

Axion EFT and Dense Axion Stars

Eric Braaten

& Abhishek Mohapatra & Hong Zhang

Ohio State University

arXiv:1512.00108, arXiv:1604.00669



U.S. DEPARTMENT OF
ENERGY



Axion EFT and Dense Axion Stars

- axions
- dilute axion stars
- Axion Effective Field Theory [arXiv:1604.00669](#)
- dense axion stars [arXiv:1512.00108](#)
- questions?

Axion

strongly motivated candidate for dark matter particle

strong CP problem of QCD can be solved

by spontaneous breaking of a U(1) symmetry

Peccei & Quinn (1977)

pseudo-Goldstone boson: “axion”

spin-0 particle with tiny mass and extremely weak interactions

Weinberg (1978), Wilczek (1978)

produced in early universe by nonthermal mechanisms

vacuum misalignment

Preskill, Wise & Wilczek (1983)

Abbot & Sikivie (1983)

Dine & Fischler (1983)

cosmic string decay

Davis (1986)

Hararie & Sikivie (1987)

Axion

axions produced in early universe:

vacuum misalignment

highly nonrelativistic

high occupation numbers

coherent

cosmic string decay

highly nonrelativistic

high occupation numbers

incoherent

gravitational interactions can thermalize **axions**
so they remain in a **Bose-Einstein condensate**

Sikivie & Yang (2009)

Axion

Relativistic Field Theory for axions

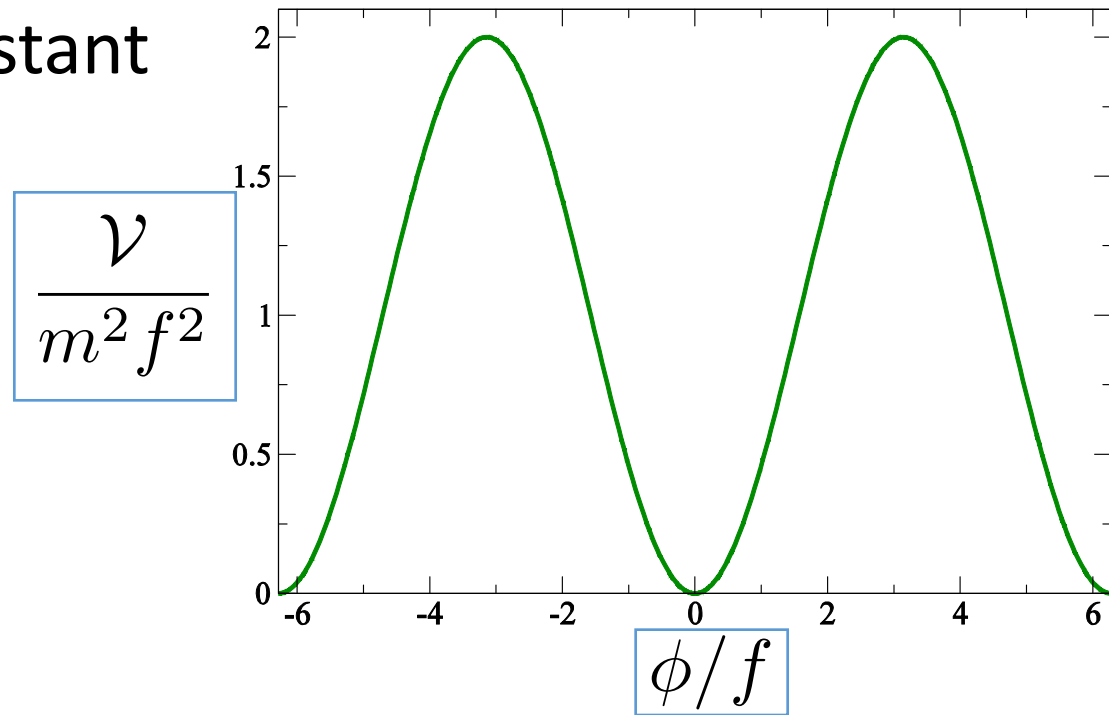
real scalar field $\varphi(x)$

simple model potential*:

$$\mathcal{V}(\phi) = m^2 f^2 [1 - \cos(\phi/f)]$$

m : axion mass

f : decay constant



* more accurate potential depends on up/down quark mass ratio [Di Vecchia & Veneziano 1980](#)

Axion

relativistic axion potential

$$\mathcal{V}(\phi) = m^2 f^2 [1 - \cos(\phi/f)]$$

constraint from QCD $m f = (80 \text{ MeV})^2$

axion decay constant

astrophysics + cosmology $\Rightarrow 10^8 \text{ GeV} < f < 10^{13} \text{ GeV}$

axion mass $10^{-6} \text{ eV} < m < 10^{-2} \text{ eV}$

specific choice: $m = 10^{-4} \text{ eV}$

Dilute Axion Stars

Axion star

stable configuration of axions bound by gravity

Tkachev (1981)

previously known axion stars are dilute: $|\varphi(x)| \ll f$

Barranco & Bernal (2011)

kinetic pressure of axions
balances attractive forces
from gravity and
from axion pair interactions

lowest-energy state of axion star
is Bose-Einstein condensate!



