PROBLEMS ASSOCIATED WITH REMOTELY SENSING WIND SPEED

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IMA Preprint Series # 2285

(October 2009)

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Team 4: Problems Associated with Remotely Sensing Wind Speed

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August 14, 2009

Abstract

Atmospheric turbulence combined with wind causes optical scintillations. By measuring this optical scintillation we can gain insight into our turbulent atmosphere. The main objective of this research area is to measure the average wind speed along the path of a laser beam and also to determine the refractive index structure parameter C_n^2. We propose Brownian model for the refractive index structure parameter C_n^2. Also a perturbation analysis is described for refractive index structure parameter C_n^2 and the velocity $v(z)$. Next we compare the cross covariance for spherical and Gaussian waves.

1 Introduction

Atmospheric turbulence, which is statistically nonstationary and inhomogeneous, affects the propagation of light . The optical scintillations caused by the atmospheric turbulence effects the astronomical observation, the twinkling of stars is an example [3]. Recognizing the effects caused by atmospheric turbulence on the optical waves propagating can help us to measure the nature of our atmosphere.

Our main problem is to use optical scintillation to determine the speed of wind. In the early 1970s Lawrence, Clifford and Ochs [15] and Lee and Harp [18] studied the variation of the intensity of light on a pair of photo detectors. The schematic is as shown in Figure 1.

In this setup we have a light source(Laser) and a pair of detectors. This laser gives off waves and the detectors measure the time varying intensity of the light. We assume the distance between the detectors and the light source anywhere from 100 m to 10 km. Our work looks at three different wavefront forms: a plane wave, a spherical wave, or a Gaussian wave. Most references we have found have looked into the plane and spherical wave. Plane and spherical waves are used in astronomy. But in our application, our distances are short compared to those used in astronomy so we want to use Gaussian waves. The Gaussian wave is more complicated but more

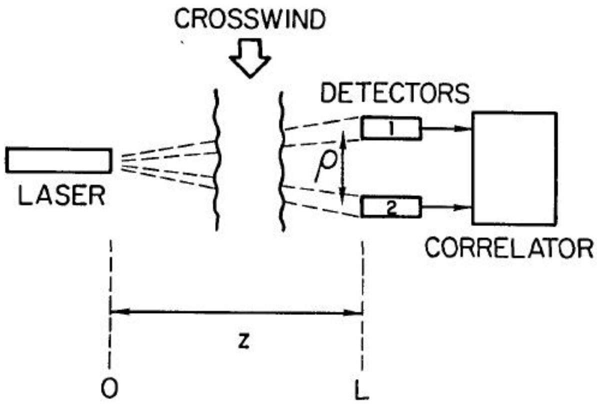

Figure 1: Schematic

accurate. We are doing work with both the spherical and Gaussian wave. The laser considered is monochromatic and the wavelength is anywhere from 400 nm to 2000 nm.

1.1 Atmospheric Turbulence Model

The main source of optical scintillations is caused by inhomogeneity of the temperature in the atmosphere. To obtain the wind speed we measure the cross-correlation of the light intensity measured at the detectors. To derive the relation between this cross-correlation and the wind speed we need the following assumptions:

1. Kolmogorov power-law spectrum
The corresponding spatial power spectrum that we assume is the Kolmogorov power-law spectrum [3] which is given by

$$\phi(K,z) = .033 C_n^2(z) K^{-11/3}$$

The strength of turbulence is predicted by the refractive index structure parameter C_n^2, which will vary as a function of propagating distance, altitude, location and time of the day. For our scheme altitude remains constant and the experiments are conducted for a short duration of time. Hence we write C_n^2 as a function of the propagating distance z.

2. Taylor's frozen turbulence hypothesis Taylor's hypothesis is an assumption that advection contributed by turbulent circulations themselves is small and that therefore the advection of a field of turbulence past a fixed point can be taken to be entirely due to the mean flow; also known as the Taylor "frozen turbulence" hypothesis [3]. In other words turbulence causes the formation of eddies of many different length scales and these eddies are not changing as they travel through the beam. There are two time scales:

1. Due to the motion of the atmosphere across the path of the observation which is of the order of 1 s.

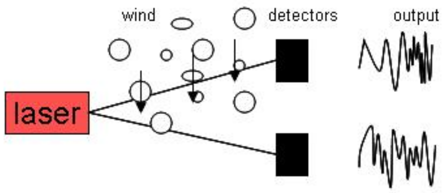

Figure 2: Eddy Balls

2. Due to dynamics of the turbulence, i.e., eddies, and this is of the order of 10 s.

Hence the second time scale can be neglected in comparison with the mean wind flow. This assumption is required to draw a connection between two types of statistical averaging, namely spatial and temporal averages. Using [22] we can explain what Taylor's frozen flow hypothesis means. For a point $\boldsymbol{x}$ the intensity of laser beam at time $t_2 > t_1$, $I(\boldsymbol{x}, t_2)$ is related to intensity of a laser beam at time t_1, $I(\boldsymbol{x}, t_1)$ by

$$I(\boldsymbol{x}, t_2) = I(\boldsymbol{x} - \boldsymbol{v}(t_2 - t_1), t_1),$$

where $\boldsymbol{v}$ is the velocity of the wind. The cross covariance of the intensity of a laser beam measured at the detectors, with a time delay τ, is given by

$$\begin{aligned} C(\rho, \tau) &= E(I_1(\boldsymbol{x}, t) I_2(\boldsymbol{x}, t + \tau)) \\ &= E(I_1(\boldsymbol{x}, t) I_2(\boldsymbol{x} - \boldsymbol{v}\tau, t)). \end{aligned}$$

Therefore using Taylor's frozen flow hypothesis we can obtain a relationship between the temporal covariance and the spatial covariance.

