Vortices in Neutron Superfluids

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with **Kota Masuda** (U.Tokyo/ RIKEN) nuclear phys

Takeshi Mizushima (Osaka U.) cond-mat phys

Masuda & MN, Phys.Rev.C [arXiv:1512.01946 [nucl-th]]
arXiv:1602.07050 [nucl-th]

Mizushima, Masuda & MN, arXiv: 1605.***** [cond-mat]

Neutron Star Matter

Keio University
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Topological Materials Science

Keio U.
@ Yokohama

in greater Tokyo

Topological Science Project

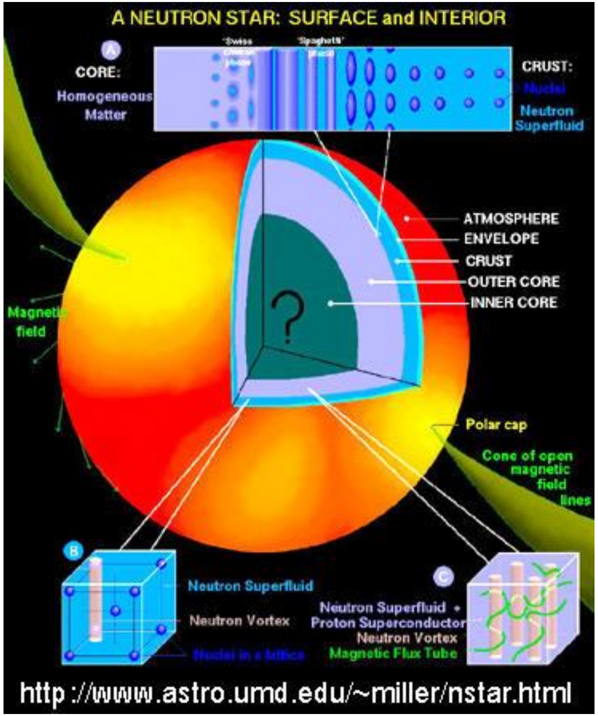
Aimed to understand all subjects of physics in terms of Topology

5 years, 11 postdocs

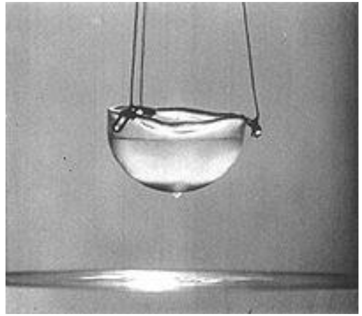
Anyone here is very welcome to visit!!

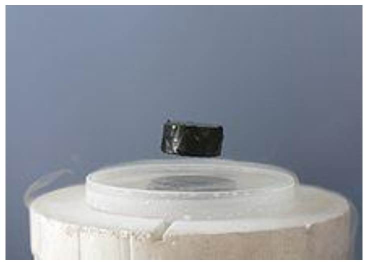
Neutron Stars

Core Nuclear matter



Neutron superfluid





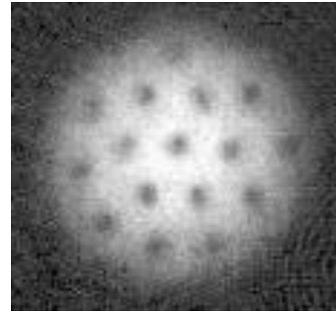
Proton super-conductor

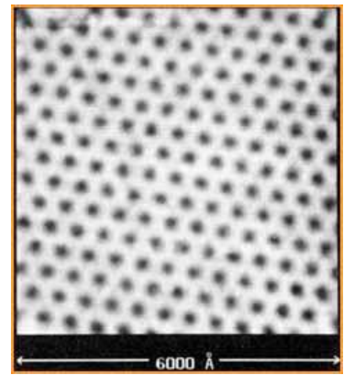
Rotation

Magnetic field

Baym&Pines ('60s)
Anderson&Itoh ('75)

Superfluid vortices

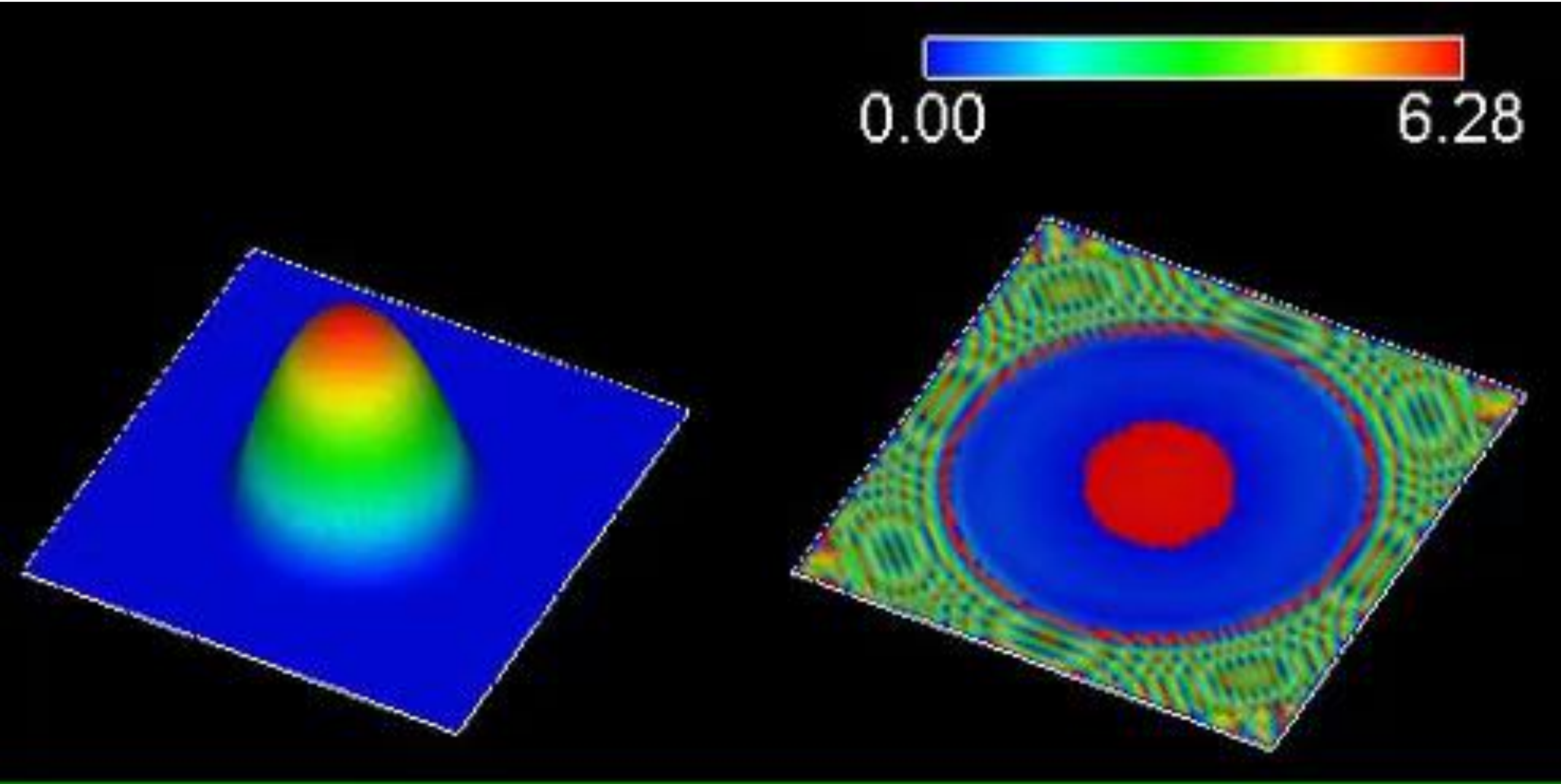




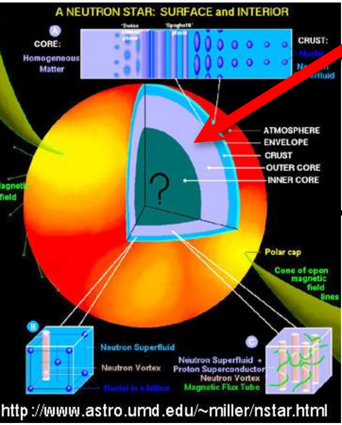
vortices (Flux tubes)

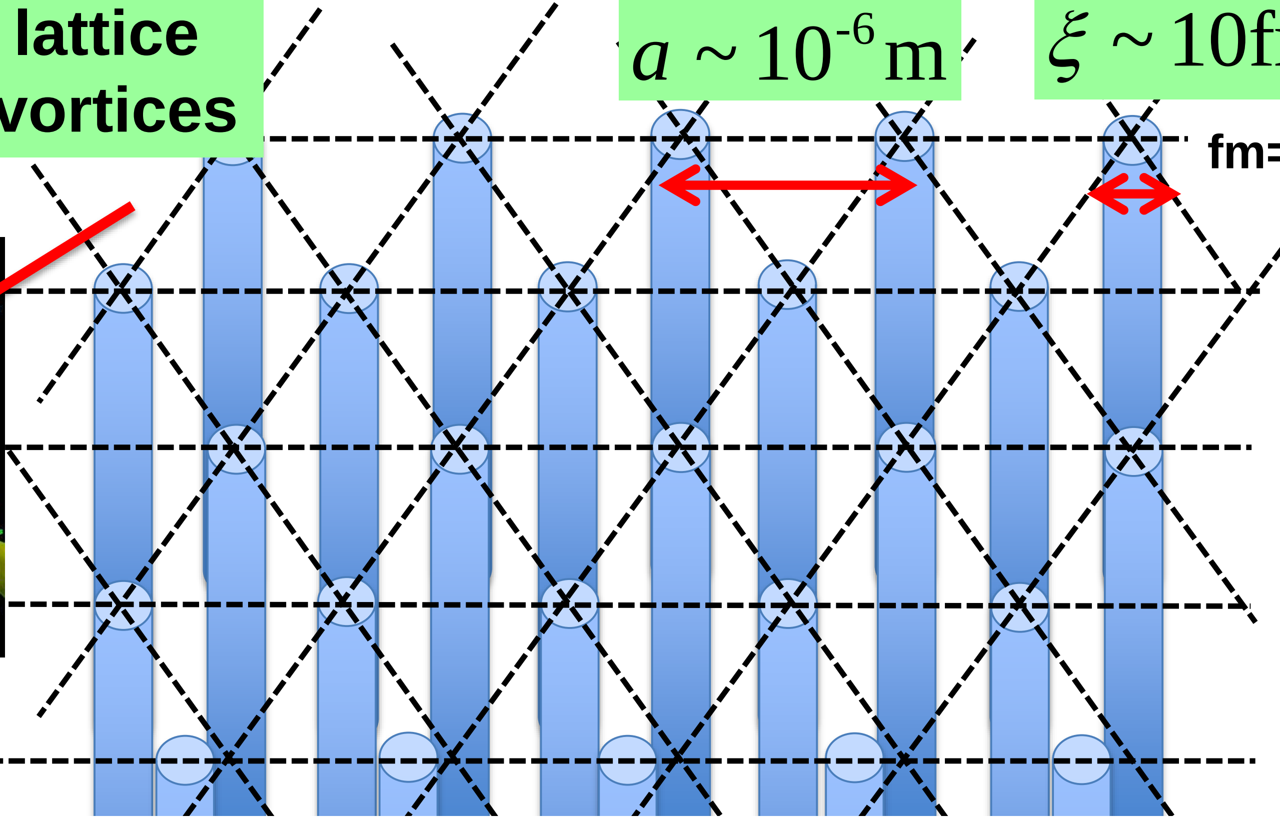
Vortex lattice in a rotating superfluid (Bose-Einstein condensate)

