Low-Energy Effective Action of CP^{N-1} Model at large N

Pavel A. Bolokhov

University of Minnesota Duluth

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Vortices

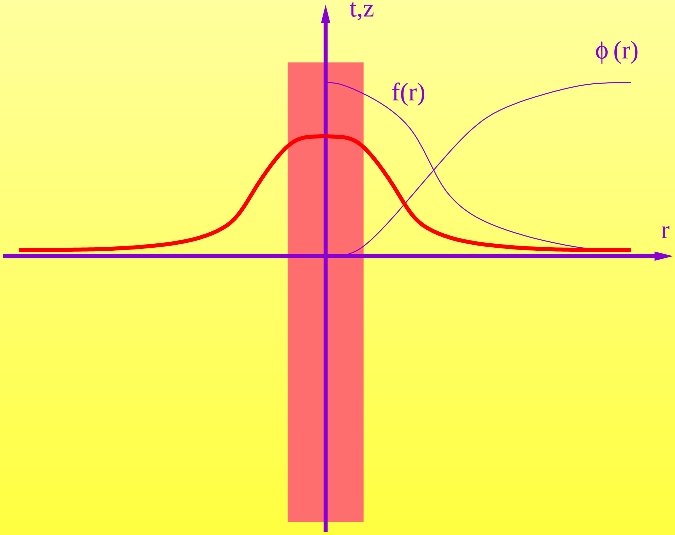
Non-Abelian vortices

Non-abelian vortices appear in $\mathcal{N} = 2\, U(N)$ SQCD, $N_c \;=\; N_f$

[M.Shifman, A.Yung] [A.Hanany, D.Tong] [N.Dorey]

$$
\begin{aligned}
V(q^A, \widetilde{q}_A, a^a, a) \;&=\; \frac{g_2^2}{2}\left(\frac{1}{g_2^2} f^{abc}\overline{a}^b a^c \;+\; \overline{q}_A T^a q^A - \widetilde{q}_A T^a \overline{\widetilde{q}}^A\right)^2 \\
&+\; \frac{g_1^2}{8}\left(\overline{q}_A q^A \;-\; \widetilde{q}_A \overline{\widetilde{q}}^A \;-\; N\,\xi_3\right)^2 \\
&+\; 2\,g_2^2\left|\widetilde{q}_A T^a q^A\right|^2 \;+\; \frac{g_1^2}{2}\left|\widetilde{q}_A q^A \;-\; \frac{N}{2}\,\xi\right|^2 \\
&+\; \frac{1}{2}\sum_{A=1}^{N}\left\{\left|\left(a \;+\; \sqrt{2} m_A \;+\; 2T^a a^a\right) q^A\right|^2\right. \\
&\qquad\left. +\; \left|\left(a \;+\; \sqrt{2} m_A \;+\; 2T^a a^a\right) \overline{\widetilde{q}}^A\right|^2\right\} .
\end{aligned}
$$

The Z_N string solution



The Z_N string solution

$$
\begin{aligned}
\varphi &= \begin{pmatrix} \phi_2(r) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \phi_2(r) & 0 \\ 0 & 0 & \dots & e^{i\alpha}\phi_1(r) \end{pmatrix} \\
A_i^{\mathrm{SU(N)}} &= \frac{1}{N} \begin{pmatrix} 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & -(N-1) \end{pmatrix} \times \\
&\qquad \times (\partial_i \alpha) \Big(-1 \; + \; f_{NA}(r) \Big) \\
A_i^{\mathrm{U(1)}} &= \frac{1}{N} (\partial_i \alpha) \Big(1 \; - \; f(r) \Big)
\end{aligned}
$$

In the singular gauge, it is the gauge field that winds, now around the origin

$$A_i^{\mathrm{SU(N)}} = \frac{1}{N}\, U \begin{pmatrix} 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & -(N-1) \end{pmatrix} U^{-1} \times \\ \times\, (\partial_i \alpha)\, f_{NA}(r)$$

$$A_i^{\mathrm{U(1)}} = -\frac{1}{N}\, (\partial_i \alpha)\, f(r)$$

The rotation matrix U provides the *orientation* of the string in the $SU(N)$ space

The string solution breaks

$$\frac{SU(N)}{SU(N-1)\times U(1)} \;\sim\; CP(N-1)\,.$$

The string orientation U can be unambigiously parametrized by the modulus $n^l \in C$:

$$\frac{1}{N}\, U \begin{pmatrix} 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & -(N-1) \end{pmatrix} U^{-1} \;\; = \;\; -\, n^i\, \overline{n}_l \;\; + \;\; \frac{1}{N} \cdot \mathbf{1}^i{}_l$$

with a condition

$$\overline{n}_l \cdot n^l \;\; = \;\; 1.$$

Thus n^l are *orientational* collective coordinates

This defines $2(N-1)$ degrees of freedom, since CP^{N-1} theory can be obtained from a gauge theory, and one phase can be removed