1.2 Covariance

Lawrence, Clifford and Ochs [15] derive an expression for the time-lagged covariance using the spatial covariance from [18]. Using the Kolmogorov spectrum $\phi(K, z) = .033 C_n^2(z) K^{-11/3}$ and Taylor's flow hypothesis that the turbulent eddies do not change significantly during the time they drift through the light beam. The expression which relates the normalized time lagged cross covariance $C_{\chi N}$ and $v(z)$ given by [15] is:

$$C_{\chi N}(\rho, \tau) = 2.33 (kL)^{5/6} \frac{\int\limits_0^L dz C_n^2(z) \int\limits_0^\infty dK K^{-8/3} \sin^2 \left[\frac{K^2 z(L-z)}{2kL}\right] J_0 \left[K\left(\frac{\rho z}{L} - v(z)\tau\right)\right]}{\int\limits_0^L dz C_n^2(z) [z(L-z)]^{5/6}} \tag{1}$$

where

$\tau -$ is the time lag for the covariance
$L -$ is the distance the wave beam has propagated
$z \in [0, L]$
$k -$ wave number $- \dfrac{2\pi}{\lambda}$
$\lambda - wavelength$
$K -$ reciprocal of the size of the turbulent eddy ball
$\rho -$ spacing between detectors
$v(z) -$ wind speed parallel to the detectors
$C_n^2(z) -$ turbulence strength
$J_0 -$ is the $0th$ order Bessel function.

We measure this normalized time lagged cross covariance $C_{\chi N}$ and want to solve for $v(z)$. One way of solving, as described in [15], is to find the slope of (1) with respect to τ, where $\tau = 0$, and perform an inverse problem. The slope of $C_{\chi N}(\rho, \tau)$ is represented by M_N,

$$M_N = \frac{\int_0^L dz C_n^2(z) v(z) W(z)}{\int_0^L dz C_n^2(z) [z(L-z)]^{5/6}} \tag{2}$$

where $W(z)$ is the path-weighting function

$$W(z) = \int_0^\infty dK K^{-5/3} \sin^2[K^2 z(L-z)/(2kL)] J_1(K\rho z/L).$$

1.3 Refractive Index Structure Parameter C_n^2

To solve for the average wind speed, $v(z)$, many people treat the scintillation coefficient, C_n^2, as a constant. If that is the case, one can see below that the $C_n^2(z)$ does not play a role in our time-lagged covariance function because this coefficient is both in the numerator and denominator of equation (1). There is a cancelation and can solve for $v(z)$.

From [16], we see that the scintillation coefficient is not constant, but

$$C_n^2 \propto \frac{C_T^2}{T^4},$$

where T is temperature and C_T is a function depending on temperature.

2 Brownian Model for C_n^2

Based on the above explanation, we know that C_n^2 varies through time. It would be interesting to see what happens when a noise term is added to C_n^2. To this end, we write $C_n^2 = C_0 + \alpha W_z$, where C_0 is a constant value for the scintillation coefficient, $\alpha \in \mathbb{R}^+$, and define W_z as a white noise term, i.e. $W_z = \frac{dB_z}{dz}$, where B_z is Brownian motion. Substituting this into the covariance equation(1), we obtain:

$$C_{\chi N}(\rho,\tau) = 2.33(kL)^{5/6} \frac{\int_0^L dz(C_0+\alpha W_z)\int_0^\infty dK K^{-8/3}\sin^2\left[\frac{K^2 z(L-z)}{2kL}\right] J_0\left[K\left(\frac{\rho z}{L} - v(z)\tau\right)\right]}{\int_0^L dz(C_0+\alpha W_z)[z(L-z)]^{5/6}}.$$

If we define the deterministic functions f and g by

$$\begin{aligned} f(z) &= \int_0^\infty K^{-8/3}\sin^2\left[\frac{K^2 z(L-z)}{2kL}\right] J_0\left[K\left(\frac{\rho z}{L} - v(z)\tau\right)\right] \\ g(z) &= [z(L-z)]^{5/6}, \end{aligned}$$

then the expression can be written compactly as

$$C_{\chi N}(\rho,\tau) = \frac{C_0\int_0^L dz f(z) + \alpha\int_0^L dB_z f(z)}{C_0\int_0^L dz g(z) + \alpha\int_0^L dB_z g(z)}.$$

Note that both f and g are zero at the endpoints of the interval $[0, L]$. So using a simple form of Itô's formula, we can write this as

$$C_{\chi N}(\rho,\tau) = \frac{C_0\int_0^L dz f(z) - \alpha\int_0^L dz B_z f'(z)}{C_0\int_0^L dz g(z) - \alpha\int_0^L dz B_z g'(z)}.$$

Let

$$\begin{aligned} P(K,z) &= K^{-8/3}\sin^2\left(\frac{K^2 z(L-z)}{2kL}\right), \\ Q(K,z) &= K\left(\frac{\rho z}{L} - v(z)\tau\right). \end{aligned}$$

Then performing the derivatives in the above expression and using the fact that $J_0'(z) = -J_1(z)$ and $J_1' = \frac{1}{2}(J_0(z) - J_2(z))$, yields

$$\begin{aligned} C_{\chi N}(\rho,\tau) = &\frac{\int_0^L dz\int_0^\infty dK P(K,z) J_0[Q(K,z)]\left[C_0 + \alpha B_z \frac{K^2}{2L}(2z-L)\cot\left(\frac{K^2 z(L-z)}{2kL}\right)\right]}{\int_0^L dz[z(L-z)]^{5/6}\left[C_0 + \frac{5}{6}\alpha B_z \frac{2z-L}{z(L-z))}\right]} \\ &+ \frac{\int_0^L dz\int_0^\infty dK K\cdot P(K,z) J_1[Q(K,z)]\left[\alpha B_z\left(\frac{\rho}{L} - \frac{d}{dz}v(z)\tau\right)\right]}{\int_0^L dz[z(L-z)]^{5/6}\left[C_0 + \frac{5}{6}\alpha B_z \frac{2z-L}{z(L-z))}\right]}. \end{aligned}$$

It is unclear whether the Brownian motion term plays a significant impact in the covariance from this form. Later, in the perturbation analysis, we'll see that if the α is sufficiently small, then the affects of the noise could be controlled. However, this is not clear solely based on this equation.