K.Kasamatsu



Abrikosov lattice of integer vortices





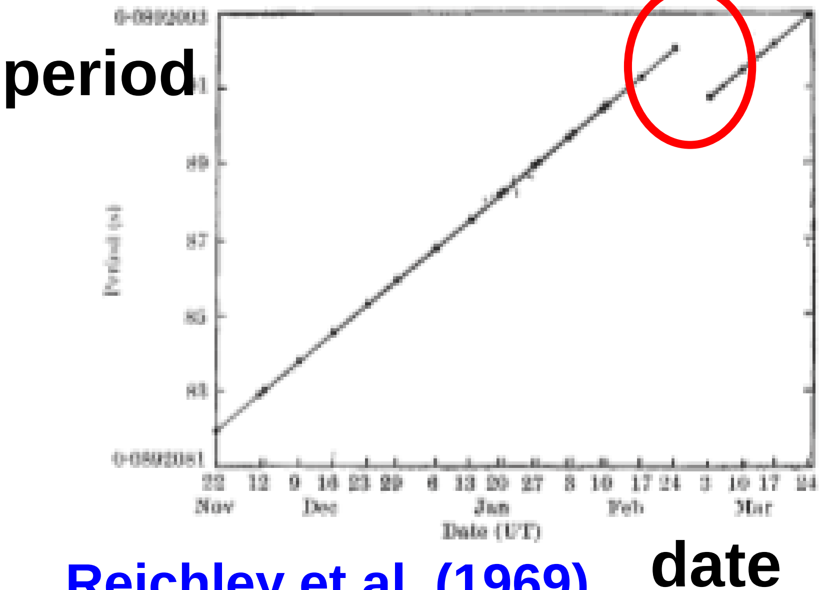
A huge # of vortices!!

$$N_v \equiv 1.9 \times 10^{19} \left(\frac{1\mathrm{ms}}{P}\right) \left(\frac{M}{900\mathrm{MeV}}\right) \left(\frac{R}{10\mathrm{km}}\right)$$

Evidences of neutron superfluidity in observation

Pulsar Glitch

sudden speed up of rotation



Reichley et.al. (1969)

Long relaxation time after glitch implies the existence of superfluid

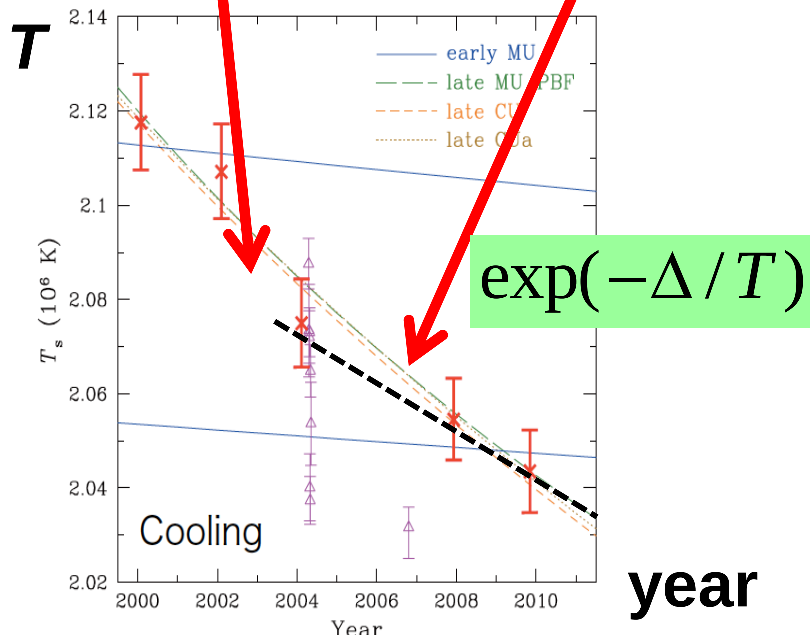
Baym&Pines ('60s)

Cooling by neutrino emission

Superconducting Gap Δ

⇒ rapid cooling ⇒slow cooling

(gap is formed)



Heinke & Ho (2010)

Evidences of neutron superfluidity in observation

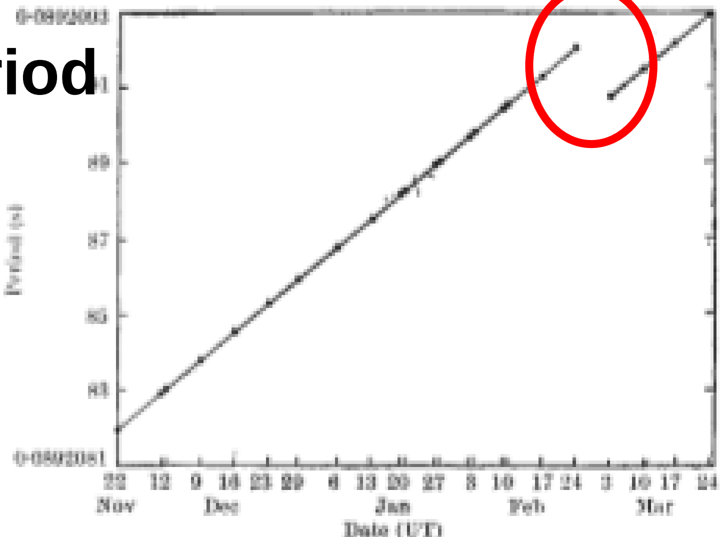
Pulsar Glitch

sudden speed up of rotation

= releasing large # of vortices

Anderson&Itoh ('75)

period



date

Reichley et.al. (1969)

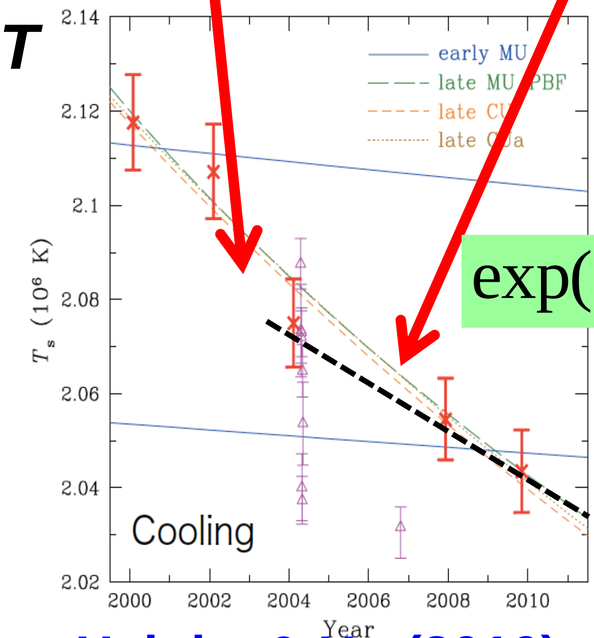
Cooling by neutrino emission

Superconducting Gap Δ

⇒ rapid cooling ⇒slow cooling

(gap is formed)

T

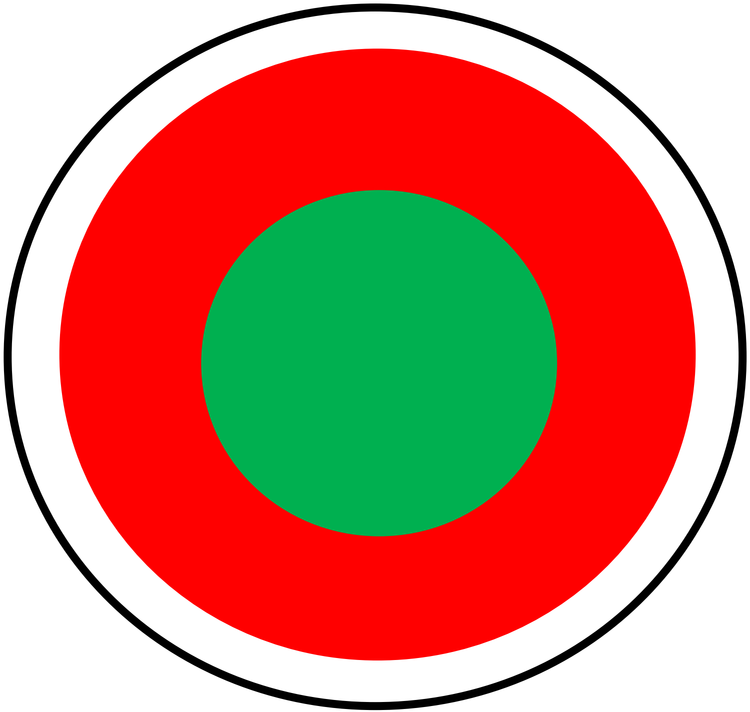


$\exp(-\Delta / T)$

year

Heinke & Ho (2010)

Neutron star

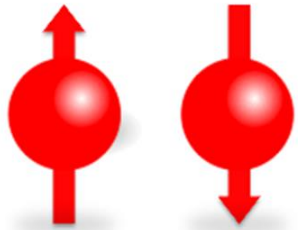


Tamagaki & Takatsuka (1970-72)

$\rho \sim \rho_0 (10^{14} \sim 10^{15} \mathrm{g/cm^3})$

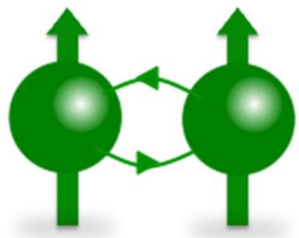
: Transition from $^1S_0$ to $^3P_2$

$^1S_0$ pairs



@outer core

$^3P_2$ pairs



@ inner core

$^3P_2$

spin s=1

angular l=1

total j=2

order parameter

A

complex scalar

(Goldstone model, Gross-Pitaevskii)

$A_{\mu i}$

spin angular

traceless symmetric 3x3 complex tensor

fields = complex 5 (of SO(3))

Plan of My Talk

§ 1 Introduction (4p)

§ 2 GL Theory for $^3P_2$ and Ground State (4p)

§ 3 Vortices (6p)

§ 4 Topological Superfluidity (3p)

§ 5 Summary

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Ginzburg-Landau effective theory for $^3P_2$ superfluid

order parameter $A_{\mu i}$ (spin: μ, angular: i) **traceless symmetric 3x3 complex matrix**

 # fields = complex 5 (of SO(3))

$$G = U(1) \times SO(3)_{\mathrm{S+L}} \quad A \to e^{i\alpha} O A O^T \quad O \in SO(3)_{\mathrm{S+L}} \quad e^{i\alpha} \in U(1)$$

$^3P_2$

spin s=1, angular l=1, total j=2

If A was 5 rep of *internal* SO(3)

gradient $\boxed{K(\partial_i A_{\mu\nu} \partial_i A^\dagger_{\mu\nu})}$

interaction $\boxed{f_4 = \alpha \mathrm{Tr} A A^\dagger + \beta[(\mathrm{Tr} A A^\dagger)^2 - \mathrm{Tr} A^2 A^{\dagger 2}]}$

The same with Gross-Pitaevskii energy functional for spin-2 BECs

Ginzburg-Landau effective theory for $^3P_2$ superfluid

order parameter $A_{\mu i}$ **traceless symmetric 3x3 complex matrix**

spin angular

 # fields = complex 5 (of SO(3))

$$G = U(1) \times SO(3)_{\mathrm{S+L}} \quad A \to e^{i\alpha} O A O^T \quad O \in SO(3)_{\mathrm{S+L}} \quad e^{i\alpha} \in U(1)$$

gradient $\boxed{K(\partial_i A_{\mu\nu} \partial_i A^\dagger_{\mu\nu}} + \partial_i A_{\mu i} \partial_j A^\dagger_{\mu j} + \partial_i A_{\mu j} \partial_j A^\dagger_{\mu i})$

internal

potential $\boxed{f_4 = \alpha \mathrm{Tr} A A^\dagger + \beta[(\mathrm{Tr} A A^\dagger)^2 - \mathrm{Tr} A^2 A^{\dagger 2}]}$

$$f_6 = \gamma[-3(\mathrm{Tr}AA^\dagger)|\mathrm{Tr}AA|^2 + 4(\mathrm{Tr}AA^\dagger)^3 + 12(\mathrm{Tr}AA^\dagger)(\mathrm{Tr}AA^\dagger)^2 + 6(\mathrm{Tr}AA^\dagger)\mathrm{Tr}(A^2A^{\dagger 2}) \\ + 8\mathrm{Tr}(AA^\dagger)^3 + 12\mathrm{Tr}[(AA^\dagger)^2 A^\dagger A] - 12\mathrm{Tr}[AA^\dagger A^\dagger A^\dagger AA] - 12\mathrm{Tr}AA(\mathrm{Tr}AA^\dagger AA)^*]$$