Dilute Axion Stars

vacuum misalignment mechanism

spacial fluctuations in axion field \Rightarrow

gravitationally bound “miniclusters” of axions

Hogan & Rees (1988), Kolb and Tkachev (1993)

typical masses around $10^{-12} M_{\odot}$

gravitational thermalization \Rightarrow

dilute axion star

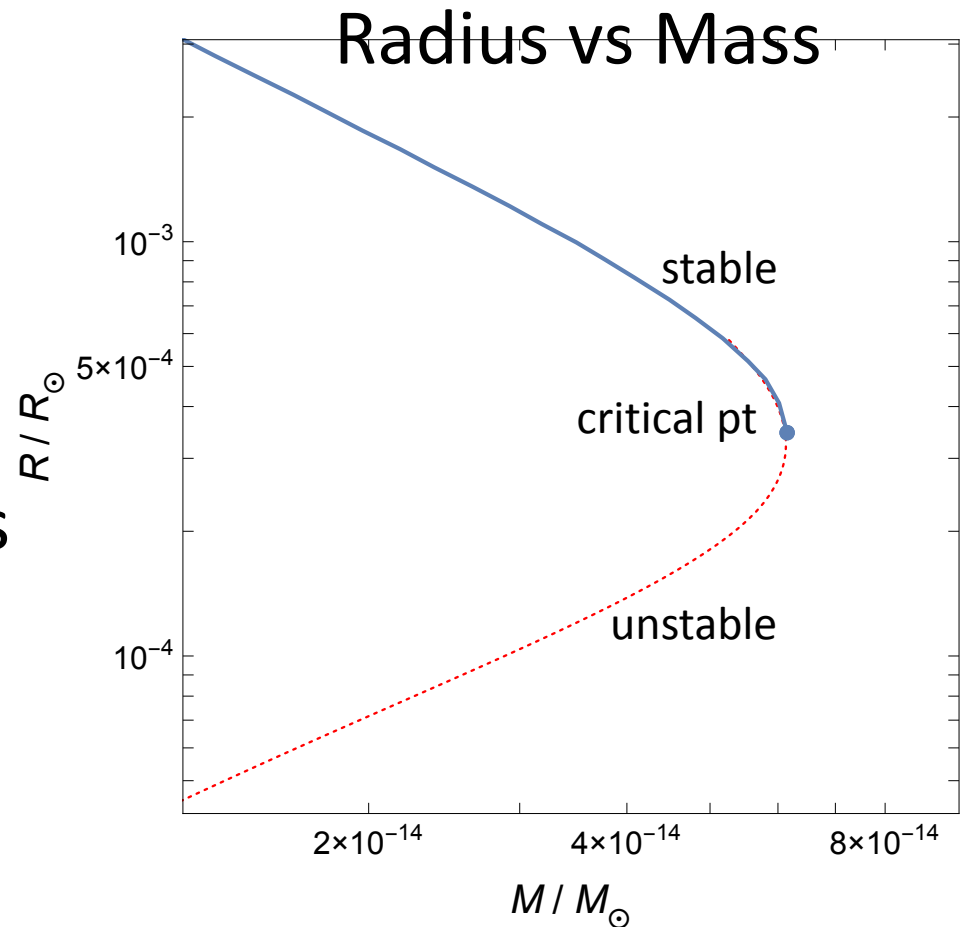


instabilities prevent much larger coherence lengths

Guth, Herzberg & Prescod-Weinstein 2015

Dilute Axion Stars

as **axion star** accretes more and more axions, its Mass increases and its Radius decreases



critical mass M_* beyond which kinetic pressure cannot balance attractive forces

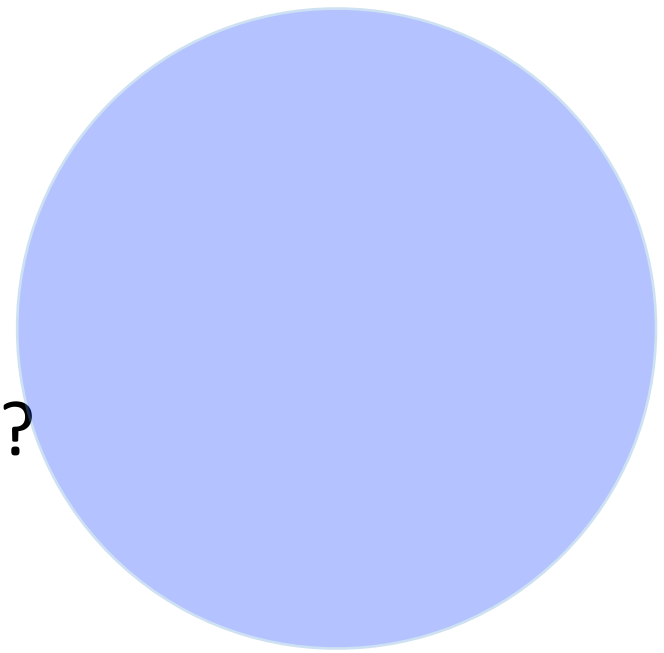
$$\begin{aligned} M_* &= 10.2 f/G^{1/2}m \\ &= 6 \times 10^{-14} M_{\odot} \end{aligned}$$

Chavanis and Delfini (2011)

$$R_* = 3 \times 10^{-4} R_{\odot} = 200 \text{ km}$$

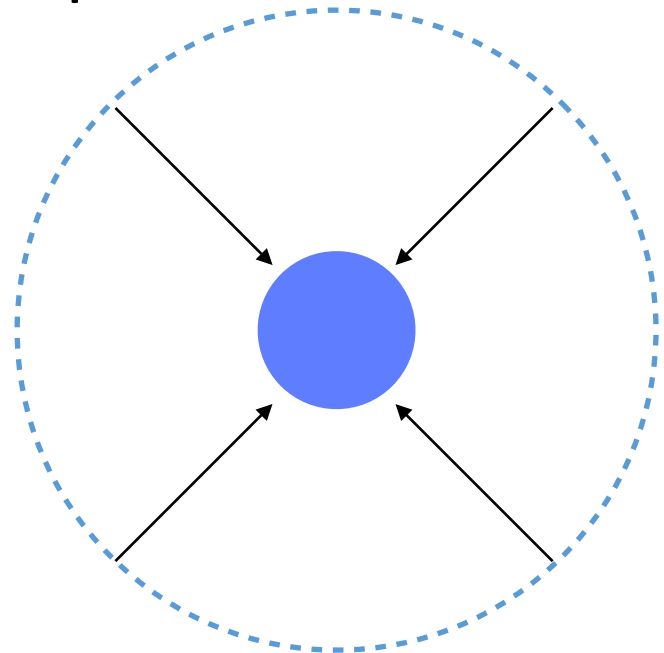
Dilute Axion Stars

What happens to **dilute axion star** with the critical mass when it accretes additional axions?



