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of

Committee on Thesis

The undersigned, acting as a Committee of the Graduate School, have read the accompanying thesis submitted by Hugo William Wahlquist for the degree of Electrical Engineer. They approve it as a thesis meeting the requirements of the Graduate School of the University of Minnesota, and recommend that it be accepted in partial fulfillment of the requirements for the degree of Electrical Engineer.

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AN INVESTIGATION OF THE MAGNETIC FIELD OF IRON AND AIR-CORE SOLENOIDS OF VARIOUS LENGTHS EXCITED BY ALTERNATING CURRENT.

A THESIS SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL OF THE UNIVERSITY OF MINNESOTA,

by

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PREFACE.

Since the electrical industry has always and will always depend almost wholly upon electromagnetic phenomena, it seemed desirable to find out something more specific and quantitative concerning the laws of the magnetic field. Solenoids have been the center of interest of many investigators but always in the more practical than in the theoretical sense. With a view to expanding the knowledge regarding the magnetic circuit, we have carried on experiments with the hope that we might reduce to mathematical laws some ofthe phenomena of the magnetic field, or at least to discover some characteristic behaviors or reactions of that field.

After some months of investigation, it was determined to select from the enormously large field suggested by our subject, the specific problem of the investigation of the fields of solenoids, giving particular heed to the variation of reluctance and leakage of the magnetic path, which might be varied by bending the solenoid into various positions. The air path of the magnetic flux may be made to vary by bending the core into arcs with varying radii. That there is a definite relation between the length of air gap or the radius of the bent core and the variation of reluctance of the resulting magnetic circuits is a thing to be ascertained by experiment and by reducing these relations to positive mathematical expressions.

There is, of course, the fundamental and basic equation from which all mathematical expressions in the realm of magnetism must spring, namely,

$$\text{Total Flux} = 1.257 \times N \times I \times \mu \times A \div L$$

which is the equation for a simple magnetic circuit. A ring, uniformly wound, obeys the laws expressed in the above equation. When we leave ideal cases, however, and consider magnetic circuits consisting of a combination of iron and of long distances of air, then simple mathematics fail, simple variationsffail, and variables of staggering number enter into the problem. Small air gaps in the magnetic circuit have been treated with satisfaction by Ewing and others, but the effect of long air paths on the magnetic circuit involves conditions that render the problem rather difficult to attack mathematically. The mathematics involved is by far the most complex in electrical engineering, because of the manyvvariables as well as the variations among the variables. C.H.Livens, of the University of Sheffield, in his masterly volume, "The Theory of Electricity" says, after 706 pages of the most intricate maze of mathematical discussion of the theory of electricity, "Although the present exposition is essentially a mathematical one, much of the purely analytical mathematics usually associated with the subject has been omitted."

On this particular subject, namely, the variation of reluctance and changes in leakage with varying positions of the bent core, nothing has been said by Ewing, Underhill and others, who seem to have been particularly interested in the investigations of magnetic nature. The specific problem in hand seems to have been outside of the scope of their work, since they were unable to surmount the difficulties of experimental nature. Because the apparatus at our disposal was peculiarly adapted to the task in hand, we were enabled to obtain definite data on this subject.

PREPARATION.

The investigation of a magnetic field lead us finally to examine the variation of reluctance of the air path and leakage of the magnetic field of cores that were excited by alternating current and were bent into positions determined by various definite radii of circles. The pursuit of the subjectof magnetism and the examination of what others had done in that field lead us naturally into the work done by that master, Ewing. His scholarly book, "Magnetic Properties of Iron and Other Metals", influenced us at first to emulate his method of magnetic investigation by means of the ballistic galvanometer. This lead us into a maze of work consisting of eliminating vibrations, calibrating the instrument every day, determining constants,and other details of similar and tedious nature. We found that this method was particularly useless for our work, because of the fact that every time the core was bent into a new position, the permeability changed and was hopelessly inconstant. The fact has been pointed out by Ewing that iron changes its permeability veryconsiderably when it is subjected to shocks or taps. Because of the very nature of our problem, we were compelled to lift the coil, move it to a different position and take a reading. The result of disturbing the core and forcing it into a new position was to change the permeability and magnetic conditions, so that no uniform observations could be obtained.