Magnetic field *H* $f_B = g' H^2 \mathrm{Tr}(AA^\dagger) + g H_\mu (AA^\dagger)_{\mu\nu} H_\nu$

Fujita&Tsuneto ('72), Richardson('72), Sauls('78)

Ginzburg-Landau effective theory for $^3P_2$ superfluid

order parameter $A_{\mu i}$ **traceless symmetric 3x3 complex matrix**

spin angular

 # fields = complex 5 (of SO(3))

$$G = U(1) \times SO(3)_{\mathrm{S+L}} \quad A \to e^{i\alpha} O A O^T \quad O \in SO(3)_{\mathrm{S+L}} \quad e^{i\alpha} \in U(1)$$

gradient $K\left(\partial_i A_{\mu\nu} \partial_i A^\dagger_{\mu\nu} + \boxed{\partial_i A_{\mu i} \partial_j A^\dagger_{\mu j} + \partial_i A_{\mu j} \partial_j A^\dagger_{\mu i}}\right)$

specific to $^3P_2$

potential $f_4 = \alpha \mathrm{Tr} A A^\dagger + \beta[(\mathrm{Tr} A A^\dagger)^2 - \mathrm{Tr} A^2 A^{\dagger 2}]$ the same as ^{3}He

$$\boxed{\begin{aligned} f_6 =& \gamma[-3(\mathrm{Tr}AA^\dagger)|\mathrm{Tr}AA|^2 + 4(\mathrm{Tr}AA^\dagger)^3 + 12(\mathrm{Tr}AA^\dagger)(\mathrm{Tr}AA^\dagger)^2 + 6(\mathrm{Tr}AA^\dagger)\mathrm{Tr}(A^2A^{\dagger 2}) \\ &+ 8\mathrm{Tr}(AA^\dagger)^3 + 12\mathrm{Tr}[(AA^\dagger)^2A^\dagger A] - 12\mathrm{Tr}[AA^\dagger A^\dagger A^\dagger AA] - 12\mathrm{Tr}AA(\mathrm{Tr}AA^\dagger AA)^*] \end{aligned}}$$

Magnetic field $\boldsymbol{H}$ $f_B = g' H^2 \mathrm{Tr}(AA^\dagger) + g H_\mu (AA^\dagger)_{\mu\nu} H_\nu$

Fujita&Tsuneto ('72), Richardson('72), Sauls('78)

Ginzburg-Landau effective theory for $^3P_2$ superfluid

order parameter $A_{\mu i}$ traceless symmetric 3x3 complex matrix

spin angular

fields = complex 5 (of SO(3))

GL parameters in the weak coupling (BCS) limit

TABLE I. Coefficients in BCS limit.

α	K	β	γ	g
$\frac{N(0)}{3}\frac{T-T_c}{T}k_F^2$	$\frac{7\xi(3)}{240m^2}\frac{N(0)}{(\pi T_c)^2}k_F^4$	$\frac{7\xi(3)}{60}\frac{N(0)}{(\pi T_c)^2}k_F^4$	$-\frac{31}{16}\frac{\xi(5)}{840}\frac{N(0)}{(\pi T_c)^4}k_F^6$	$\frac{7\xi(3)}{24}\frac{N(0)}{(\pi T_c)^2}\frac{(\gamma\hbar)^2}{2(1+F)^2}H^2k_F^2$

Fujita&Tsuneto ('72), Richardson('72), Sauls('78)

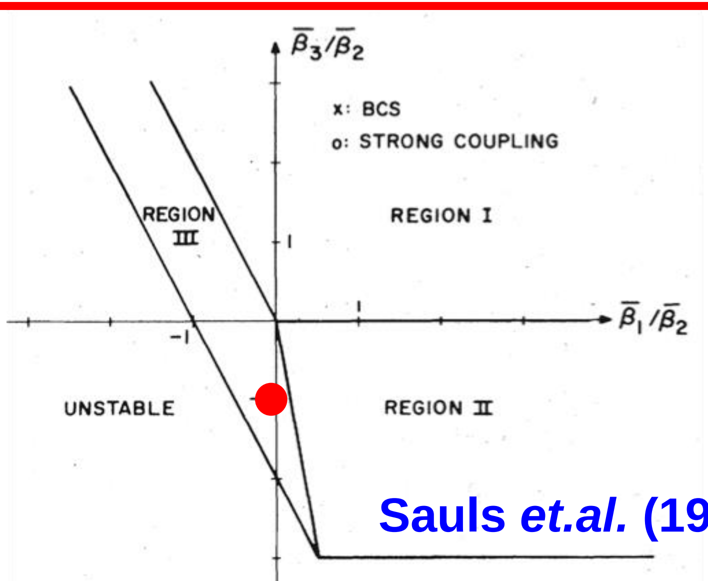
Ground state of $^3P_2$ GL up to 4th with no magnetic field H

$$A \sim \begin{pmatrix} r & 0 & 0 \\ 0 & -1-r & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$-1 \le r \le -1/2$$

continuous degeneracy r

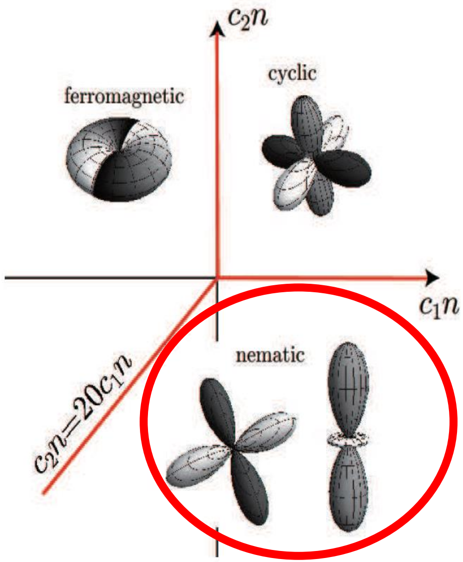
nematic phase @ BCS limit



Sauls *et.al.* (1978)

1:1

Ultracold atomic Spin-2 Bose-Einstein condensate



Kawaguchi and Ueda (2010)

Ground state of $^3P_2$ GL up to 4th with no magnetic field H

$$A \sim \begin{pmatrix} r & 0 & 0 \\ 0 & -1-r & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$-1 \le r \le -1/2$$

continuous degeneracy r

$$G = U(1) \times SO(3)_{\mathrm{S+L}}$$

H

G/H **Order Parameter Manifold (OPM)**

$r = -1/2$

Uniaxial

$O(2)$

$$U(1) \times \frac{SO(3)}{O(2)} = U(1) \times \mathbf{R}P^2$$

$-1 < r < -1/2$

D_2 biaxial

~ biaxial nematic liquid

D_2

$$U(1) \times \frac{SO(3)}{D_2}$$

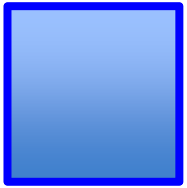
The whole mfd is

$$\frac{U(1) \times S^4}{\mathbf{Z}_2}$$

$r = -1$

D_4 biaxial

D_4



$$\frac{U(1) \times SO(3)}{D_4}$$

Cf) quasi-NG modes
Uchino-Kobayashi-MN-Ueda
PRL105,230406 ('10)

Degeneracy is lifted by 6^{th} order &/or magnetic field H

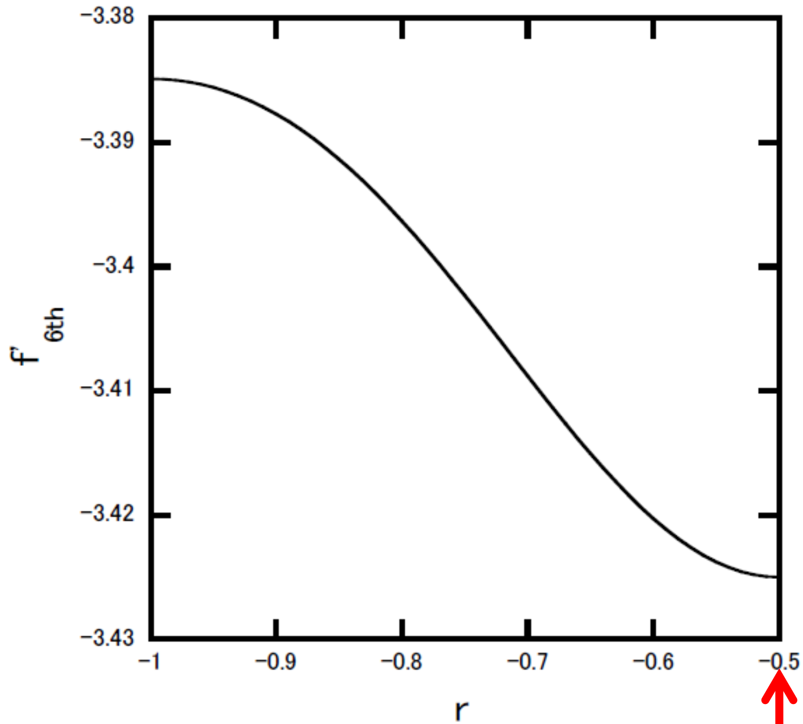
6^{th} order term → Uniaxial $r=-1/2$

Masuda & MN
arXiv:1512.01946
[nucl-th]

$r=-1/2$
Uniaxial

$-1<r<-1/2$
D_2 Biaxial

$r=-1$
D_4 Biaxial



Degeneracy is lifted by 6th order &/or magnetic field H

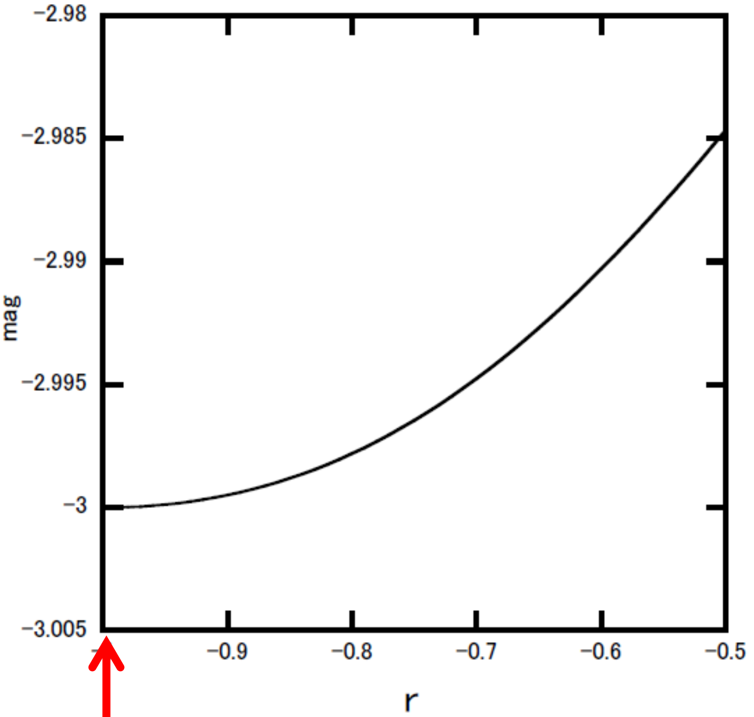
Masuda & MN
arXiv:1512.01946
[nucl-th]

magnetic field ⟶ D_4 Biaxial $r=-1$

$r=-1/2$
Uniaxial

$-1<r<-1/2$
D_2 Biaxial

$r=-1$
D_4 Biaxial



Degeneracy is lifted by 6th order &/or magnetic field *H*

6th order term → **Uniaxial** $r=-1/2$

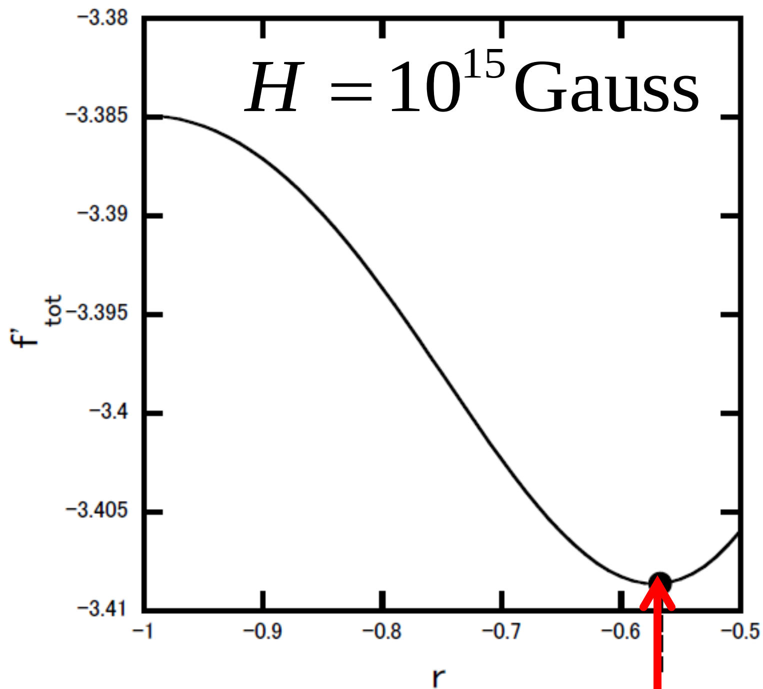
magnetic field → ***D*$_4$ Biaxial** $r=-1$