Kinetic pressure cannot prevent collapse

What is the remnant after the collapse?



Dilute Axion Stars

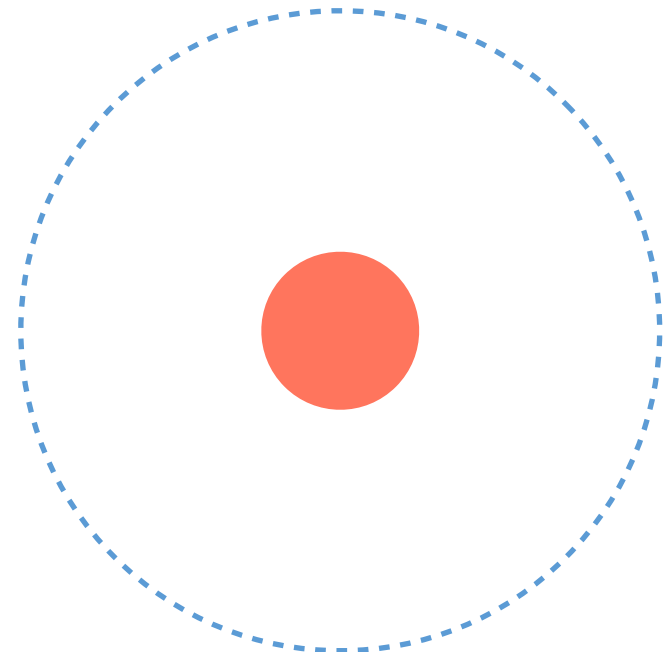
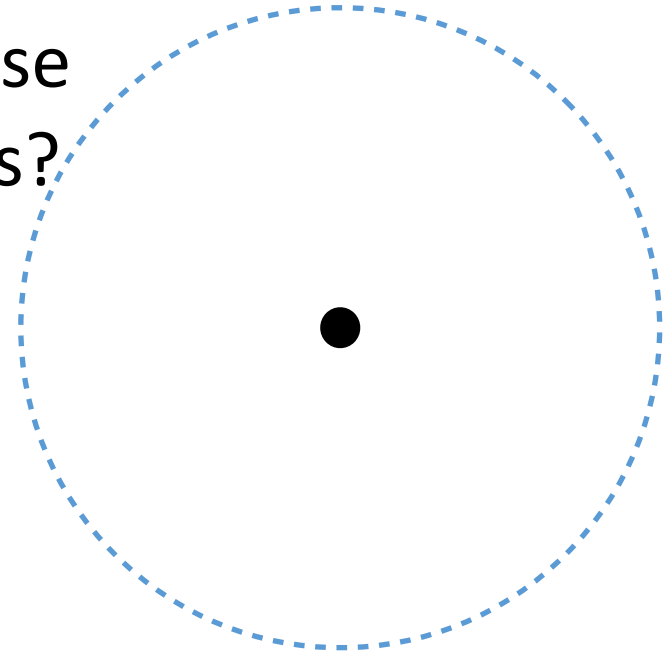
What is the remnant after the collapse of an **axion star** with the critical mass?

black hole?

Schwarzschild radius is smaller
by 72 orders of magnitude

dense axion star?

if radius decreases by 5 orders
of magnitude, ϕ is of order f
higher order terms in axion potential
are important



Axion EFT

Axions can be described by a relativistic field theory with a real scalar field $\phi(x)$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \mathcal{V}(\phi)$$

Low-energy axions can be described by a nonrelativistic Effective Field Theory (“axion EFT”) with a complex scalar field $\psi(r,t)$ [arXiv:1604.00669](https://arxiv.org/abs/1604.00669)

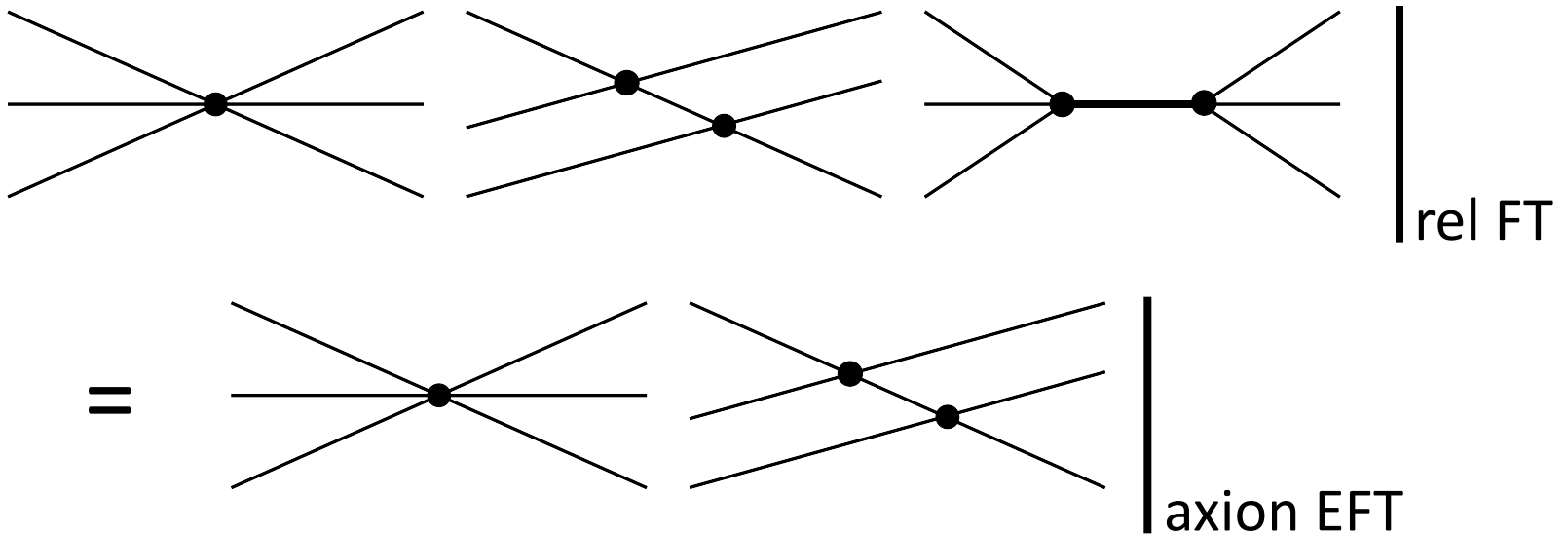
$$\mathcal{H} = \frac{1}{2m} \nabla \psi^* \cdot \nabla \psi + \mathcal{V}_{\text{eff}}(\psi^* \psi)$$