Another method which we quickly discarded was the magnetometric. This method was unsuited to investigating closed rings and was, moreover, subject to the same limitations as was the ballistic method. We also abandoned the potentiometer method of measuring the alternating voltages of minute amounts because of troubles and inaccuracies due to phase lag and wave distortion. "After a bitter month of puttering with the galvanometer, which from the very outset was entirely inadequate and unsuited to our needs, we were encouraged to carry out the investigation by the fact that there was in the possession of the Electrical Engineering Department an electrostatic voltmeter capable of measuring as low as one volt and as high as seven volts of alternating current. This was the very thing that made the pursuit of our problem possible.

The Ayrton and Mathers electrostatic voltmeter, it was decided, was to be used in our work to record voltages in the secondary of an exploring coil that enclosed the primary windings of the solenoids. The connections used are shown on Plate VI. It was planned to send an alternating current of constant value under 110 volt pressure through the winding of the solenoid. A secondary coil of a sufficient number of turns to get a good reading on the electrostatic voltmeter would be connected in series with the meter. Any change in reluctance or any leakage would then be registered by the voltmeter, which, of course, changed in proportion to the change in flux or effective lines of force.

It was our next task to reduce this extremely sensitive instrument to definiteness of action and reliability of indication. Some difficulties were encountered which at first seemed to promise nothing but tribulations. There was, for example, great trouble experienced due to the mechanical vibrations of the instrument. The proximity of the Corliss engine in the powerhouse caused a considerable trembling of the instrument. To remedy this, we mounted the instrument on iron stakes which were driven directly into the ground. On these stakes we mounted the instrument as shown on Plate I, figure IV. No further troubles due to mechanical vibrations were experienced.

We found, in the second place, that on connecting the instrument to a coil which was in turn connected inductively to a source of alternating current, that the moving element would be strongly affected by the movement of a person walking across the room at some distance from it. The dragging of a machine across the floor directly above would cause the moving element to disappear for some thirty seconds until the accumulated charge could escape. The motion of the shaper upstairs and the running of the Fort Wayne alternator in the laboratory also rendered it inaccurate. The instrument, depending as it does, on the electrostatic difference of potential between the two vanes or surfaces, is easily and strongly influenced by the slightest electromagnetic disturbance. To eliminate the disturbances, we proceeded in a definite manner. First, we twisted the leads which were made from #30 d.c.c. copper wire. Static and electromagnetic disturbances were still in evidence, though less marked. It was found that when taking voltage readings, there was a certain point where the capacity and inductance of the circuit were in the exact proportion to produce resonance. At this point, the moving element would, of course, move violently off scale. After some study and consultation, it was found that the insertion of a resistance into the circuit would obviate the resonance difficulty. A mathematical solution showed that this should be a resistance of about a megohm and one half. A grid was made from India ink and paper which when inserted into the circuit eliminated the source of annoyance.

Accumulated charged still vitiated our readings. It was quickly found that if a free passage was afforded over which charges would leak off, the instrument was unaffected except by unusually large disturbances such as the sudden starting of a nearby induction motor or by the tremendous fieldset up by the wireless transmission set. As shown on Plate I, we also grounded the case. By means of the connection shown in Figure III on Plate I, we had no difficulty from disturbances excepting that of the wireless station. It was by the latter that we were most constantly annoyed, because the station was frequently operated. During laboratory hours, it might be stated, there was a destructive variation of voltage that made investigation impossible. However, the instrument was made to behave beautifully during certain periods, and when conditions were favorable, it rendered dependable and faithful service. Calibration curves were made from time to time to check the constancy of the meter. It remained surprisingly constant, following exactly the curve plotted on Plate II.

