Large-N QCD, Veneziano Amplitude, and Holography

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A. A., Edwin Ireson, work in progress

Introduction

The 1968 Veneziano amplitude

$$\mathcal{A}(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$$

marks the birth of string theory.

It has some positive and phenomenological appealing properties, most importantly linear Regge trajectories of the form $J = \alpha(0) + \alpha' s$ and their daughter trajectories.

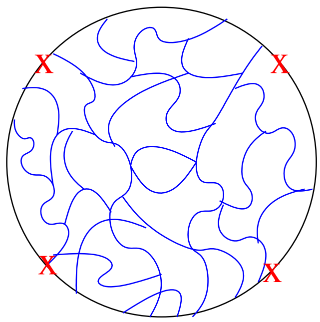
The amplitude suffers from several bad properties such as the UV behavior ($s, t \to \infty$ with s/t fixed) where the amplitude decreases exponentially $\mathcal{A}(s,t) \sim \exp -\alpha' s$.

It is then interesting to ask, what is the relation between the Veneziano amplitude and QCD?

Is there a limit of QCD where meson scattering is described by the Veneziano amplitude?

An old field theory argument in favor of the amplitude

The first attempt to relate between the Veneziano amplitude and field theory was made in 1970 by Nielsen and Olesen and by Sakita and Virasoro. They argued that a dense perturbative contribution to the scattering amplitude forms a fishnet. As the holes close at high orders the fishnet resembles a string worldsheet



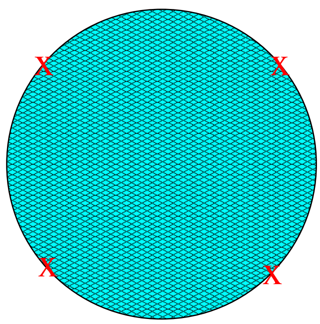
This nice picture, which was developed before QCD (the authors used ϕ^3 theory) cannot be the full story, not even if QCD is used, since the phenomenon of confinement is at the heart of the Veneziano amplitude. Confinement with a string tension cannot be obtained from naive perturbative considerations.

The relation between the amplitude and QCD

The topology of the string amplitude, the disk, suggests that we should take the 't Hooft limit of QCD.

$$\mathcal{A}(x_1, x_2, x_3, x_4) = \langle \bar{q}q(x_1)\bar{q}q(x_2)\bar{q}q(x_3)\bar{q}q(x_4)\rangle$$

Indeed, as we shall see, in the limit $N_c \to \infty$, fixed $g^2 N_c$ and fixed N_f the QCD amplitude does not contain internal fermionic loops (windows) or handles.



Let us write

$$\left(\det(i \not{D})\right)^{N_f} = \exp(N_f \Gamma[A_\mu]) \, ,$$

with

$$\Gamma[A_\mu] = -\frac{1}{2}\int_0^\infty \frac{dT}{T}$$

$$\times \int \mathcal{D}x^\mu \mathcal{D}\psi^\mu \;\exp\left\{-\int_\epsilon^T d\tau\,\left(\frac{1}{2}\dot{x}^\mu\dot{x}^\mu + \frac{1}{2}\psi^\mu\dot{\psi}^\mu\right)\right\}$$

$$\times \mathrm{Tr}\,\mathcal{P}\exp\left\{i\int_0^T d\tau\,\left(A_\mu\dot{x}^\mu - \frac{1}{2}\psi^\mu F_{\mu\nu}\psi^\nu\right)\right\}$$

expanding the exponent $\exp N_f\Gamma$ in powers of N_f/N_c yields the following expression for the scattering amplitude

$$\mathcal{A}(x_1,x_2,x_3,x_4) =$$

$$\frac{1}{2}\int_0^\infty \frac{dT}{T}\int \mathcal{D}x\;\exp\left(-\int d\tau\,\tfrac{1}{2}\dot{x}_\mu^2\right)\langle W(x_1,x_2,x_3,x_4)\rangle_{\mathrm{YM}}$$

where the worldline fermions were omitted for the brevity of writing.

Thus, in the 't Hooft limit *the scattering amplitude* $\mathcal{A}(x_1,x_2,x_3,x_4)$ *is given by sum over all sizes and shapes of Wilson loops that pass via the points* x_1,x_2,x_3,x_4.