Axion EFT

effective potential for axion EFT: V_{eff}

expansion of V_{eff} in powers of $\psi^*\psi$ can be calculated by matching low-energy scattering amplitudes

[arXiv:1604.00669](https://arxiv.org/abs/1604.00669)



$$V_{\text{eff}} = m\psi^*\psi - \frac{1}{16f^2}(\psi^*\psi)^2 - \frac{1}{288mf^4}(\psi^*\psi)^3 + \dots$$

Axion EFT

If axion Bose-Einstein condensate is **dense**
with $\psi^* \psi$ of order mf^2 ,
must keep terms in \mathcal{V}_{eff} with all powers of $\psi^* \psi$

Naive nonrelativistic reduction

$$\phi(\vec{r}, t) = \frac{1}{\sqrt{2m}} [\psi(\vec{r}, t)e^{-imt} + \psi^*(\vec{r}, t)e^{+imt}]$$

insert into relativistic Hamiltonian

$$\frac{1}{2} \dot{\phi}^2 + m^2 f^2 [1 - \cos(\phi/f)]$$

drop terms with rapidly oscillating phases

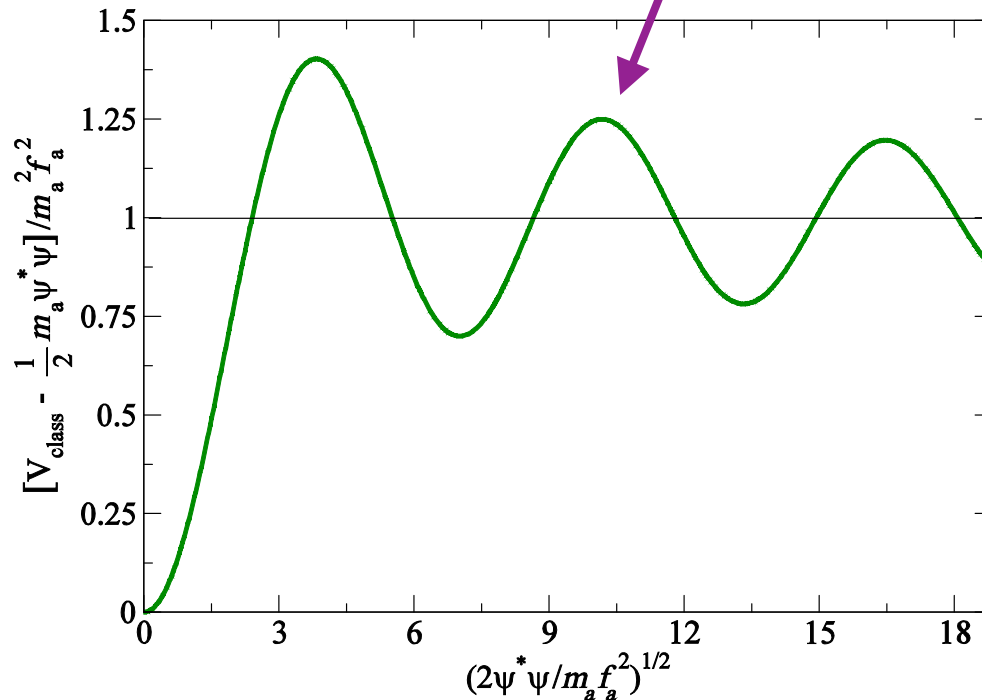
$$\mathcal{V}_{\text{eff}} = \frac{1}{2} m \psi^* \psi + m^2 f^2 [1 - J_0(\hat{n}^{1/2})]$$

Eby, Suranyi, Vaz & Wijewardhana $\hat{n} = 2\psi^* \psi / mf^2$

Axion EFT

naive effective potential from nonrelativistic reduction

$$\hat{n} = 2\psi^* \psi / m f^2$$
$$\mathcal{V}_{\text{eff}} = \frac{1}{2} m \psi^* \psi + m^2 f^2 [1 - J_0(\hat{n}^{1/2})]$$



Axion EFT

effective potential V_{eff} for $\psi^*\psi$ of order mf^2
can be obtained by matching low-energy amplitudes
for $n \rightarrow n$ scattering for all n

