Applying Prandtl's Lifting-Line Theory to Formation Flight

Peter J. Humbert

Department of Aerospace Engineering and Mechanics, University of Minnesota

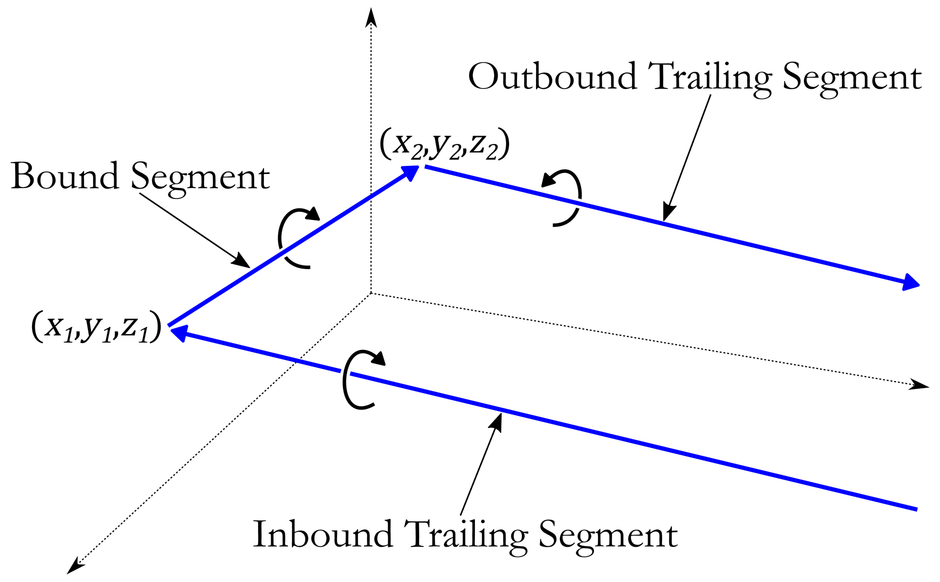
Background

The motivation for this project was threefold:

- to obtain predictions for formation flight forces and moments without the computational resource requirements of current methods;
- to optimize aircraft design for the greatest combination of safety and energy efficiency; and
- to study how aircraft in formation must be flown.

These motivations can be addressed by simulating the influence an aircraft has on the aerodynamics of nearby aircraft. Several approaches exist for creating such a simulation. For this project, a form of Prandtl's lifting-line theory is used despite Prandtl's version not traditionally being used for multiple bodies.

In lifting-line theory, a lifting surface is defined by one or more horseshoe vortices. Each vortex, in turn, is composed of a bound segment (along the lifting-line) and two trailing segments (in the direction of the airflow), the collection of which form the lifting surface's wake.[1] The physical manifestation of these vortices and a diagram of a single horseshoe vortex are shown at right.



This project's lifting-line simulator can solve a system of equations to yield the strengths of these vortices, with the results thus far agreeing with established data. These strength values are then used to directly solve for the aerodynamic forces and moments acting on the lifting surfaces.

References

[1] McBain, G. D. *Theory of Lift: Introductory Computational Aerodynamics in MATLAB/OCTAVE*. Chichester, West Sussex, U.K.: Wiley, 2012. Print.

[2] Phillips, W. F.; Snyder, D. O., "Modern Adaptation of Prandtl's Classic Lifting-Line Theory," Journal of Aircraft, Vol. 37, No. 4, 2000, pp. 662-670.