3 Perturbation Analysis

To analyze how controlled perturbations to the wind velocity and $C_n^2(z)$ affect the measured covariance, we assume that they can be written as

$$\begin{aligned} C_n^2(z) &= C^{(0)}(z) + \varepsilon C^{(1)}(z) + \varepsilon^2 C^{(2)}(z) + o(\varepsilon^3) \\ v(z) &= v^{(0)}(z) + \varepsilon v^{(1)}(z) + \varepsilon^2 v^{(2)}(z) + o(\varepsilon^3). \end{aligned}$$

Then we can plug these expressions into the original normalized covariance to obtain a perturbed expression, expanded in powers of ε. The first step is to expand the Bessel function term $\widetilde{J}(\varepsilon) = J_0[K(\rho z/L - (v^{(0)}(z) + \varepsilon v^{(1)}(z) + \dots)\tau)]$ in terms of ε. Writing out the formal power series, and using $Q(K,z) \equiv K\left(\frac{\rho z}{L} - v^{(0)}(z)\tau\right)$, we have

$$\widetilde{J}(\varepsilon) = J_0[Q(K,z)] + \varepsilon K\tau v^{(1)}(z) J_1[Q(K,z)] + \frac{\varepsilon^2}{4} K^2\tau^2 (v^{(1)})^2 (J_0[Q(K,z)] - J_2[Q(K,z)]) + O(\varepsilon).$$

Rewriting equation 1 using our perturbed values gives us a quotient of two series in ε. Using long division, we can write out the entire expression as a single series in ε. The result of doing this, up to order ε, is

$$\begin{aligned} C_{\chi N}^{(\varepsilon)}(\rho,\tau) = {} & \frac{2.33(kL)^{5/6}}{R} \int_0^L dz C^{(0)}(z) \int_0^\infty dK P(K,z) J_0(Q(K,z)) \\ & + \varepsilon \left\{ \frac{2.33(kL)^{5/6}}{R} \int_0^L dz C^{(1)}(z) \int_0^\infty dK P(K,z) J_0(Q(K,z)) \right. \\ & + \frac{2.33(kL)^{5/6}}{R} \int_0^L dz C^{(0)}(z) \tau v^{(1)}(z) \int_0^\infty dK P(K,z) \cdot K \cdot J_1(Q(K,z)) \\ & \left. + \frac{2.33(kL)^{5/6}}{R} \int_0^L dz C^{(0)}(z) \int_0^\infty dK P(K,z) J_0(Q(K,z)) \left(\frac{\int_0^L dz C^{(1)}(z)[z(L-z)]^{5/6}}{\int_0^L dz C^{(0)}(z)[z(L-z)]^{5/6}} \right) \right\} \\ & + o(\varepsilon^2), \end{aligned}$$

where

$$\begin{aligned} P(K,z) &= K^{-8/3} \sin^2\left(\frac{K^2 z(L-z)}{2kL}\right), \\ Q(K,z) &= K\left(\frac{\rho z}{L} - v^{(0)}(z)\tau\right), \end{aligned}$$

and R is the normalizing constant (for structure constant $C^{(0)}$), given by

$$R = \int_0^L dz C^{(0)}(z)[z(L-z)]^{5/6}.$$

If we assume that $C^{(0)}$ and $C^{(1)}$ are comparable, then both the first and the last terms in the braces are comparable to the original covariance for $C^{(0)}$. The interesting term is the middle term, which is similar to the slope of the covariance at $\tau = 0$ given in the Lawrence, Ochs and Clifford paper.

To see if the perturbation affects the measurement of the wind using the slope technique, we differentiate this expression with respect to τ and set $\tau = 0$. This yields

$$\begin{aligned}\left.\frac{dC_{\chi N}(\rho,\tau)}{d\tau}\right|_{\tau=0} &= \frac{\int_0^L dz C^{(0)} v^{(0)}(z) W(z)}{R} + \varepsilon \frac{\int_0^L dz C^{(1)} v^{(0)}(z) W(z)}{R} \\ &+ \varepsilon \left(\frac{\int_0^L dz C^{(1)}(z)[z(L-z)]^{5/6}}{\int_0^L dz C^{(0)}[z(L-z)]^{5/6}} \right) \frac{\int_0^L dz C^{(0)} v^{(0)}(z) W(z)}{R} \\ &+ \varepsilon \cdot 2.33 (kL)^{5/6} \frac{\int_0^L C^{(0)} v^{(1)}(z) \int_0^\infty dKK \cdot P(K,z) J_1[\frac{K\rho z}{L}]}{R},\end{aligned}$$

which can be written more succinctly as

$$M_n = \frac{\int_0^L \widetilde{C}(z) v^{(0)}(z) W(z)}{\int_0^L dz C^{(0)}[z(L-z)]^{5/6}} + \varepsilon \frac{\int_0^L C^{(0)} v^{(1)}(z) W(z)}{\int_0^L dz C^{(0)}[z(L-z)]^{5/6}},$$

with

$$\widetilde{C}(z) = C^{(0)} + \varepsilon C^{(1)}(z) + \varepsilon \left(\frac{\int_0^L dz C^{(1)}[z(L-z)]^{5/6}}{\int_0^L dz [z(L-z)]^{5/6}} \right)$$

and

$$W(z) = 2.33(kL)^{5/6} \int_0^\infty dK K^{-5/3} \sin^2 \left(\frac{K^2 z(L-z)}{2kL} \right) J_1(K\rho z/L).$$

If we assume that $C^{(1)} = C^{(2)} = 0$, then we see that the expression for the slope M_n coincides with the expression given in Lawrence, Ochs and Clifford, regardless of the perturbation on v. Thus we conclude that fluctuations in the wind do not affect the covariance term.