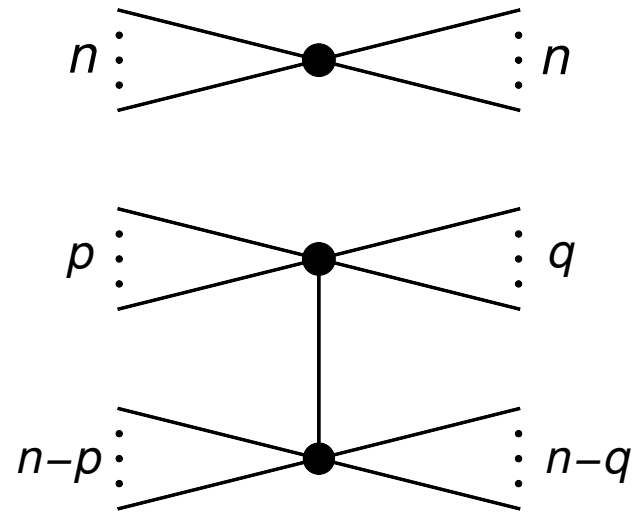
naive effective potential from nonrelativistic reduction

$$\mathcal{V}_{\text{eff}} = \frac{1}{2}m\psi^*\psi + m^2 f^2 [1 - J_0(\hat{n}^{1/2})]$$

match diagrams with
no virtual propagators

1st improvement

also match diagrams with
one virtual propagator



Dense Axion Stars

- axions in **axion star** are nonrelativistic

⇒ use **Axion EFT**

use **naive effective potential**

$$\mathcal{V}_{\text{eff}} = \frac{1}{2} m \psi^* \psi + m^2 f^2 [1 - J_0(\hat{n}^{1/2})]$$

- **gravity** in **axion star** is weak

⇒ approximate **General Relativity**
by **Newtonian gravity**

- time independent
- spherically symmetric

Dense Axion Stars

coupled equations for axion field $\psi(r)$
and gravity potential $\phi(r)$

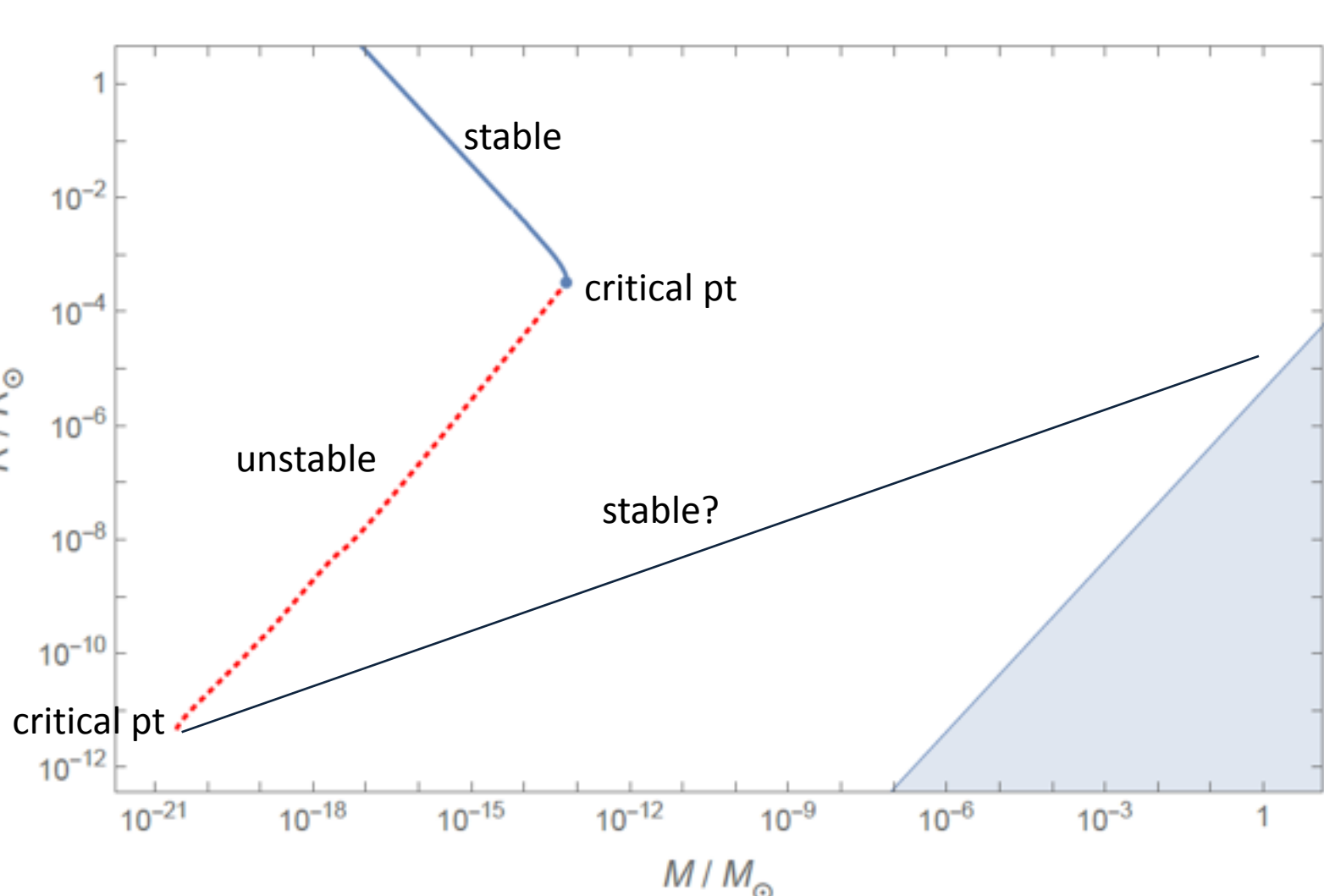
$$\begin{aligned}\nabla^2\psi &= -2m[\mu - (\mathcal{V}'_{\text{eff}}(\psi^*\psi) - m) - m\phi]\psi \\ \nabla^2\phi &= 4\pi Gm\psi^*\psi.\end{aligned}$$