PROCEDURE.

The first move, after good working conditions had been obtained with the instrument, was to make a long core, which will be referred to as core #1 which could be considered to be of infinite length as far as end demagnetizing effects were concerned. A wire was annealed electrically and was cut into 200 cm. lengths and gathered into a diameter of three tenths of an inch or .762 cm. This was wound with a layer of flat copper wire .015" x .098", making a total of 810 turns. Coils of 300 turns of #28 and one of 765 turns of #36 wire were wound so that they would fit snugly over the primary core. The drawings are shown on Plate III.

After the solenoid had been connected as shown on Plate VI, a magnetization curve was obtained which enabled us to determine just where the permeability suffered the greatest change. Plate IV, data table II. Having obtained the point at whichthe iron underwent the greatest change of permeability, a piece of apparatus was laid out on which the cores could be bent with ease and precision. On a standard platform were laid out curves of radii varying from infinity to 9 cm. The curves were lined with pegs which held the core in any desired position. This bending plate is shown clearly on Plate V. On this plate was placed the core in the straight positionm i.e. where the tradius equals infinity. At each radius readings were means of the uneven flux distribution along the core when a leakage test was made. Curve on Plate VII. The readings were erratic and demonstrated that this type of core would not do for the work. Subsequent work with this core showed that either the investigation must be carried on with an air core that would be free from these magnetic evils or that a core must be built in such a manner that no effects due to twist or eddy currents would occur.

To eliminate eddy currents and twist distortion from the cores was the next problem. With this end in view some electric transformer sheet steel was obtained and cut into pieces .5" x 29.5". Eleven of these strips were used for the core of the new solenoid. Each strip was separated by an oiled linen cloth. When the solenoid #2 (descriptions of which will be found on Plate III) was assembled and wound, it was found that each lamination was insulated perfectly for 110 volts and that bending could be done smoothly and without much friction. A magnetizing curve was made and shown on Plate X, data Table VI. That eddy currents were missing will be seen from the smoothness of the leakage curve on Plate VIII, curve #I, data Table IV.

Now it was possible to get readings of leakage and of changes of voltage with different bent positions of the core. The core was first placed in a straight position with radius equal to infinity, and readings were taken placing the exploring coil at various positions along the core. These readings gave the flux densities as well as the relative voltage changes due to variations in reluctance. The curve on Plate VIII, curves #2 and #3, data Table IV, shows the variation of flux and voltage as the core was bent from a straight position to the positions of 30 and 12 cm. and to such a position that the air gap was .25". This flux distribution was also examined with a higher density of excitation. The curves are shown on Plate IX, data Table V.

Still another core, known as #3 and shown on Plate III, was made up of a single lamination of the sheet steel. This core, which was far from being free of troubles due to twist and bending, wassomewhat of an improvement in that respect and was free to a large extent from eddy currents. Readings were taken of this core with the exploring coil in the middle of radius, of curvature, of amperes, and of electrostatic voltmeter deflections. The curve on Plate XI, data Table VII, shows the variation of voltage and flux and reluctance with the variation of radius. This core gave no distortions in leakage and seemed to be singularly free of eddy current losses. The curves show quite distinctly that the reluctance increases quite sharply as the core assumes a circular position. The reluctance increases up to a radius of 17.5 cm. at which point it drops off sharply again. That it does not assume the same reluctance as it had at arradius of infinity is due partially to the fact that there is always an air gap no matter how hard the ends are presses together and partly to the fact that the stress caused by bending lowers the permeability of the iron.

To calculate the flux density and the reluctance from readings of voltage, the following formulas and processes were followed.

(1) $E = 4.44\, f\, N \phi_M 10^{-8}$ (For sine wave current)

Where f = frequency of alternation

N = number of effective turns in secondary coil.

E = effective volts.

ϕ_M = maximum flux.

