Lessons from Numerical Holography

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based on work with Paul Chesler

Continuous Advances in QCD, Minneapolis, May 13, 2016

gauge/string duality

- exact mapping between theories
- easiest to exploit in large N, large λ limit

$$\boxed{\mathcal{N}=4 \text{ SYM}} = \boxed{\text{IIB strings}} \xrightarrow[N \to \infty]{\lambda \gg 1} \boxed{\text{classical gravity}}$$

- has enabled much insight into strongly coupled theories
  - E.g.: thermodynamics, η/s, higher order transport coefficients, quasinormal modes, probe dynamics, chiral hydrodynamics, holographic models of superconductors, strange metals, topological insulators, ...
  - mostly equilibrium and near-equilibrium properties
  - what about far-from-equilibrium dynamics?

“shut up and calculate”

- nontrivial QFT dynamics ⬌ non-trivial but *classical* gravitational dynamics in asymptotically anti-de Sitter spacetimes

- nontrivial QFT initial state ⬌ non-trivial gravitational initial data

  - physically motivated initial data = “gedanken” experiments: scattering experiments or time dependent external fields

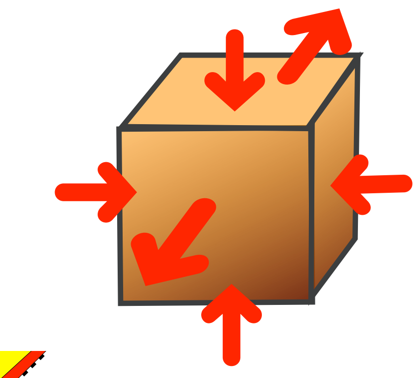
    - E.g.: models of heavy ion collisions, quantum quenches

- requires numerical solution of gravitational initial value problem

complexity timeline

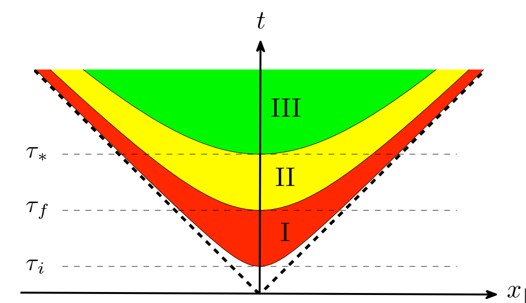
- 2008: homogeneous isotropization

  5D GR → 1+1D PDEs



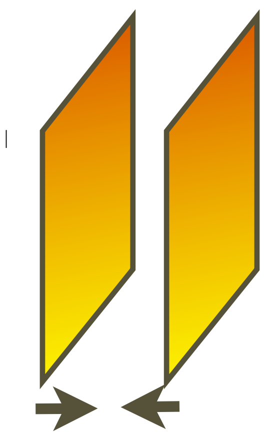
- 2009: boost-invariant expansion

  5D GR → 1+1D PDEs



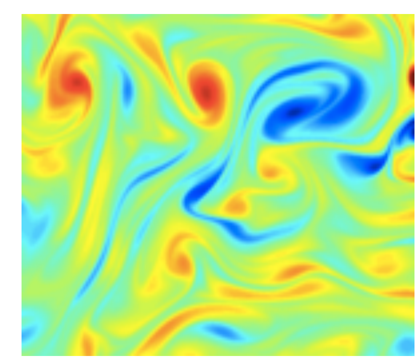
- 2010: colliding planar shock waves

  5D GR → 2+1D PDEs



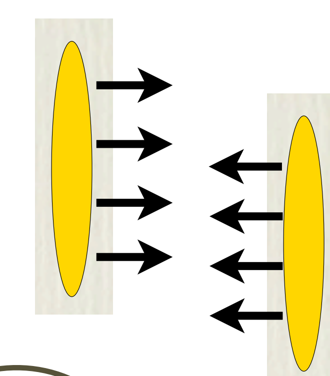
- 2013: 2D turbulence

  5D GR → 3+1D PDEs



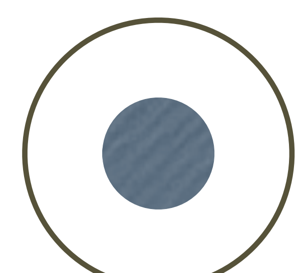
- 2014: off-center, colliding localized shocks

  5D GR = 4+1D PDEs



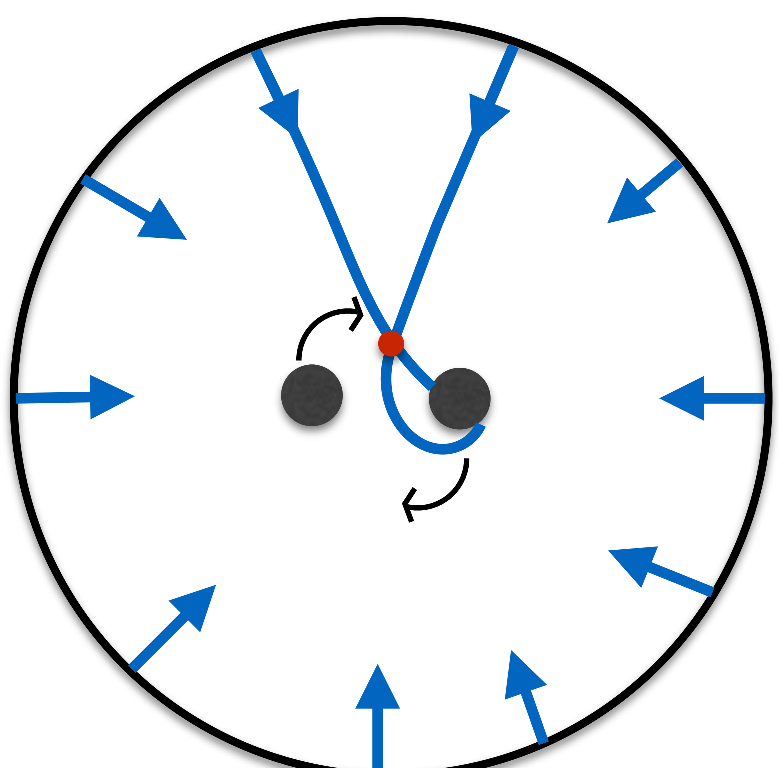
- 2016?: unstable black holes with deformed S^5

  10D GR → 2+1D PDEs



computational lessons (1)

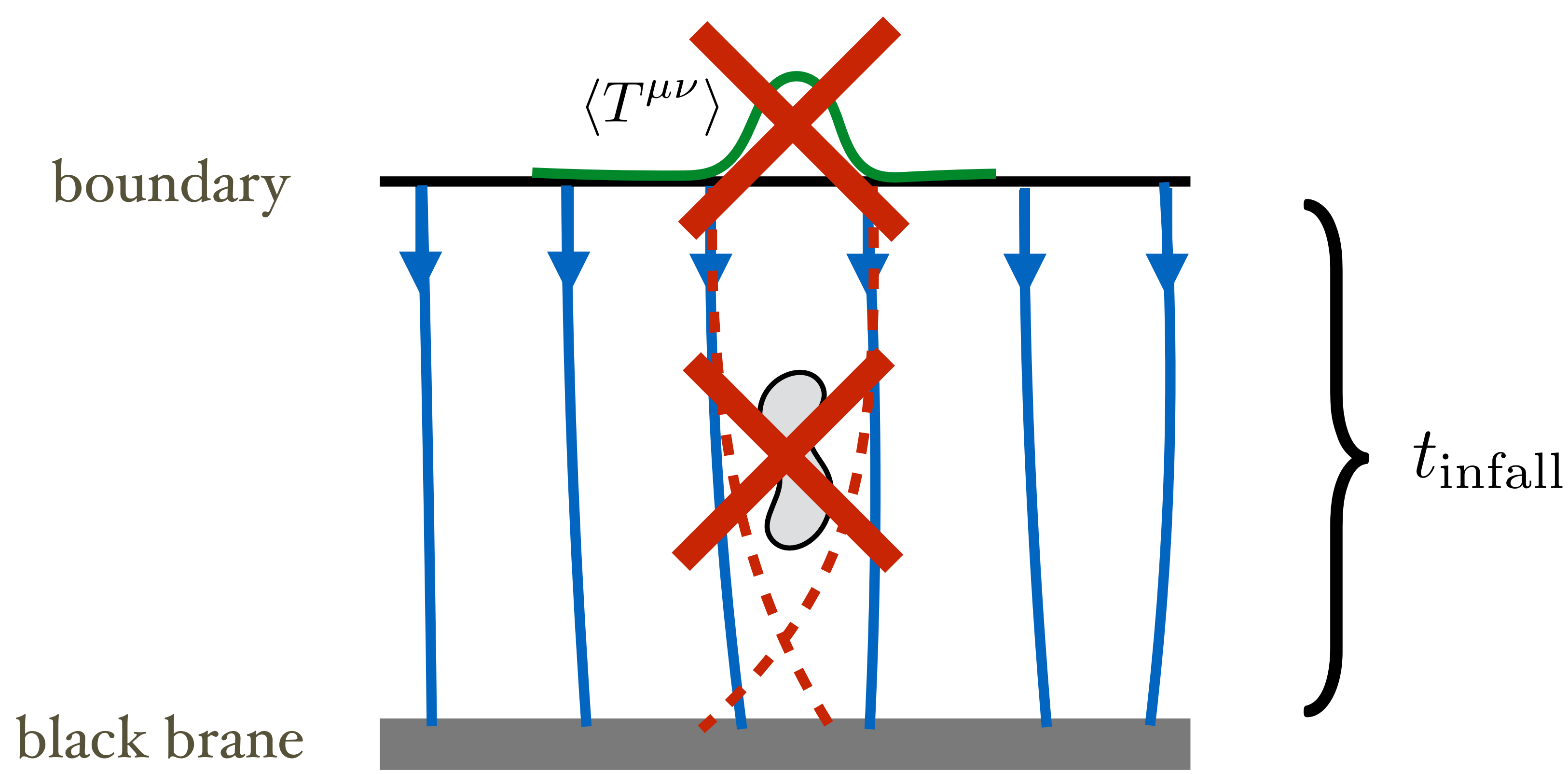
- characteristic formulation *works* for many problems
  - coordinates tied to congruence of infalling null geodesics
  - regular across future horizons
  - coupled non-linear PDEs → nested linear radial ODEs
  - but ... rarely used in asymptotically flat GR due to caustics



