			2	2			4	2			3	2			5	2
# of Failures in Month 11			4	2			2	3			3	2			3	1			4	2
# of Failures in Month 12			4	2			2	2			3	3			4	2			5	2
Blue: Pumpjack																				
Green: Gear Box																				

Figure 2: Failure Simulation Results for Type I and Type II

We need a standard by which to judge our output. Since ideally we want the highest availability possible for our customers, we must be very conservative in how we interpret the results of our Monte

Carlo simulation. We choose to take the maximum number of failures per month and per day; in other words, we assume the worst case scenario from our results in order to plan for the highest number of failures that could occur in the service period.

From our simulation results, we make the following assumptions:

1. There will never be more than three failures in one day.
2. There will never be more than ten failures in a five day period.
3. There will never be more than four consecutive days with a failure.
4. In case of failures in consecutive days, there never occurs three on one of those days.
5. If there are three failures on one day, the two days before and the two days after that triple failure will have no failures.

3 Modeling Cost

3.1 The Total Cost Function

The cost of providing maintenance of the oils wells depends on the number of technicians as well as the number of failures in each year.

The cost function is the sum of the spare parts cost p_i , fuel cost u_i , labor cost l_i and automobile cost a_i for year i of the service period y

$$C = \sum_{i=1}^y (p_i + u_i + l_i + a_i)$$

The following table describes the dependencies of each part of the cost function.

Cost Component	Variable	Dependency
spare parts cost	p_i	# of failures
fuel cost	u_i	# of failures
labor cost	l_i	# of failures, # of technicians
automobile cost	a_i	# of technicians

Contract Types for Technicians

We consider two types of contracts for technicians: Contract Type I are yearly contracts with 6 weeks of vacation, Contract Type II are monthly contracts with no vacation for the technicians. It could be beneficial to hire more technicians during months with scheduled maintenance or during vacation months in order to speed up time to finish. We make the following assumptions for this problem:

1. Monthly contracts are more expensive than annual ones, so each technician would sign a contract for a monthly salary of w^{mo} .
2. A month consists of four forty-hour weeks.
3. Technicians still work a maximum of eight hour days and work no more than five out of seven days in a week.

For Contract type I, we have the cost function given as:

$$C^I = \sum_{i=1}^y (p_i + f_i + l_i + a_i)$$

The cost function for Contract II, C^{II} , is similar to Contract type I, but now the components of the cost function are indexed by the month j and the year i .

$$C^{II} = \sum_{i=1}^y \left[\sum_{j=1}^{12} (p_{i_j} + u_{i_j} + l_{i_j} + a_{i_j}) \right]$$

Here p_{i_j} denotes the spare parts cost for month j in year i , u_{i_j} denotes the fuel cost for month j in year i , l_{i_j} denotes the labor cost for month j in year i , and a_{i_j} denotes the automobile cost for month j in year i .

For the entire service period the cost is

$$C = \beta \left(\sum_{i=1}^y C_i^I \right) + (1 - \beta) \left(\sum_{i=1}^y C_i^{II} \right)$$

where β is a boolean variable that takes a value of 1 if we use Contract type I and 0 if we choose Contract type II.

3.2 Spare Parts Cost

To perform maintenance on the oil wells, we must have spare parts. The cost for spare parts for year i is the sum of the corrective spare parts cost, p_i^c , and the scheduled spare parts cost, p_i^{sch} .

Corrective Spare Parts Cost

There are two types of failures being considered and we will denote variables pertaining to the different failure types by subscripts with $m \in \{1, 2\}$. Each time a well fails, the parts must be ordered at a cost of s^m per well for Type m failures. The corrective parts cost is

$$p_i^c = \sum_{m=1}^2 s^m * f_i^m$$

Scheduled Spare Parts Cost

The cost of spare parts for scheduled maintenance for all five years is calculated from the cost for ordering scheduled parts, s^{sch} , multiplied by the number of wells, n , multiplied by the number of rounds of scheduled maintenance, r .

$$p_i^{sch} = s^{sch} * n * r_i$$

For the entire service period, we have:

$$p^{sch} = \sum_{i=1}^y p_i^{sch}$$

3.3 Fuel Cost Function

In order to calculate the fuel cost, we must first determine the distance our teams must travel to perform maintenance.

Corrective Distance Traveled

The distance traveled each time corrective maintenance must be performed is estimated from the mean distance from home base to any oil well. We regard the wells as being in a grid layout with home base given index $(0, 0)$, the top right corner $(10, 9)$, etc (where (i, j) represents the oil well in the i th row and j th column).