Then

$$\phi_M = \frac{E \times 10^8}{4.44\, f\, N} \quad \text{and} \quad \phi = \frac{M.M.F.}{R} = \frac{.4\pi N I}{R}$$

$$\phi_M = \frac{.4\pi N_p I_M}{R} \quad \text{or Reluctance} = \frac{M.M.F.}{\phi_M}$$

$$R = \frac{.4\pi N_p I_M \times 4.44 \times f \times N_s}{E \times 10^8}$$

In all calculations of reluctance, the M.M.F. per cm. of length, of course, was kept constant. In calculating the total reluctance it was kept in mind that the total reluctance consisted of the reluctance of the external circuit as well as of the reluctace of the internal or core reluctance.

Thus

$$\phi = \frac{1.257\ N\ I}{R_i + R_e} = \frac{M.M.F.}{\Sigma_i R}$$

where

R_i = internal reluctance

R_e = external reluctance

The current was kept constant and readings of current and galvanometer deflectionswwere taken simultaneously. A carbon resistor was inserted in the circuit and proved to be an excellent means for making the slight adjustments necessary to compensate for irregular line voltage. A magnetic-vane ammeter, carefully calibrated, was used thruout the experimental work, thereby eliminating discrepancies which might accompany changing instruments.

Since the calculations for flux involve the frequency of the line, it was necessary to have some accurate means of determining it. The best instrument available for use was a vibrating reed type which indicated variations of $\frac{1}{4}$ cycle per second and by interpolation the frequency could be determined to within .5%. It was found that the line frequency remained remarkably constant, only at one occasion did it fall to 59.5 cycles per second and never went above 60 cycles per second. This is to be expected when we consider that the Minneapolis General Electric Company's lines, the source of our current, are interconnected to form a transmission system covering the entire state, and that slight variations in frequency would produce enormous surges of current in the line. Because of this great constancy, it was found unnecessary to take readings of frequency, the only precaution taken was to check it from time to time. The value 60 cycles per second was used in all computations.

It will be noted that the relation: $E = 4.44 \times f \times N \times \phi_M \times 10^{-8}$ is based on a sine wave current. To determine the exact nature of the current furnished by the M.G.E. lines, an oscillographic picture was taken in the laboratory by means of a General Electric Company three element oscillograph. The height of the curve obtained was measured and a sine curve ($y = \sin x$), was plotted on coordinate paper at such a scale that its maximum ordinate was equal to the maximum ordinate of the photographic curve. It was found that the curve on the film coincided with the calculated curve when it was superimposed on it. This being the case, we felt justified in using the above formula assuming a sine wave current.

To check the conclusions arrived at with the iron cores, an air core was made as shown in Figure 4, Plate III. This core would obviously be entirely free from eddy currents, hysteresis losses, and bending distortions. Some difficulty was experienced in getting an exploring coil of sufficient number of turns to give an appreciable reading on the electrostatic voltmeter. This was finally solved by placing around the primary a coil of 7270 turns obtained from the secondary of a high tension spark coil.

A flux distribution curve was made from the air core as shown on Plate XII, data Table VIII. The distribution, as will be seen, is even and without variations, increasing rapidly from a density of 40 gauss at the ends to a maximum of 90 gauss in the center.

Starting with this core 115 cm. in length, it was desired to examine the variation of reluctance with the variation of solenoid length. To do so, the winding and core was cut off to the desired length and then, keeping the secondary coil in the center, readings were taken of the amperes, the deflection of the voltmeter, and of the length of the solenoid. From these data calculations could be made of reluctance, using the formulae given above. Plate XIII, data Table IX shows the variation of reluctance and flux density with a constant magnetomotive force per unit length. As will be seen from the curves, the flux density and the reluctance remain constant up to 30 cm. when the reluctance increases sharply and the flux density drops off.

