AN ANALYSIS OF VISUAL ADAPTATION AND CONTRAST PERCEPTION FOR A FAST TONE MAPPING OPERATOR

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An analysis of visual adaptation and contrast perception for a fast tone mapping operator

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***Abstract*— Tone Mapping is the problem of compressing the radiance range embedded in a High Dynamic Range image into a displayable Low Dynamic Range. In this paper we follow the philosophy of perceptually inspired tone mapping methods: the final image should produce a sensation as close as possible to the sensation produced by the real world scene to an observer. We translate this idea as the proper reproduction of color and contrast. In line with the current literature, we will develop the compression process using the Naka-Rushton equation which describes the response of retinal photoreceptors to light stimuli. We will discuss its influence on chromatic reproduction and provide a version that better reproduces color. Moreover, we will underline that the Naka-Rushton equation misrepresents overall contrast and, thence, we will propose a suitable modification. The main contribution of this paper is the discussion of the modified Naka-Rushton formula in terms of perceptual contrast and the analysis of two suitable equations derived from perceptual contrast measures. The new formulae provide a real-time and artifact-free tone mapping. Tests and comparisons with other tone mapping models are provided and commented.**

I. Introduction and State of the Art

High Dynamic Range (HDR) images constitute a powerful tool to store real-world radiance values and have become more and more accessible during the last fifteen years. In photography, the great popularity of these images is motivated by the possibility to easily create a HDR image by combining multiple photos of a static scene taken with different time exposures (see, e.g. [1]). In computer graphics, the use of HDR images has provided a much realistic synthesis of artificial scenarios. However, real-world radiance can span many orders of magnitude, while common displays and printers can only span up to two. So, a further 'compression step', called 'Tone Mapping' (TM) is required to properly visualize the information stored in the HDR images.

In this paper we will follow the TM philosophy proposed by Ward et al. [2] (p.2): '*We consider the following two criteria most important for reliable TM: 1) Visibility is reproduced. You can see an object in the real scene if and only if you can see it in the display. Objects are not obscured in the under- or over-exposed regions, and features are not lost in the middle 2) Viewing the image produces a subjective experience that corresponds with viewing the real scene*'. In our own words, we consider that the main purpose of TM is to emulate as much as possible the perception of contrast and color produced by the real-world scene. Ideally, a perfect model of the Human Visual System (HVS) would satisfy these tasks, however the knowledge about human vision is still too vague to allow building a precise or complete HVS model. Therefore, works in this direction usually consider a simplified HVS description. In line with this idea, in the present paper we will analyze the role of visual adaptation and relate it to the reproduction of visibility.

The visual adaptation phase occurs mostly in the retina, where photoreceptors (cones and rods) strongly compress the light range. In controlled conditions and for small light ranges, this compression process can be described by the Naka-Rushton (NR) equation [3] which has been widely used as a global TM operator (see subsection I-A). We will underline that this formula fails to reproduce properly the overall contrast of the scene, which is one of the most important issues in TM. Therefore, in this paper we analyze the contrast problem and propose a new method based on the NR equation that is able to reproduce overall perceptual contrast. Finally, we will present some results of our method and show that they compare well to the state of the art.

A. State of the art in TM techniques

Many tone mapping operators have been proposed in the literature; for a thorough review and analysis of the state of the art until 2005, we refer to the excellent book [4]. Here we just want to give a very brief overview of the different schools of thought that have been proposed so far.

We can roughly divide the TM literature in two groups: perceptual-based operators and gradient-based operators. In the first category, we can distinguish between spatially global and local TM transformations. The former are, in general, very fast and do not introduce halos or artifacts, but their contrast rendition tends to be poor. The first works in this category used Stevens' law [5], [6] to achieve range compression [7]–[9]. A rational function very close to NR's formula was used in [10] showing improvements with respect to Stevens' law. The global NR formula has been exploited in [11]–[13]. More sophisticated vision models were also proposed, taking into account time-adaptation [14] or Weber-Fechner's law [2], [15].

The spatially-variant operators show more detailed images, but halos and artifacts usually appear next to edges [16], [17]. Within the perceptually-based spatially-varying TM operators we also find a modified version of the Retinex model of color vision [18]. The anchoring theory, best known in the image processing literature as white-patch or gray-world methods, has been used in [19] after a suitable subdivision of the original image into layers of similar luminance.

Given the difficulties of avoiding halos and artifacts when dealing with spatially-variant methods, we propose to use a

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global approach but with a luminance-varying formula based on the NR equation modified in order to be consistent with the laws of contrast perception. Local contrast can be improved without producing artifacts given that, in natural images, luminance locality is often related with spatial locality.

On the other hand, gradient-based TM operators rely on the idea of shrinking large intensity gradients while preserving small fluctuations, which correspond to fine details. The way in which this compression is achieved can vary: in [20] the authors used a hierarchical method based on a Partial Differential Equation (PDE) inspired by anisotropic diffusion. [21] and [13] obtained improved results by using techniques inspired on bilateral filtering. In [22] and [23], spatially varying compression factors were used to implement suitable manipulations of the gradient field.

Finally, in [24] and [25] interactive methods for TM were proposed in order to produce pleasant results for particular luminance zones selected by a user.

This paper is organize as follows: in section II we will present the original NR formula. The application of this formula to HDR images presents two problems that will be addressed in section III: color rendition and contrast reproduction. Three NR-based functions are discussed and evaluated in section IV, results and comparisons with other TM methods are presented in section V. Finally, we summarize the paper in section VI.