CP^{N-1} Model

Bosonic theory

The non-supersymmetric CP^{N-1} describes a complex vector

$$n^l \,, \qquad\qquad l \;=\; 1\,,\,\ldots\,,\,N$$

subject to the identification

$$\vec{n} \;\sim\; \lambda\,\vec{n}\,, \qquad\qquad \lambda \;\in\; \mathcal{C}$$

The *gauge* formulation for such a theory was introduced by Witten

$$\mathcal{L} \;=\; \frac{1}{4e^2}\,F_{kl}^2 \;+\; \frac{1}{2e^2}\,D^2 \;+\; \big|\nabla n\big|^2 \;+\; iD\big(\,\big|n^l\big|^2 - 2\beta\,\big)$$

where

$$\nabla_k\, n^l \;=\; (\,\partial_k \;-\; i\,A_k\,)\; n^l$$

In the limit $e \to \infty$ resolution of A_k and D imposes the CP^{N-1} constraint $\vec{n} \;\sim\; \lambda\,\vec{n}$
One of n^l components can be expressed in terms of the other $N-1$, and put to an arbitrary phase — *e.g.* set *real*

$$\mathcal{L} \;=\; \big|\,\partial\, n\,\big|^2 \;+\; \big(\,\overline{n}\,\partial_k\, n\,\big)^2, \qquad\qquad l \;=\; 1\,,\,\ldots\,,\,N-1$$

$\mathcal{N} = (2,2)$ Supersymmetric Theory

$$
\begin{aligned}
\mathcal{L}_{(2,2)} \;=\;& \frac{1}{4e^2} F_{kl}^2 + \frac{1}{e^2}\left|\partial_k \sigma\right|^2 + \frac{1}{2e^2} D^2 + \\
+\;& \frac{1}{e^2}\,\overline{\lambda}_R\, i\partial_L\, \lambda_R + \frac{1}{e^2}\overline{\lambda}_L\, i\partial_R\, \lambda_L + \\
+\;& \left|\nabla n\right|^2 + |\sqrt{2}\sigma|^2 \left|n^l\right|^2 + iD\left(\left|n^l\right|^2 - 2\beta\right) + \\
+\;& \overline{\xi}_R\, i\nabla_L \xi_R + \overline{\xi}_L\, i\nabla_R \xi_L + i\sqrt{2}\sigma\, \overline{\xi}_R \xi_L + i\sqrt{2}\overline{\sigma}\, \overline{\xi}_L \xi_R + \\
+\;& i\sqrt{2}\,\overline{\xi_{[R}\,\lambda_{L]}}\, n - i\sqrt{2}\,\overline{n}\,\lambda_{[R}\,\xi_{L]}\,, \qquad l = 1, \ldots N
\end{aligned}
$$

The Exact Superpotential

This theory is known to have an exact *Veneziano-Yankielowicz* type superpotential

$$\int d\theta_R\, d\bar{\theta}_L \left(\sqrt{2}\Sigma \log \sqrt{2}\Sigma \;-\; \sqrt{2}\Sigma \right)$$

also known as *Witten* superpotential, where Σ is a *twisted* superfield

$$\Sigma \;=\; \sigma \;-\; \sqrt{2}\,\theta_R \overline{\lambda}_L \;+\; \sqrt{2}\,\overline{\theta}_L \lambda_R \;+\; \sqrt{2}\,\theta_R \overline{\theta}_L \left(D \;-\; i\,F_{03} \right)$$

and

$$\Sigma \;=\; \frac{i}{\sqrt{2}}\, D_L\, \overline{D}_R\, V$$

We show that at large N one can do better than just superpotential

The Effective Action

The Effective Scalar Potential

In M.Shifman, A.Yung arXiv:0803.0698 the effective scalar potential was found at large N,

$$-V_{\text{eff}} \;\propto\; \left(|\sigma|^2 + iD\right)\log\left(|\sigma|^2 + iD\right) \;-\; iD \;-\; |\sigma|^2\log|\sigma|^2$$

This clearly does not fit into the $\Sigma\,\log\Sigma$ picture!