Acknowledgements

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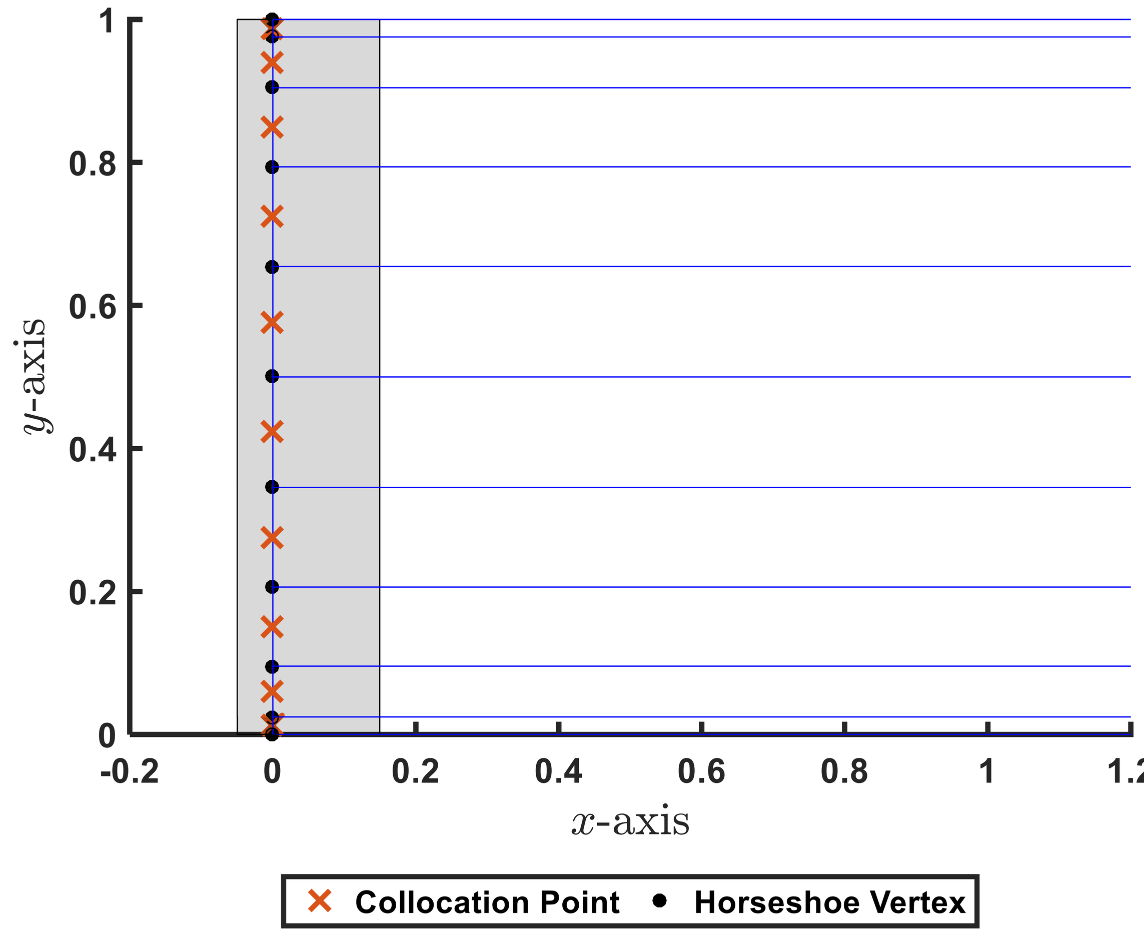
Process

Implementing the lifting-line simulator began with deciding which programming language to use; MATLAB was selected for its versatility and high-level features. The requirements for implementing the simulator were:

1. defining a data structure containing all the requisite geometric features of the wing to be simulated;
2. calculating quantities dependent on the wing's geometry; and
3. creating a routine that finds the strength of each horseshoe vortex.

The figure below is the result of the first two requirements and contains the following features:

- The grey area represents the lifting surface, a flat plate rotated 10 degrees with respect to the airflow.
- The blue lines parallel to the x-axis represent the unbound horseshoe vortex segments (equivalently, the wake).
- The blue line along which the collocation points and horseshoe vertices lie represents the lifting-line (placed at the quarter-chord line).