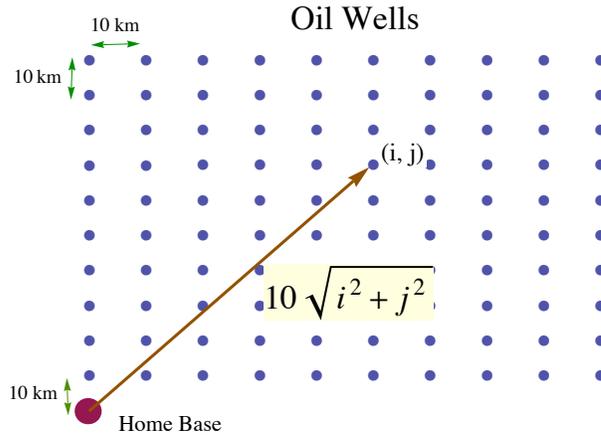


Figure 3: Calculation of Mean Distance

The mean distance is calculated by taking the distance from home base to each oil well, $10\sqrt{i^2 + j^2}$, and taking the average of these n values. This gives us our mean distance for traveling to one oil well (AV_1).

$$AV_1 = \frac{\sum_{i=1}^{10} (\sum_{j=0}^9 (10\sqrt{i^2 + j^2}))}{n}$$

If we have two corrective stops to make, we must estimate the distance traveled to two wells. We estimate this distance from the sum of the average distance to one well, AV_1 , and the average distance from one of the four oil wells near the center of the grid to any other well. A similar approach as before is used, with the grid split into quadrants, so this time with a 5×5 grid. The average distance is the sum of the distances from one of the corner wells to the rest of the wells in its quadrant plus the distance to the remaining 3 wells divided by the number of wells other than the first well we traveled to. We would not travel to the same well twice in a row to perform corrective maintenance. Thus the distance to travel to two wells (AV_2) is:

$$AV_2 = \frac{\sum_{i=1}^5 (\sum_{j=1}^5 (10\sqrt{i^2 + j^2})) * 4 + 20 + 10\sqrt{2}}{n - 1}$$

Since our teams can only work up to 10 hours in one day, one team can perform at most two corrective maintenance tasks in one day, assuming that travel time is negligible. We assume the technicians travel back to home base on the same route they took to the wells, so the total distance traveled is twice these estimates.

$$d_{(1)}^c = 2AV_1$$

$$d_{(2)}^c = 2(AV_1 + AV_2)$$

Scheduled Distance Traveled

We assume that no team would perform both scheduled and corrective maintenance on the same day. Given the time for scheduled maintenance per well to be 2 hours we can perform scheduled maintenance on four wells per day per team. We group the wells into pods of 4. In addition, we assume:

1. In an eight-hour work day, one team would drive out, complete maintenance on a pod of four wells and drive home.
2. The team drives to the pod nearest to home base first.

The distance traveled between the four wells in any one pod is 40 km (10 km between each well).

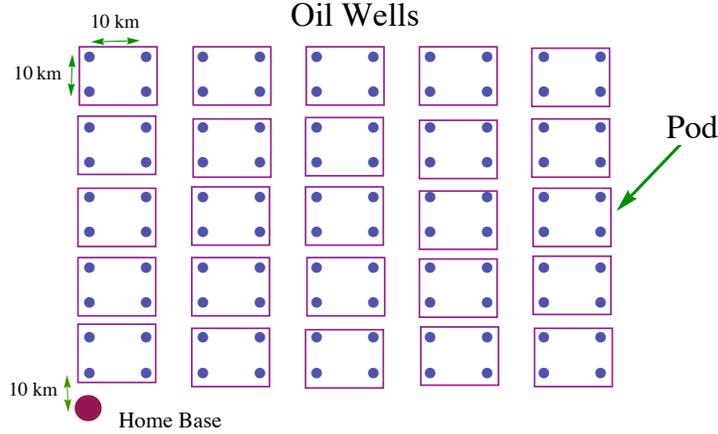


Figure 4: Pod Visualization

For one team to perform scheduled maintenance on one pod of 4 wells, the total distance traveled is 40 km (for all 25 pods) plus twice the distance to home base.

We assume that when traveling to the pods, the teams would travel only to the closest well in that group of four wells. To calculate the distance traveled to complete one round of maintenance, we sum the distance from home base to the well in the bottom left corner of each pod, i.e.

$$d^{sch} = 40 * 25 + 2 * \sum \left(\sum (10\sqrt{i^2 + j^2}) \right)$$

where these sums are taken over i from 1 to 10 in increments of 2 and j from 0 to 9 also in increments of 2.

3.3.1 Fuel Cost

Total fuel costs for year i will be

$$u_i = u_i^c + u_i^{sch}$$

Corrective Fuel Costs

The cost for fuel for corrective maintenance is calculated from the distance traveled for corrective maintenance $d_{(j)}^c$, ($j = 1, 2$) multiplied by the price of fuel, z . For one stop, we have:

$$u_{(1)}^c = d_{(1)}^c * z$$

For two stops

$$u_{(2)}^c = d_{(2)}^c * z$$

Total fuel cost for corrective maintenance in one year then comes to the sum of the costs for one stop trips and the costs for two stop trips. We must also add the cost of fuel for triple failures.