$$\mathcal{V}_{\text{eff}} = \frac{1}{2}m\psi^*\psi + m^2 f^2 [1 - J_0(\hat{n}^{1/2})]$$

eigenvalue equations for chemical potential μ

Dense Axion Stars

equations imply new branch of dense axion stars
radius increases with mass



Dense Axion Stars

mass range: from about $10^{-20} M_{\odot}$ to about M_{\odot}

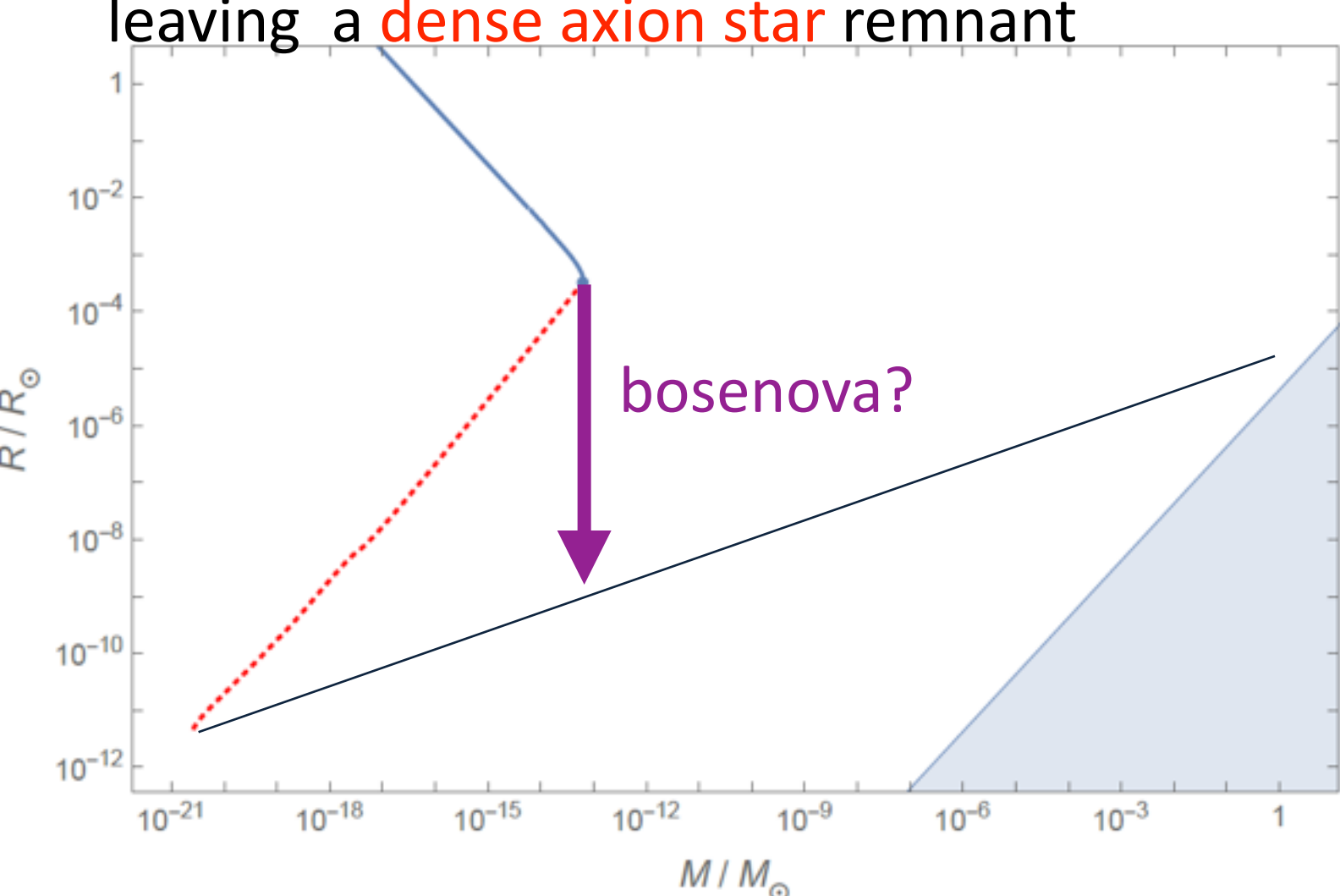
through most of mass range, solutions can be simplified using **Thomas-Fermi approximation**:
neglect kinetic term for axions

$$\cancel{\nabla^2 \psi} = -2m [\mu - (\mathcal{V}'_{\text{eff}}(\psi^* \psi) - m) - m\phi] \psi$$
$$\nabla^2 \phi = 4\pi G m \psi^* \psi.$$

attractive force from **gravity** is balanced by
mean-field pressure of **axion Bose-Einstein condensate!**

Dense Axion Stars

when dilute axion star with critical mass collapses,
it could produce a bosenova
leaving a dense axion star remnant