Masuda & MN
arXiv:1512.01946
[nucl-th]

$r=-1/2$
Uniaxial

$-1<r<-1/2$
***D*$_2$ Biaxial**

$r=-1$
***D*$_4$ Biaxial**



$r_{\mathrm{tot}}\sim-0.572$

Degeneracy is lifted by 6th order &/or magnetic field *H*

6th order term → **Uniaxial** $r = -1/2$

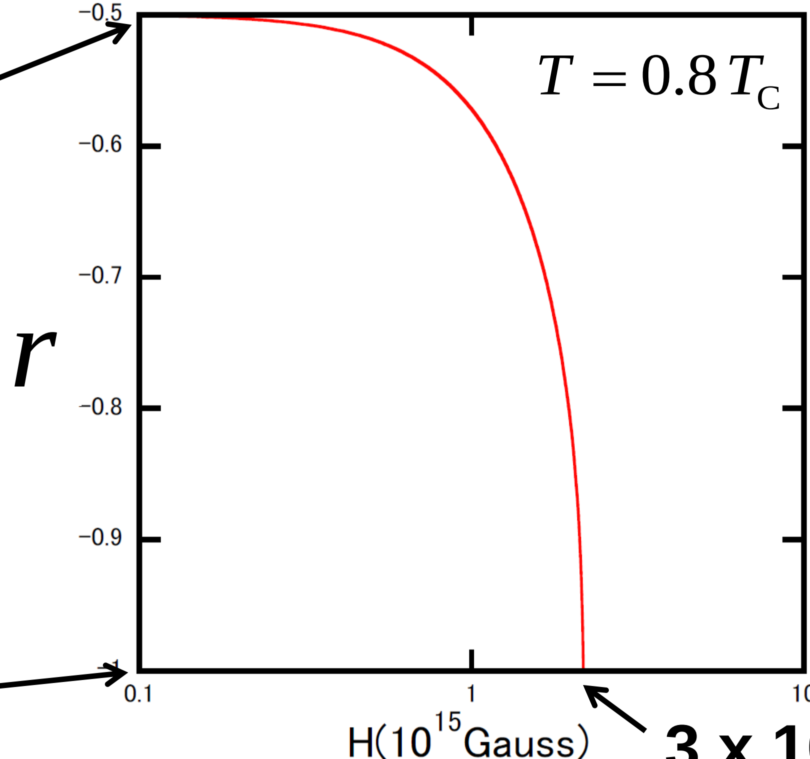
magnetic field → D_4 **Biaxial** $r = -1$

Masuda & MN
arXiv:1512.01946 [nucl-th]

$r = -1/2$
Uniaxial

$-1 < r < -1/2$
D_2 **Biaxial**

$r = -1$
D_4 **Biaxial**



3 x 10^{15}G

Degeneracy is lifted by 6th order &/or magnetic field *H*

6th order term → **Uniaxial** $r=-1/2$

magnetic field → **D_4 Biaxial** $r=-1$

Masud… MN

…12.0194…

$r=-1/2$

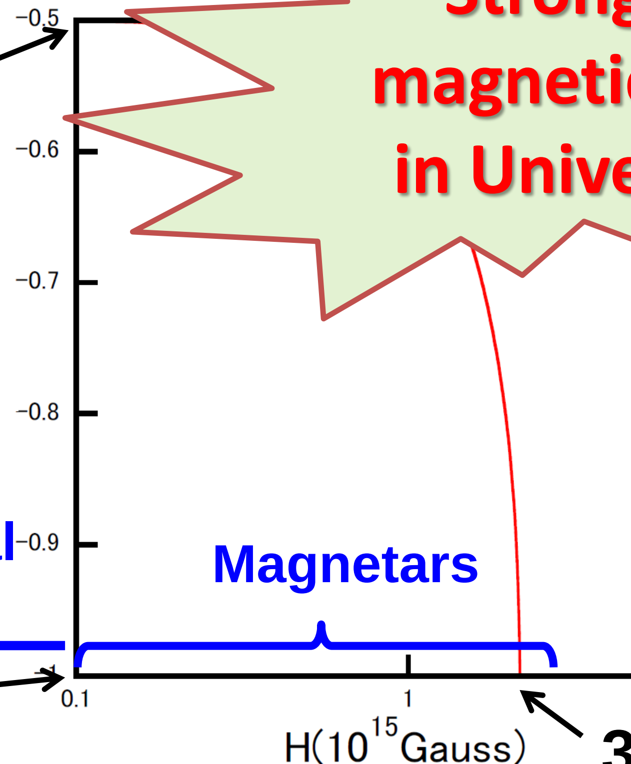
Uniaxial

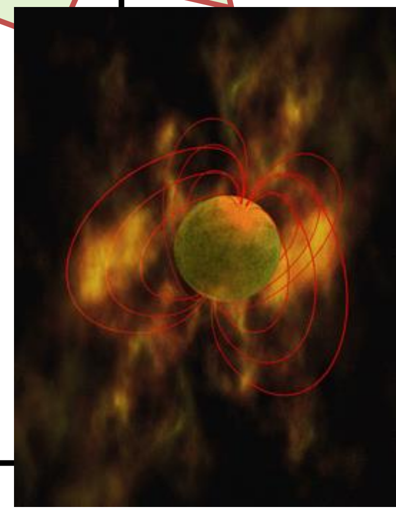
$-1<r<-1/2$

D_2 Biaxial

$r=-1$

D_4 Biaxial



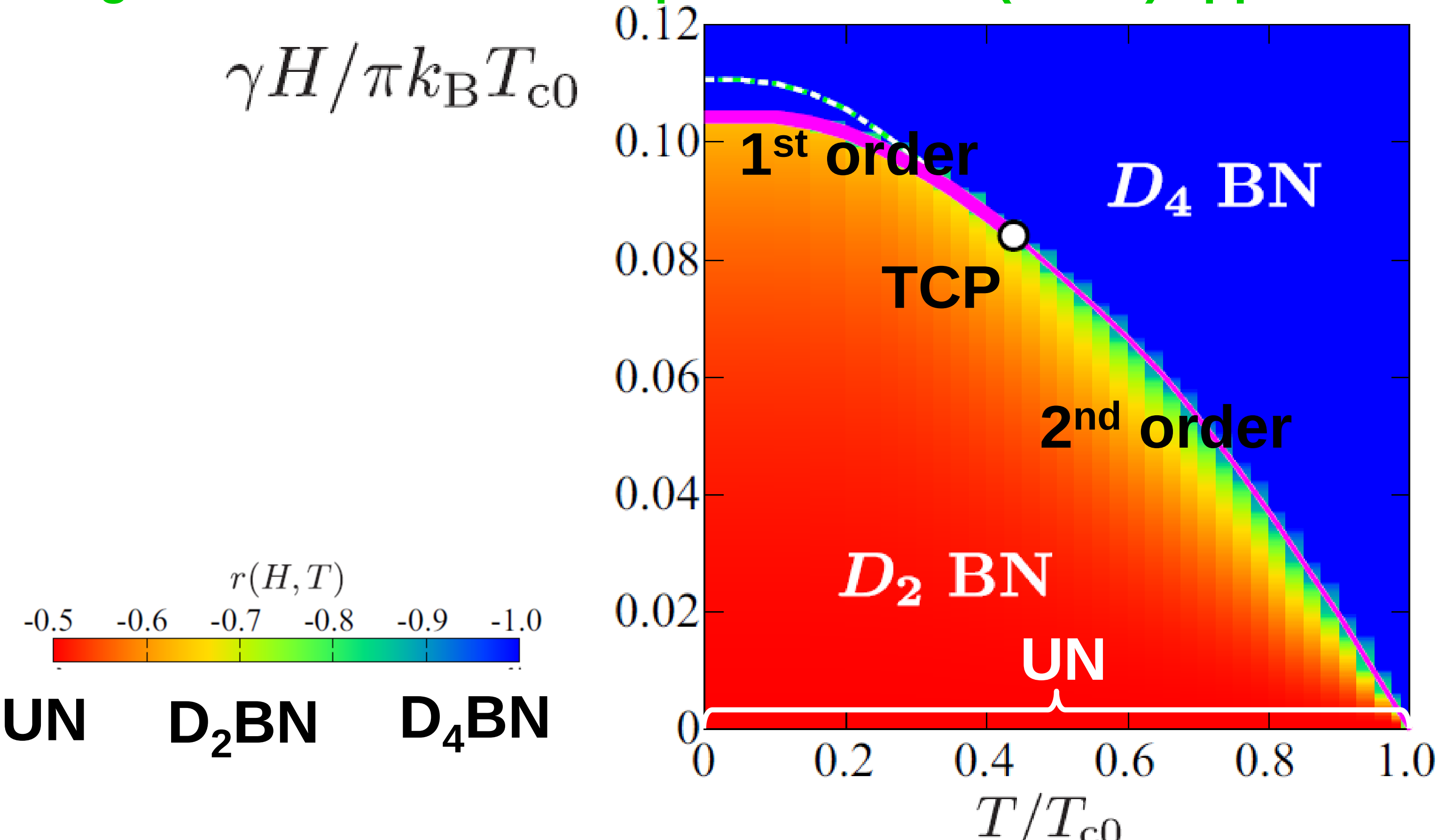


3 x 10^{15}G

(T,H) phase diagram *beyond GL*

Mizushima, Masuda & MN (in prep)

Bogoliubov-de Gennes + quasi-classical(linear) approx



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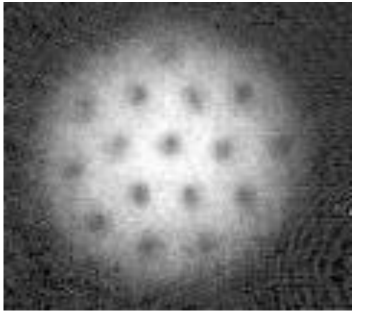
How are voritces created?