We will write f_{ij}^{sm} to be the number of single failures of type m in month j of year i , f_{ij}^{dm} the number of double failures of type m in month j of year i , and f_{ij}^{tm} the number of triple failures of type m in month j of year i .

The variable f_i^{sm} is the number of single failures of Type m in the entire year i , and similarly, f_i^{dm} is the number of double failures of Type m in the entire year i , and f_i^{tm} is the number of triple failures of Type m in the entire year i .

The variable f_i^m denotes the total number of failures (single, double, and triple all included) of Type m in year i .

Since it takes 8 hours (an entire workday) to fix one Type I failure, we must send one team to each failure. They make only one stop. The fuel cost for Type I failures for year i , u_i^{c1} , is

$$u_i^{c1} = u_{(1)}^c * f_i^{s1} + 2u_{(2)}^c * f_i^{d1} + 3u_{(1)}^c f_i^{t1}$$

One team can fix two failures of type II each day. In order to minimize cost, if there are enough teams, one team will be sent to fix one well each. If there are two failures and we have only one team, that team will be sent to fix both wells in one day, in order to maximize availability. But if we have more than one team, then two teams will be sent to fix one each. Triple failures will be treated similarly. If we have one team, the team will fix two wells one day and one the next. If we have two teams, then one will fix two wells and the other team will fix the remaining well (on the same day). If we have three or more teams, in order to minimize the cost, we will send one team to each failed well.

$$u_i^{c2} = \begin{cases} u_{(1)}^c f_i^{s2} + u_{(2)}^c * f_i^{d2} + (u_{(1)}^c + u_{(2)}^c) * f_i^{t2} & \text{if } t_i = 2 \\ u_{(1)}^c * f_i^{s2} + 2 * u_{(1)}^c * f_i^{d2} + (u_{(1)}^c + u_{(2)}^c) * f_i^{t2} & \text{if } t_i = 4 \\ u_{(1)}^c * f_i^2 & \text{if } t_i > 4 \end{cases}$$

The total corrective fuel cost for year i would then be

$$u_i^c = \sum_{m=1}^2 u_i^{cm}$$

Scheduled Fuel Costs

Scheduled fuel cost is found by taking the distance traveled for one round of scheduled maintenance, d^{sch} , multiplied by the number of rounds, r_i , multiplied by the cost for fuel, z .

$$u_i^{sch} = d^{sch} * r_i * z$$

For the entire service period, we have:

$$u^{sch} = \sum_{i=1}^5 u_i^{sch} = d^{sch} * r * z$$

3.4 Labor Cost

The labor costs for year i will be found from the annual wage for each technician, w^a , times the number of technicians in year i , t_i , plus the weekend wage, w^w times the weekend hours, o_i^w . The technicians are hired on a yearly contract, so t_i might change each year.

$$l_i = w^a * t_i + w^w o_i^w$$

For Contract type II, the labor cost is:

$$l_{i,j} = w^{mo} t_{i,j} + w^w o_{i,j}^w$$

Weekend Hours

Technicians will only have to work on the weekend if there are failures during a month with scheduled maintenance. We assume during these months, the teams are doing this scheduled maintenance all 5 days of their workweek. A failure will take priority over scheduled maintenance, and the scheduled maintenance will be done on the weekend.

The weekend hours required depends on the year (due to different scheduled maintenance requirements for the first year), on the number of failures (single, double, triple), and on the number of technicians.

If we have more than one team, a team can work on the scheduled maintenance while others work on the corrective. In this case, the teams will be able to finish all scheduled maintenance before the 5 week deadline. So we will not have to delay scheduled maintenance to the weekend and $o_i^w = 0$, for more than one team.

We consider the worst case, that any failures in the month with the scheduled maintenance happen in the beginning of the month. This is the worst case because of the longer (close to one whole month) shipping time for Type I spare parts. So, in this case, any failures in the month with scheduled maintenance will delay scheduled maintenance.

If there is only one team, each time a Type I failure occurs, the team must be sent to fix the failure. This repair takes the entire day. So a double failure takes two days and a triple failure three days to repair all the failed wells. We must pay the technicians for two weekend days (because there are 2 people per team) times the number of days it takes to complete the repair.