II. Representing visual adaptation: the NR formula

Let us begin this section by recalling how the retina responds to light stimuli. The range of radiances over which the HVS can operate is very large: from 10^{-6}cd/m^2 (scotopic limit) to 10^6 cd/m^2 (glare limit) [26]. The automatic process that allows the HVS to operate over such a huge range is called visual adaptation [27]. It is important to stress that the HVS cannot operate over its entire range simultaneously. Rather, it adapts to an average intensity and handles a smaller magnitude interval.

Neuroscience experiments show that visual adaptation occurs mainly in the retina. The experiments to measure this behavior were performed using very simple, non-natural images: on a uniform background were superimposed brief pulses of light with intensity $\mathcal{I}$. When a photoreceptor absorbs $\mathcal{I}$, the electric potential of its membrane changes accordingly to the empirical law known in vision research literature as NR's equation [3], [27], [28]:

$$r(\mathcal{I}) = \frac{\mathcal{I}}{\mathcal{I} + \mathcal{I}_s}, \tag{1}$$

where $r(\mathcal{I})$ is the normalized electric response of the photoreceptor to $\mathcal{I}$ and $\mathcal{I}_s$ is the light level at which the photoreceptor response is half maximal, called *semi-saturation level* and which is usually associated with the level of adaptation. [1]

The graph of the function $r(\mathcal{I})$ is depicted in Fig. 1. Let us notice that, since $\mathcal{I}$ and $\mathcal{I}_s$ are intensity levels, they are both positive and therefore $r(\mathcal{I}) \in [0, 1]$. From a mathematical point of view, the utility of this formula for TM is threefold: first of all, it allows to compress every given range of light stimuli into $[0, 1]$, it is monotonically increasing, so it preserves the order of the level lines, and, finally, it has a non-linear behavior that allows us to reproduce details along a wide luminance range by enhancing the differences in dark areas and reducing them in brighter zones. Moreover, this formula has been related to sensation in controlled conditions [27], so it is a natural candidate to be used for our TM purposes.

[1]Even though with this setting this equation is identical to the so-called Michaelis-Menten's equation [28] (pag. 301), we will maintain the name NR's equation to follow the custom in tone mapping and computer vision literature.

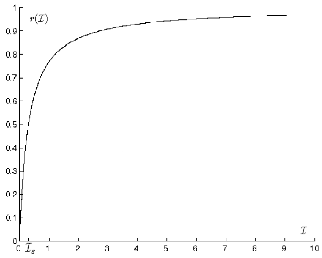


Fig. 1. Graph of the NR function $r(\mathcal{I})$ vs. $\mathcal{I}$, with $\mathcal{I}_s = 0.1$ in linear scale.

The representation of light absorption by photoreceptors as described by this formula is valid only for a small range of values of $\mathcal{I}$, since it saturates too quickly the light levels $\mathcal{I}$ significantly far from $\mathcal{I}_s$. This effect is called 'saturation catastrophe' in [27]. To properly describe the response of the photoreceptors for arbitrary ranges of $\mathcal{I}$,the NR function has to be modified. In [27], the authors show that the easiest way to properly modify this function while maintaining its analytic structure and modeling properties is to substitute the constant $\mathcal{I}_s$ by a function $f_{\mathcal{I}_s}(\mathcal{I})$:

$$r_f(\mathcal{I}) = \frac{\mathcal{I}}{\mathcal{I} + f_{\mathcal{I}_s}(\mathcal{I})}. \tag{2}$$

In order to apply this formula to HDR images, we must correctly identify which features correspond to the variables appearing in eq. (2).

III. The NR formula in the framework of HDR images

Let us here introduce the notation that we will use throughout the paper. Let $\vec{I} : \mathfrak{I} \to (0, +\infty)^3$ be the radiance map representing the input HDR image, $\mathfrak{I}$ being its spatial domain $\subset \mathbb{Z}^2$. We denote with I_c the generic value of the scalar chromatic components of $\vec{I}$, $c \in \{R, G, B\}$, with $x = (x_1, x_2) \in \mathfrak{I}$ the spatial position of an arbitrary pixel in the image, and with $I_c(x)$ the intensity value of the pixel x in the c channel. To avoid singularities when operating on $\vec{I}$ with ratios or logarithms, we add a small positive constant to all values: 10^{-12}. Finally, we denote with $\lambda(x) = \frac{1}{3}\left[I_R(x) + I_G(x) + I_B(x)\right]$ the luminance of the pixel $x \in \mathfrak{I}$ and with λ a generic luminance value, i.e. $\lambda \in [\lambda_{\min}, \lambda_{\max}] \subset (0, +\infty)$, where $\lambda_{\min}$ and $\lambda_{\max}$ are the extreme luminance values of the image.

Notice that, since the HDR image represents the radiance map of a scene, it is natural to let $\vec{I}$ play the role of $\mathcal{I}$.

Moreover, it makes sense, for a *natural scene*, to identify the semi-saturation level as an average value, which we denote by μ. In the literature there is no agreement about which expression of μ must be used: arithmetic average μ_a, geometric average μ_g, median μ_{med}, or combinations of them [4]. Furthermore, neurophysiological data involving complex scenes do not give a precise indication about which value of μ is most suitable. For these reasons, we choose to let unspecified, for the moment, the formal expression of μ.