The simulator is based on the "Modern Adaptation of Prandtl's Classic Lifting-Line Theory" developed by Phillips and Snyder,[2] the central tenet of which is:

$$\underbrace{2\left|\left(\vec{v}_\infty + \sum_{j=1}^{N} \vec{v}_{ji} G_j\right) \times \vec{\zeta}_i\right| G_i}_{\text{Aerodynamic Force at Station } i} = \underbrace{C_{Li}(\alpha_i, \delta_i)}_{\text{Sectional Lift Coefficient at Station } i}$$

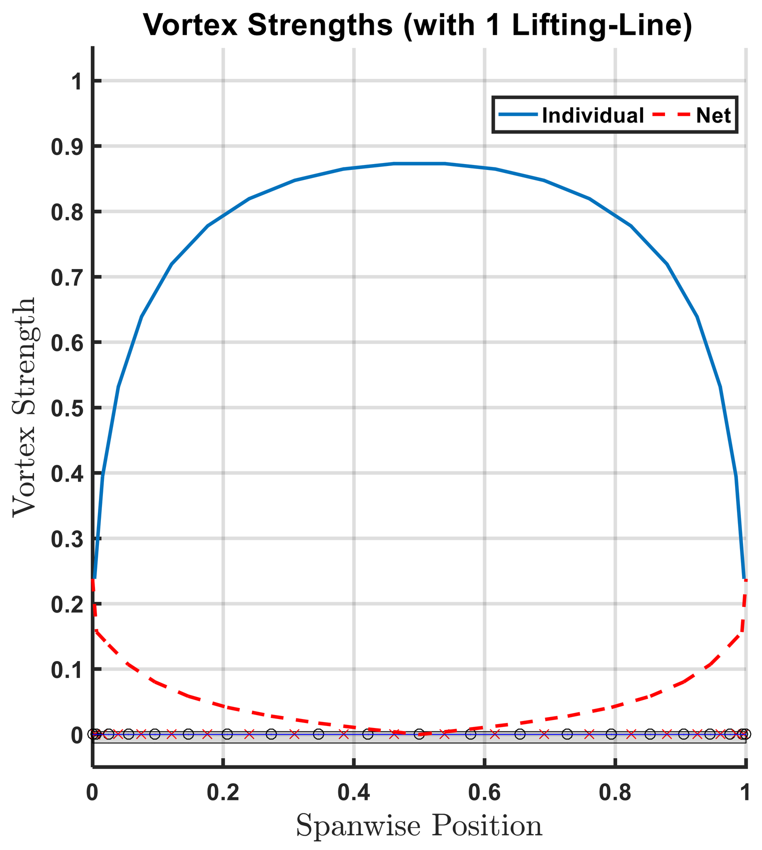
With the geometry definition in place, each variable in the above equation could be defined. The third requirement for implementing the simulator could then readily be fulfilled, and each value of G_i, the dimensionless horseshoe strength at station i, could be found.