For Type II, one team can fix up to two failures in one day. Each single or double failure adds only one weekend day while each triple failure adds two.

$$o_1^w = \begin{cases} \sum_{j \in \{2,5,11\}} \{2(f_{1_j}^{s1} + 2f_{1_j}^{d2} + 3f_{1_j}^{t1}) + 2(f_{i_j}^{s2} + f_{1_j}^{d2} + 2f_{i_j}^{t2})\} & \text{if } t_i = 2 \\ 0 & \text{if } t_i \geq 4 \end{cases}$$

$$o_{i \in \{2,3,4,5\}}^w = \begin{cases} \sum_{j \in \{5,11\}} \{2(f_{1_j}^{s1} + 2f_{1_j}^{d2} + 3f_{1_j}^{t1}) + 2(f_{i_j}^{s2} + f_{1_j}^{d2} + 2f_{i_j}^{t2})\} & \text{if } t_i = 2 \\ 0 & \text{if } t_i \geq 4 \end{cases}$$

3.5 Automobile Cost

Each team of two technicians requires one car. We are given a^r as the rental cost for cars. The cost for automobiles is

$$a_i = a^r * \frac{t_i}{2}$$

3.6 Spare Parts Storage Optimization

3.6.1 Spare Parts Storage Model

Another problem to consider is whether having a storage facility for spare parts would be optimal. We reformulated the model to include spare parts storage and looked at whether this would increase our availability or lower the cost.

We assume:

1. We buy spare parts storage space once and use the same amount of space for all five years of service.
2. If we have more failures than spare parts in the storage space, we must buy more spare parts with a shipping delay of δ^m , for each type m failure.
3. Spare parts for storage are purchased twice a year.

3.6.2 Spare Parts Storage Cost Function

We break down the cost into 6 month periods each year. Let f_l be the number of failures in the l th 6 month period ($l = 1, 2, \dots, 10$). At the beginning of the five year period, we can purchase enough space to store σ spare parts at a set up cost of s^{su} per part. The cost for the l th 6 month period is:

$$\gamma_l = s^{st} \sigma_l + s^c * \max\{(f_l - \sigma_l), 0\}$$

We purchase area to store σ parts with a set up cost of s^{su} . This is only purchased once, at the beginning of the five year period. The total cost spare parts and spare parts storage for all five years, γ is

$$\gamma = s^{su} \sigma + \sum_{l=1}^{2y} \left[\sum_{m=1}^2 (s^{c_m} f_l^m - \sum_{f_l^m > \sigma_l^m} s^{st_m} \sigma_l^m) \right]$$

given the constraints that

$$\begin{aligned} \sigma &\leq \max_{l=1, \dots, 2y} \{f_l^1 + f_l^2\}, \\ \sigma_l^m &\leq f_l^m \quad \text{for each } l \text{ and each } m, \\ \text{and} \\ \sigma_l^1 + \sigma_l^2 &\leq \sigma \quad \text{for each } l. \end{aligned}$$

3.6.3 Cost Optimization

This optimization problem was coded and solved in glpk.

4 Modeling Availability

Our main goal is to maximize the availability of the oil wells. We define availability, A , to be the the fraction of the oil wells uptime, U , over the maximal possible uptime, U_{max} . The maximum possible uptime is defined to be the service period minus the downtime for scheduled maintenance.

$$A = \frac{U}{U_{max}}$$

4.1 Availability for Contract type I

We calculate the maximum available days for each year, MA_i , by subtracting the time needed for scheduled maintenance, which equals the number of rounds r_i of maintenance in year i .

$$MA_i = (365 \text{ days} - r_i)n$$

The total maximum uptime is then

$$U_{max} = \sum_{i=1}^y MA_i$$

We factor in 6 weeks of vacation for the technicians. We assume the following:

1. Each team of two technicians takes vacation together.
2. Each technician is entitled to three two-week periods of vacation in a calendar year.
3. Vacation cannot be taken during the five week scheduled maintenance period.
4. If more than one team is contracted in a year, the teams may not take vacation at the same time.

For Type I failures, only one can be fixed per team per day. If we have one team, each double failure of Type I will take two days to repair. Each triple failure will take three days to repair; one well will be delayed by one day, and another well will be delayed by two days. Thus, each triple failure adds three days of delay.

If we have two teams, only triple failures of Type I will delay maintenance by one day. For three or more teams, there will be no additional delay for multiple failures.

For Type II failures, if we have only one team, we can only perform up to two corrective repairs in one day. Each time there are three failures, one of those failures must be delayed one day before it can be repaired. So for each triple failure, we have one less day that the third well is available.

Let δ^m denote the spare parts delivery time (or shipping delay) for failures of Type m . For each failure of Type m , we will always have δ^m days of delay time to ship and 1 more day of the well being shut down to perform maintenance. Multiple failures delay repair if there are only one or two teams.

If $\delta^m < 14$, the days to ship the parts for Type m failures is not more than the 14 days of vacation, and so we assume the worst case that we have to delay repair for all 14 days.