When we assume that $v^{(1)} = v^{(2)} = 0$ and focus on the perturbations of C_n^2, what we see is that there is one additional term, corresponding to the final term in $\widetilde{C}(z)$, i.e.

$$M_n = \frac{\int_0^L dz (C^{(0)} + \varepsilon C^{(1)}(z)) v(z) W(z)}{\int_0^L dz C^{(0)}[z(L-z)]^{5/6}} + \frac{\int_0^L dz v(z) \varepsilon \left(\frac{\int_0^L dz C^{(1)}[z(L-z)]^{5/6}}{\int_0^L dz [z(L-z)]^{5/6}} \right) W(z)}{\int_0^L dz C^{(0)}[z(L-z)]^{5/6}}.$$

If we know that $C^{(0)}$ and $C^{(1)}$ are comparable, then this term will be very close to constant, and as such can be incorporated into the other terms. So, although there is some additional information given by the analysis, it appears that small perturbations in the C_n^2 term do not greatly affect the slope of the covariance at $\tau = 0$. However, if $C^{(1)}$ is significant, then the measurement of the slope can be altered significantly.

As a particular example, we look at the case where the wind velocity is not perturbed and the C_n^2 term is perturbed by a characteristic function, i.e.

$$C_n^2(z) = b\mathbf{1}_{[a_1,a_2]}.$$

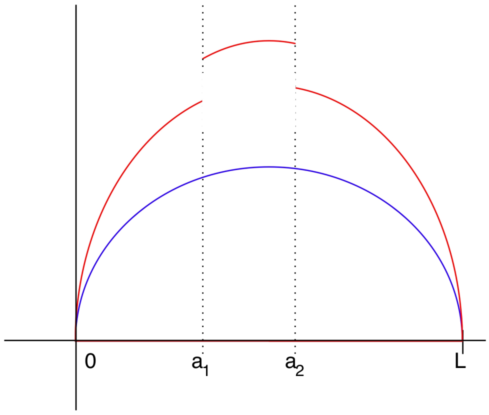

Figure 3: Perturbed and Unperturbed Path Weighting Functions. Blue curve is the graph of Unperturbed path weighting function assuming C_n^2. Red curve represents Perturbed path weighting function.

In this case, the slope of the perturbed covariance at $\tau = 0$ is given by the expression

$$M_n = \left(1 + \frac{\varepsilon b}{C^{(0)}}\alpha\right) \frac{\int_0^L dz v(z) W(z)}{\int_0^L dz [z(L-z)]^{5/6}} + \frac{\varepsilon b}{C^{(0)}} \frac{\int_{a_1}^{a_2} dz v(z) W(z)}{\int_0^L dz [z(L-z)]^{5/6}},$$

where

$$\alpha = \frac{\int_{a_1}^{a_2} dz [z(L-z)]^{5/6}}{\int_0^L dz [z(L-z)]^{5/6}} \leq 1.$$

When compared to the slope of the non-perturbed covariance, the path-weighting function is modified slightly. It is multiplied by a constant factor over the entire path, and over the region where C_n^2 is perturbed, the weight is also perturbed. If b and ε are large in relation to $C^{(0)}$, then this perturbation can have a significant impact on the measurement of the wind velocity.

4 Comparison of Spherical and Gaussian Wave Equations

4.1 Spherical Wave Equation

In this part, we show that for a spherical wave, the formulation for the covariance of the log-amplitude fluctuations presented by Andrews and Phillips [3] is the same as that presented by Lawerence, et. al.[15] (and Lee and Harp [18]).

The formulation presented by Andrews and Phillips[3] is

$$B_\chi(\tau, L) \approx \frac{1}{4} B_{\{I,sp\}} = 2\pi^2 k^2 L \int_0^1 \int_0^\infty \kappa \Phi_n(\kappa) J_0(\kappa V_\perp \tau) \times \left\{1 - \cos\left[\frac{L\kappa^2}{k}\xi(1-\xi)\right]\right\} d\kappa d\xi, \tag{3}$$

where

L is the distance the wave beam has propagated
τ is the time lag for the covariance
$V_\perp$ is the mean transverse wind velocity
J_0 is the 0th order Bessel function
k is the wavenumber of the beam
$\Phi_n(\kappa)$ is a given refractivity spectrum.

By Taylor's hypothesis, we can make the association $\xi p = V_\perp \tau$. Using this, the substitution $s = L\xi$, the trigonometric identity $\cos 2x = 1 - 2\sin^2 x$, and a little simiplifications we can get

$$\begin{aligned} B_\chi(\tau, L) &= 2\pi^2 k^2 \int_0^L \int_0^\infty \kappa \Phi_n(\kappa) J_0\left(\frac{\kappa s \rho}{L}\right) \left\{1 - \cos\left[\frac{s\kappa^2}{k}\left(1 - \frac{s}{L}\right)\right]\right\} d\kappa ds \\ &= 4\pi^2 k^2 \int_0^\infty \int_0^L \kappa \Phi_n(\kappa) J_0\left(\frac{\kappa s \rho}{L}\right) \sin^2\left[\frac{s\kappa^2(L-s)}{2kL}\right] ds d\kappa \end{aligned}$$

This is the same result as presented by Lawrence [15] without the normalized term. The outer integral here goes from 0 to ∞, while Lee and Harp [18] have bounds from 0 to k.