On Plate XIV, the total reluctance and reluctance per cm. are shown. The total reluctance, as will be seen, is a constantly increasing value, increasing as the length of the core increases except at the vicinity of the origin where it has a bend upward. This upward bend is due to the fact that the total reluctance consists of two parts, the internal and the external, and the internal being constant is having added to it the constantly increasing external reluctance. The external reluctance increases because of the fact that the area of the path is growing less and less and since the reluctance varies directly as the length and as the square of the radius ofthe area, although the length is growing less, the area is growing less still faster, causing an increase in reluctance.

On Plate XV it will be noticed that the total reluctance is plotted together with a tangent drawn to the total reluctance curve at such a point that the external reluctance can be neglected. At such a point the reluctance can be considered as due entirely to the internal path in the core, since the area of the external path is infinite. The graph of the air core reluctance is a straightbline passing thru the origin, since the reluctance willvary directly as the length, the area being constant. The total reluctance curve together with the air core reluctance curve are shown plotted to a large scale to bring out the effect of external reluctance. It will be noticed that the total reluctance curve recedes gradually from the air core reluctance curve as the latter approaches the origin. The difference between the ordinates of these two curves gives the reluctance of the external circuit.

In order to show the relation existing between the external reluctance and length of solenoid these differences were found and a curve was plotted, (Pl. 17), showing the percent external reluctance for various lengths of solenoids. From this curve it will be noticed that at a length of 22 cms. or 11 diameters, the external reluctance is only about 2 % of the total. The data obtained for this curve was subject to inaccuracies because the secondary coil extended too near the ends of the solenoid, so that there was considerable leakage flux which failed to cut all the turns. Since the external reluctance increases very rapidly when $\frac{L}{D}$ is small, it was found necessary to design a solenoid having a large diameter which would permit accurate determinations of the percent external reluctance. By increasing the diameter the total flux increased as the square and fewer secondary turns were necessary, which made a small compact coil possible. A solenoid was designed with the following demensions:

Mean diameter -------- 8.60 cms.

Length of winding ---- 17.0 cms.

Primary turns -------- 89 turns # 14 S.C.C.

For maximum accuracy the secondary coil had to be designd so that an allowable current in the primary would produce a full scale deflection on the static voltmeter.

The number of turns necessary to accomplish this were was determined as follows:

Area of core equals $\frac{\pi D^2}{4} = \frac{3.1416 \times (8.60)^2}{4} = 58.1$

Primary turns per cm. $\frac{N}{L} = \frac{89}{17} = 5.25$ turns.

Allowable current equals 12 amperes.

Then, $H = .4\pi N \times I = 1.257 \times 5.25 \times 12 = 78.8$ gauss.

And, $\phi = H \times A = 78.8 \times 58.1 = 4583$ maxwells.

$$\phi_m = \frac{\phi}{.707} = 6480 \text{ maxwells.}$$

$$\phi_m = \frac{E \times 10^8}{4.44 \times f \times N} \quad \text{or} \quad N = \frac{E \times 10^8}{4.44 \times f \times \phi_m} \qquad \begin{pmatrix} E = 7 \text{ volts} \\ f = 60 \end{pmatrix}$$

Then, $N = \frac{7 \times 10^8}{4.44 \times 60 \times 6480} = 406$ turns.

Hence for along solenoid (N) would be 406 turns. However, since the solenoid in question was short, $\frac{L}{D}$ being only 1.98, a correction had to be made before the true value for the secondary turns could be found. The curve on plate XVII gave the necessary data for determining this correction. For $\frac{L}{D}$ 1.98, the curve shows the external reluctance to be 18.5 % of the total. Then:

$$R_i = .815 \, R_t \quad \text{or} \quad R_t = \frac{R_i}{.815} = 1.23 \, R_i$$

Since O varies inversely with R and N varies inversely with O , the number of turns required would be:

$$409 \times 1.23 = 502 \text{ turns.}$$

Asecondary coil containing 500 turns of # 36 wire was made to fit snugly over the solenoid, and the primary winding was marked at centimeter intervals so that the coil could be accurately located with respect to the ends of the solenoid. A flux distribution curve for the above solenoid is shown on plate XVIII. An inspection of this curve will reveal the marked difference between it and the curve for the long solenoid shown on plate XII. The flux falls off rapidly from the center to the ends insteads of gradually as with the long solenoid.

