Moment of Inertia Estimation Using a Bi-filar Pendulum

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Objective

The objective of this project is to investigate experimental methods for estimating rotational moments of inertia.

Motivation

The moments of inertia of an aircraft are important in understanding its aerodynamic properties and thus its translational and rotational motion during flight. A current method used by researchers at the University of Minnesota in the Unmanned Aerial Vehicle (UAV) Laboratory to estimate these moments of inertia includes a bifilar pendulum.

Figure I: Research aircraft in the University of Minnesota UAV Lab.

An investigation of the bifilar pendulum includes determining the accuracy of the experiment. The results of the investigation will determine if the bifilar pendulum is sufficient for continued use by the UAV Lab, or if further research must be done to develop a more accurate method. Additionally, it will allow overall knowledge to be gained about its experimental process.

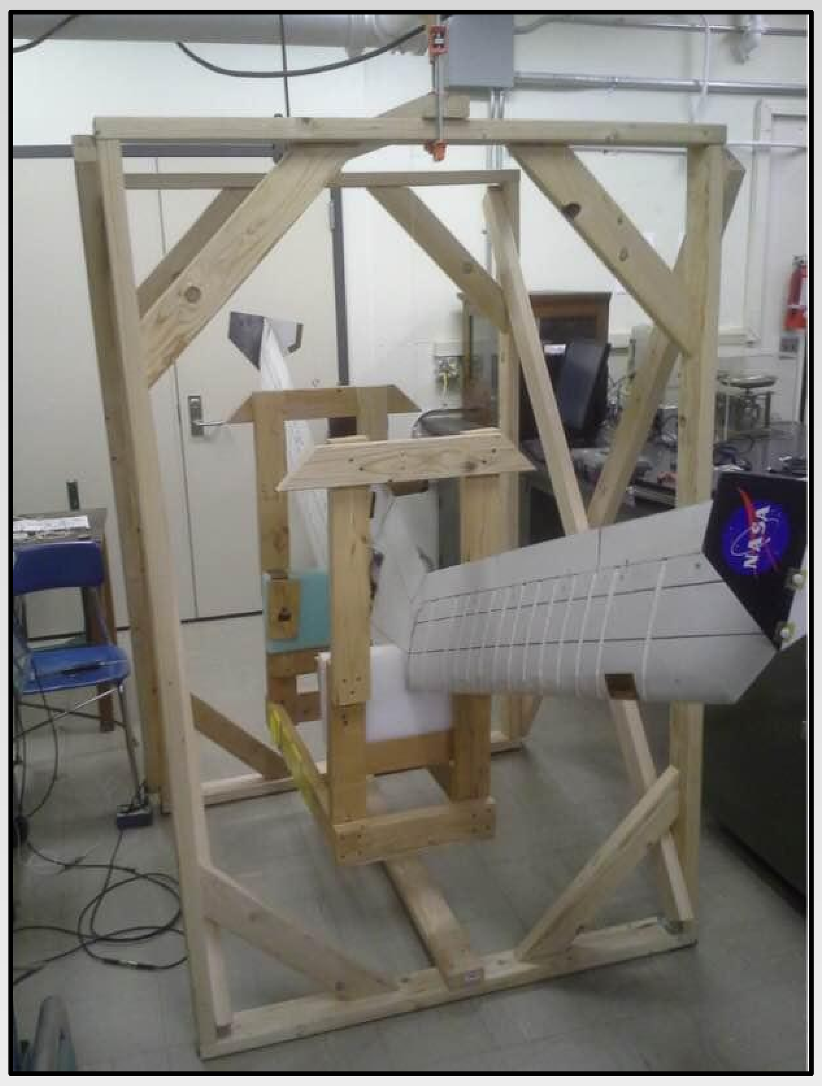


Figure II: A bifilar-pendulum experiment. Courtesy of the UAV Lab.