To calculate A_i^I , the days when the oil wells are available for year i with Contract type I, is

$$A_i^I = \begin{cases} MA_i - \{ \sum_{k \neq v} [(1 + \delta^1) * f_{i_k}^1 + f_{i_k}^{d1} + 3f_{i_k}^{t1} + (1 + \delta^2)f_{i_k}^2 + f_{i_k}^{t2}] \\ + \sum_{k=v} [(1 + \max\{\delta^1, 14\}) * f_{i_k}^1 + f_{i_k}^{d1} + 3f_{i_k}^{t1} + (1 + \max\{\delta^2, 14\})f_{i_k}^2 + f_{i_k}^{t2}] \\ + (s_{2i} + s_{2i-1})\delta^1 + (p_{2i} + p_{2i-1})\delta^2 \} & \text{if } t_i = 2 \\ MA_i - \{ \sum_{k \neq v} [(1 + \delta^1)f_{i_k}^1 + f_{i_k}^{t1} + (1 + \delta^2)f_{i_k}^2] \\ + \sum_{k=v} [(1 + \max\{\delta^1, 14\})f_{i_k}^1 + f_{i_k}^{t1} + (1 + \max\{\delta^2, 14\})f_{i_k}^2] \\ + (s_{2i} + s_{2i-1})\delta^1 + (p_{2i} + p_{2i-1})\delta^2 \} & \text{if } t_i = 4 \end{cases}$$

where v indexes the months technicians take vacation $v \in \{1, 7, 10\}$.

s_{2i-1}, s_{2i} are storage of type 1 and p_{2i-1}, p_{2i} are storage of type II for each semi annual year i .

We are interested in the availability for each year. The availability (for Contract type I) for year i is

$$\alpha_i^I = \frac{A_i^I}{MA_i}$$

The total uptime (in days) for the entire service period is

$$U^I = \sum_{i=1}^y A_i^I$$

4.2 Availability for Contract type II

We now calculate the uptime (or available days) of the wells per month. Now since we are calculating on a monthly contract, we have a maximum monthly availability of

$$MA_{i_j} = (30 - r_{i_j}) * 100$$

where we assume there are 30 days in a month and we have a field of 100 oil wells.

The number of days the oil wells are available for Contract type II is calculated by the following equation:

$$\sum_{j=1}^{12} A_{i_j}^{II} = \begin{cases} \sum_{j=1}^{12} (MA_{i_j}) - \sum_{j=1}^{12} [(1 + \delta^1) * f_{i_j}^1 + f_{i_j}^{d_1} + 3f_{i_j}^{t_1} + (1 + \delta^2)f_{i_j}^2 + f_{i_j}^{t_2}] \\ \quad + (s_{2i} + s_{2i-1})\delta^1 + (p_{2i} + p_{2i-1})\delta^2 & \text{if } t_i = 2 \\ \sum_{j=1}^{12} MA_{i_j} - \sum_{j=1}^{12} [(1 + \delta^1)f_{i_j}^1 + f_{i_j}^{t_1} + (1 + \delta^2)f_{i_j}^2] \\ \quad + (s_{2i} + s_{2i-1})\delta^1 + (p_{2i} + p_{2i-1})\delta^2 & \text{if } t_i = 4 \end{cases}$$

For year i , the availability (for Contract type II) is

$$\alpha_i^{II} = \frac{\sum_{j=1}^{12} A_{i_j}^{II}}{\sum_{j=1}^{12} MA_{i_j}}$$

For the entire service period, with Contract type II,

$$A^{II} = \frac{\sum_{i=1}^y \alpha_i^{II}}{U_{max}}$$

4.3 Spare Parts Storage Availability

We were only able to include spare parts storage into the Contract type I situation. Since we subtracted a day for each failure when calculating the number of days the wells are available, we add back the number of failures that will be taken care of with storage.

The new function for the number of available days for Contract type I (including spare parts storage) is:

$$A_i^{Ist} = A_i^I + \sum_{m=1}^2 (\delta^m + 1) \left(\sum_{l=2i-1}^{2i} \sigma_l^m \right)$$

For the entire service period, we have

$$A^{Ist} = A^I + \sum_{m=1}^2 (\delta^m + 1) \left(\sum_{l=1}^{2y} \sigma_l^m \right)$$

4.4 Full Availability Function

The number of days the oil wells are available for each year i , A_i , is

$$A_i = \beta A_i^{Ist} + (1 - \beta) A_i^{II}$$

The availability for each year is

$$\alpha_i = \frac{A_i}{MA_i}$$

The availability for the entire service period is

$$A = \frac{\sum_{i=1}^y A_i}{U_{max}}$$

5 Results

We used the inputs and values given in Appendix A for our calculations. Certain costs are fixed, meaning they do not depend on how many failures occur or the number of technicians hired.