The length of the solenoid was varied by baring the turns at intervals of one centimeter and making connections at the various points along the winding. With the secondary coil in the center of each length the lengths were varied and readings were taken for voltage and current. Plate XIX shows the total reluctance curve plotted from these readings, also the air core reluctance curve. The lengths were reduced to $\frac{L}{Dl}$ and plotted against percent external reluctance. Plate XX.

By combining this curve with the curve on plate XVII, an accurate percentage curve was obtained for cores up to 8.5 diameters, at which point the external reluctance becomes almost negligible. This means that for solenoids in which $\frac{L}{D}$ is greater than 8.5 the relation :

$$H = \frac{.4\pi N I}{L}$$

is accurate enough for all ordinary purposes.

After the investigation of solenoids in the straight position had been made, the effect of bending air core solenoids from the straight position to a closed loop was determined. A constant current was allowed to flow thru the long air core solenoid #4 as it was bent from the straight position to a closed loop. Readings of the voltage induced in a secondary coil at the center were taken for the limitihg and intermediate positions of the solenoid. Theoretically the voltage reading should have decreased slightly for large radii and then increased until the closed loop position was reached, at which point the voltage reading should be a maximum. The difference between the reading in the loop and straight positions would be proportional to the difference between the total reluctances at the points in question. It has already been pointed out that this difference for longcores is exceedingly small and therefore requires very accurate measurements if consistent readings indicative of the true nature of the changes involved are to be obtained. Take a core of twenty diameters for instance. The external reluctance of such a core represents less than one-tenth per cent of the total reluctance, which means that the voltage readings at the straight and closed positions will differ by less than one-tenth per cent. Such a small change is exceedingly difficult to detect, especially when there are changes due to the mechanical operation of bending the core, which render constant conditions impossible. Some of the difficulties encountered in performing such an experiment might be mentioned. In the first place, the solenoid must be constructed in such a manner that it can be readily bent from the straight position to the closed loop. This particular difficulty can be readily overcome by winding the solenoid on a rubber core of constant diameter. But here another difficulty presents itself. It is necessary that either the length of the core remain constant thruout the bending operation or else some accurate means of determining any change in length must be possible. Now, since the turns of wire are in contact with one another in the straight position, it means that in bending the solenoid to form a torus, the inner circumference must be equal to the length of the solenoid. The mean length of the core is therefore increased, which increase can be readily calculated if there has been no crowding together of the turns at the inner circumference of the torus. At the external circumference the turns must separate since the length has increased, and unless this separation is absolutely uniform, the readings will be vitiated. This uniformity ishard to obtain and great care must be exercised in winding the core to keep a constant tension on the wire. A slight variation in tension will not only change the separation when the core is bent, but also introduces changes in the area which further vitiate the results. When we consider that the total theoretical change is less than one-tenth per cent for a solenoid ofthe above mentioned dimensions, and that some of the supposedly constant conditions are subject to variations of one per cent or more, the difficulty of securing consistent readings is apparent.

It might be suggested that a solenoid with a small $\frac{L}{D}$ be used, thereby introducing a much larger change in reluctance. Suppose we choose a solenoid which will give a five percent change in reluctance from the straight to the closed loop position. Consulting curve B plate XX, we find in such a core that $\frac{L}{D}$ equals 4.5. Now if we usea diameter of say one inch, the length will be four and one-half inches. It would be very difficult to bend such a core to form a torus without distorting the winding. Furthermore, the size of the secondary coil wouldbe so great as to prohibit the bending of the coil. If a diameter of say five inches was used instead of one inch, the secondary coil would require only one twenty-fifth as many turns and this difficulty would be overcome. It is evident, however, that such a solenoid could not be constructed on a rubber core without introducing difficulty in bending.