caustics (outside horizon)
= coordinate singularities

computational lessons (1)

- asymptotically AdS GR *easier* than asymptotically flat
  - black brane ➡ dissipative physics
  - infall time = dissipative timescale
  - horizon hides caustics *provided* dissipative is shortest relevant length



computational lessons (2)

- spectral methods *vastly* outperform traditional discretization methods
  - (pseudo)spectral approximation = real space implementation of truncated Fourier or Chebyshev basis set expansion
  - exponential convergence
  - can handle boundary conditions at regular singular points
- all work to date: only desktop computational resources

selected results (& puzzles)

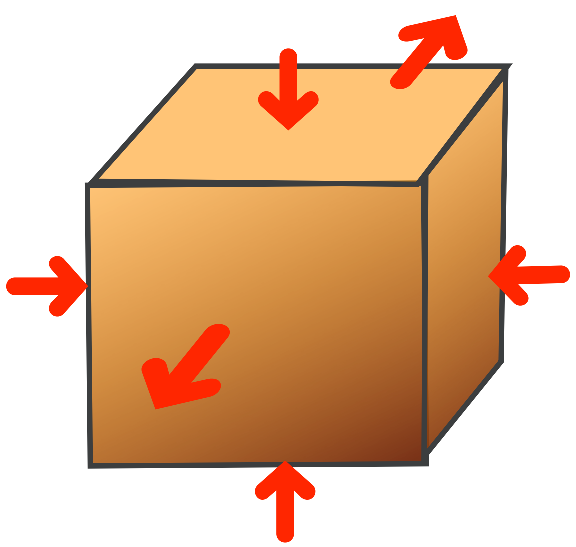
- homogeneous isotropization: chemical potential and magnetic field sensitivity
  PC & LY; J. Fuini & LY

- planar shock collisions: universal rapidity dependence
  PC & LY; PC, N. Kilbertus, W. van der Schee

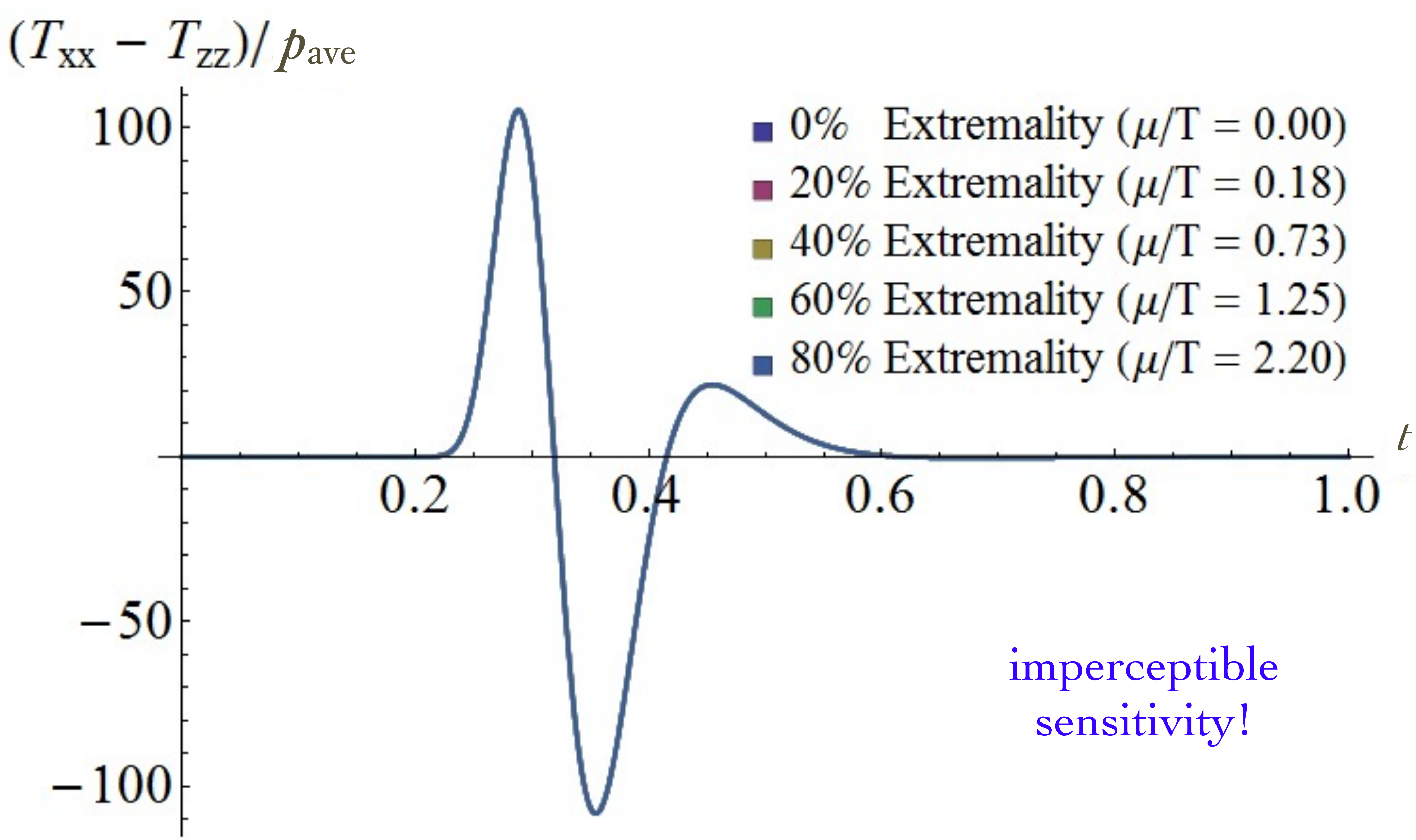
- colliding "nuclei": early flow, extreme hydrodynamics
  LY & PC; PC