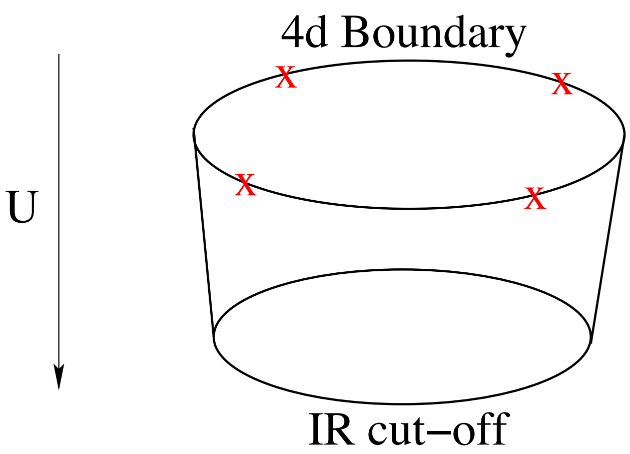
See related discussion by Makeenko and Olesen.

Holography and Wilson loops

The advantage of expressing the amplitude as a sum over Wilson loops is that since we know how to calculate expectation values of Wilson loop via holography, we can relate the field theory expression to string theory (and eventually derive the Veneziano amplitude).

The holographic prescription (Maldacena) is to find a string worldsheet which terminates on the AdS boundary. The worldsheet boundary is the contour of the Wilson loop.

The present calculation requires a contour that passes through x_1, x_2, x_3, x_4, hence a typical worldsheet looks like



Holography and Wilson loops

We propose the following expression for the amplitude

$$\frac{1}{2}\int_0^\infty \frac{dT}{T}\int \mathcal{D}x^\mu(\tau)\times$$

$$\exp\left(-\int d\tau\,\tfrac{1}{2}\dot{x}_\mu^2\right)\langle W(x_1,x_2,x_3,x_4)\rangle =$$

$$\int \mathcal{D}g^{\alpha\beta}\,\mathcal{D}x^M\times$$

$$\exp\left(-\frac{1}{2\pi\alpha'}\int d^2\sigma\,\sqrt{g}g^{\alpha\beta}\partial_\alpha x^M\partial_\beta x^N G_{MN}\right)|_{\{x_1,x_2,x_3,x_4\}}$$

The amplitude is given by a sum over all string worldsheets that terminate on the AdS space and pass through the points x_1, x_2, x_3, x_4.

The above expression holds for any gauge/gravity pair. The information about the gauge theory is encoded in the metric G_{MN}.

Witten's model

We are interested in the gravity dual of pure Yang-Mills theory. Such a dual does not exist, but Witten's model of compactified D4 branes on a thermal circle contains the essential ingredients: confinement and a mass gap.

The metric is

$$ds^2 = (U/R)^{3/2}(\eta_{\mu\nu}dx^\mu dx^\nu$$

$$+f(U)d\tau^2) + (R/U)^{3/2}(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2)$$

with

$$f(U) = 1 - U_{KK}^3/U^3$$

Due to the metric singularity at $U = U_{KK}$, large Wilson loops exhibit an area law $\langle W \rangle = \exp -(\Sigma A)$ with a string tension $\Sigma = \frac{1}{2\pi\alpha'}(\frac{U_{KK}}{R})^{\frac{3}{2}}$.

It is therefore anticipated that if Δx_i is large the sum over Wilson loops will be dominated by configurations that exhibit an area law.