With these difficulties in mind, the experiment was carried thru using solenoid #4, Plate III. Bebinning with a length of one hundred centimeters, the core was bent into a closed loop and readings were taken for voltage current and radius of curvature. The length was then reduced to eighty centimers, fifty centimeters, thirty centimeters and twenty centimeters, respectively, and similar readings were taken. The readings obtained were somewhat erratic due to the variations cited above and no changes which could be attributed to a varying reluctance were perceived. Since the twenty centimeter length represents nine diameters, only a one percent variation couldhave been expected. The distortion due to bending introduced changes in length and diameter which were far greater than this, so that the true state of conditions existing could not be interpreted from the readings of the voltmeter.

Since the bending of air core solenoids into the closed loop position failed to throw any light on the variations of reluctance which took place, it was decided to use an iron core, which, because of a very high permeability, causes most of the reluctance to fall in the external circuit. Now, as this external air circuit is shortened until the ends of the core come into contact, the total reluctance should decrease materially from the intermediate positions, especially if short solenoids are employed.

Here again the difficulty of constructing a core, capable of being bent in a circle, presented itself. Since the core was to be used on alternating current, eddy currents would prevent the use of a solid core and laminations of some sort must be used.

A core similar to No. I, consisting of a bundle of small wires, introduced, it has already been pointed out, sufficient eddy currents of a variable nature to vitiate the readings materially. Furthermore, since stresses in iron affect greatly its permeability, it was necessary to build a core which would be as free as possible from tensile and compressive stresses, when the core was bent. The construction of core No. II was such that it practically eliminated eddy currents and reduced to a small value internal stresses in the iron. The bending experiment was performed on this core with gratifying results.

Beginning with a length of seventy centimeters, the core was bent from the straight position to a closed loop. Readings were obtained for radius of curvature, volts induced in secondary coil, and current. The core was then cut off and a similar set of readings was taken, and so on, until the length of the core became such as to prevent bending into a loop. The complete set of readings is shown in TableXIV and the curves plotted therefrom on plates XXII and XXIII. Referring to Plate XXII, it will be seen how the reluctance varies for different lengths and radii. For the seventy centimeter length, the reluctance is a maximum when the radius is infinity and increases gradually until a radius of fifteen centimeters is reached, at which point it gradually decreases. The curve is nearly straight, however, which indicates that practically all reluctance exists in the iron core for this length. As the length of the core is decreased, the external reluctance becomes much greater because of the rapidly diminishing area, thereby decreasing the flux for the initial or infinity-radius points. Just before the closed loop position, the total reluctance decreases sharply and tends to approach the value for the seventy centimeter core, where the external reluctance was almost zero. If a perfect metallic contact could have been produced in the closed loop position, the reluctance per centimeter for all the lengths would have been constant and all the curves would terminate on a horizontal line representing the reluctance per centimeter for a perfect magnetic circuit. For the ten and fifteen centimeter lengths, bending was impossible and readings for only the infinity-radius position were possible. They indicate relative fluxes set up in the core for this length.

On plate XXIII, the curves are reversed, since they are plotted in relative flux values instead of reluctances. The curves on plate XXIV show the flux, total reluctance, and reluctance per centimeter for the iron core solenoid used above as the length was varied. To obtain these curves only the readings at infinity-radius were taken. The reluctance per centimeter is seen to increase gradually until thirty-five centimeters is reached, at which point it increases sharply and at five centimeters the reluctance is four times the value at seventy centimeters. This increase in reluctance is due to the decreasing area of the airpath since the iron reluctance is practically constant.

CONCLUSIONS.