- confinement/deconfinement dynamics: small BHs in global AdS
  A. Buchel, PC, LY

homogeneous isotropization

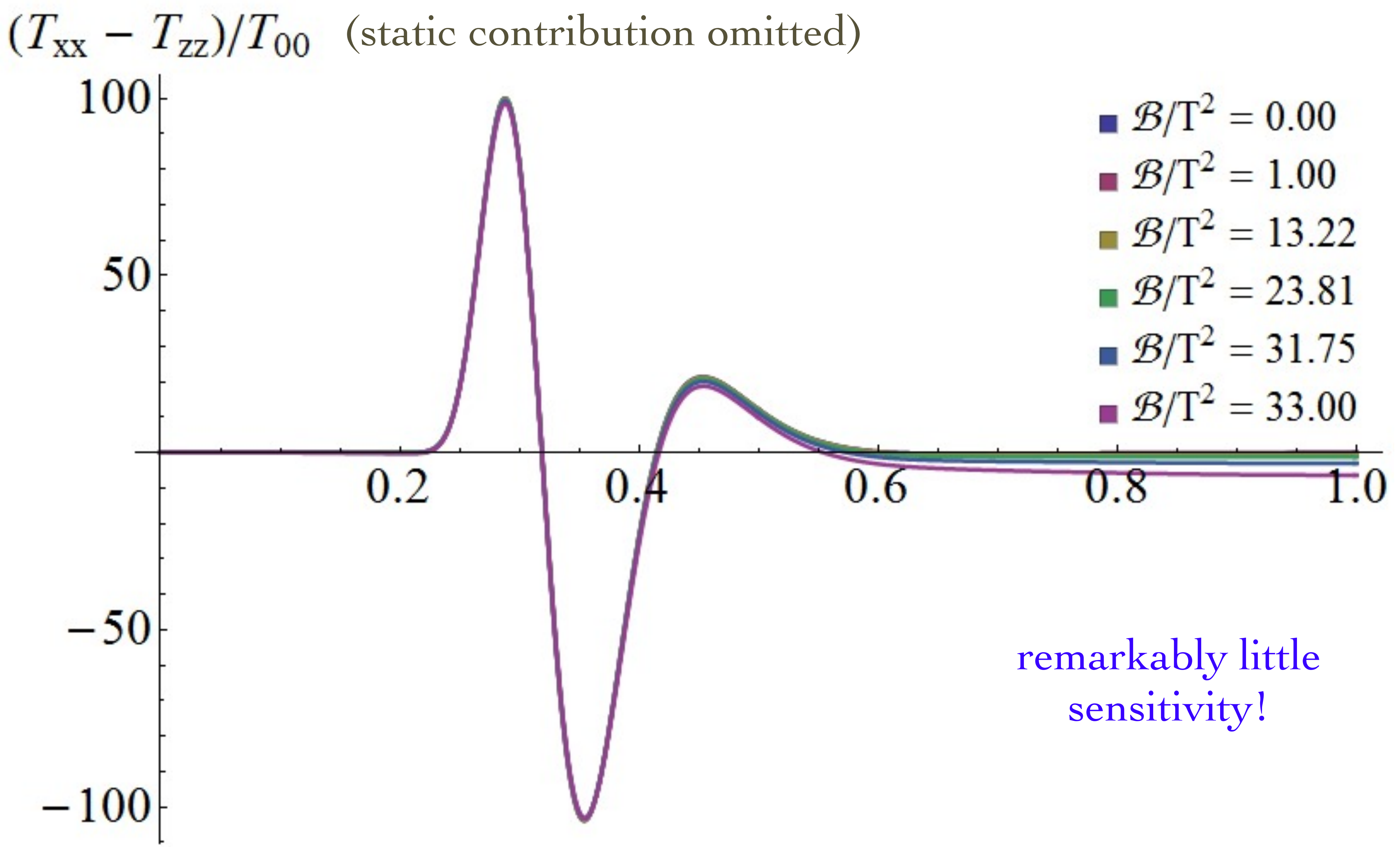


homogeneous isotropization: non-zero flavor charge density



imperceptible sensitivity!

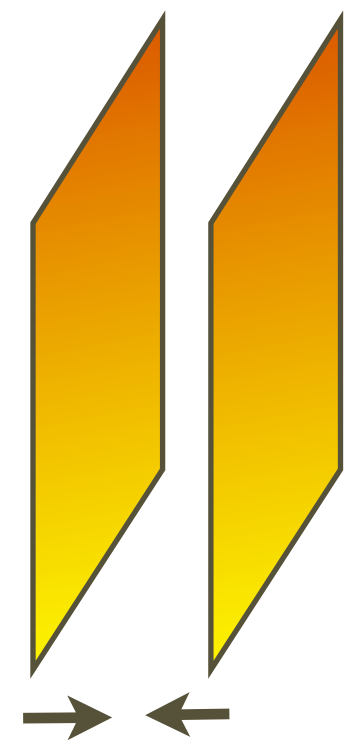
homogeneous isotropization: non-zero magnetic field



homogeneous isotropization: lessons

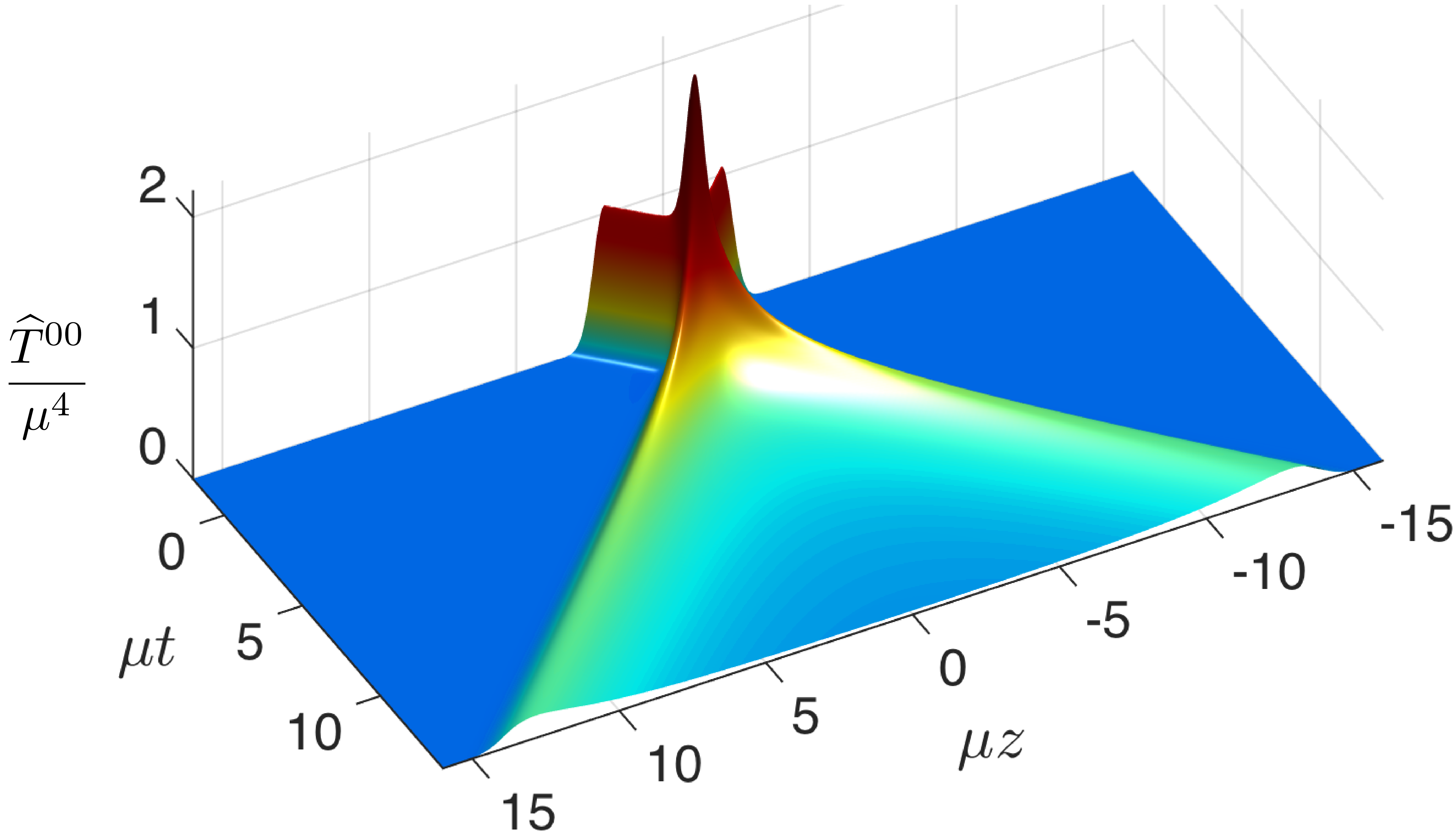
- relaxation time scale = gravitational infall time
- remarkably little sensitivity to plasma constituents
- remarkably little sensitivity to added magnetic field
- dynamics, as probed by boundary observables, is close to linear even far from equilibrium!