Dense Axion Stars

Bosenova in cold atoms

stable Bose-Einstein condensate of atoms

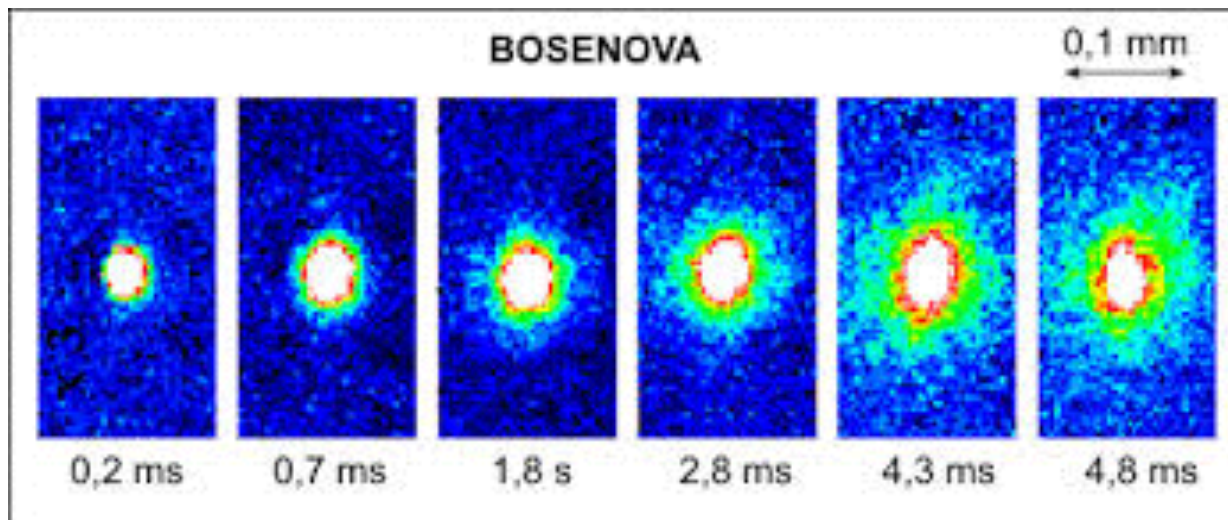
with positive scattering length

sign of scattering length is suddenly reversed

condensate collapses

burst of energetic atoms from inelastic reactions
(3-body recombination)

cold remnant cloud of atoms



JILA group
2001

Dense Axion Stars

Bosenova in axion star

dilute axion star with critical mass
accretes more axions

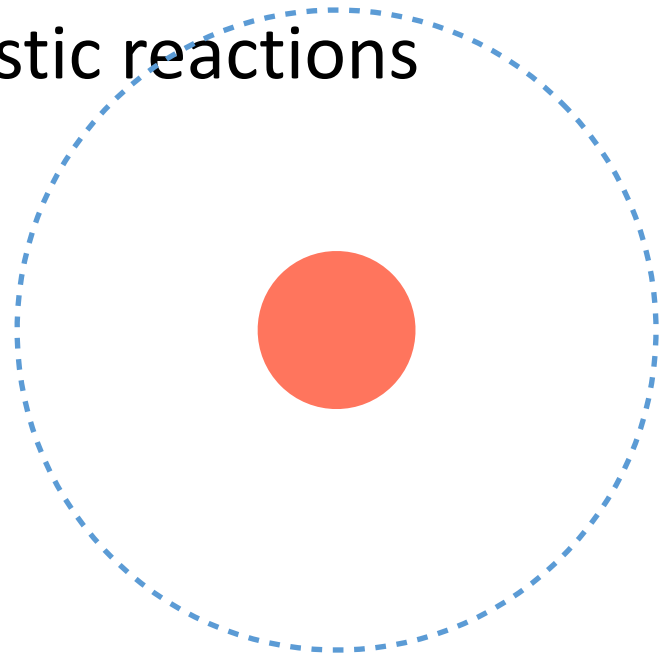
axion star collapses

(radius decreases by 5 orders of magnitude)

burst of relativistic axions from inelastic reactions

($4 \rightarrow 2$ scattering, etc.)

remnant **dense axion star**?



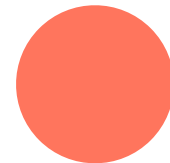
Questions

What is the effective potential for **Axion EFT**?

How can it be calculated accurately?

Are there **dense axion stars**?

Is V_{eff} from the naive nonrelativistic reduction
qualitatively correct?



What happens to a **dilute axion star**

when it exceeds the critical mass and collapses?

Is the remnant a **dense axion star**?

Solve the time dependent classical field equations of Axion EFT.

Questions

What is the mass distribution for axion dark matter?

What fraction is in the form of gas of axions?

dilute axion stars?

dense axion stars?

Can dilute and dense axion stars be observed?

bosenova from collapse of dilute axion star?

fast radio burst from collapse of dilute axion star? Tkachev 2014

fast radio burst from collision with neutron star? Iwazaki 1999

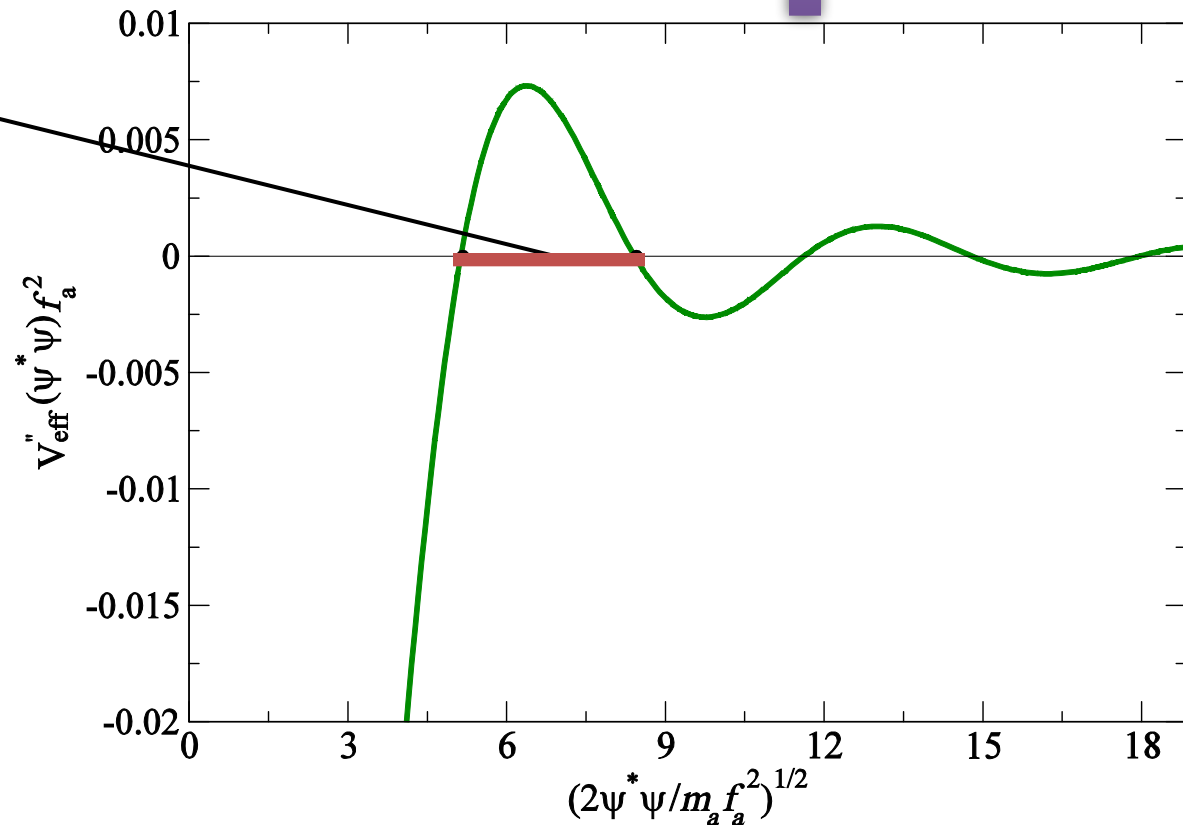
gravitational microlensing?