With the notation just introduced, the expression of the modified NR equation is the following:

$$r_f(\vec{I}) = \frac{\vec{I}}{\vec{I} + f_\mu(\vec{I})}. \tag{3}$$

As stated in the Introduction, the two criteria that we are interested in for a good TM operator are color rendition and visibility preservation. The first one is related to a good reproduction of hue and saturation, while the second one is related to a proper luminance mapping. In the following subsections we will discuss different options to implement these two properties and select the most suitable analytic form of the modified NR formula.

A. *Color rendition with the Naka-Rushton formula*

In the literature we find, at least, three different options on how to apply the NR formula to a color HDR image.

1) Non-linear luminance proportion: The first one consists in the application of the popular formula (see e.g. [20], [22]):

$$r_{\eta,f}(I_c(x)) \equiv \left(\frac{I_c(x)}{\lambda(x)}\right)^\eta r_f(\lambda(x)), \tag{4}$$

where $c \in \{R, G, B\}$ and $\eta \in [0, 1]$ is a parameter that controls the saturation. In order to understand the effect of η, let us make explicit the saturation of the pixel x in the original image $\vec{I}$ as $\vec{S}(\vec{I}(x)) \equiv \left(\frac{I_R(x)-\lambda(x)}{\lambda(x)}, \frac{I_G(x)-\lambda(x)}{\lambda(x)}, \frac{I_B(x)-\lambda(x)}{\lambda(x)}\right)$. After straightforward computations, it can be verified that the c-th component of the saturation of pixel x in the output image $\vec{r}_{\eta,f}$ is:

$$S_c(\vec{r}_{\eta,f}(x)) = \frac{I_c(x)^\eta - \frac{1}{3}\sum_{c\in\{R,G,B\}} I_c(x)^\eta}{\frac{1}{3}\sum_{c\in\{R,G,B\}} I_c(x)^\eta}. \tag{5}$$

Note that when $\eta = 1$ the saturation of the output coincides with that of the original image, which our tests have shown to be excessive. On the other extreme, if $\eta = 0$ the saturation is zero, i.e. we have a greyscale image. It is very difficult, or in some cases impossible, to find an intermediate value of η between 0 and 1 that gives good saturation in both dark and bright image zones (for a recent proposal of a different formula see [29]).

2) Independent color channels: The second method [12] consists in applying the modified NR formula eq.(3) separately to the three chromatic channels R,G and B:

$$r_f(I_c(x)) := \frac{I_c(x)}{I_c(x) + f_\mu(I_c(x))}, \tag{6}$$

3) Interdependence between color channels and luminance: Finally, we propose a formula inspired by a similar equation presented in [12]:

$$r_f(I_c(x)) := \frac{I_c(x)}{I_c(x) + f_\mu(\lambda(x))}. \tag{7}$$

Notice that this equation mixes the chromatic components of $\vec{I}$ with the corresponding luminance λ.

We have observed that the choice of f_μ affects the detail visibility, but not color rendition, therefore let us postpone the correct identification of f_μ and consider it here constant, i.e. $f_\mu \equiv \mu$, in order to understand the effect of the different formulas (4), (6) and (7) on the chromatic rendition. The images shown in Fig. (2) exhibit how eq. (4) produces over-saturated colors, eq. (6) tends to desaturate the image, and, finally, eq. (7) produces a satisfactory saturation.

Fig. 2. From left to right, results obtained using eq.(4) with $\eta = 0.8$, eq. (6) and eq.(7). In every equation $f_\mu(\lambda) \equiv \mu_{\text{geom}}$.

This behaviour was consistent for all the tests that we have performed. For the rest of the paper we will implement the NR equation on color images considering eq. (7).

Remark: A common assumption in many color processing algorithms is that color channels are correlated, in the sense that they share the same geometry, which also coincides with the geometry of the luminance image [30]. In other words, it is reasonable to assume that color channels and luminance image share the same family of level lines. Now, with this assumption, if two pixels x, y are in the same isointensity line for the channel I_c, then they will also be in the same isointensity line of $r(I_c)$. This may partially explain the good geometry preservation properties of the mapping produced by eq.(7), without halos or artifacts of any kind.

B. *Visibility and contrast perception in the modified NR equation*

Now that we have selected the NR formula that best represents chromatic rendition, let us focus our attention on the second condition for a good TM: detail visibility. Since this feature is related to light intensity changes, we are going to analyze it by considering a simplified model that takes into account only the luminance λ:

$$r_f(\lambda) = \frac{\lambda}{\lambda + f_\mu(\lambda)} \tag{8}$$

Later on we will use the results of this section on the full color image.

(a) (b)

Fig. 3. Results obtained using the Naka-Rushton function eq.(8) setting f_μ to **(a)** the geometric average and **(b)** the arithmetic average of the luminance. These results are an example of the "saturation catastrophe": if we use a value for μ that enables to reasonably tone map dark areas (a), the bright ones are completely burned out. On the other hand, if we increase the value of μ to avoid saturation of bright areas (b), we completely lose visibility of the dark ones. Thus, we cannot expect to improve the overall visibility of the final image by solely changing the value of μ.

As previously remarked, detail visibility can be studied by introducing the function $f_\mu(\lambda)$. If $f_\mu(\lambda)$ were a constant, then we would return to an equation of the same type as eq. (1), which suffers from the "saturation catastrophe" (see Fig. 3). Therefore, we need a function that varies with λ. Since we are interested in a perceptually-sound detail visibility, we are going to take into account HVS properties to determine $f_\mu(\lambda)$.