colliding planar shocks



colliding planar shocks

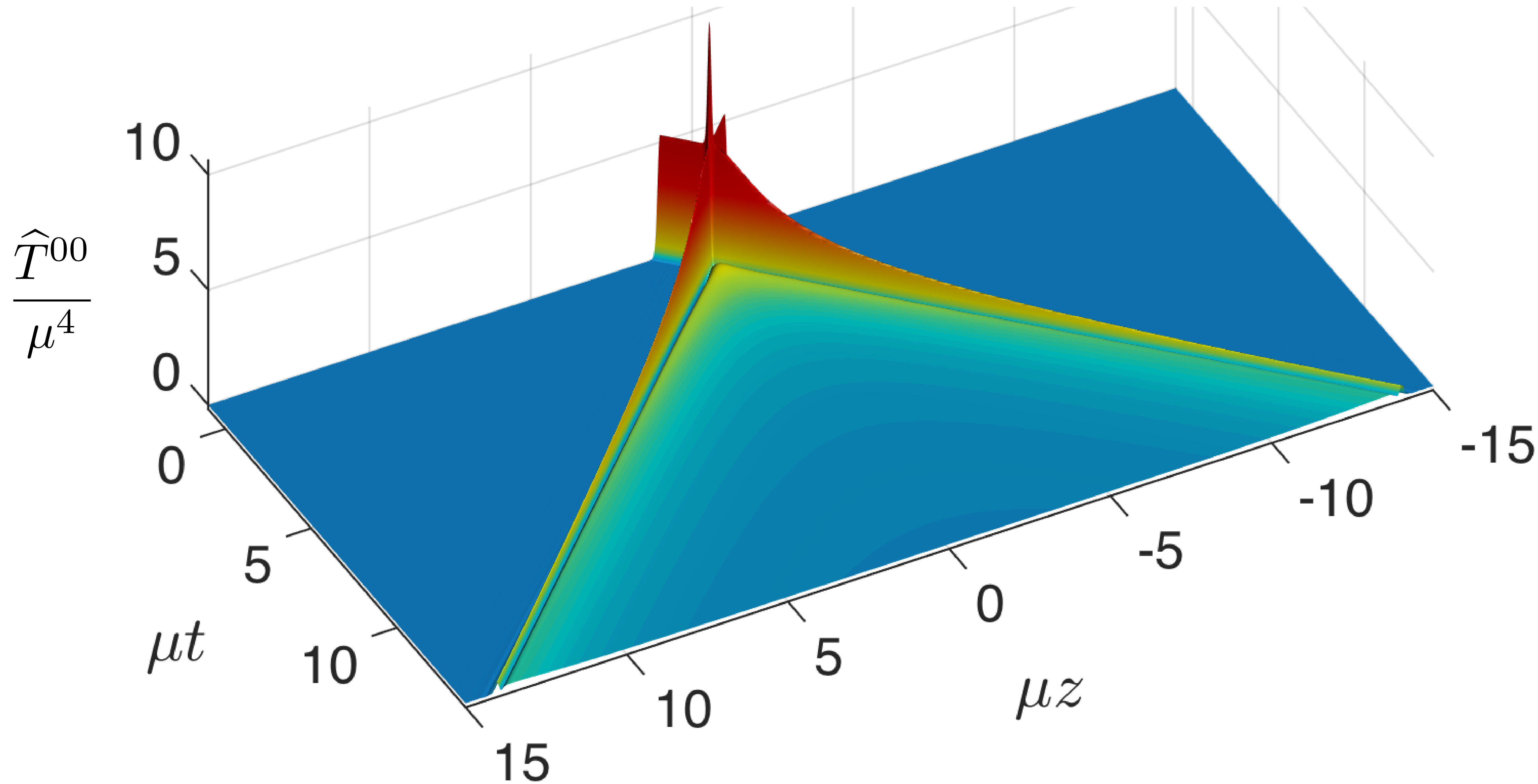
wide shocks



P. Chesler, N. Kilbertus, W. van der Schee, 1507.02548

colliding planar shocks

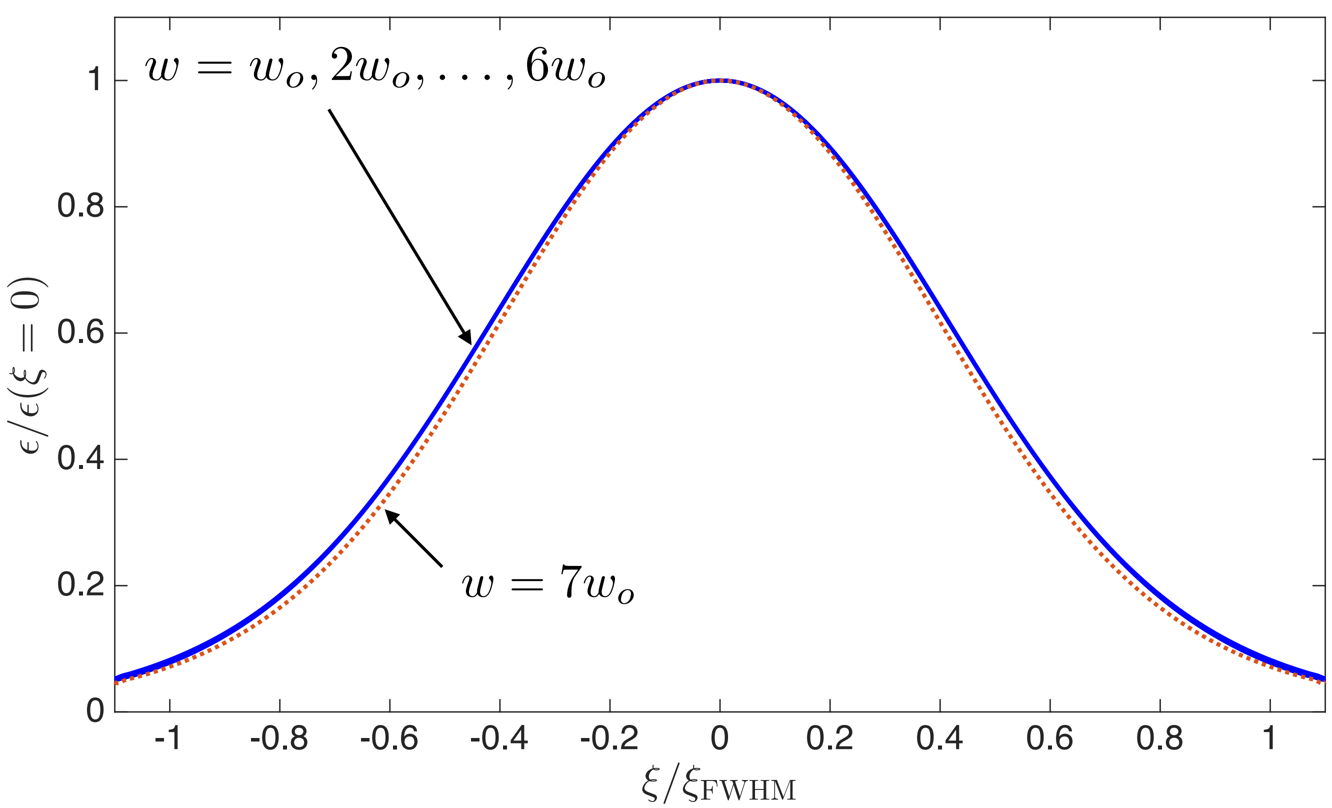
narrow shocks

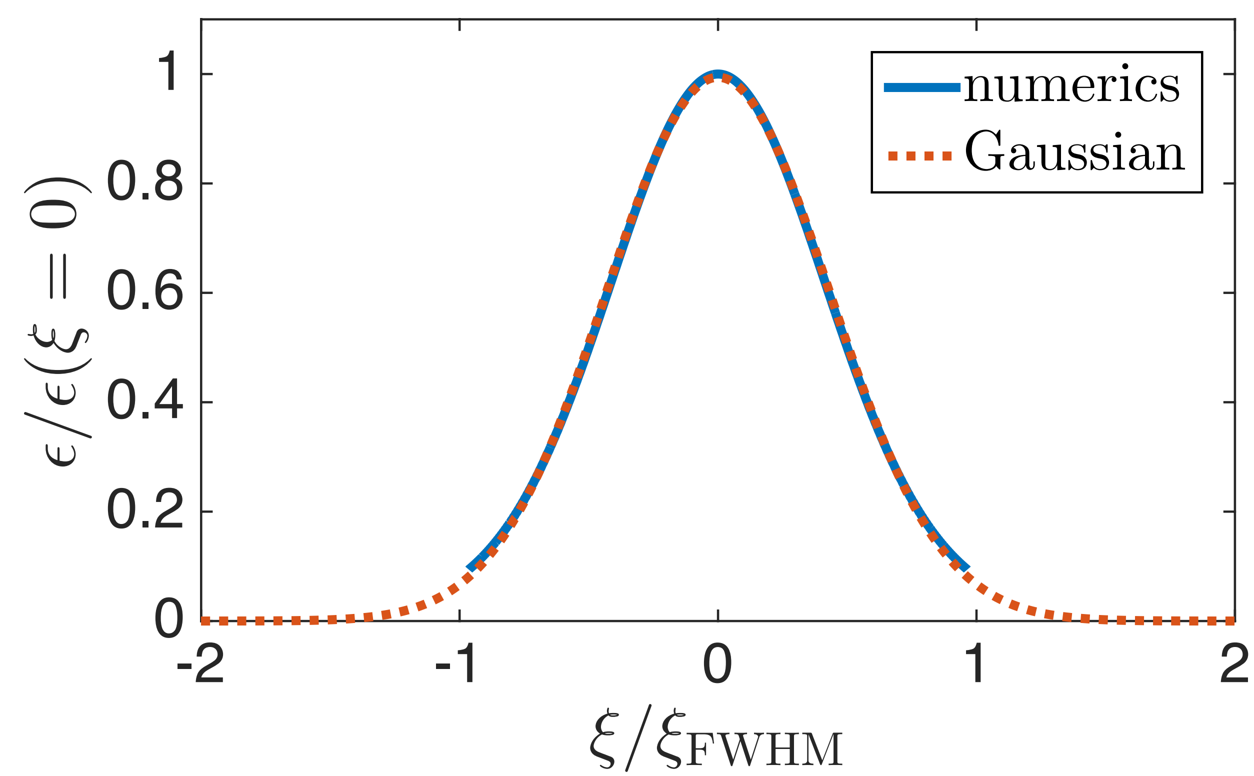


P. Chesler, N. Kilbertus, W. van der Schee, 1507.02548

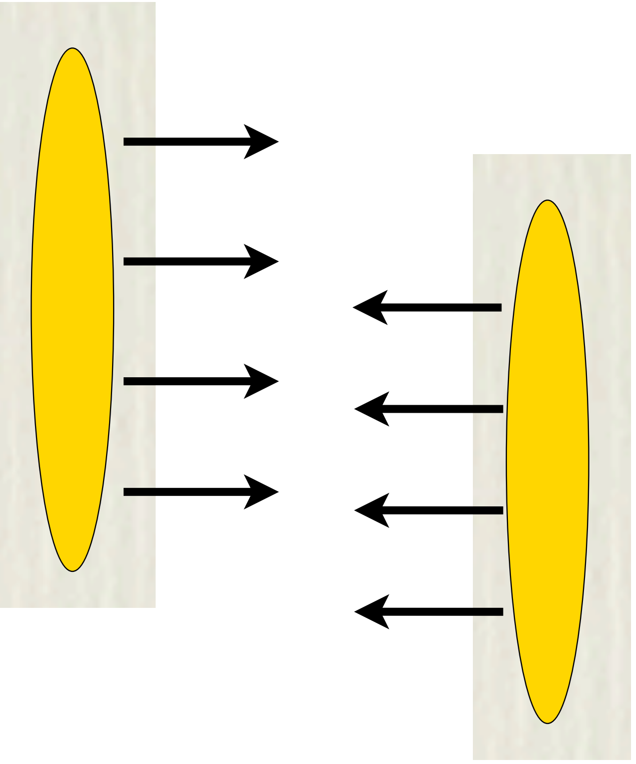
planar shocks: lessons

- no surviving remnants on lightcone
- no significant difference between wide & narrow shocks
- universal rapidity dependence $\epsilon(\xi, w)|_{\tau=\tau_{\text{init}}} = \mu^4 A(\mu w) f\left(\frac{\xi}{\xi_{\text{FWHM}}(\mu w)}\right)$