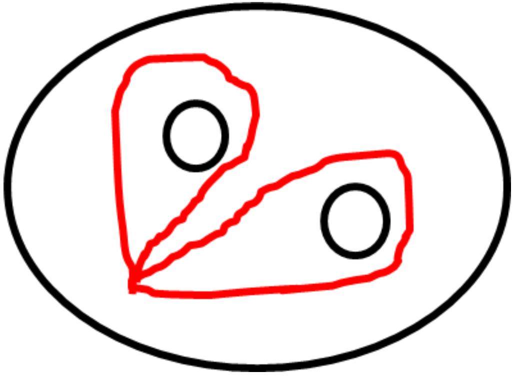
(1) Rotation of superfluid ⇒ vortex lattice



(2) Phase transition (Kibble-Zurek mechanism)

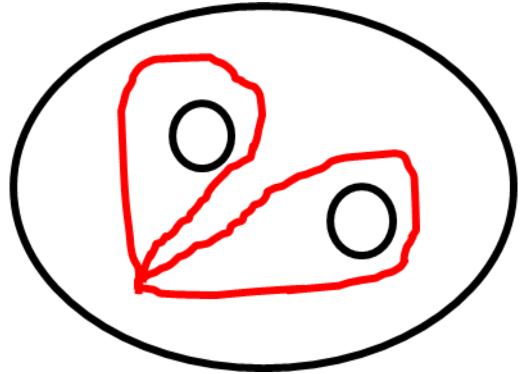
⇒ vortex network

Possible vortices can be determined by topology (1st homotopy) of order parameter space



Elements of 1st homotopy groups

$$A \sim \begin{pmatrix} r & 0 & 0 \\ 0 & -1-r & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad -1 \le r \le -1/2$$



1st homotopy

	G/H OPM	$\pi_1(G/H)$	
$r = -1/2$ **Uniaxial**	$U(1) \times \mathbf{R}P^2$	$\mathbf{Z} \oplus \mathbf{Z}_2$	**Integer vortex, Z_2 Spin vortex**
$-1 < r < -1/2$ ***D_2* Biaxial**	$U(1) \times \dfrac{SO(3)}{D_2}$	$\mathbf{Z} \oplus D_2^* = \mathbf{Z} \oplus \mathbf{Q}$	**Integer vortex, Quaternionic spin <u>non-Abelian vortex</u>**
$r = -1$ ***D_4* Biaxial**	$\dfrac{U(1) \times SO(3)}{D_4}$	$\boxed{\mathbf{Z} \oplus_h D_4^*}$	**Half-integer(quantized) <u>non-Abelian vortex</u>**

Elements of homotopy groups

$$A \sim \begin{pmatrix} r & 0 & 0 \\ 0 & -1-r & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad -1 \le r \le -1/2$$

higher homotopy

monopole
2d Skyrmion

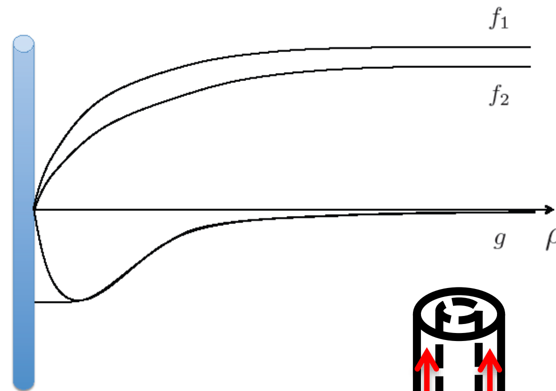
3d Skyrmion

instanton

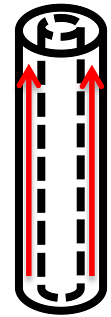
	G/H OPM	π_0	π_1	π_2	π_3	π_4
$r = -1/2$ **Uniaxial**	$U(1) \times \mathbf{R}P^2$	0	$\mathbb{Z} \oplus \mathbb{Z}_2$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{Z}_2$
$-1 < r < -1/2$ **D_2 Biaxial**	$U(1) \times \frac{SO(3)}{D_2}$	0	$\mathbb{Z} \oplus \mathbb{Q}$	0	$\mathbb{Z}$	$\mathbb{Z}_2$
$r = -1$ **D_4 Biaxial**	$\frac{U(1) \times SO(3)}{D_4}$	0	$\mathbb{Z} \times_h D_4^*$	0	$\mathbb{Z}$	$\mathbb{Z}_2$

Integer vortex without *H*

$$A = e^{i\theta} R(n\theta) \begin{array}{c} \begin{pmatrix} f_1(\rho) & ig(\rho) & 0 \\ ig(\rho) & f_2(\rho) & 0 \\ 0 & 0 & -f_1 - f_2 \end{pmatrix} \\ \begin{matrix} x & \quad y & \quad z \end{matrix} \end{array} R(n\theta)^T$$



$$\rightarrow e^{i\theta} R \begin{array}{c} \begin{pmatrix} r & 0 & 0 \\ 0 & -1-r & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ \begin{matrix} x & \quad y & \quad z \end{matrix} \end{array} R^T \qquad R(n\theta) = \begin{pmatrix} \cos n\theta & -\sin n\theta & 0 \\ \sin n\theta & \cos n\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad n \in \mathbf{Z}$$



$n=0$ x y z

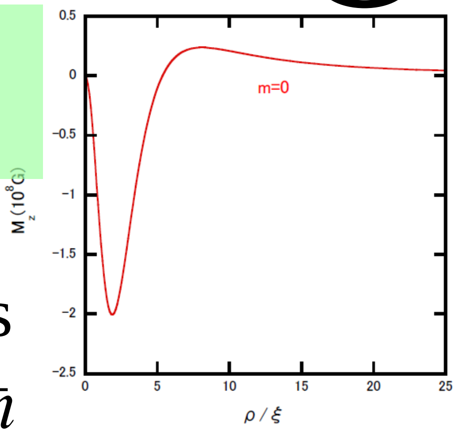
$n=1$ ρ θ z

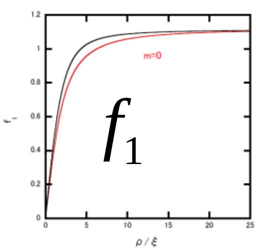
magnetization @ vortex core

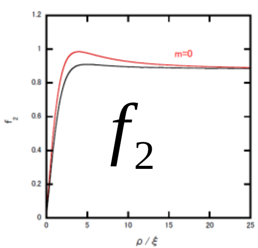
$$\propto g(f_1 - f_2)$$

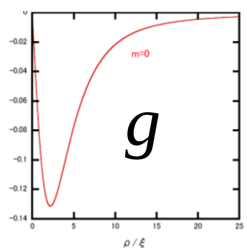
$$\sim 10^8 \,\mathrm{Gauss}$$

neutron anomalous magnetic moment $\gamma\hbar$









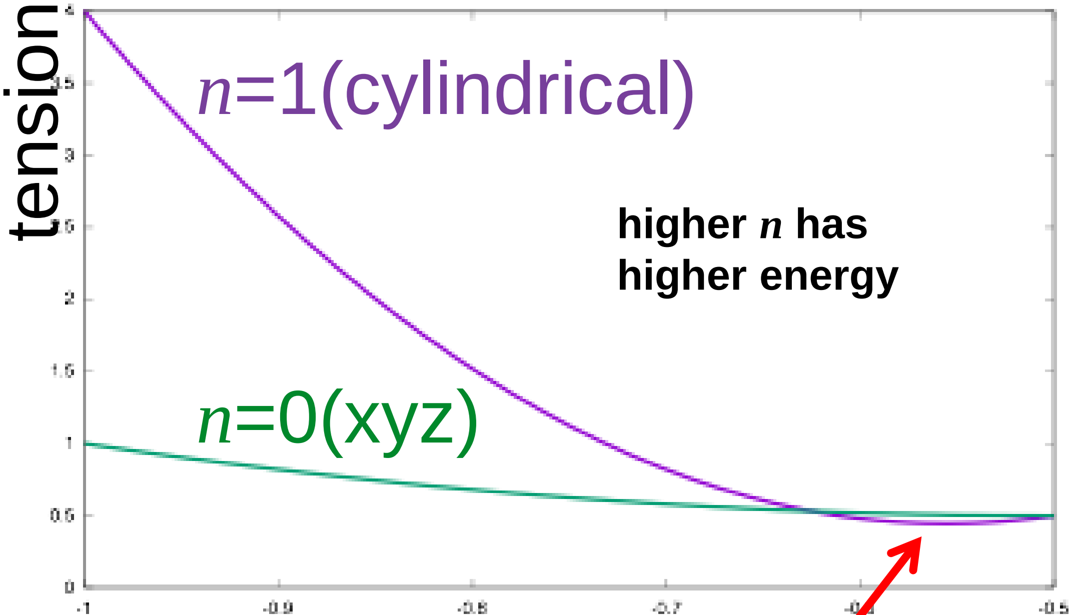
Integer vortex without *H*

Tension

$$T = c \log \Lambda / \xi$$

$$\begin{array}{cccl} \multicolumn{3}{c}{\begin{pmatrix} r & 0 & 0 \\ 0 & -1-r & 0 \\ 0 & 0 & 1 \end{pmatrix}} & \\ x & y & z & n=0 \\ \rho & \theta & z & n=1 \end{array}$$

In which coordinate basis, the matrix is diagonalized.



$r = -1$ r **minimum** $r = -1/2$

***D*$_4$ Biaxial** $r_{\mathrm{grad}} = 6 - \sqrt{43}$ **Uniaxial**

Characteristic feature of tensor order parameter

$$K(\partial_i A_{\mu\nu} \partial_i A^\dagger_{\mu\nu} \boxed{+ \partial_i A_{\mu i} \partial_j A^\dagger_{\mu j} + \partial_i A_{\mu j} \partial_j A^\dagger_{\mu i})}$$

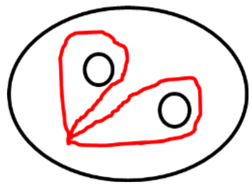
D_4 BN: Half-quantized non-Abelian vortices@magnetars

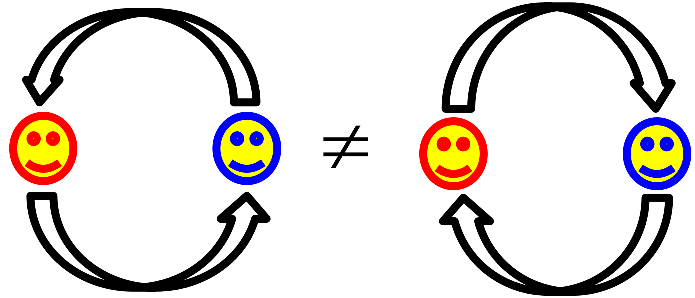
Circulation is quantized by half quanta

$$\int d^2x \, \mathrm{rot}\mathbf{v}_{\mathrm{eff}} = \oint d\mathbf{r} \cdot \mathbf{v}_{\mathrm{eff}} = \frac{\hbar}{M} k$$

$\pi_1 \sim D_4$ non-Abelian

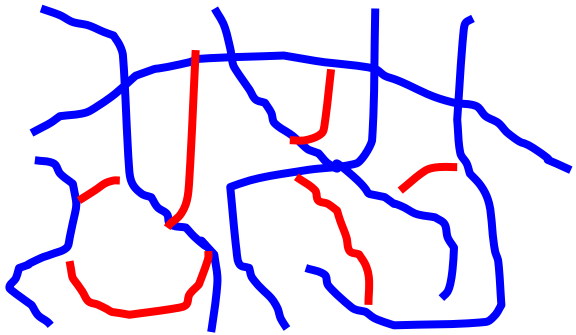
What consequence of non-Abelianilty?