The most well-known psychophysical experiments on contrast perception and visibility where developed by E.H. Weber. He presented to a subject a set of non-natural images that consisted in a uniform background with a small superimposed patch of a different intensity. The subject had to say when the small patch was distinguishable from the background, i.e. when he/she was perceiving a Just Noticeable Difference (JND). These experiments were the first ones to describe the perception of contrast and the minimum condition for visibility and led to psychophysical laws that we are now going to use in order to determine the function $f_\mu(\lambda)$. We do this by imposing $r(\lambda) = s(\lambda)$, where $s(\lambda)$ is a function that models contrast sensation. The function $f_\mu(\lambda)$ obtained by solving this equation, will be incorporated into eq. (7).

1) Modification based on Weber-Fechner's law: Let us now introduce Weber's law. Given an intensity difference $\Delta\lambda$, the contrast sensation Δs that it induces depends on the background intensity λ as follows [31] (pag. 490):

$$\frac{\Delta\lambda}{m+\lambda} = k\Delta s, \tag{9}$$

where $k > 0$ is a constant and $m > 0$ is a quantity often interpreted as internal noise in the visual mechanism [32] (pag. 859). Following Fechner's work, we consider infinitesimal changes in eq. (9), integrate the corresponding equation and derive the following functional expression:

$$s_{\mathrm{WF}}(\lambda) \equiv C + k\log(m+\lambda), \tag{10}$$

where $C \in \mathbb{R}$ is an integration constant. We will refer to this expression as the Weber-Fechner's (WF) law.

This function s_{WF} is monotonically increasing, but not bounded in $[0, 1]$. So, if we want to use it to find $f_\mu(\lambda)$ through the equation $r_f(\lambda) = s_{\mathrm{WF}}(\lambda)$, we must properly normalize s_{WF}.

Let us impose these constraints: $s_{\mathrm{WF}}(\mu) = 1/2$, $s_{\mathrm{WF}}(\lambda_{\min}) = 0$ and $s_{\mathrm{WF}}(\lambda_{\max}) = 1$. The first constraint is imposed following the so-called "grey world" hypothesis [33] by setting the normalized sensation response relative to μ in the *middle* of the unit interval. With straightforward algebraic computations it can be proven that the expressions of k and m that satisfy these constraints are:

$$k \equiv \frac{1}{\log\left(\frac{m+\lambda_{\max}}{m+\lambda_{\min}}\right)} \text{ and } m \equiv \frac{\mu^2 - \lambda_{\max}\lambda_{\min}}{\lambda_{\max}+\lambda_{\min}-2\mu}. \tag{11}$$

Now, if we impose $r_f(\lambda) = s_{\mathrm{WF}}(\lambda)$ for all λ, we get the following functional expression for f:

$$f_{\mathrm{WF}}(\lambda) \equiv \frac{\lambda}{\frac{1}{2} + k\log\left(\frac{m+\lambda}{m+\mu}\right)} - \lambda. \tag{12}$$

with k, m set as in Eq. (11).

Remark: The WF law and the NR equation were related before in [4] (pag. 205), where the authors conclude that the NR equation satisfies the WF law in certain conditions.

Furthermore, Ward et al. propose [2] performing tone mapping with a constrained Histogram Equalization (HE). More precisely, they introduce an upper bound on the derivative of the tone map, so that the output contrast is never *higher* than the WF contrast. The main reason to introduce this constraint was not to make visible in the final image objects that were not visible in the original scene.

Our approach is rather different: we are not using Weber-Fechner's law as an upper bound, but as an exact contrast measure. This is consistent with the visibility principle stated by Ward since we are mapping intensity differences above the JND into visible distances in the final tone mapped image.

2) Modification based on Stevens' law: In 1960's, Stevens developed psychophysical experiments([6]) from a comparative perspective, concluding that the HVS perceives physical stimulus following a power-law behavior:

$$s_{\mathrm{S}}(\lambda) \equiv C + k\lambda^\gamma, \tag{13}$$

where $C, k \in \mathbb{R}$ are constants and $\gamma \in \mathbb{R}$ is the exponent that controls the concavity of the function $s_{\mathrm{S}}(\lambda)$. Stevens found out that the perception of each human sense corresponds to a precise value of γ. In vision, this value varies between $\gamma = 0.33$ and $\gamma = 0.5$, depending on the illuminant conditions.

As we did for function s_{WF}, we impose to the monotonically increasing function s_{S} these constraints: $s_{\mathrm{S}}(\mu) = 1/2$, $s_{\mathrm{S}}(\lambda_{\min}) = 0$ and $s_{\mathrm{S}}(\lambda_{\max}) = 1$. The value of C that satisfies the first constraint is $C = 1/2 - k\mu^\gamma$. With straightforward algebraic computations it can be proven that the parameters which satisfy the last two constraints are, respectively, $k_0 = 1/[2(\mu^\gamma - \lambda_{\min}^\gamma)]$ and $k_1 = 1/[2(\lambda_{\max}^\gamma - \mu^\gamma)]$. The only value of μ for which $k_0 \equiv k_1$ is $\mu \equiv [(\lambda_{\max}^\gamma + \lambda_{\min}^\gamma)/2]^{1/\gamma}$.