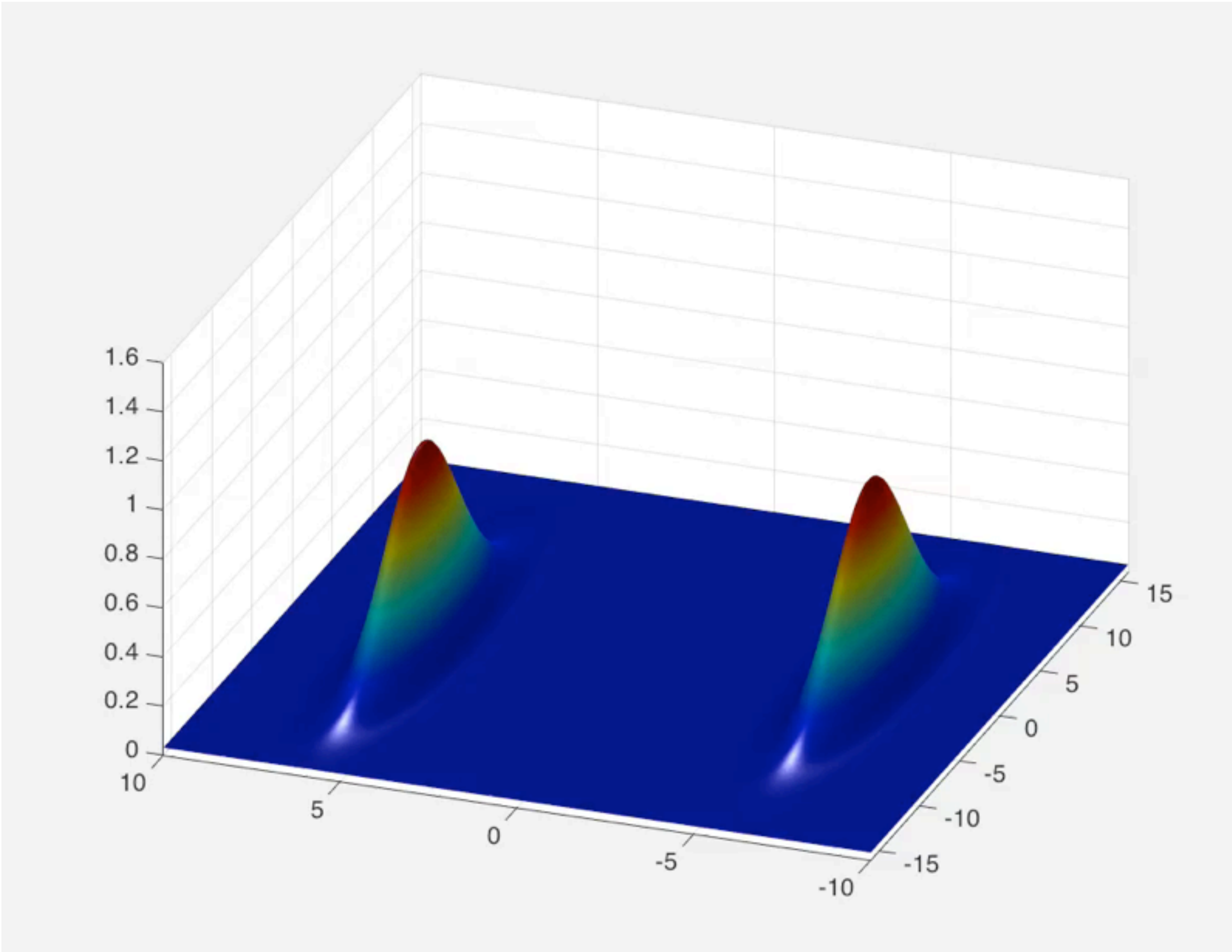




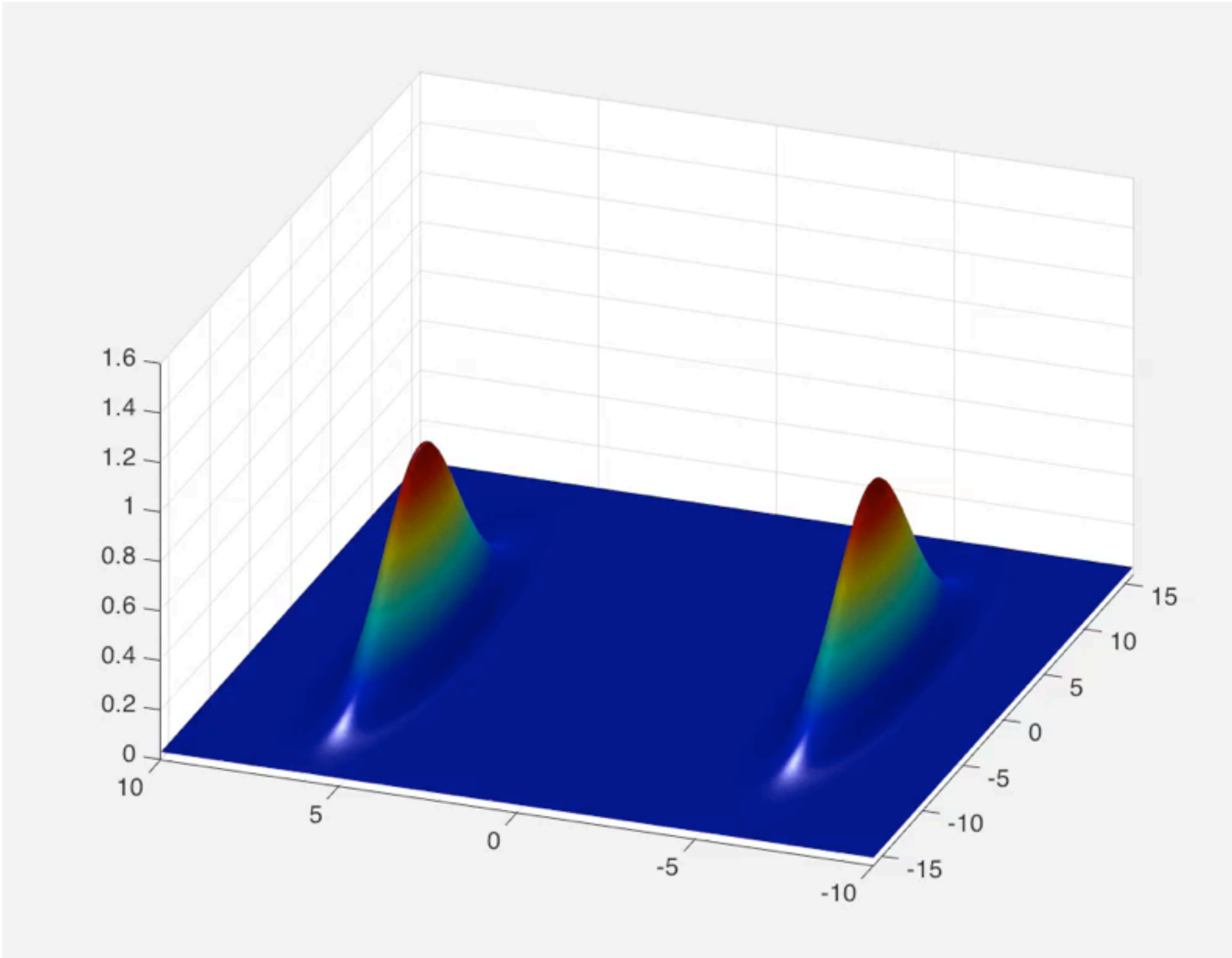
colliding localized shocks ("nuclei")