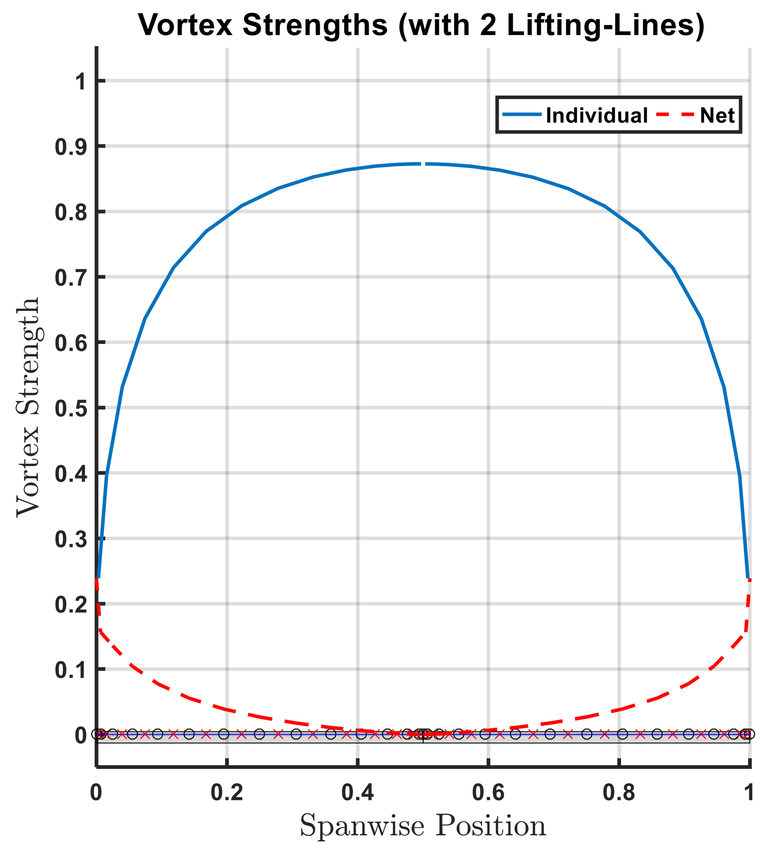
Results

The plots below show the following calculated values for a flat plate ($AR = 5$, $AOA = 5°$):

- horseshoe vortex strengths plotted by collocation point; and
- the resulting net vortex strengths (i.e., the differences in horseshoe strengths between adjacent vortices) plotted by horseshoe vertex.

The left plot uses a single lifting-line with twenty horseshoe vortices while the right plot uses two lifting-lines with fourteen horseshoe vortices each.





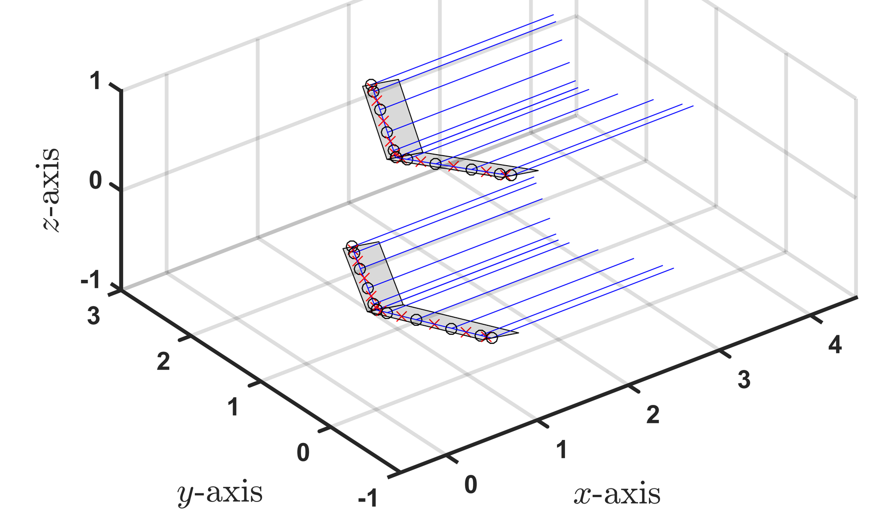
The close agreement between the two cases validates the simulator's handling of multiple lifting surfaces. The decrease in net strength toward the middles of the spans is the result of interior trailing horseshoe segments overlapping and therefore counteracting each other's effects. The cancelling effects result in the greatest vorticity being located at the wingtips, providing consistency with other experimental results (such as those from the NASA study shown in the *Background* section).

These results are noteworthy because the strength values converge with approximately 20 horseshoes; other methods may require millions of elements. Further, the computational complexity of other methods is often proportional to the spacing between lifting surfaces. In Phillips and Snyder's version of lifting-line theory, no such dependence exists.

Future Plans

This research project will continue with the following objectives:

- make scenario data (e.g., wing geometries) more customizable;
- expand the handling of multiple lifting surfaces (see figure at right); and
- optimize the lifting surface positions and geometries to maximize efficiency and stability (i.e., the system's ability to return to its equilibrium position after being slightly displaced).



Interested in learning more about this research? Contact Peter Humbert at *humbe037@umn.edu*