Now, by setting $r_f(\lambda) = s_{\mathrm{S}}(\lambda)$ for all λ, we get the following functional expression for f:

$$f_{\mathrm{S}}(\lambda) \equiv \frac{\lambda}{\frac{1}{2} + k(\lambda^\gamma - \mu^\gamma)} - \lambda, \tag{14}$$

with μ as above and k as any of the equivalent expressions k_0, k_1.

3) Modification based on histogram equalization: In order to show the importance of using a perceptual contrast measure, we will also introduce a purely mathematical contrast measure related to HE, showing that the corresponding results are, in general, not satisfactory.

In image processing theory, a well known contrast measure is the entropy. The transformation that maximizes this contrast function is HE, which amounts to setting $\lambda = H(\lambda)$ for all λ, where H is the *normalized cumulative histogram* of the image. We notice that H is a monotonically increasing function bounded in $[0, 1]$, so it makes sense to set $r_f(\lambda) = H(\lambda)$, for all λ. This implies the following expression for f:

$$f_{\mathrm{HE}}(\lambda) \equiv \frac{\lambda}{H(\lambda)} - \lambda. \tag{15}$$

Note that the contrast rendition produced by this function depends, by definition of HE, on the histogram of the original image, thus the final contrast will not be related, in general, to human perception.

Now that we have presented the three functions f_{WF}, f_{S} and f_{HE}, let us see their effect on contrast rendition.

IV. The modified NR equations on color HDR images

Until now we have separately discussed color and contrast rendition, obtaining eq.(7) from the color discussion and the functions f_{WF}, f_{S} and f_{HE} from the contrast analysis. In this section, we will fuse this information to obtain three TM operators on color images. Finally, we will set the missing parameters, present results obtained by each TM operator and analyze the results.

Let us start by reminding the reader that applying the NR formula as in eq.(7) where f_μ can be f_{WF}, f_{S} or f_{HE} does not correspond (respectively) to a logarithm, a gamma transformation or a HE: this would only happen in the case that $I_c(x) = \lambda(x)\ \forall x \in \mathfrak{I}$. In fact, if we introduce in eq.(7) the explicit representation of f_{WF} we have:

$$r_{f_{\mathrm{WF}}}(I_c(x)) = \frac{I_c(x) r_{f_{\mathrm{WF}}}(\lambda(x))}{I_c(x) r_{f_{\mathrm{WF}}}(\lambda(x)) + \lambda(x)\left[\frac{1}{2} - k\log\left(\frac{m+\lambda}{m+\mu}\right)\right]}, \tag{16}$$

and similarly for $f_{\mathrm{S}}, f_{\mathrm{HE}}$.

We can provide a colorimetric interpretation of the previous formula by considering the c-th component of the chrominance of a pixel x, defined as $\mathrm{Cr}_c(x) \equiv I_c(x) - \lambda(x)$. After straightforward computations, we can rewrite $r(I_c(x))$, for $c = \{R, G, B\}$, as:

$$r_f(I_c(x)) = r_f(\lambda(x))\, \frac{I_c(x)}{r_f(\lambda(x))\mathrm{Cr}_c(x) + \lambda(x)}. \tag{17}$$

Note how this formula differs from s_{WF}, s_{WF} and H since it is a nonlinear combination of chrominance, luminance and pixel channel intensity. However, a complete understanding of the formula is still lacking, so we leave it as a matter of further research.

A. Parameter setting

Before showing some results, let us first empirically define some variables that were left undetermined for f_{WF} and f_{S}. The parameters of these functions depend on $\lambda_{\max}$ and $\lambda_{\min}$, which can be outliers and alter the final result. Taking into account this fact, we propose a fast method to redefine $\lambda_{\min}$ and $\lambda_{\max}$. We compute the histogram of the luminance image in the Log scale and define a window of V orders of magnitude. We slide the window over the Log-histogram and for each position we compute the number of pixels that fall into the window. Finally, we select the position where the number of pixels inside the window is maximum. The values $\lambda_{\min}$ and $\lambda_{\max}$ are taken as the end points of this window. All values above $\lambda_{\max}$ are set to $\lambda_{\max}$ and all values under $\lambda_{\min}$ are set to $\lambda_{\min}$. A value for V that gives overall good results is $V = 5$, all the results that we will present in this paper will use this setting for V.

Now, regarding f_{WF}, a parameter that has to be empirically defined is μ. As we already said, there is no complete agreement in the definition of a proper value of the average light level. Therefore, we propose an empirical value given by a convex linear combination in the logarithmic domain between the arithmetic (μ_a) and geometric (μ_g) averages: $\mu(\rho) = \mu_a^\rho \mu_g^{1-\rho}$, where $\rho \in [0, 1]$. The best results were achieved using values of ρ that vary between 0.5 and 1. The effect of varying ρ is a modification in the overall brightness of the final image: the bigger the value of ρ the darker the image. Since the choice of ρ is a matter of preference of the user (who in some cases may want a darker or a lighter result) it is left as the only parameter of f_{WF}.

Finally, the function f_{S} has a free parameter: γ. The influence of γ can be perceived mainly on the overall contrast of the final image. Since contrast is proportional to intensity changes, the derivative is a natural contrast descriptor. Now, r'_{f_S} directly depends on γ, so changing γ will produce contrast variations. Hence, the main drawback of this method is that the selection of γ is not obvious. Our tests show that the best results were obtained, in general, setting γ in the range 0.01 to 0.5.