4.2 Gaussian Wave Equation

Andrews and Phillips [3] goes further and provides us with a hypergeometric approximation of the time-lagged cross covariance,

$$B_\chi(\tau, L) \approx 3.87\sigma_R^2 Re\left[0.40 i^{5/6} {}_1F_1\left(\frac{-5}{6}; 1; \frac{9ikV_\perp^2\tau^2}{8L}\right) - 0.47\left(\frac{kV_\perp^2\tau^2}{L}\right)^{5/6}\right],$$

where ${}_1F_1(a; c; z) = \sum_{n=0}^{\infty} \frac{(a)_n z^n}{(c)_n n!}$, $|z| < \infty$.

We can also find a relationship between the plane wave, spherical wave, and Gaussian wave cross covariance expressions. The Gaussian wave formulation given by Andrews and Phillips is represented by

$$\begin{aligned} B_I(r, L) = 8\pi^2 k^2 L \int_0^1 \int_0^\infty & \kappa \Phi_n(\kappa) \exp\left(-\frac{\Lambda L \xi^2 \kappa^2}{k}\right) J_0(\kappa V_\perp \tau) \\ & \times \left\{I_0(2\Lambda\kappa\xi r) - \cos\left[\frac{L\kappa^2}{k}\xi(1 - \bar{\Theta}\xi)\right]\right\} d\kappa d\xi, \end{aligned}$$

where, in addition to before,

$\bar{\Theta} = -\frac{L}{F_0}$
F_0 − is the phase front radius of curvature
$\Lambda = \frac{2L}{kW_0^2}$
W_0 − is the phase front beam radius
r − is a distance from the emitter towards the receivers
ρ − is the distance between r_1 and r_2
I_0 − is 0th order modified Bessel function $I_0(x) = J_0(ix)$

Andrews and Phillips [3] states that when $\Theta = 1$ and $\Lambda = 0$, then the gauss equation becomes the representation for the plane wave. When $\Theta = \Lambda = 0$, we obtain the spherical formula. This implies that the Gaussian cross covariance equation is the general expression for all the waves considered.

Because of complications arising from a more difficult formulation, Gaussian beams are often approximated by spherical waves. Therefore, it is of interest to make a comparison of Gaussian and spherical waves. We did not have time to make this comparison. Below, we present a strategy for developing a temporal formulation of the temporal cross covariance for the Gaussian beam. From this, we would compute the wind speeds for a Gaussian and spherical beam using the slope method and compare the results.

For the Gaussian beam, Andrews and Phillips provide a formulation for the spatial cross covariance. From this, we assume some specific beam parameters, and attempted to derive a formulation for temporal cross covariance. In particular, we assume

- The phase front radius of curvature (F_0) to be infinity
- The beam radius (W_0) to be between 0 and 10 cm
- The distance of wave propagation (L) to be between 100 m and 10 km
- The wavelength of the monochromatic light (λ) to be between 200 and 20,000 nm.

With these assumptions, the formulation provided by Andrews and Phillips (p.279) can be reduced to

$$\begin{aligned} C_\chi(\vec{p},\vec{r}) = 2\pi^2 k^2 L \int_0^1 \int_0^\infty & K\Phi_n(K)\exp\left(-\frac{\Lambda\xi^2 LK^2}{k}\right) \\ \times \operatorname{Re}\Bigg\{ & J_0\left[K\left|(1-\bar{\Theta}\xi)\vec{p} - 2i\Lambda\xi\vec{r}\right|\right] \\ - & \exp\left[-\frac{iLK^2\xi}{k}\right] J_0\left[(1-\bar{\Theta}\xi - i\Lambda\xi)K\rho\right]\Bigg\} dK\,d\xi \end{aligned} \tag{4}$$

where J_0 is the zero-order Bessel function, $\Lambda = \frac{\Lambda_0}{1+\Lambda_0^2}$, $\Lambda_0 = \frac{2L}{kW_0^2}$, $\bar{\Theta} = \frac{\Lambda_0^2}{1+\Lambda_0^2}$, and $\Phi_n(K)$ is the spectrum. The vector $\vec{p}$ is the vector between the two detectors and $\vec{r}$ is the vector from the transmitter to the midpoint of the detectors. Note that $|\vec{p}| = \rho$. Our plan is to take this spatial formulation of the covariance and use Taylor's hypothesis to form a temporal formulation so that we may apply the slope method to find the wind speed.

Each of the terms in the real-part expression of the cross covariance contain a $\vec{p}$ or its norm ρ, and thus will be affected by the flow of an eddy ball by Taylor's hypothesis. Because $F_0 = \infty$, the z coordinate of where a given eddy ball crosses into the beam influences what will be received.

For the second term in the real-part expression, Andrews and Phillips show that the expression $J_0\left[(1-\bar{\Theta}\xi - i\Lambda\xi)K\rho\right]$ becomes $J_0[\vec{v}_\perp\tau]$, for $\vec{v}_\perp$ the wind speed and

τ the time lag. When applying the slope method, we take the derivative and set $\tau = 0$. It can easily be seen that this term will thus go to 0.

The first term in the real-part expression, however, is more troublesome. Andrew and Phillips invoke a property of Bessel functions (Andrews, p.284), giving an approximation. However, this does not approximate the derivative of the expression well. By Taylor's hypothesis, we get that

$$[1 - (\bar{\Theta} + i\Lambda)\xi]\vec{p} = \vec{v}_\perp \tau \tag{5}$$

$$[1 - \bar{\Theta}\xi]\vec{p} = \vec{v}_\perp + i\Lambda\xi\vec{p}. \tag{6}$$

The first term in the real-part expression becomes

$$J_0[K|(1 - \bar{\Theta}\xi)\vec{p} - 2i\Lambda\xi\vec{r}|] = J_0[K|\vec{v}_\perp\tau + i\Lambda\xi\vec{p} - 2i\Lambda\xi\vec{r}|]. \tag{7}$$

Instead of making an approximation, the derivative with respect to τ could be directly computed, and the slope method could then be applied to find the wind speed.