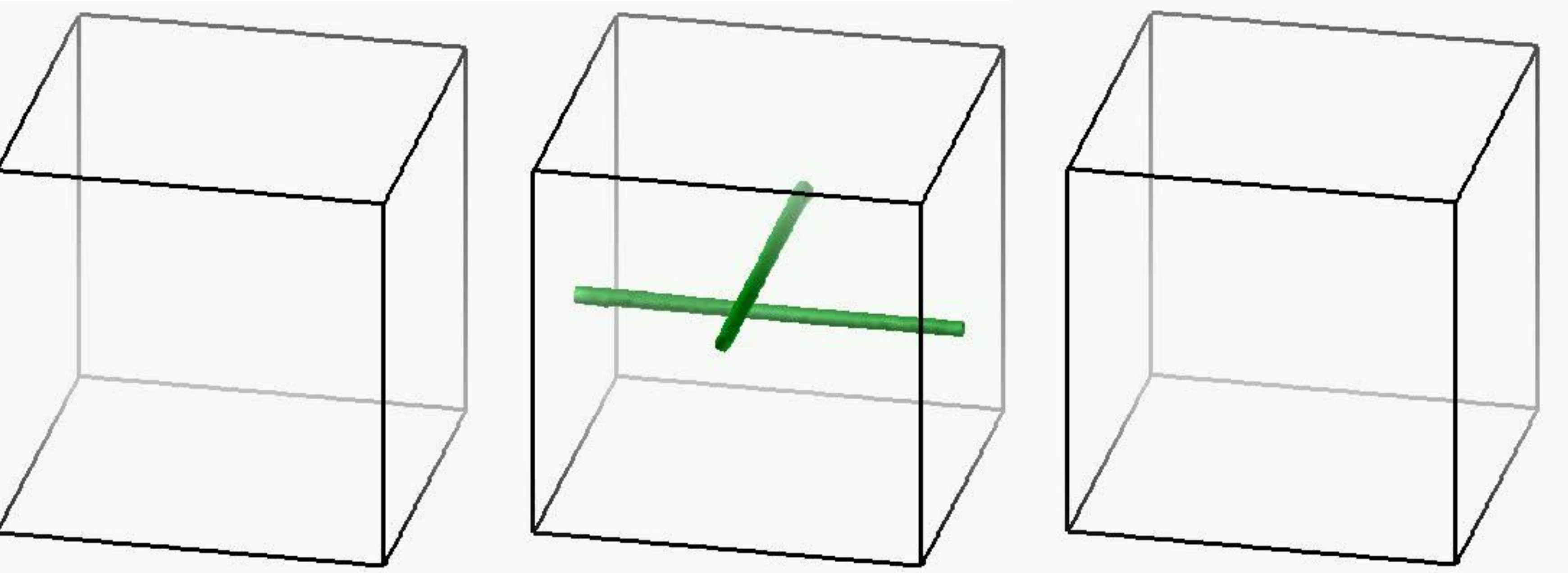
2 dim Non-commutativity

3 dim vortex network are entangled

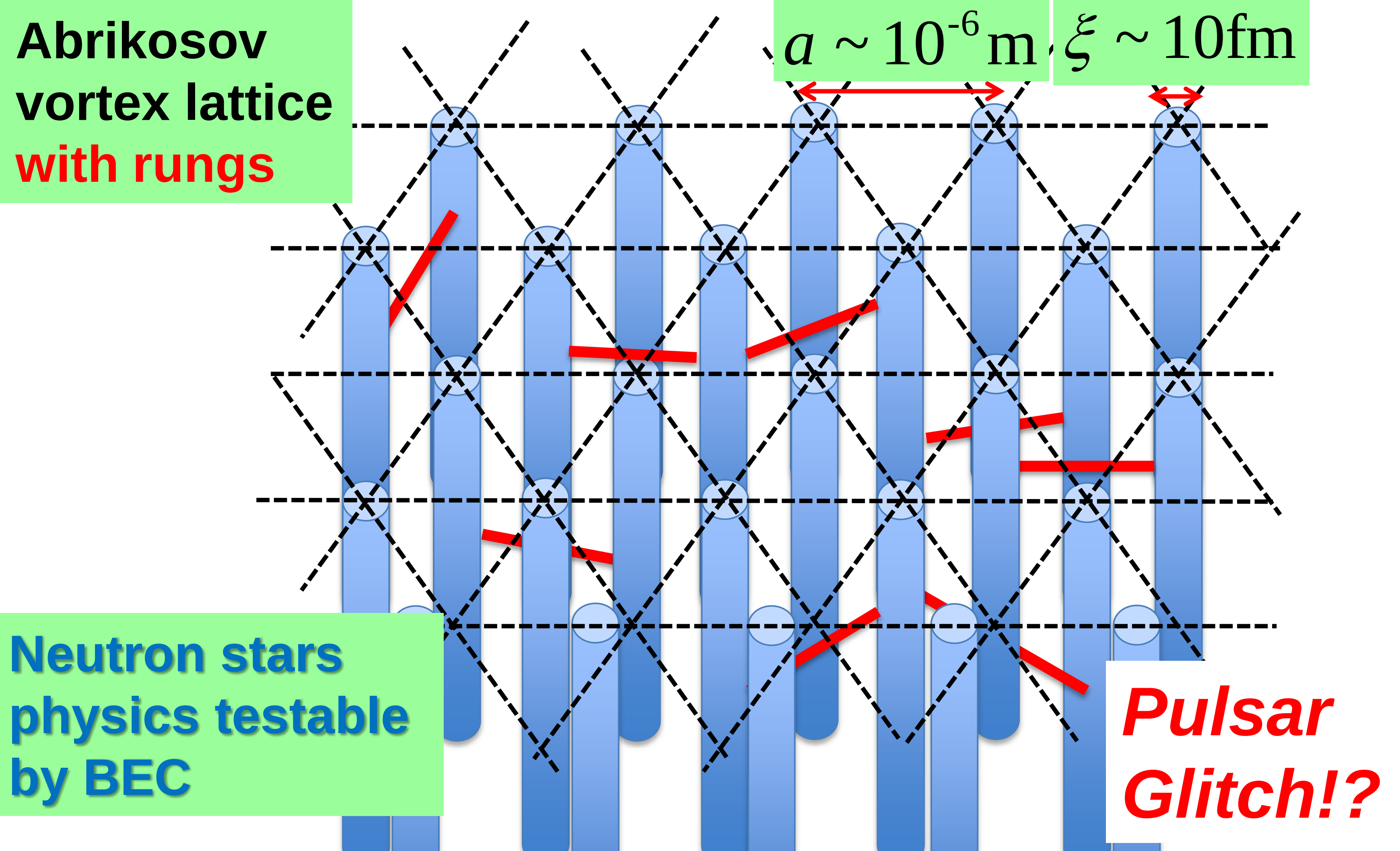


Non-Abelian vortex collision in spin-2 BEC (cyclic phase)

M.Kobayashi,Kawaguchi,MN,Ueda, Phys.Rev.Lett. 103 (2009) 115301

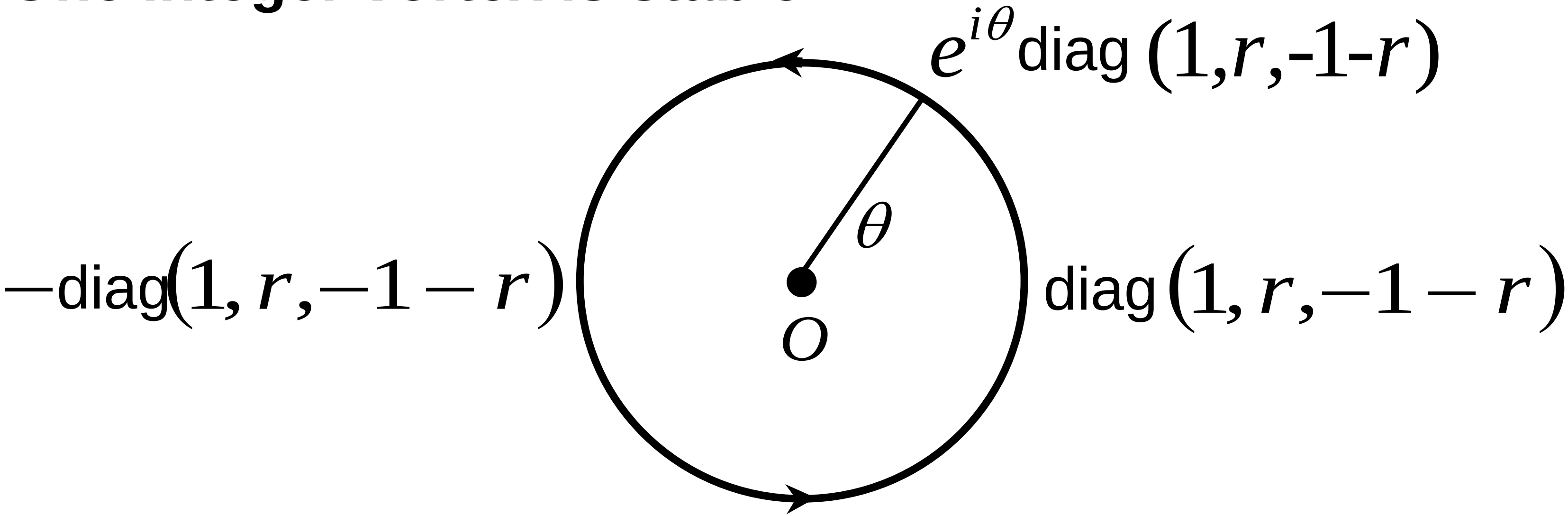


reconnection	passing	rung formation
$\left[a,a\right]=0$	$\left[a,b\right]=0$	$\boxed{\left[a,b\right]=c\neq 0}$
Abelian	**Abelian**	**Non-Abelian**


Abrikosov
vortex lattice
with rungs
$a \sim 10^{-6}\mathrm{m}$
$\xi \sim 10\mathrm{fm}$
Neutron stars
physics testable
by BEC
Pulsar
Glitch!?

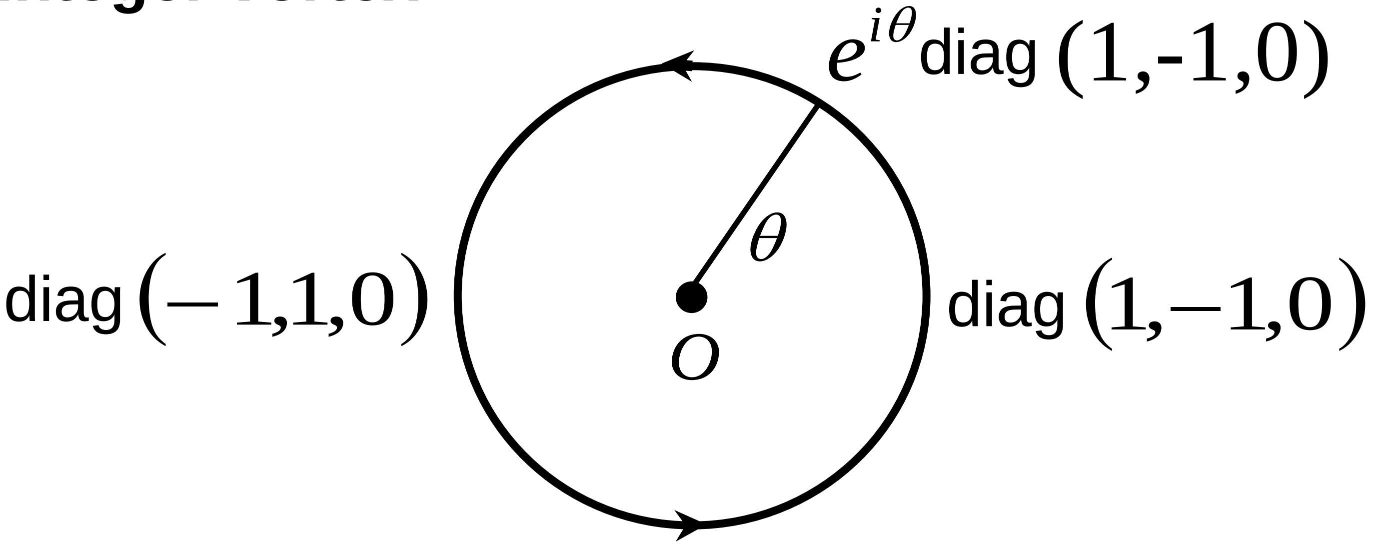
For H < 3 x 10^{15}G **(Ordinary neutron stars)**