B. Evaluation of the modified NR equations

Now that we have defined all the parameters appearing in f_{WF} and f_{S}, let us evaluate the effects of the corresponding TM operators on a representative image. In Fig. 4, we can check a set of images produced with f_{WF}, f_{S} and f_{HE}. Note that the image corresponding to f_{HE} has a non-realistic contrast, while the other two appear closer to human perception. This happens very commonly with the contrast function f_{HE}. Even though HE is a widely used technique to improve contrast in LDR images, it has been reported to produce unnatural results for images with well defined modes in its histogram. This problem is even bigger in HDR images due to their large range of intensities. The main reason is that

(a) (b) (c)

Fig. 4. Results of the 'Cars' image obtained with **(a)** f_{WF} ($\rho \equiv 1$) **(b)** f_{S} ($\gamma = 0.1$) **(c)** f_{HE}.

the contrast rendition produced by HE is not defined on a perceptual basis, but depends on the actual histogram shape of the image. Since we are interested in producing images with a realistic contrast, we will focus in the remaining part of this paper on the functions f_{WF} and f_{S}.

The computation just described is point-wise, i.e. it does not depend on the surround of each pixel. Thus, the corresponding algorithm has minimal computational cost and can be run in real time.

V. Results and comparison with the state of the art

Although a thorough comparison is out of the scope of this paper, we would like to give an indication of the quality of our results by presenting a comparison with the output of two other tone mapping methods which belong to the state of the art: [16] and [23]. The results presented here were produced by the code provided by the authors in the first case, and by the software pfstmo (http://www.mpi-inf.mpg.de/resources/tmo/) in the case of [23].

Recently, some authors proposed metrics to evaluate tone mapping results: [29], [34]–[37]. The problem in defining such a metric is that it is very complicated to compare the real scene sensation with the tone mapped HDR image. In fact, the image size and the ambient conditions are, in general, very different. Up to this date, none of these measures have become universally accepted or standard, so we will just provide visual comparisons, as it is usually done in the TM literature.

In the authors' opinion, the proposed method compares well and, in some cases, outperforms the state of the art methods in terms of color reproduction and overall contrast visibility. It does not produce spurious colors nor over-saturation as can be checked in the sky of Fig. 5 and in the glass window area of Fig. 6 and in Fig. 7. Regarding contrast, the presented method does not over-enhance or diminish contrast as can be seen in Fig. 5 (see the area under the table and on the shutter) and in Fig. 6. For more results see www.gpi.upf.edu/static/sira/Sira_Ferradans/Tone_Mapping.html

A. An optional contrast enhancement and chromatic adaptation post-processing step

If we interpret a HDR image as a measure of the scene radiance, the main aim of tone mapping should be to create a LDR image that reproduces the perception of brightness produced to an observer by such a scene. The range compression step is important but the perception of brightness also involves, at least, these other two features: an enhancement of local contrast and a chromatic adaptation. The first feature is related to the HVS ability to magnify details. Human color constancy or chromatic adaptation is the ability to *partially* discard the illumination color of a scene, an essential feature of the HVS which makes it very robust in object detection in color images, even under strong light changes [38].

We propose to add a further step that introduces these two phenomenological features. A good example of such a procedure is the variational algorithm presented in [39], a post-processing method able to perform a perceptually inspired color correction and local contrast enhancement.

For images that do not present fine details or strong color cast, this variational contrast enhancement step is not necessary. We remark this fact with the detail presented in Fig. 8(a) and Fig. 8(b) where we can see that the action of the variational algorithm is almost inappreciable.

However, images with strong color cast or highly textured regions can be greatly improved by the action of this algorithm. A good example of color cast removal can be seen in Fig. 8(c) and Fig. 8(d). The output of the tone mapping stage Fig. 8(c) preserves the original color cast, while the final result Fig. 8(d) is free from color cast and has sharper details. The effect of color cast removal is particularly visible on the white napkin, while local detail enhancement is shown in the image 'Trees' (Fig. 8) (f), specially visible around the bushes.

VI. Conclusions and perspectives

We have presented a tone mapping operator for HDR images inspired by a basic stage of the human vision: visual adaptation. We have implemented this feature through a modified NR's equation, proposing two new analytical expressions that allow to compress the range of the HDR image into $[0, 1]$ in accordance to perceptual contrast. Because this step does not significantly alter the ordering of the level lines, it is free from the typical halos and artifacts produced by spatially local methods. Moreover, this algorithm is real-time and has only one user-dependent parameter. We have also considered including in the tone mapping method a post-processing step that implements local contrast enhancement and color cast removal, obtaining more realistic results.