Overview

A bifilar pendulum consists of suspending an aircraft from two-parallel filar wires that allow it to rotate freely about a given axis. The experiment is to measure the moment of inertia for the axis parallel to these filars. A small moment is applied to the aircraft to measure its period of oscillation, which allows further calculation of its angular frequency as denoted by omega (ω). The moment of inertia can then be calculated using the following equation,

$$I = mgd^2 / 4L\omega$$

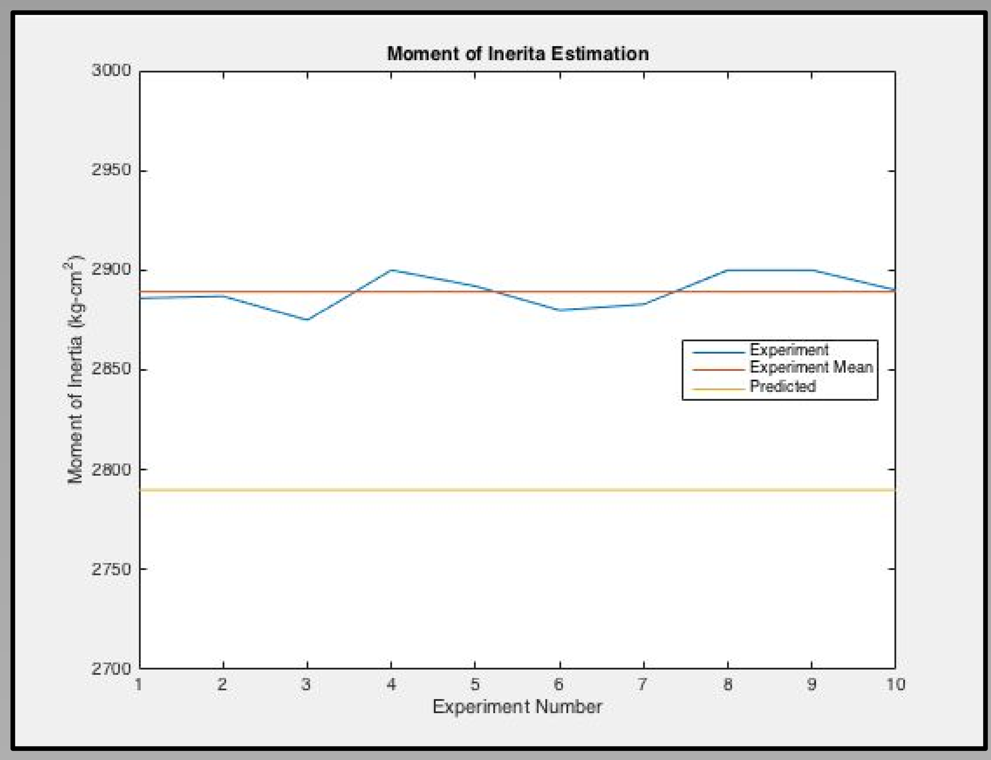
where m is the mass of the aircraft, g is the acceleration due to gravity, d is the distance between the filars, and L is the length of the filars. It is important to verify the center of gravity (C.G.) of the aircraft is within the plane of the filars so that the accuracy of the experiment is not compromised.

Procedure

The experimental model used was constructed out of PVC pipe and configured as a cross. SOLIDWORKS was used to create an analytical model for comparative analysis. Hooks were screwed into the model parallel to and on opposite sides of the C.G. as attachment points for the filars. The filars were attached above to pivot points consisting of a metal screw atop a metal plate, which allowed frictional forces to be minimized. A level tool was used to verify the location of the C.G. of the model. Per experiment, the period of oscillation was measured and the average period for 10 trials was used. To reduce random error, filar lengths were adjusted to different heights per experiment. A total of 10 experiments were conducted for the investigation.

Figure III: Experimental set-up

Results



Graph I: Moment of inertia estimation per experiment

	Moment of Inertia ($kg\text{-}cm^2$)	Percent Error
Experimental	Mean: 2890 Std.(σ): 8.81	+3.6%
Analytical	2790	

Table I: Moment of inertia results

Discussion

A bias occurs between the experimental mean and analytical values and can be seen in Graph I. The source of the bias is assumed to be due to the non-uniform density of the PVC pipes used in the experimental model. In a further analytical analysis, it was found that the moment of inertia followed a linear relationship for small variations in the mass distribution. Additionally, it was found experimentally that flipping the orientation of the pipe-arms yielded a significant change in the moment of inertia. Both of these analyses suggest that the PVC is in fact non-uniform.

Conclusions

The variance for 10 given experiments proves to be small allowing confidence to be had when estimating moments of inertia of given aircraft. However, it should be noted that uncertainty in aircraft properties can affect the comparative analysis between the analytical and experimental results. Additionally, this investigation provides insight into the experimental process of moment of inertia estimation and motivates future research in the area, which includes an experimental method for estimating moment of inertia tensors.

Acknowledgements

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References

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