Fixed Cost Component	Cost
Scheduled Maintenance Spare Parts Cost	\$550,000
Fuel Cost for Scheduled maintenance	\$5,457.43

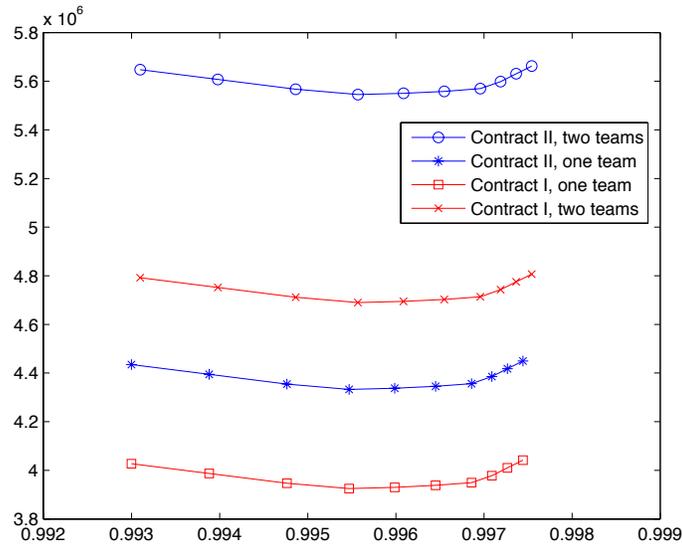


Figure 5: Availability vs Total Cost for One and Two Teams, Contract type I and Contract type II

From figure 5, we can see that hiring two teams costs much more but has a negligible influence on the availability. Since the total number of failures is low, hiring more than two teams will increase the costs significantly but will not have a significant raise in availability. Comparing these two different contracts, we would suggest hiring one team with Contract Type I.

Two teams	Availability	Total costs		One team	Availability	Total costs
s=8	0.9931	4.79E+06		s=8	0.993	4.03E+06
s=9	0.994	4.75E+06		s=9	0.9939	3.99E+06
s=10	0.9949	4.71E+06		s=10	0.9948	3.95E+06
s=11	0.9956	4.69E+06		s=11	0.9955	3.93E+06
s=12	0.9961	4.69E+06		s=12	0.996	3.93E+06
s=13	0.9965	4.70E+06		s=13	0.9964	3.94E+06
s=14	0.997	4.71E+06		s=14	0.9969	3.95E+06
s=15	0.9972	4.74E+06		s=15	0.9971	3.98E+06
s=16	0.9974	4.77E+06		s=16	0.9973	4.01E+06
s=17	0.9975	4.81E+06		s=17	0.9974	4.04E+06

Figure 6: Availability vs Total Cost for One and Two Teams, Contract type 1

One team	Availability	Total costs		Two teams	Availability	Total costs
s=8	0.993	4.43E+06		s=8	0.9931	5.65E+06
s=9	0.9939	4.39E+06		s=9	0.994	5.61E+06
s=10	0.9948	4.35E+06		s=10	0.9949	5.57E+06
s=11	0.9955	4.33E+06		s=11	0.9956	5.55E+06
s=12	0.996	4.34E+06		s=12	0.9961	5.55E+06
s=13	0.9964	4.35E+06		s=13	0.9965	5.56E+06
s=14	0.9969	4.36E+06		s=14	0.997	5.57E+06
s=15	0.9971	4.39E+06		s=15	0.9972	5.60E+06
s=16	0.9973	4.42E+06		s=16	0.9974	5.63E+06
s=17	0.9974	4.45E+06		s=17	0.9975	5.66E+06

Figure 7: Availability vs Total Cost for One and Two Teams, Contract type II

6 Future Work

Given more time, we would like to generalize our model to more than two different failure types.

Moreover, we could have added multiple levels of technicians. This might be more realistic as different technicians may not be qualified to repair all types of failures.

7 Acknowledgements

We would like to acknowledge the Institute for Mathematics and its Applications, the University of Calgary, PIMS, NSERC, mprime, and the Government of Alberta.

8 Appendices

8.1 Appendix 1 General Input for Oil Well Problem

Example Values for 5 year Service

Number of oil wells, n	100
Distance between oil wells	10 km
Distance to Home Base	10 km
Service Period, y	5 years
Technician per team	2

Scheduled Maintenance Information:

Intervals: 1 month, 3 months, half-annual

Rounds of Scheduled Maintenance	Total
r_1	3
r_i for $i = 2, 3, 4, 5$	2
$r = \sum_{i=1}^5 r_i$	11

Scheduled maintenance spare parts cost, s^{sch}	\$500 per well
Duration for scheduled maintenance	2 hours

Corrective Maintenance:

	Type I	Type II
Spare Parts Cost, s^{cm}	\$30,000	\$6000
Technicians Needed	2	2
Repair Duration (in hours)	8	4
Spare Parts Delivery Time, δ^m , (in days)	21	9

Wages:

Annual Wage for each Tech, w^a	\$83,200
Weekend Wage, w^w	\$480 per tech per day
Maximum Hours of Overtime on Weekends	8 hours
Monthly Wage, w^{mo}	\$10,400

Automobile Information:

Yearly Rental cost for transportation, a^r	\$4000
Fuel cost per 100 km, z	\$11
Capacity per car	2 technicians

Spare Parts Storage

Costs for spare parts for corrective maintenance, s^{stm}	\$15,000
Costs for spare parts for scheduled maintenance	\$250
Set up costs, s^{su}	\$500,000
Stock keeper costs per year	\$1,380

Distances

Estimated distance to one well, AV_1	76.8072
Estimated distance to a second well, AV_2	32.4016
Estimated distance traveled in one stop, $d_{(1)}^c$	153.6144
Estimated distance traveled in two stops, $d_{(2)}^c$	218.4176
Distance traveled for scheduled maintenance, d^{sch}	4510.3

Fuel

Fuel cost for traveling one stop, $u_{(1)}^c$	\$16.90
Fuel cost for traveling two stops, $u_{(2)}^c$	\$24.03
Fuel cost for scheduled maintenance, u^{sch}	\$5457.43

8.2 Appendix 2

Failure rates for the Oil Well Problem

Table 1 shows the input used for simulation for the failures of Type I and Type II failures.

Failures rates for Type I and Type II are shown in Figures 8 and 9

To ensure more accurate results, we do this simulation 1,000 times for all 100 wells for a $y=5$ year period. We limited the simulation to 1,000 times because of computation time constraints.

Table 1: Failure Rates Input Data for Type I and Type II Failures

Year (Age of Well)	Failure Rate Type I	Failure Rate Type II
0.5 (6 months)	0.089	0.007
1.0 (12 months)	0.044	0.005
1.5 (18 months)	0.026	0.007
2.0 (24 months)	0.010	0.009
2.5 (30 months)	0.011	0.007
3.0 (36 months)	0.014	0.004
3.5 (42 months)	0.018	0.003
4.0 (48 months)	0.023	0.004
4.5 (54 months)	0.028	0.004
5.0 (60 months)	0.033	0.005

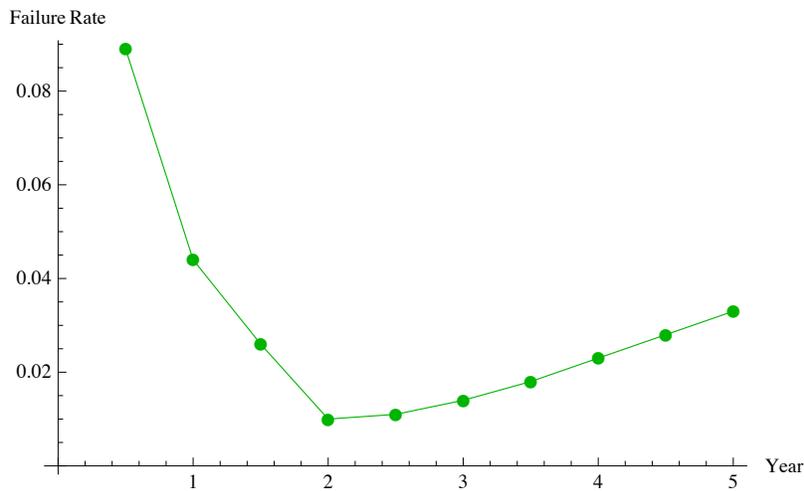


Figure 8: Failure Rates Type I versus Time

8.3 Appendix 3 Code for Monte Carlo Simulation

Monte Carlo code

```
% Here, well1 and well2 are matrices that contain the simulation
results for 100 wells in a six month period. Also, prob1 and
prob2 represent the probabilities for the two different types
of failure.
```

```
for j = 1:1000
    well1 = zeros(100,nday);
    well2 = zeros(100,nday);
    rng shuffle;
    for k = 1:nday

        for i = 1:100
            x = rand ;
            toss = (x < prob1 );
```

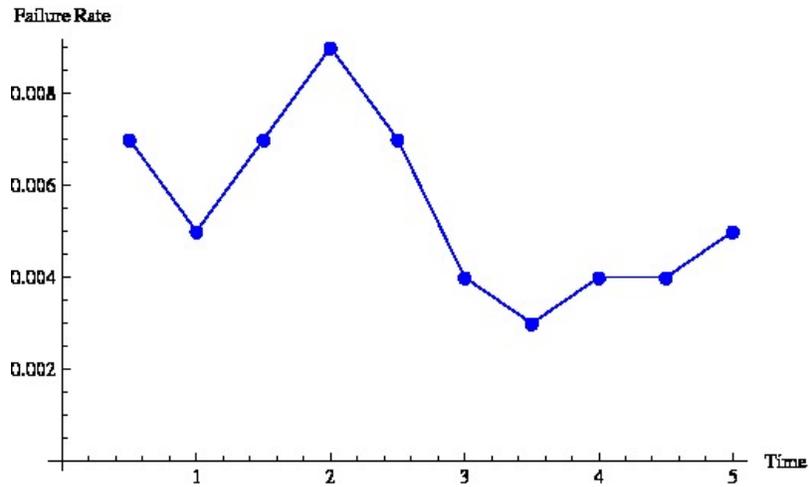


Figure 9: Failure Rates Type II versus Time

```

        well1(i,k) = toss;
    end
end

for k = 1:nday

    for i = 1:100
        x = rand ;
        toss = (x < prob2 );
        well2(i,k) = toss;
    end
end