5 A Different Model

Here we describe a very general method for determining the wind velocity profile that was developed recently by V.A. Banakh and D.A. Marakasov. Advantages of this method are that it does not require multiple receiver planes and light sources and that the wind profile reconstruction is independent of the variation of the structure function C_n^2 along the propagation path.

Let an optical wave propagate in a turbulent atmosphere along the x'-axis, starting from a point $x' = x_0$. The complex amplitude of this wave satisfies the following parabolic equation [4]

$$\begin{gathered} 2ik\frac{\partial U(x', \boldsymbol{\rho}, t)}{\partial x'} + \Delta_{\boldsymbol{\rho}} U(x', \boldsymbol{\rho}, t) + 2k^2\varepsilon_1(x', \boldsymbol{\rho}, t)U(x', \boldsymbol{\rho}, t) = 0 \\ U(0, \boldsymbol{\rho}, t) = U_0(\boldsymbol{\rho}), \end{gathered} \tag{8}$$

where $\Delta_{\boldsymbol{\rho}} = \partial^2/\partial y^2 + \partial^2/\partial z^2$ and $\varepsilon_1(x', \boldsymbol{\rho}, t)$ is the fluctuating part of the dielectric permittivity in air. Assume that the initial wave is specified by a Gaussian laser beam

$$U_0(\boldsymbol{\rho}) = U_0 \exp\left[-\frac{\boldsymbol{\rho}^2}{2a^2} - ik\frac{\boldsymbol{\rho}^2}{2F}\right].$$

The solution of (8) can be written in the form [5, 6]

$$U(x, \mathbf{r}, \tau) = \int U_0(\rho')G(x, \mathbf{r}; x_0, \rho', \tau)\, d\rho', \tag{9}$$

where

$$G(x, \mathbf{r}; x_0, \rho') = \frac{k}{2\pi i(x - x_0)} \exp\left[\frac{ik}{2(x - x_0)(\mathbf{r} - \rho')^2}\right] G_R(x, \mathbf{r}; x_0, \rho'; \tau) \tag{10}$$

is the Green function, $\mathbf{r}$ is a vector in the plane perpendicular to the x'-axis at the point $x' = x$ and ρ' is a vector on the source plane (i.e. the plane perpendicular to the x'-axis at the point $x' = x_0$). The random component of the Green function, G_R, takes into account the turbulent fluctuations. The exact form of G_R can be found, for example, in [5, 7, 6]. Based on the Green function representation of the solution $U(x, \mathbf{r}, \tau)$ Banakh and Marakasov have developed a powerful technique for calculating the wind velocity from the spatiotemporal intensity correlation function. This technique applies to Gaussian waves and, in particular, for spherical and plane waves. It was shown how to use these new ideas in more complicated situations, specifically, when the laser reflects off a random surface and is detected on the plane of the source [6] or in the focal plane of a telescope placed behind the source [7]. Below we describe the main idea for the case of a Gaussian wave and detector surface located at the point $x' = x$, i.e. without reflection. We follow the paper [5]. The spatiotemporal intensity correlation function is

$$K(x, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2; \tau) = < I(x, \boldsymbol{\rho}_1, 0) I(x, \boldsymbol{\rho}_2, \tau) > - < I(x, \boldsymbol{\rho}_1, 0) >< I(x, \boldsymbol{\rho}_2, \tau) >,$$

where the intensity $I(x_0, \boldsymbol{\rho}; \tau) = U(x_0, \boldsymbol{\rho}; \tau) U^*(x_0, \boldsymbol{\rho}; \tau)$. Using the form (9) results in the following long and complicated expression, which is significantly simplified in the cases of spherical and plane waves.

$$\begin{aligned} K(\mathbf{R}, \boldsymbol{\rho}, \tau) = \frac{\pi k^2 x U_0^4 \Omega^4}{2g^4} \exp\left\{ -\frac{k\Omega}{2xg^2}(\boldsymbol{\rho}^2 + 4\mathbf{R}^2) \right\} \times \\ \int_0^1 d\xi \int d\kappa C_n^2(\xi) \Phi_\varepsilon(K) e^{iKV(\xi)\tau} \sum_{n=1}^{4} (-1)^n \exp\{-t_n K^2 + \mathbf{q}_n \kappa\}, \end{aligned} \tag{11}$$

where $\mathbf{R} = (\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2)/2,\ \boldsymbol{\rho} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$.

In the cases of spherical and plane waves the forms of $K(x_0, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2; \tau)$ coincide with the results of Wang, Ochs and Lawrence. As usual the assumptions of Taylor's hypothesis of frozen turbulence, Kolmogorov's spectrum and weak fluctuation regime are made. Taylor's hypothesis gives $\varepsilon_1(x', \boldsymbol{\rho}; \tau) = \varepsilon_1(x', \boldsymbol{\rho} - \mathbf{V}(x')\tau; 0)$ and this is how the wind velocity $\mathbf{V}(x')$ enters the equation.

Here is a very rough description of the algorithm for finding the wind velocity profile $V(x')$.

- Normalize the correlation function $K(\mathbf{R}, \boldsymbol{\rho}, \tau) \rightarrow f(\mathbf{R}, \boldsymbol{\rho}, \tau)$ and take the difference $D(\mathbf{R}, \boldsymbol{\rho}, \tau) = f(\mathbf{R}, \boldsymbol{\rho}, \tau) - f(-\mathbf{R}, \boldsymbol{\rho}, \tau)$.

- Take the Fourier transform w.r.t. $\boldsymbol{\rho}$ and $\tau \Rightarrow \widetilde{D}$ has the form

  $$\widetilde{D} = ...\delta\left(\omega + \frac{\mathbf{q}\mathbf{V}(\xi)}{\gamma(\xi)(1-\xi)}\right)$$

  Here ω is the Fourier dual variable to τ, and $\mathbf{q}$ is the dual of $\boldsymbol{\rho}$.