One integer vortex is stable



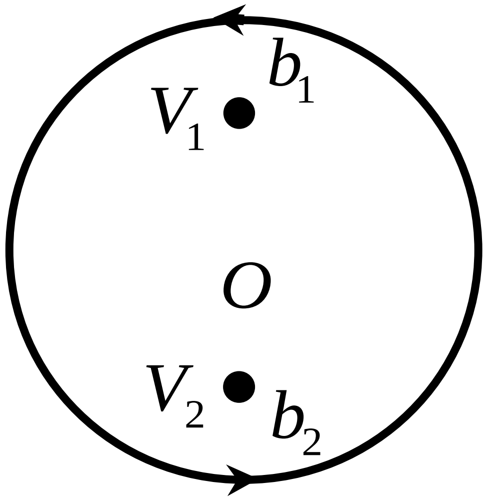
For H > 3 x 10^{15}G **(Magnetars)**

One integer vortex



For H > 3 x 10^{15}G **(Magnetars)**

One integer vortex is split into 2 half-quantized vortices



$\mathrm{diag}\left(-1,1,0\right)$

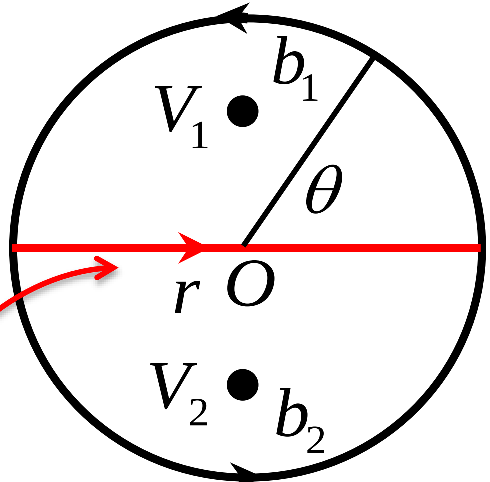
$\mathrm{diag}\left(1,-1,0\right)$

For H > 3 x 10^15^G (Magnetars)

One integer vortex is split into 2 half-quantized vortices

encircling $b_1 + r$
and $b_2 - r$

diag$(-1,1,0)$

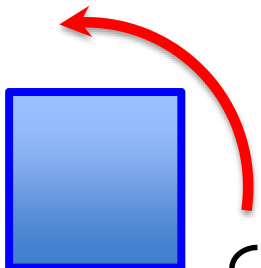


diag$(1,-1,0)$

$$E \sim k^2 \log \Lambda$$

$$1^2 > \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

$\frac{\pi}{2}$ **rotation**



along r

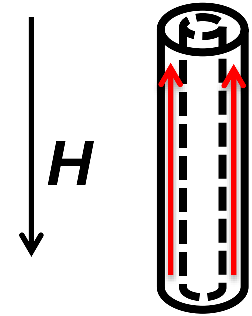
$$R(y) = \begin{pmatrix} \cos\alpha(y) & -\sin\alpha(y) & 0 \\ \sin\alpha(y) & \cos\alpha(y) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\alpha(y) \to \begin{cases} 0, & y \to -\infty \\ \frac{\pi}{2}, & y \to +\infty \end{cases}$$

Half-quantized vortex with *H* > 3 x 10^15^G

$$A = e^{i\theta/2} R(\theta) \begin{array}{c} \begin{pmatrix} f(\rho) & ig(\rho) & 0 \\ ig(\rho) & -f(\rho) & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \begin{matrix} x \quad & \quad y \quad & \quad z \end{matrix} \end{array} R(\theta)^T$$

$$\to e^{i\theta} R \begin{array}{c} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\ \begin{matrix} x \quad & \quad y \quad & \quad z \end{matrix} \end{array} R^T \qquad R = \begin{pmatrix} \cos\theta/4 & -\sin\theta/4 & 0 \\ \sin\theta/4 & \cos\theta/4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

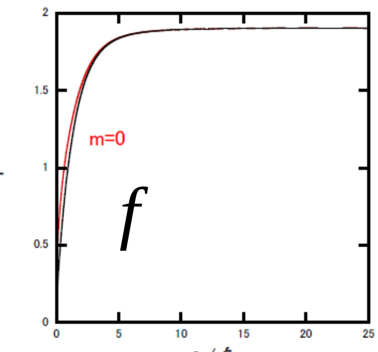


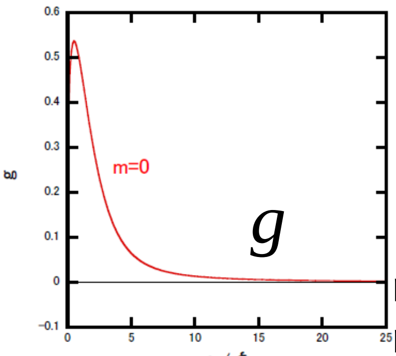
Anti-ferro magnetization @ vortex core

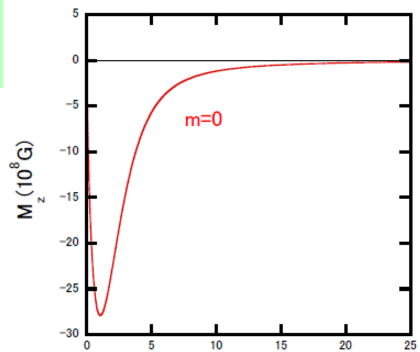
$\propto gf$

$\sim 10^9 \, \mathrm{Gauss}$

neutron anomalous magnetic moment $\gamma\hbar$







Plan of My Talk

§ 1 Introduction (4p)

§ 2 GL Theory for $^3P_2$ and Ground State (4p)

§ 3 Vortices (6p)

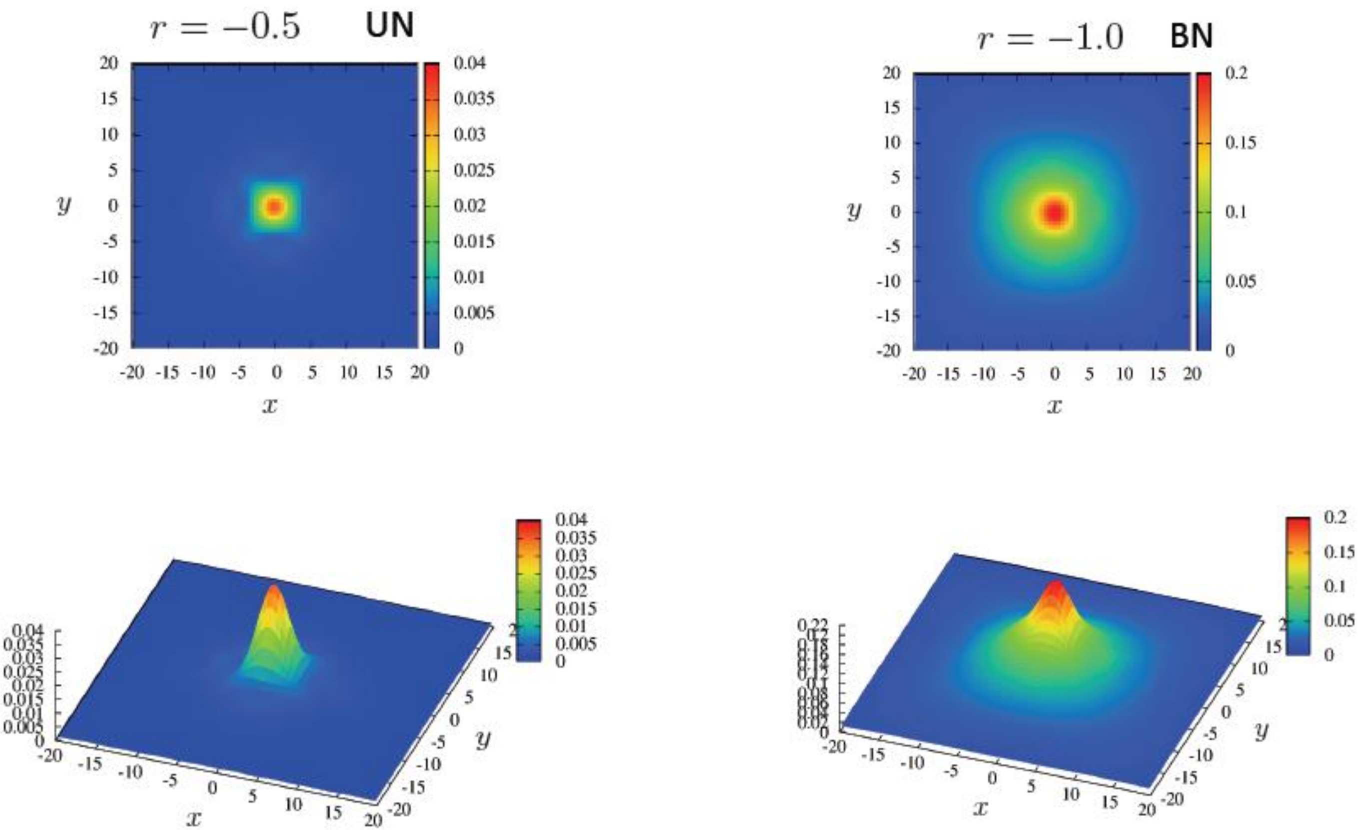
§ 4 Topological Superfluidity (3p)

§ 5 Summary

Mizushima, Masuda & MN, in preparation

Topological Superfluidity Mizushima, Masuda & MN, in prep

Majorana zero mode in a vortex core



Plan of My Talk

§ 1 Introduction **(4p)**

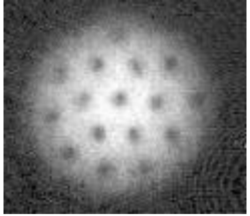
§ 2 GL Theory for $^3P_2$ and Ground State **(8p)**

§ 3 Vortices **(11p)**

§ 4 Topological Superfluidity **(3p)**

§ 5 Summary

Summary



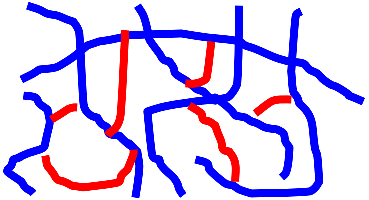
Neutron $^3P_2$ superfluid in a neutron star

1. **Ground state**
   - **Uniaxial nematic** $H \le 10^{14}\mathrm{G}$
   - **D_2 Biaxial nematic** $10^{14}\mathrm{G} \le H \le 3\times 10^{15}\mathrm{G}$
   - **D_4 Biaxial nematic** $H \ge 3\times 10^{15}\mathrm{G}$

2. **Integer vortices -- ordinary neutron stars**

   Half-quantized non-Abelian vortices -- magnetars

   Magnetization $10^{8\sim 9}$G @core



3. **Topological Superfluidity**

   Majorana fermion on a surface – anisotropic magnetization

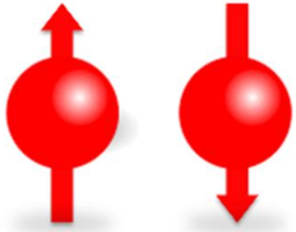
   Majorana fermion in a vortex core

future: a novel non-Abelian anyon: two origins of non-commutativity

order parameter

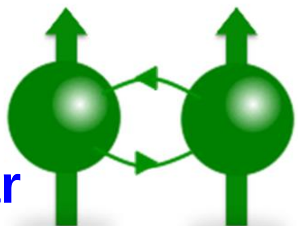
$A_{\mu i}$

$^1S_0$ pairs



@outer core

$^3P_2$ pairs



$A_{\mu i}$

spin angular

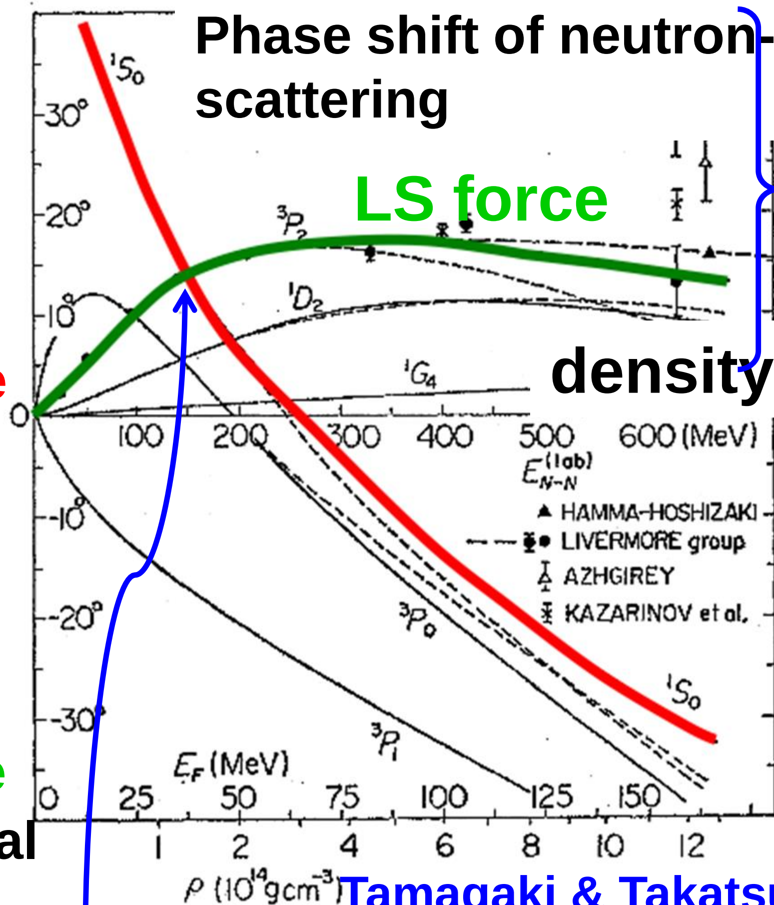
traceless symmetric
3x3 complex matrix
fields
= complex 5 (of SO(3))