```

```

% This finds the maximum run length of consecutive days of failure.
q = diff([0 (sumwell2w1+sumwell2w2) 0] > 0 );
v = find(q == -1) - find( q == 1);
if ~isempty(max(v))
maxv(j) = max(v);
end

```

8.4 Appendix 4

Code for Optimization of Spare parts Problem

```

/* Decision variables */
#var p >=0; /* soldier */
var p1 >=0; /* train */
var p2 >=0; /* train */
var p3 >=0; /* train */
var p4 >=0; /* train */
var p5 >=0; /* train */
var p6 >=0; /* train */
var p7 >=0; /* train */

```

```

var p8 >=0; /* train */
var p9 >=0; /* train */
var p10 >=0; /* train */
var s >=0; /* soldier */
var s1 >=0; /* train */
var s2 >=0; /* train */
var s3 >=0; /* train */
var s4 >=0; /* train */
var s5 >=0; /* train */
var s6 >=0; /* train */
var s7 >=0; /* train */
var s8 >=0; /* train */
var s9 >=0; /* train */
var s10 >=0; /* train */
/* Objective function */
minimize z: 30000*100+50000*s - 15000*(s1+s2+s3+s4+s5+s6+s7+s8+s9+s10)+6000*43 -
3000*(p1+p2+p3+p4+p5+p6+p7+p8+p9+p10);
/* Constraints */
s.t. Finishing : s <= 26;
#s.t. Carpentry : x1 + x2 <= 80;
s.t. Demand1 : s1 <=20;
s.t. Demand2 : s1+p1 <=s;
#s.t. Demand : s1 <= s;

s.t. Demand3 : s2 <= 14;
s.t. Demand4 : s2+p2 <=s;
s.t. Demand5 : s3 <=8;
s.t. Demand6 : s3+p3 <=s;
s.t. Demand7 : s4 <=5;
s.t. Demand8 : s4 +p4<=s;
s.t. Demand9 : s5 <=6;
s.t. Demand10 : s5+p5 <=s;
s.t. Demand11 : s6 <=8;
s.t. Demand12 : s6+p6 <=s;
s.t. Demand13 : s7 <=7;
s.t. Demand14 : s7+p7 <=s;
s.t. Demand15 : s8 <=11;
s.t. Demand16 : s8+p8 <=s;
s.t. Demand17 : s9 <=11;
s.t. Demand18 : s9+p9 <=s;
s.t. Demand19 : s10 <=10;
s.t. Demand20 : s10+p10 <=s;
#s.t. Finishing1 : p <= 5;
#s.t. Carpentry : x1 + x2 <= 80;
s.t. Demand21 : p1 <=5;
#s.t. Demand22 : p1 <=p;
#s.t. Demand : s1 <= s;
s.t. Demand23 : p2 <=5;
#s.t. Deman24 : p2 <=p;
s.t. Demand25 : p3 <=5;

```

```
#s.t. Demand26 : p3 <=p;  
s.t. Demand27 : p4 <=6;  
#s.t. Demand28 : p4 <=p;  
s.t. Demand29 : p5 <=4;  
#s.t. Demand30 : p5 <=p;  
s.t. Demand31 : p6 <=4;  
#s.t. Demand32 : p6 <=p;  
s.t. Demand33 : p7 <=3;  
#s.t. Demand34 : p7 <=p;  
s.t. Demand35 : p8 <=3;  
#s.t. Demand36 : p8 <=p;  
s.t. Demand37 : p9 <=4;  
#s.t. Demand38 : p9 <=p;  
s.t. Demand39 : p10 <=4;  
#s.t. Demand40 : p10 <=p;  
#s.t. Demand41 : p+s <=26;  
end;
```

References

- [1] R. E. Barlow and F. Proschan, *Mathematical Theory of Reliability*, SIAM, Philadelphia, PA, 1996.
- [2] G. L. Nemhauser and L. A. Wolsey, *Integer and Combinatorial Optimization*, John Wiley and Sons, New York, 1988.
- [3] M. H. Kalos and P. A. Whitlock, *Monte Carlo Methods*, John Wiley and Sons, New York, 1986.