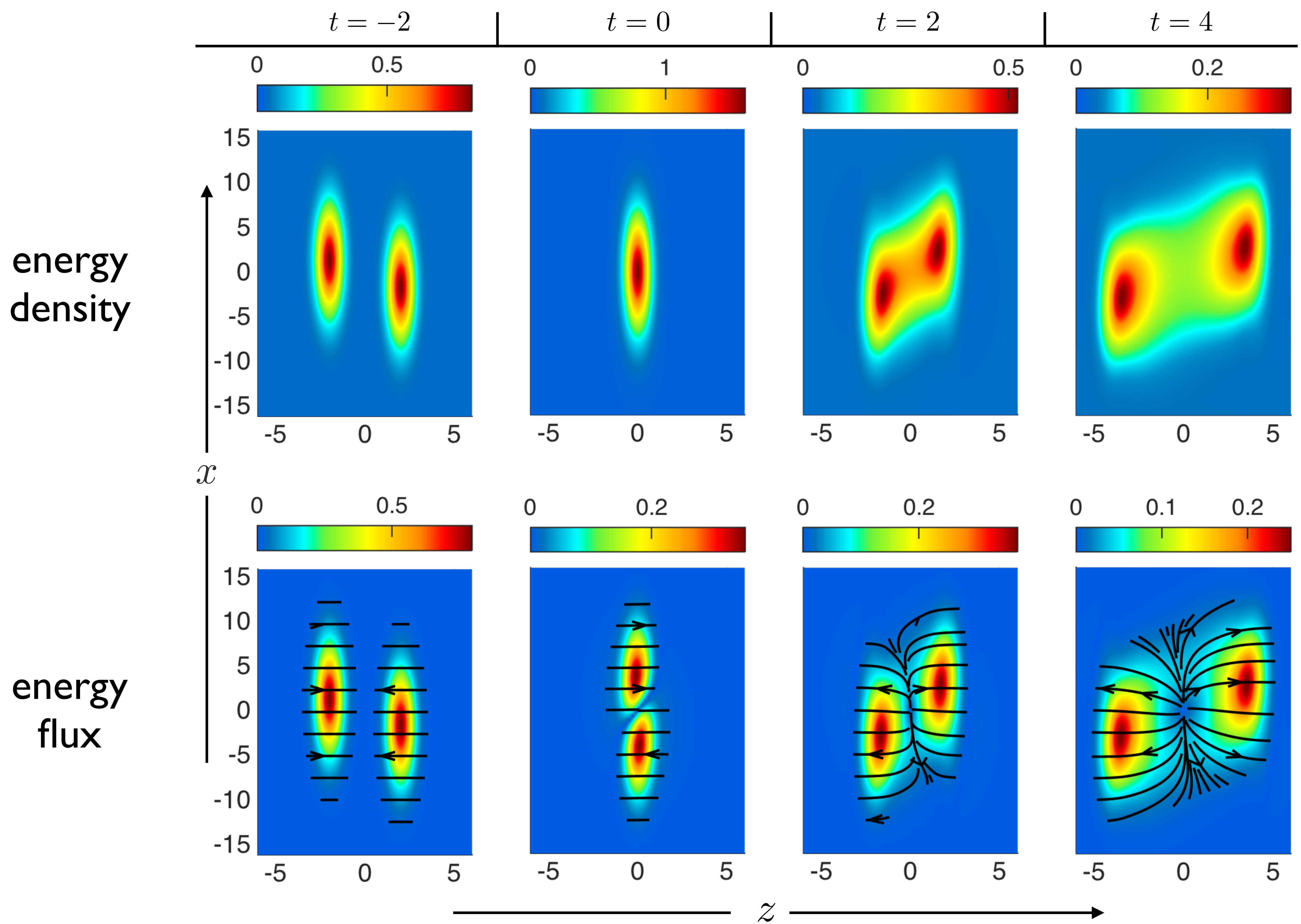
off-center collisions



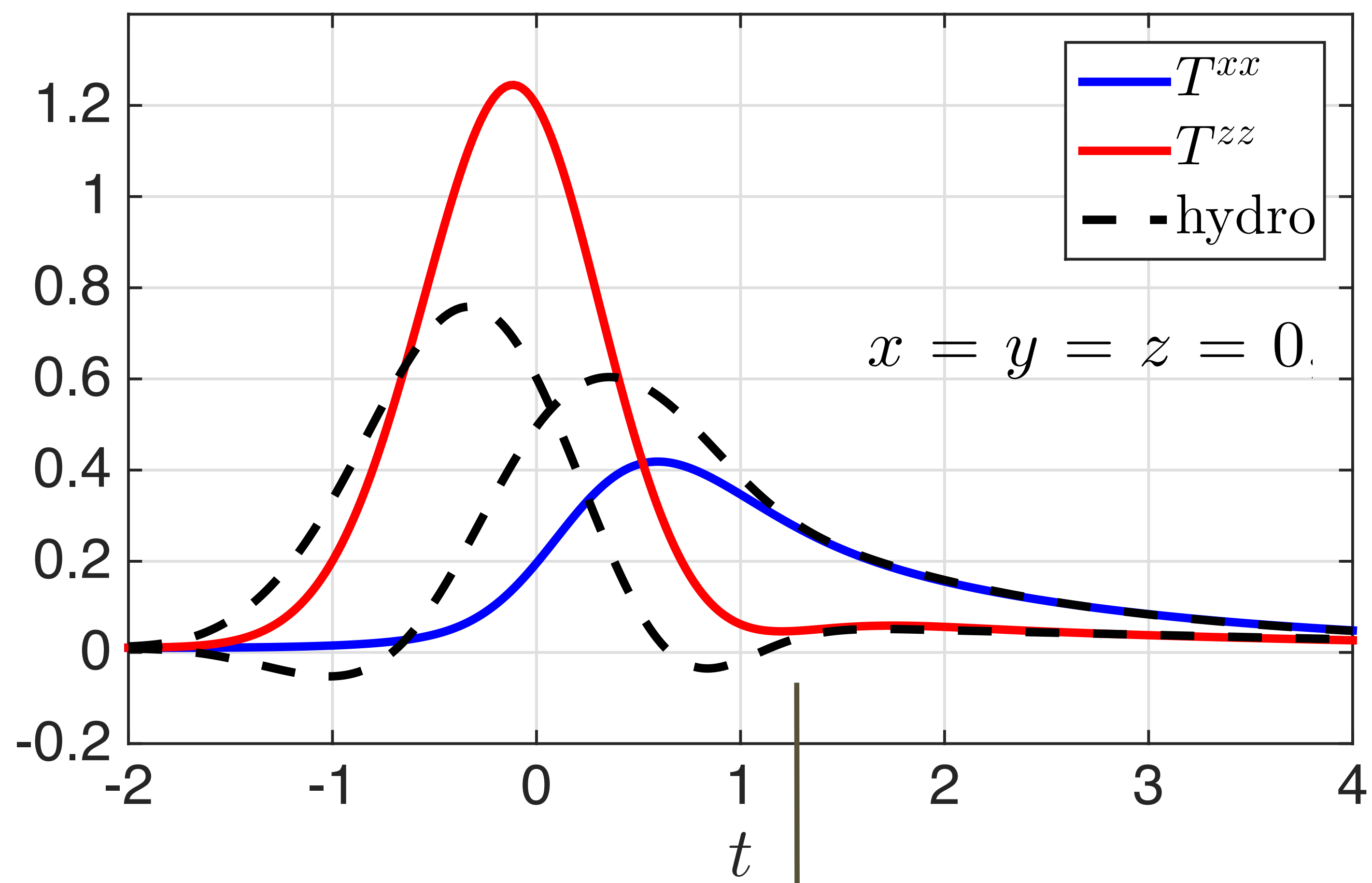
off-center collisions



snapshots



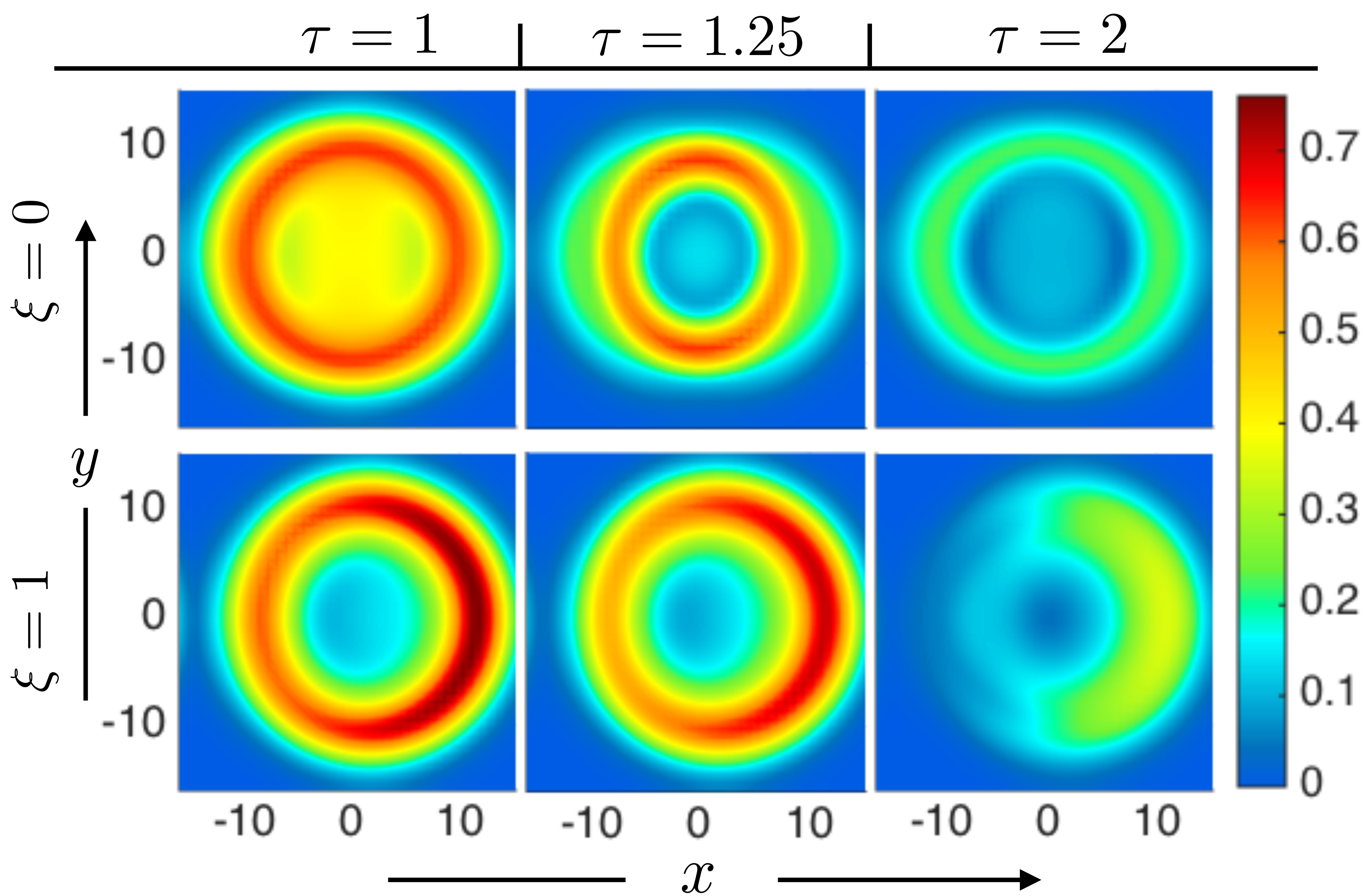
transverse & longitudinal pressure



$t_{\text{hydro}} \approx 1.25$

hydro onset ≈ 30% faster than for planar shocks