- Assume that the vector $\mathbf{q}$ is directed along one of the coordinate axis, i.e. $\mathbf{q} = q\mathbf{e}_i$. The condition $\widetilde{D} \neq 0$ results in $V_i(\xi) = (\omega/q)\,[(1-\xi)\gamma(\xi)]$. Fix a value $\alpha = \omega/q$.

- Find a set of points ξ_j s.t. $V(\xi_j) = \alpha$. This is done by defining a function g, which can be computed from the data we have, and which has δ-peaks exactly at the points ξ_j such that $V_i(\xi_j) = \alpha$.
- Iterate for different values of α.

An important feature of this algorithm is that one can resolve the wind velocity profile without any information about the variations of C_n^2 along the propagation path.

This method, with certain modifications, is applied to situations where the laser beam is reflected off a surface and then detected in the plane of the source [6]. In this case the reciprocity of the Green function is used to write the wave arriving at the detector plane:

$$U_R(x_0, \boldsymbol{\rho}) = \int U_0(\rho')V(\mathbf{r}, \mathbf{r}')G(x, \mathbf{r}; x_0, \rho')G(x, \mathbf{r}'; x_0, \boldsymbol{\rho})\, d\mathbf{r}d\mathbf{r}'d\rho',$$

where $V(\mathbf{r}, \mathbf{r}')$ is the reflection coefficient of the surface. The computations that follow are more involved but the general idea remains the same.

6 Conclusion

We started by investigating the cross covariance expression given by [15]. Most of the references assume that C_n^2 is constant. We analyzed the variations of C_n^2 by considering a Brownian model and small perturbations of a constant C_n^2. Also, perturbation analysis for the velocity $v(z)$ is presented. A lot of the literature considers spherical waves, but for our application we use Gaussian Laser beams. Hence we compared the cross covariance expressions for both the spherical and Gaussian waves.

7 Bibliography

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2. L.C.Andrews, " An analytical model for the refractive index power spectrum and its application to optical scintillations in the atmosphere", J. Mod. Opt., **39**, 1849 - 1853, 1992.

   A spectral model for the refractive index fluctuataions, showing characteristic bump at high wavenumbers, is proposed. Using this spectrum, analutic expressions are derived for the variance of log-amplitude that show good agreement with numerical results based on the Hill spectrum.

3. L.C.Andrews and R.L.Phillips , *Laser Beam Propagation through Random Media, Second Edition.* SPIE Press, 2005. This book gives an extensive treatement of the effect of atmospheric turbulence on the propagation of laser beam. Most of the references cited in this report are obtained from this book. They also provide closed form solutions to cross-correlation expressions for both spherical and Gaussian waves.

4. V.A. Banakh, D.A. Marakasov, "Wind velocity profile reconstruction from intensity fluctuations of a plane wave propagating in a turbulent atmosphere", *Optic Letters*,**32**, 2007.

   This is the first paper in the series of papers by these authors, in which they deal with propagation of waves in turbulent atmosphere. This paper is the simplest of all that follow and the method used for finding the wind speed is well described.

5. Banakh,V.A. Marakasov,D.A., "Wind profiling based on the optical beam intensity statistics in a turbulent atmosphere", *J. Opt. Soc. Am. A*, **24**, 3245-3254, 2007

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8. Bernt Oksendal, *Stochastic Differential Equations, Fifth Edition*, Springer-Verlag, Berlin Heidelberg New York, 1998.

   The book is an introduction to Stochastic Differential Equations, including formulations of Brownian motion and Itô's formula.

9. J.T.Beyer, M.C.Roggemann, L.J.Otten, T.J.Schultz, T.C.Havens, and W.W.Brown, "Experimental estimation of the spatial statistics of turbulence-induced index of refraction fluctuations in the upper atmosphere", *Appl/ Opt.*, 908 - 921 2003.

   In this paper the authors conducted some experiments to estimate the parameters of homogeneous, isptropic optical turbulence in the upper atmosphere. The ballon borne experiment made high resolution temperature measurementsat seven points on a hexagonal grid for altituded from 12,000 to 18,000 m.

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An expression for the refractive index structure parameter C_n^2 is given in terms of the temperature and water vapur pressure structure parameters. Different expressions are given acoustic, optical and microwave radiations. In case of optics the the refractive index structure parameter C_n^2 is shown to be depending only on temperature.

11. D.L.Fried "Remote Probing of the Optical Strength of Atmospheric Turbulence and of Wind Velocity", *Proc. IEEE*, **57**, 415 - 420, 1969.

    Fried developes a procedure to determine the optical strength of turbulence of the atmosphere C_n^2 and wind velocity $v(z)$ by measuring the spatial and temporal covariance of scintillation. For determining C_n^2 a linear integral equation is developed, however to determine $v(z)$ a non-linear integral equation is developed. This procedure applies to plane waves, but a similar analysis can be developed that applies to spherical waves.

12. A.E.Green, K.J.McAneney, and M.S.Astill, "Surface layer scintillation measurements of daytime heat and momentum fluxes", *Boundary-Layer Meteoralogy*, **68**, 357-373, 1994.

    Line averaged measurements of the structure parameter of the refractive index (C_n^2) were made using a semiconductor laser diode scintillometer above tow different surfaces during hours of positive net radiation. Atmospheric stability ranged from nearly neutral and strongly unstable. The temperature structure parameter C_T^2 computed from the optical measurements over four decades agreed to within 5%of those determined from temperature spectra in the inertial subrange of frequencies.,

13. R.J.Hill, "Corrections to Taylor's froaen turbulence approximation", *Atmos. Res.*, **40**, 153 - 175, 1996.

    The author uses Lumley's two term approximation which gives corrections for the effect of fluctuating convection velocity. Such corrections are given for every turbulence statistic. The statistic may be a tensor of any rank, and may be a correlation or structure function or spectrum.