String theory calculation

Let us ignore the compact directions (the four-sphere and τ). This is a reasonable assumption. These direction are more of an artefact than a feature of Yang-Mills theory. Moreover, the contribution to the path integral from compact directions is expected to be small. We therefore approximate the amplitude by

$$\int \mathcal{D}x^\mu \mathcal{D}U \, \exp\left(-\frac{1}{2\pi\alpha'}\int d^2\sigma \, \{(\partial_\alpha x^\mu \partial^\alpha x^\nu G_{\mu\nu}) + (\partial_\alpha U \partial^\alpha U G_{UU})\}\right)\Big|_{\{x_1,x_2,x_3,x_4\}}$$

with the 5d metric

$$ds^2 = (U/R)^{3/2}\eta_{\mu\nu}dx^\mu dx^\nu + (R/U)^{3/2}(\frac{dU^2}{f(U)})\,.$$

Holographic Calculation of the Amplitude

Due to the metric singularity, the path integral is dominated by U in the vicinity of U_{KK}. This is the case for large worldsheets, where the string "sits" at the horizon. Thus, let us insert $\delta(U - (U_{KK} + \epsilon))$ into the path integral, suppressing quantum fluctuations in the U directions. Later on we will discuss what is expected to happen if we omit the delta function and allow fluctuations in the U direction. The modified path integral reads

$$\int \mathcal{D}x^{\mu} \mathcal{D}U \, \delta(U - U_{KK} - \epsilon) \times$$

$$\times \exp\left(-\frac{1}{2\pi\alpha'} \int d^2\sigma \, \{(\partial_\alpha x^\mu \partial^\alpha x^\nu G_{\mu\nu}) + (\partial_\alpha U \partial^\alpha U G_{UU})\}\right) |_{\{x_1,x_2,x_3,x_4\}} =$$

$$\int \mathcal{D}x^{\mu} \exp\left(-\frac{1}{2\pi\alpha'} \int d^2\sigma \, \partial_\alpha x^\mu \partial^\alpha x^\nu \hat{G}_{\mu\nu}\right) |_{\{x_1,x_2,x_3,x_4\}}$$

with $\hat{G}_{\mu\nu}$ the flat 4d metric

$$ds^2 = (U_{KK}/R)^{3/2} \eta_{\mu\nu} dx^\mu dx^\nu$$

Holographic Calculation of the Amplitude

It is easier to calculate the amplitude in momenta space by inserting vertex operators of the form

$$V \sim \int dy \, \exp(ik \cdot x)$$

The resulting expression is

$$\mathcal{A}(k_1, k_2, k_3, k_4) = \int \mathcal{D}x^\mu \prod_{i=1,..,4} \int dy_i \exp\left(ik_i x(y_i)\right)$$

$$\times \exp\left(-\Sigma \int d^2\sigma \, \partial_\alpha x^\mu \partial^\alpha x_\mu\right)$$

We obtain the Koba-Nielsen expression for the scattering amplitude

$$\mathcal{A}(k_1, k_2, k_3, k_4) \sim \delta(\sum_i k_i) \times$$

$$\int \prod_i dy_i \prod_{j<l} (y_j - y_l)^{2\alpha'(R/U_{KK})^{3/2} k_j \cdot k_l}$$

which yields the Veneziano amplitude, with $\alpha'_{eff} = \alpha'(R/U_{KK})^{3/2}$.

Conclusions

- In the 't Hooft limit meson scattering can be expressed as a sum over Wilson loops.
- Using holography the sum over Wilson loops is mapped into a sum over string worldsheets.
- By using an unjustified and crude approximation where both the compact directions and the holographic direction are neglected, the path integral become gaussian.
- The gaussian integration yields the Veneziano amplitude.
- The most critical approximation, which is not valid in QCD, is equivalent to taking the IR cut-off to the UV cut-off. In field theory language it means that Wilson loops of all sizes (including small Wilson loops) admit an area law.

Work in progress

It is natural to relax some of the crude assumptions that I've made.

In particular to incorporate the effect of the holographic coordinate, U.

Adding the fifth coordinate should bring us closer to real QCD.

On Sunday my PhD student Edwin Ireson will present results with

$$\mathcal{A}(k_1, k_2, k_3, k_4) = \int \mathcal{D}x^\mu \mathcal{D}\hat{U} \prod_{i=1,..,4} \int dy_i \exp\left(ik_i x(y_i)\right)$$

$$\times \exp(-\Sigma \int d^2\sigma \, (\partial_\alpha x^\mu \partial^\alpha x_\mu + \partial_\alpha \hat{U} \partial^\alpha \hat{U} +$$

$$\lambda \hat{U}^2 \partial_\alpha x^\mu \partial^\alpha x_\mu))$$

Please stay tuned!