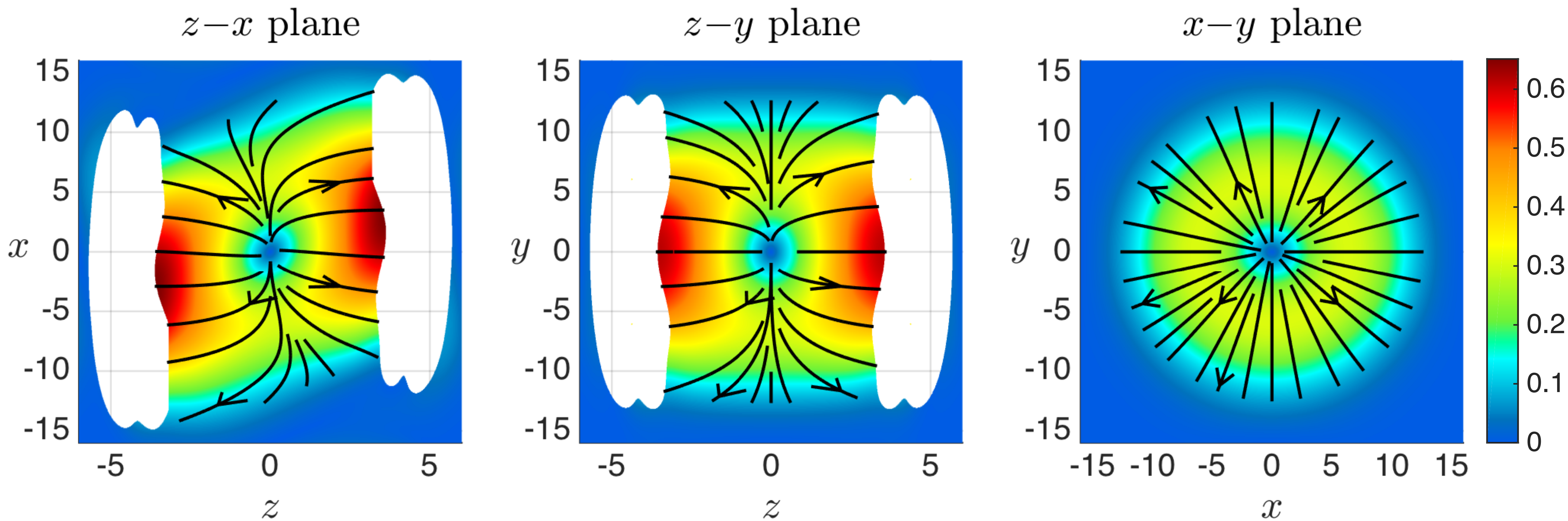
hydrodynamic residual



$\Delta \equiv (1/\bar{p})\sqrt{\Delta T_{\mu\nu}\Delta T^{\mu\nu}}$, $\qquad \Delta T^{\mu\nu} \equiv T^{\mu\nu} - T^{\mu\nu}_{\text{hydro}} \qquad \bar{p} \equiv \epsilon/3$

flow velocity

$t = 4$ non-hydro regions excised



substantial radial flow:

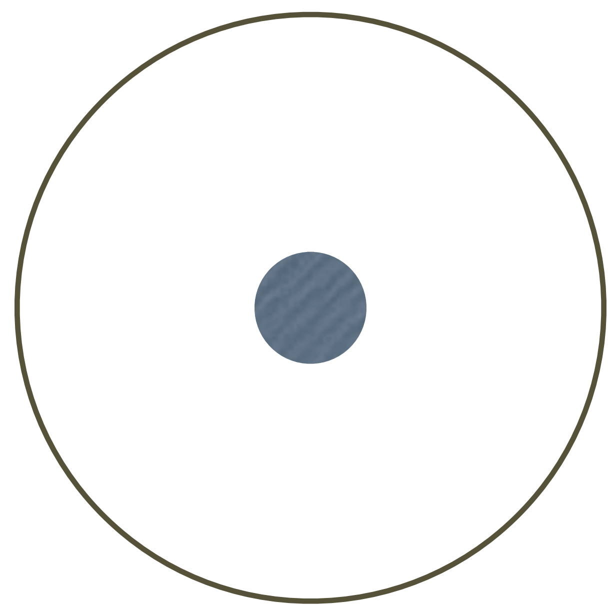
$$v_\perp(x_\perp = 5) \approx 0.3$$

$$v_\parallel^{\max} \approx 0.64$$

colliding "nuclei": lessons

- "pre-hydro" development of transverse flow
- rapid equilibration, $t_{\text{hydro}}\, T_{\text{eff}} \approx 0.3$
- extreme hydrodynamics:
  - huge anisotropy but well-behaved gradient expansion
  - works down to $R\, T_{\text{eff}} \approx 0.5\text{-}1$
    - compatible with interpretations of high multiplicity *p-p* collisions as producing deconfined quark-gluon plasma exhibiting collective flow

small AdS black holes

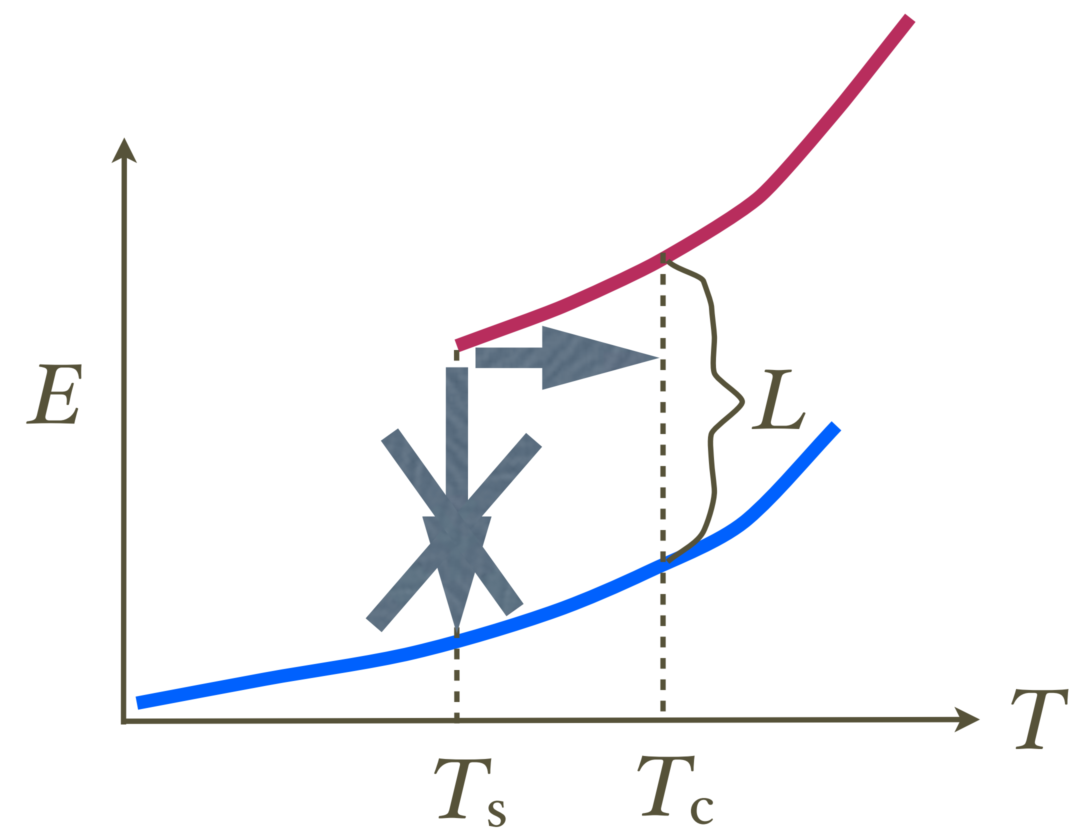


confinement dynamics

- $\mathcal{N}$=4 SYM on $S^3 \times R$:
  - $T<T_c$: confined phase, $O(N^0)$ free energy
    - dual description = "thermal" AdS
  - $T>T_c$: deconfined phase, $O(N^2)$ free energy
    - dual description = global AdS black hole
  - $T=T_c$: first order phase transition

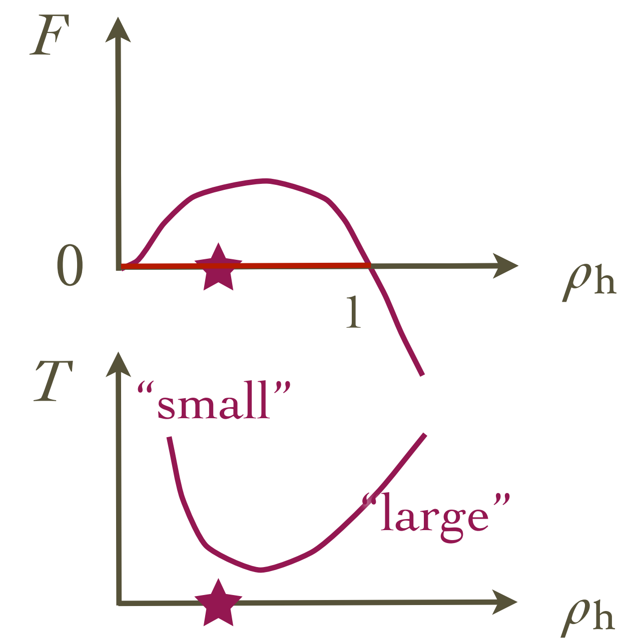
first order transitions

- T=T_c: phase coexistence, multiple equilibrium states
- latent heat:
  - $L = E^+(T_c) - E^-(T_c) > 0$
- cooling dynamics:
  - E, T, S all ↘
  - T=T_c: enter metastable supercooled phase
  - T=T_s: spinodal decomposition = limit of metastability
  - re-equilibrates to *mixed* state at T=T_c if $E^+(T_s) > E^-(T_c)$



global AdS black holes

- asymptotic geometry $AdS_5 \times S^5$
- bulk geometry: $ds^2 = -g(\rho)\, dt^2 + \dfrac{d\rho^2}{g(\rho)} + \rho^2\, d\Omega_3^2 + d\Omega_5^2\,, \qquad g(\rho) \equiv \rho^2 + 1 - (1+\rho_h^2)\, \dfrac{\rho_h^2}{\rho^2}$
- free energy $\quad F = C\,(1-\rho_h^2)\,\rho_h^2 + (\text{Casimir})$
- temperature $\quad T = \dfrac{2\rho_h^2+1}{2\pi\rho_h}$



- "large" BH branch ($\rho_h > 1$): deconfined equilibrium states
- "small" BH branch ($\rho_h < 1$): thermodynamically unstable
  - $\rho_h < 0.44$: dynamically unstable wrt. deformation of S^5

unstable small black holes

- $0.44 < \rho_h < 1$: supercooled plasma, stable at $N = \infty$
- $\rho_h = 0.44$: spinodal decomposition threshold
- $\rho_h < 0.44$: dynamical instability leads to ???
  - does system re-equilibrate to new stationary solution with broken $SO(6)_R$ symmetry?
    - known “lumpy” $S^3 \times S^5$ BH solutions have lower entropy
    - recent S^8 BH solutions have higher entropy but $T > T_c$

O. Diaz, J. Santos, B. Way

unstable small black holes

- microcanonical description *must* be consistent with canonical description in thermodynamic ($N_c \to \infty$) limit
  - what is manifold of coexisting equilibrium states at T_c?
    - extremal states = glueball gas & deconfined plasma
    - are there mixed states when thermodynamic limit = large N limit?
  - do supercooled states fail to re-equilibrate?
- in progress: find time dependent solutions numerically
  - 10D GR + self dual 5-form, SO(4) × SO(5) invariant
  - multiple towers of scalar condensates

conclusions

- numerical holography does allow exploration of interesting far-from-equilibrium dynamics
  - numerics "easier" than might have been expected
  - AdS asymptotics & dissipative dynamics helps
- has already yielded phenomenologically relevant implications for heavy ion collisions, superfluids
- many open questions, even on basic thermodynamics!