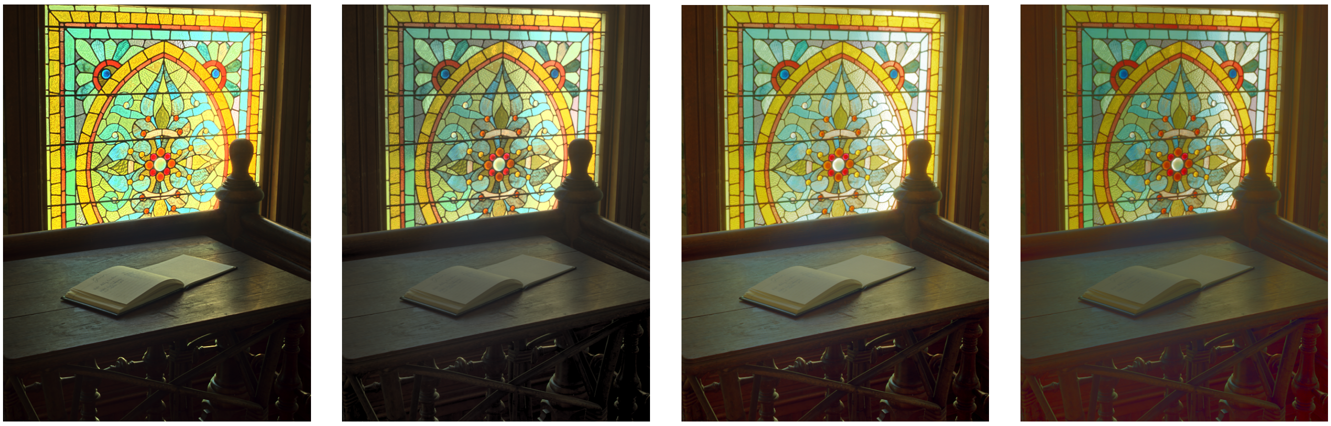
We leave as future work the use of images with calibrated data. Since very few HDR images are calibrated, there is no exact match between the parameters of this method and

(a) (b) (c) (d)

Fig. 5. Results of the 'Office' image produced by the methods based on the papers of **(a)** Reinhard et al. **(b)** Mantiuk et al. **(c)** the proposed method using $r_{f_{\mathrm{WF}}}$ with parameter set to $\rho = 0.7$ **(c)** the proposed method using $r_{f_{\mathrm{S}}}$ with parameter set to $\gamma = 0.01$

(a) (b) (c) (d)

Fig. 6. Results of the 'Memorial' image produced by the methods based on the papers of **(a)** Reinhard et al. **(b)** Mantiuk et al. **(c)** the proposed method using $r_{f_{\mathrm{WF}}}$ with parameter set to $\rho = 1$ **(d** the proposed method using $r_{f_{\mathrm{S}}}$ with parameter set to $\gamma = 0.03$.



(a) (b) (c) (d)

Fig. 7. Results of the 'Desk' image produced by the methods based on the papers of **(a)** Reinhard et al. **(b)** Mantiuk et al. **(c)** the proposed method using $r_{f_{\mathrm{WF}}}$ with parameter set to $\rho = 0.5$, **(d)** the proposed method using $r_{f_{\mathrm{S}}}$ with parameter set to $\gamma = 0.2$.

the results obtained by neuroscience experiments. It would be interesting to explore a possible refinement of the new NR equation presented in this paper when the values of the HDR images correspond to real-world cd/m^2. Further research is also needed in order to deeply understand the behavior of the modified NR formula by a colorimetric point of view.

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References

[1] P. Debevec and J. Malik, "Recovering high dynamic range radiance maps from photographs," in *Proc. of the 24th annual conf. on Computer graphics*, 1997, pp. 369–378.

Fig. 8. Details of the ‘Cars’, ‘Still life’ (taken from OpenEXR source) and ‘Trees’ images in top to bottom order obtained by eq. (16) (left column) and by eq. (16) followed by the post-processing stage in the right column (see text).