The Effective Scalar Potential

At large N, the effective action for A^μ, σ up to two derivatives was found

$$\mathcal{L}_{\rm eff} \;=\; \frac{1}{4e_\gamma^2}\,F_{03}^2 \;+\; \frac{1}{e_\sigma^2}\,\left|\partial_\mu\sigma\right|^2 \;+\; \frac{1}{e_\lambda^2}\,\overline{\lambda}_R\,i\partial_L\,\lambda_R \;+\; \frac{1}{e_\lambda^2}\,\overline{\lambda}_L\,i\partial_R\,\lambda_L$$
$$+\; V_{\rm eff}(D,\sigma) \;+\; \ldots$$

At large N these terms are just found at one-loop

None of these actually fit into Witten's potential

Supersymmetric Form

The hypothesis is

$$\frac{4\pi}{N}\,\mathcal{L}_{\text{eff}} \;=\; \int d^4\theta \,\Big|\ln\Sigma\Big|^2 \;+\; \int d^2\tilde{\theta}\,\Big(\Sigma\,\ln\Sigma \;-\; \Sigma\Big) \;+\; \int d^4\theta\, G(\Sigma,\overline{\Sigma})$$

The existence of the $\big|\ln\Sigma\big|^2$ term has been known since
A. D'Adda, A. .C. Davis, P. Di Vecchia and P. Salomonson, Nucl. Phys. B 222, 45 (1983)

Supersymmetrizing

The effective couplings depend on D along with σ

$$\frac{1}{e_\gamma^2} = \frac{1}{3}\,\frac{1}{D+|\sigma|^2} + \frac{2}{3}\,\frac{1}{|\sigma|^2}$$

$$\frac{1}{e_\lambda^2} = \frac{1}{|\sigma|^2}\,\frac{x - \ln(1+x)}{x^2}$$

where

$$x = \frac{D}{|\sigma|^2}$$

Supersymmetrizing

What is the supersymmetric form of these expressions?

They are functions of

$$\sigma \qquad\qquad \text{and} \qquad\qquad x \;=\; \frac{D}{|\sigma|^2}$$

It has been suggested that x can be promoted to superfield

$$x \;=\; \frac{D}{|\sigma|^2} \quad \longrightarrow \quad \frac{S}{\Sigma}$$

where

$$S \;=\; \frac{i}{2}\,\overline{D}_R D_L \ln \overline{\Sigma}$$

and the lowest part of $S\,/\,\Sigma$ is

$$\left.\frac{S}{\Sigma}\right| \;=\; \frac{1}{|\sigma|^2}\left(iD \,-\, F_{03} \;-\; \frac{2\,i\sigma\overline{\lambda}_R\lambda_L}{|\sigma|^2} \right)$$

So the total set of supersymmetric variables is [A. D'ADDA *et al.*]

$$\Sigma \qquad \overline{\Sigma} \qquad \frac{S}{\Sigma} \qquad \frac{\overline{S}}{\overline{\Sigma}}$$

and the remaining D-term is sought in the form

$$\int d^4\theta\, G(u,\, v)$$

where

$$u = \frac{S}{\Sigma} \qquad\qquad v = \frac{\overline{S}}{\overline{\Sigma}}$$

$G(u,\, v)$ is found by expanding

$$\int d^4\theta\, G(u,\, v)$$

and matching to the 1-loop effective action

$$\mathcal{L}_{\rm eff} \;=\; \frac{1}{4e_\gamma^2}\, F_{03}^2 \;+\; \frac{1}{e_\sigma^2}\, \big|\partial_\mu \sigma\big|^2 \;+\; \frac{1}{e_\lambda^2}\, \overline{\lambda}_R\, i\partial_L\, \lambda_R \;+\; \frac{1}{e_\lambda^2}\, \overline{\lambda}_L\, i\partial_R\, \lambda_L$$
$$+\; V_{\rm eff}(D, \sigma) \;+\; \ldots$$

Results

Physically observable is only the derivative of $G(u, v)$

$$G_{uv} \;=\; -\,\frac{1}{x^4}\,\left(1 + 2\,\frac{y^2}{x^2}\right)\int_0^x \ln(1+x)\,dx \;-\; \frac{1}{6}\,\frac{y^2}{x^3\,(1+x)}$$

where

$$x \;=\; \frac{u+v}{2} \;\propto\; \frac{D}{|\sigma|^2}$$

$$y \;=\; \frac{v-u}{2} \;\propto\; \frac{F_{03}}{|\sigma|^2}$$

Results

$$
\begin{aligned}
\frac{4\pi}{N}\,\mathcal{L}_{\rm eff} \;\; &= \;\; \int d^4\theta \left|\,\ln\Sigma\,\right|^2 \;\;+\;\; \int d^2\tilde{\theta}\,\Big(\Sigma\,\ln\Sigma \;-\; \Sigma\Big) \\
&+ \;\; \int d^4\theta\, G(\Sigma,\overline{\Sigma})
\end{aligned}
$$

Problems

Unable yet to reproduce the coupling constants e_σ, Γ of

$$\frac{1}{e_\sigma^2}\,\left|\partial_\mu \sigma\right|^2$$

$$i\,\Gamma\sigma\,\overline{\lambda}_R\,\lambda_L \;+\; i\,\Gamma\overline{\sigma\,\lambda}_L\,\lambda_R$$

Thank you