@ inner core

$^3P_2$

spin s=1 angular l=1 total j=2

Phase shift of neutron-neutron scattering



Tamagaki & Takatsuka (1970-72)

: Transition from $^1S_0$ to $^3P_2$

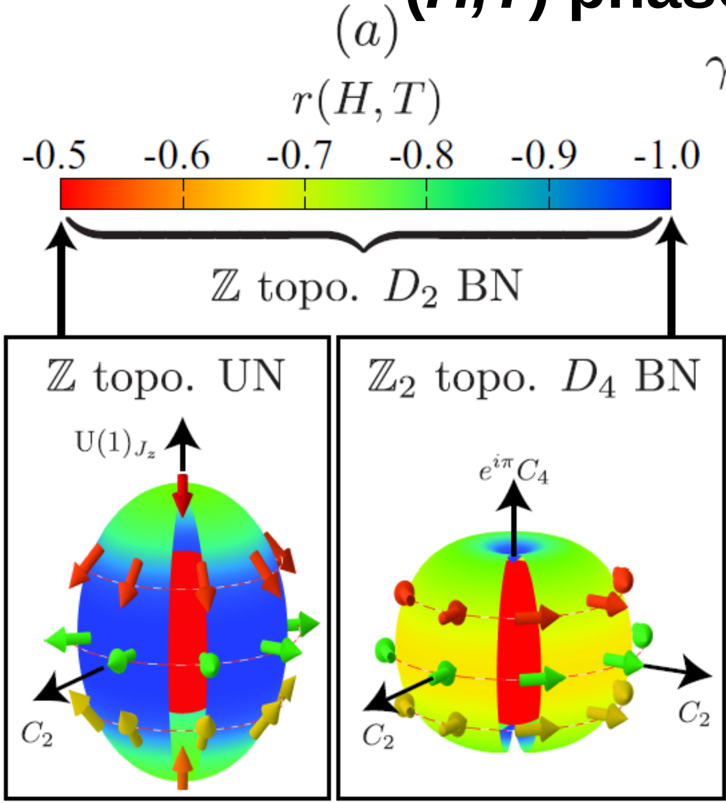
$$\rho \sim \rho_0 (10^{14} \sim 10^{15}\,\mathrm{g/cm^3})$$

Topological Superfluidity

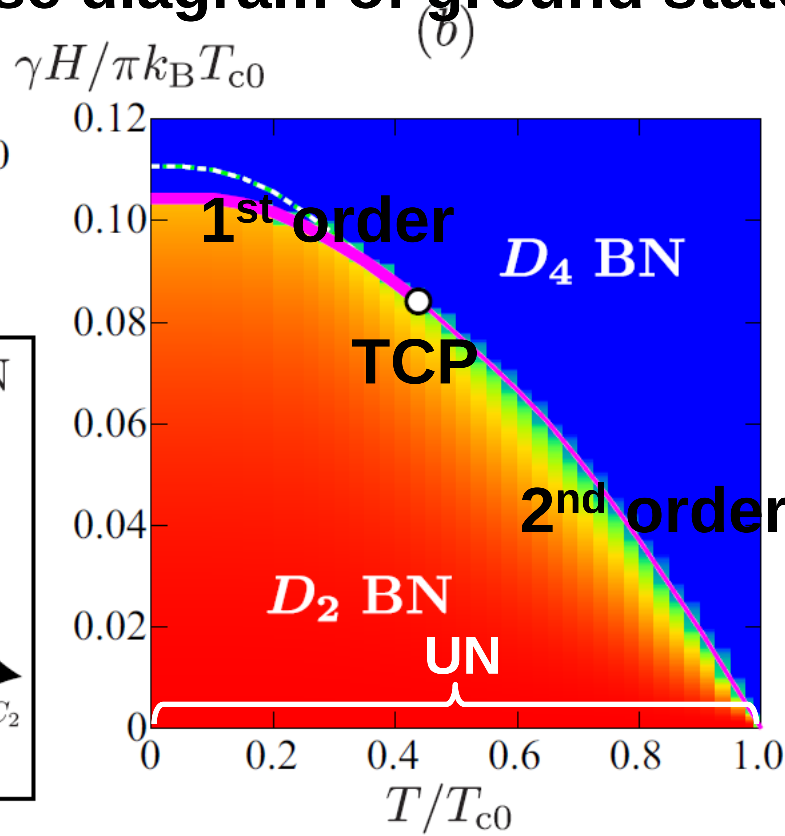
Mizushima, Masuda & MN, in prep

Bogoliubov-de Gennes equation + quasi-classical (linear) approx

(H,T) phase diagram of ground states



Gap functions



Topological Superfluidity

Mizushima, Masuda & MN, in prep

$^3P_2$ superfluids are **topological**:

cf:planar phase of 3He

no H, DIII 3d topology UN & D_2 BN: $\mathbb{Z}$ D_4 BN: $\mathbb{Z}_2$

$$w_{3\mathrm{d}} = \int \frac{d^3\boldsymbol{k}}{12\pi^3}\epsilon_{\mu\nu\eta}\epsilon_{ijkl}\hat{m}_i\partial_{k_\mu}\hat{m}_j\partial_{k_\nu}\hat{m}_k\partial_{k_\eta}\hat{m}_l = -1 \qquad \nu = (-1)^{w_{3\mathrm{d}}} = -1$$

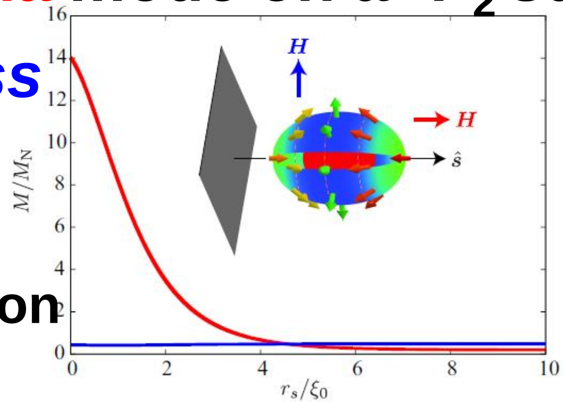
→ *gapless* **Majorana** mode on a $^3P_2$ surface

H // surface → *gapless*

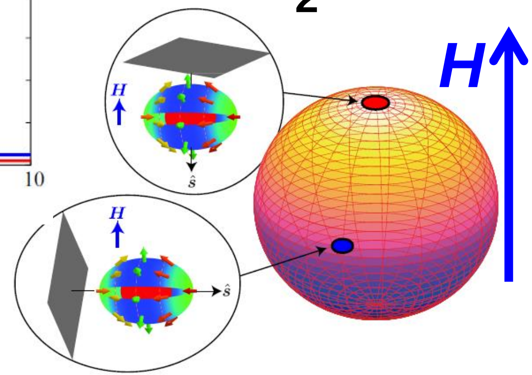
$H \perp$ surface → *gapful*

↓

magnetization

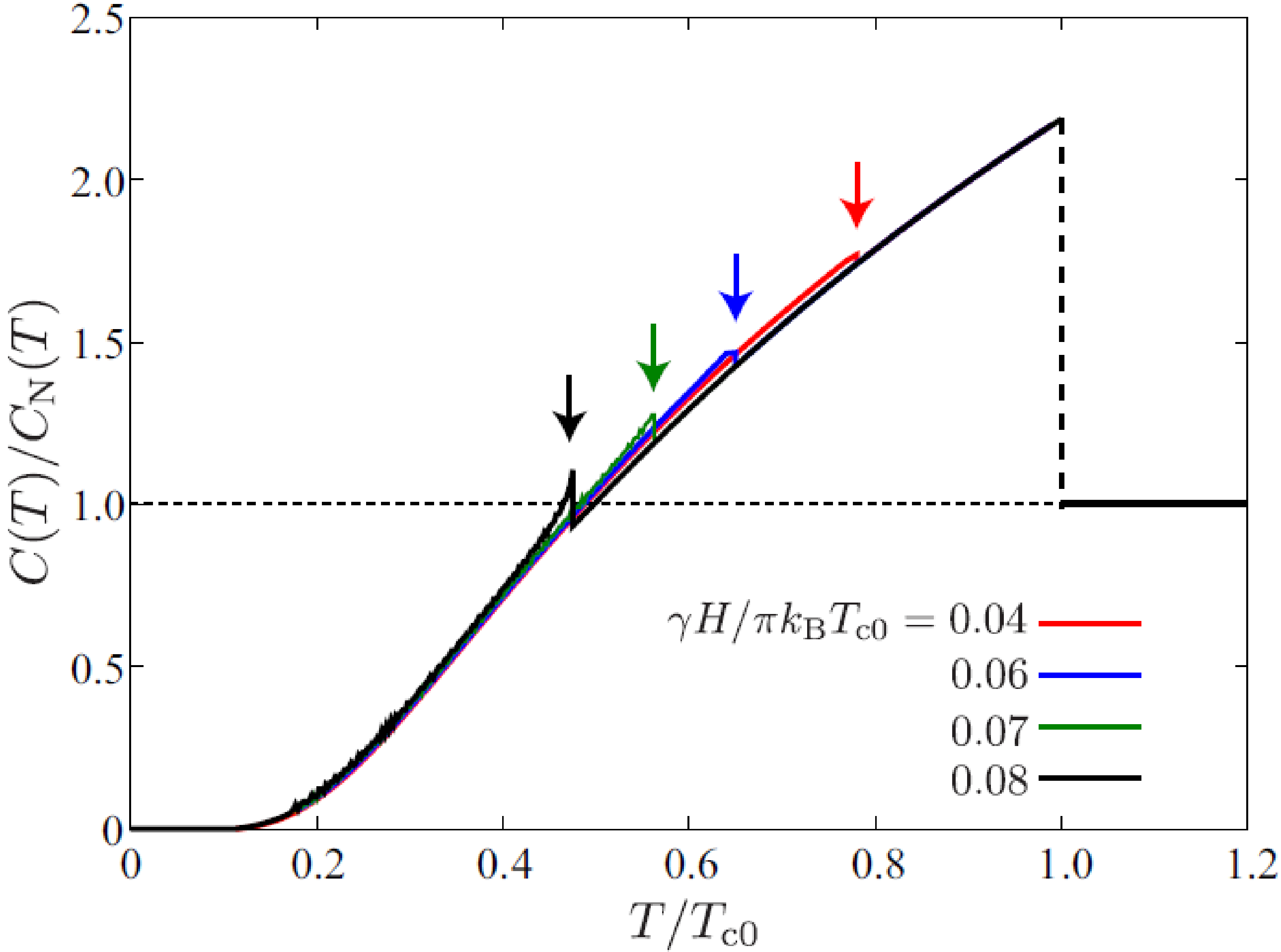


$^3P_2$ core



In a neutron star core,

anisotropic magnetization on surface


2.5
2.0
1.5
1.0
0.5
0
$C(T)/C_{\rm N}(T)$
0
0.2
0.4
0.6
0.8
1.0
1.2
$T/T_{\rm c0}$
$\gamma H/\pi k_{\rm B}T_{\rm c0} = 0.04$
0.06
0.07
0.08