[2] G. Ward, H. Rushmeier, and C. Piatko, “A visibility matching tone reproduction operator for high dynamic range scenes,” *IEEE Transactions on Visualization and Computer Graphics*, vol. 3, pp. 291–306, 1997.
[3] K. Naka and W. Rushton, “S-potentials from luminosity units in the retina of fish (cyprinidae),” *Journal of Physiology*, vol. 185, pp. 587–599, 1966.
[4] E. Reinhard, G. Ward, S. Pattanaik, and P. Debevec, *High Dynamic Range Imaging, Acquisition, Display, And Image-Based Lighting*. Morgan Kaufmann Ed., 2005.
[5] S. Stevens, “On the psychophysical law,” *Psychological Review*, vol. 64, pp. 153–181, 1957.
[6] ——, “To honor fechner and repeal his law,” *Science*, vol. 133, pp. 80–133, 1961.
[7] J. Tumblin and H. Rushmeier, “Tone reproduction for realistic images,” *IEEE Computer Graphics and Applications*, pp. 42–48, 1993.
[8] K. Chiu, M. Herf, P. Shirley, S. Swamy, C. Wang, and K. Zimmerman, “Spatially nonuniform scaling functions for high contrast images,” in *Proceedings of Graphics Interface 93*. Morgan Kaufmann, 1993, pp. 245–253.
[9] G. Ward, *A contrast-based scalefactor for luminance display*. Academic Press Professional, Inc., 1994, pp. 415–421.
[10] C. Schlick, “Quantization techniques for visualization of high dynamic range pictures,” in *Proceedings of the 5th Eurographics Workshop on Rendering Workshop*. Springer Verlag, 1994, pp. 7–20.
[11] S. Pattanaik, J. Tumblin, H. Yee, and D. Greenberg, “Time-dependent visual adaptation for fast realistic image display,” in *Proceedings of SIGGRAPH*, 2000, pp. 47–54.
[12] E. Reinhard and K. Devlin, “Dynamic range reduction inspired by photoreceptor physiology,” *IEEE Trans. on Visualization and Computer Graphics*, vol. 11(1), pp. 13–24, 2005.
[13] J. Kuang, G. Johnson, and M. Fairchild, “icam06: A refined image appearance model for hdr image rendering,” *J. Vis. Commun. Image R.*, vol. 18, pp. 406–414, 2007.
[14] J. Ferwerda, S. Pattanaik, P. Shirley, and D. Greenberg, “A model of visual adaptation for realistic image synthesis,” in *Proceedings of SIGGRAPH 96, Computer Graphics Proceedings*. Addison Wesley, 1996, pp. 249–258.
[15] M. Ashikhmin, “A tone mapping algorithm for high contrast images,” in *Eurographics Workshop on Rendering*. P. Debevec and S. Gibson Eds., 2002, pp. 1–11.
[16] E. Reinhard, M. Stark, P. Shirley, and J. Ferwerda, “Photographic tone reproduction for digital images,” *ACM Trans. on Graph.*, vol. 21, pp. 267–276, 2002.
[17] D. Tamburrino, D. Alleysson, L. Meylan, and S. Susstrunk, “Digital camera workflow for high dynamic range images using a model of retinal processing,” in *Proceedings SPIE*, vol. 6817, 2008.
[18] D. Jobson, Z. Rahman, and G. Woodell, “A multiscale Retinex for bridging the gap between color images and the human observation of scenes,” *IEEE Transactions on image processing*, vol. 6, no. 7, pp. 965–976, july 1997.
[19] G. Krawczyk, K. Myszkowski, and H. Seidel, “Lightness perception in tone reproduction for high dynamic range images,” pp. 635–645, 2005.
[20] J. Tumblin and G. Turk, “Lcis: A boundary hierarchy for detail-preserving contrast reduction,” in *SIGGRAPH: Conference Proceedings*, 1999, pp. 83–90.
[21] F. Durand and J. Dorsey, “Fast bilateral filtering for the display of high-dynamic-range images.” SIGGRAPH 2002, Proceedings of the 29th annual conference on Computer graphics and interactive techniques, 2002, pp. 257–266.
[22] R. Fattal, D. Lischinski, and M. Werman, “Gradient domain high dynamic range compression,” in *ACM Trans. Graphics*, vol. 21 (3), 2002, pp. 249–256.
[23] R. Mantiuk, K. Myszkowski, and H. Seidel, “A perceptual framework for contrast processing of high dynamic range images,” *ACM Transactions on Applied Perception (TAP)*, vol. 3 (3), pp. 286–308, 2006.
[24] A. Pardo and G. Sapiro, “Visualization of high dynamic range images,” *IEEE Transactions on Image Processing*, vol. 12, no. 6, pp. 639–647, 2003.
[25] D. Lischinski, Z. Farbman, M. Uyttendaele, and R. Szeliski, “Interactive local adjustment of tonal values,” in *SIGGRAPH ’06: ACM SIGGRAPH 2006 Papers*. New York, NY, USA: ACM, 2006, pp. 646–653.
[26] W. Pratt, *Digital Image Processing*. J. Wiley & Sons, 2007.
[27] R. Shapley and C. Enroth-Cugell, *Visual adaptation and retinal gain controls*, 1984, vol. 3, pp. 263–346.
[28] H.-C. Lee, *Introduction to Color Imaging Science*. Cambridge University Press, 2005.
[29] R. Mantiuk, R. Mantiuk, A. Tomaszewska, and W. Heidrich, “Color correction for tone mapping,” *Computer Graphics Forum (Proc. EUROGRAPHICS)*, vol. 28, no. 3, pp. 193–202, 2009.
[30] R. Kimmel, “Demosaicing: image reconstruction from color ccd samples,” *Image Processing, IEEE Transactions on*, vol. 8, no. 9, pp. 1221–1228, Sep 1999.
[31] G. Wyszecky and W. S. Stiles, *Color science: Concepts and methods, quantitative data and formulas*. John Wiley & Sons, 1982.
[32] L. Chalupa and J. E. Werner, *The visual neuroscience*. MIT Press, 2003.
[33] G. Buchsbaum, “A spatial processor model for object colour perception,” *Journal of the Franklin Institute*, vol. 310, pp. 337–350, 1980.
[34] P. Ledda, A. Chalmers, T. Troscianko, and S. H., “Evaluation of tone mapping operators using a high dynamic range display,” *Proceeding ACM Transactions on Graphics*, vol. 24, pp. 640–648, 2005.
[35] M. Ashikhmin and J. Goyal, “A reality check for tone-mapping operators,” *ACM Trans. Appl. Percept.*, vol. 3, no. 4, pp. 399–411, 2006.
[36] J. Kuang, H. Yamaguchi, C. Liu, G. M. Johnson, and M. Fairchild, “Evaluating hdr rendering algorithms,” *ACM Trans. Appl. Percept.*, vol. 4, no. 2, p. 9, 2007.
[37] T. Aydin, R. Mantiuk, K. Myszkowski, and H. Seidel, “Dynamic range independent image quality assessment,” in *ACM SIGGRAPH 2008 papers*, 2008, pp. 1–10.
[38] E. Land and J. McCann, “Lightness and Retinex theory,” *Journal of the Optical Society of America*, vol. 61, no. 1, pp. 1–11, Jan. 1971.
[39] R. Palma-Amestoy, E. Provenzi, M. Bertalmío, and V. Caselles, “A perceptually inspired variational framework for color enhancement,” *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 31, no. 3, pp. 458–474, 2009.