Axion stars have dramatic implications for the properties of axion dark matter (especially if dense axion stars exist)

Dense Axion Stars: Branches

- Balancing repulsive mean-field pressure of Bose-Einstein condensate with the attractive force from gravity:

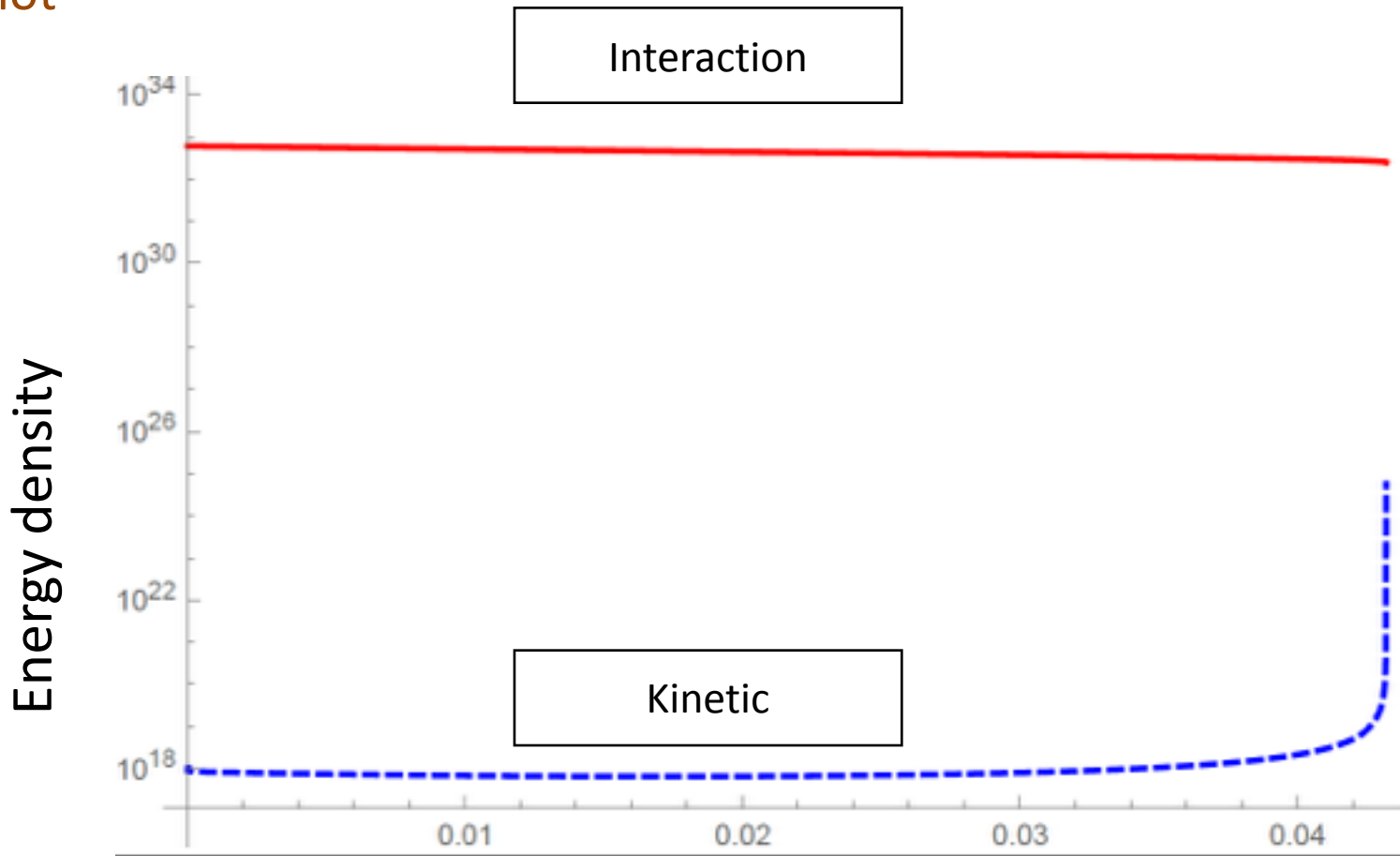
Range of central density for first branch of dense axion stars



Double Derivative of Effective Potential

Dense Axion Stars: Validity of TF Aproximation

Log plot



\hat{r}

$$\hat{r} = (Gm^2 f^2)^{1/2} r$$