14. R.J.Hill , G.R.Ochs, J.J.Wilson, " Measuring surface layer fluxes of heat and momentum using optical scintillation", *Boundary-Layer Meteorology*, **58**, 391-408,1992.

    An experiment is described showing that an optical scintillation instrument gives reliable values of heat and momentum fluxes in the surface layer, subject to the usual restrictions of homogeneity and steady state.

15. R.S.Lawrence, G.R.Ochs, and S.F.Clifford "Use of Scintillations to measure average wind across a light beam", *Appl. Opt.*, **11**, 239 - 243, 1972.

This is one of the papers that we initially started with. Here the authors derive a relationship between cross-correlation and the wind speed. Also, the slope method is explained and a prototype of an instrument to measure the wind is presented.

16. R.S.Lawrence, G.R.Ochs, and S.F.Clifford "Measurement of Atmospheric Turbulence Relevant to optical Propagation", *J. Opt. Soc. Am.*, **60**, 826 - 830, 1970.

    The authors demostrate the application of high-speed temperature sensors to the direct measurement of refractive index variations at optically important scale sizes, as small as a few millimeters. The thermometers used in pairs with spacings ranging from 3mm to 1mm, disclose that the turbulence near the ground frequently differs substantially from the Kolmogorov model, and that the temperature difference does not follow the Gaussina probability density function.

17. R.S.Lawrence, and J.W.Strohehn, "A survey of clear-air propagation effects relevant to optical communications", *Proc. IEEE*, **58**, 1523 - 1545, 1969.

    This review paper summerizes the theory and observations of the optical propagation effects of the clear turbulent atmosphere. This is mostly important to the designer of optical communication system. The phenomena considered are the variance, probability distribution, spatial covariance, aperture smoothing, and temporal power spectrum of intensity fluctuations.

18. R.W.Lee, J.C.Harp, "Weak Scattering in Random Media, with Applications to Remote Probing", *Proc. IEEE*, **57**, 375 - 405, 1969.

    This is one of the papers that we initially started with. Most of the cross correlation results that we have used come from this paper. Here the authors have derived relationship between the cross covariance and the refractive power spectrum. In this paper plane and spherical waves are considered.

19. G.R.Ochs, T.Wang, R.S.Lawrence, and S.F.Clifford "Refractive-turbulence profiles measured by one-dimensional spatial filtering of scintillations", *Appl. Opt.*, **15**, 2504 - 2510, 1976.

    Here they describe an instrument involving a 36-cm telescope and on-line mini-computer that provides, after 20 mins of observation, the refractive-turbulence profile of the atmosphere. They acheive this by calculating the variance of the filtered intensity scintillations normalized to the mean irradiance and by solving a linear integral equations for C_n^2.

20. G.R.Ochs and T.Wang, "Finite aperture optical scintillometer for profiling wind and C_n^2", *Appl. Opt.*, **17**, 3774 - 3778, 1978.

In this paper the authors describe an optical technique for measuring the path profiles of crosswind and of a refractive-index structure parameter C_n^2 along a line-of-sight path. Different sizes of transmitters and receivers are used to control the path-weighting function so that it will peak at different path locations. A prototype instrument is built and tested.

21. L.Poggio, *Use of Scintillation Measurements to Determine Fluxes in Complex Terrain, PhD Dissertation,* Swiss Federal Institute of Technology (ETH), Zurich, 1998.

    This dissertataion has an extensive list of references and detailed explanation of atmospheric turbulence. They have some simulations for varying C_n^2. The experiments they conduct are mainly in a valley region where the turbulence varies over small portion of the path.

22. M.C.Roggemann and B.M.Welsh, *Imaging Through Turbulence*, CRC Press, 1996.

    This book mainly describes turbulence effects on imaging systems. They describe some techniques to oversome the effects of turbulence, for example post processing, adaptive optics and hybrid methods. Some of the references used in this report were taken from this book

23. V.I. Tatarski, *Wave propagation in a turbulent medium.* New York: McGraw-Hill, 1961.

    The book starts with a gentle introduction to the subject of wave propagation in a turbulent medium. In chapter 6 the author derives the correlation function of amplitude fluctuations of a wave in a locally isotropic turbulent flow using the equations of geometrical optics. Inchapter 7 the same is done, this time using the wave equation and the method of small and smooth perturbations. In chapters 8 and 9 the results are extended to the case of Cn^2 that is a very smooth function. The correlation function of the amplitude fluctuations of a plane monochromatic wave is derived in chapter 8. In chapter 9 the same is done for a spherical wave.

24. T.Wang, G.R.Ochs, and R.S.Lawrence, "Wind measurements by the temporal cross-correlations of the optical scintillations", *Appl. Opt.*, **20**, 4073 - 4081, 1981.

    Authors of this paper consider various methods of cross covariance analysis to deduce the cross wind profile from a drifting scintillation pattern are described. These different methods ara compared to their immunity ot noise and their accuracy when faced with nonuniformities along the propagation path or changes in the characteristics of the turbulence C_n^2. Of the techinques considered, none is ideal. Also, they have some simulation results for the varying C_n^2 case.

25. M.L.Wesely and E.C.Alcaraz, “ Diurnal cylces of the refractive index structure function coefficient”, *J. Geop. Res.*, **78**, 6224-6232, 1973.

    An indirect method is developed in which C_n^2 is calculated from the estimates of sensible and latent heat flux components of the surface energy budgets. This indirect method is for heights less than 4 meters, because low intermittency and a near unity value of the ratio of the eddy diffusivity to that of momentum are assumed.

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