The most practiaal experience gained from the investigation was the knowledge acquired concerning the nature of the core that must be used in the work. It was demonstrated quite strikingly that the long bare wire core was utterly useless for the work. The twist and strain due to the bending of the core causedviolent changes in permeability that rendered the results absolutely worthless. The process of perfecting the core was a slow one. Each core had to be assembled carefully, the primary coil had to be carefully and accurately and closely wound and the secondary coil had to be built compactly and of just the sufficient number of turns to give the greatest effect within the range of the voltmeter. Several cores were built only to be thrown into the discard after disappointing experiments had proved their wothlessness. With the completion of the insulated, oiled laminated core and the air core, the experimentscould be carried out with a great degree of satisfaction.

The adaptation of the electrostatic voltmeter capable of low readings was also the result of careful investigation and thought. The elimination of mechanical vibration and of electrostatic disturbances rendered it usable. The fact that the potential of the secondary coil could be measured withoutin the least affecting or distorting the field was one that facilitated the investigation beyond all mention.

The progress in core preparation was one of importance. The difference between the first core #1, and core #2 was highly in evidence, making the difference between failure and suceess in the investigation. In the investigation of core #2, the laminated, oil insulated separation was the solution to the problem of eliminating eddy current losses. When flux densities were plotted of this core, it was found, as will be seen by referring to Plate VIII and IX, that as the bending approached the closed loop state, the resultant curves approached the same straight line condition that a tightly welded loop would have. This is as it should be. Since a perfectly closed magnetic ring would show a straight line, it was interesting to note how the leakage curves that were derived in this case approached this same straight line as the core approached the closed circuit position. The reason that the values in Tables IV and V of the data do not assume the original value may be ascribed to the fact that there always existed an air gap in the closed circuit loop. If the core was welded, without producing eddy current losses, we could expect to observe the fact that the flux distribution would be uniform.

From the smoothness and regularity of the air core flux distribution curves on Plate III, the air core under investigation promised to be free from the magnetic evils that attended the iron cores. The variation of the reluctance with the variation of core length is shown clearly on Plate XIII. It is apparent that the reluctance is constant up to a core length of thirty centimeters, which is a length of fifteen diameters. That this is so is due to the fact that with cores of long lengths practically the entire reluctance is within the core itself, since the path of the external reluctance has an infinite area. However, this important fact is demonstrated by this investigation that as the curve for the total reluctance approaches the origin, the curve deviates from a straight line and becomes bent. Now, the bend is due to the added increase of the external reluctance whose path has been narrowed and whose resistance has been thereby increased.

The experiment of bending iron core solenoids and examining the resultant effect onthe reluctance of the circuit is beset with so many difficulties that great accuracy is very difficult to obtain. The presence of hysteresis and eddy currents introduce so many complications that the correct interpretation of readings and curves is not an easy matter. Furthermore the permeability of iron varying as it does with changing flux densities renders the simple formulae for simple homogeneous magnetic circuits useless in obtaining absolute values. The curves plotted for iron core solenoids in this experiment show relative values onlyexcept in some cases where the exact conditions were known or where certain factors were constant. In the curves for the bending of iron cores, for instance, the hysteresis loss was assumed constant, whereas this was not the case. As the core was bent from the straight position to a closed loop. the flux density increased at the ends of the coreand since the hysteresis loss varies with the flux , an error was introduced. The same was true of the eddy current loss. A considerable error was introduced in calculating the flux in iron core solenoids excited by alternating current by using the relotion: $E = Nf10^{-8}x4.44,\phi_m$ because the flux wave is not purely sinusoidal. With a constant sinusoidal voltage impressed on the solenoid the deviation from a sinusoidal in the flux wave will depend on the resistance of the winding. If this were zero a pure sinusoidal flux wave would result but since R is always present the wave is distorted. If a constant current is employed the flux wave will be flat topped at high densities and peaked for low densities. Thus if the core is well saturated and the above formula is applied, the valves for flux will be too high.

These distortions can be minimized by using a very low resistance winding on the core and impressing a constant voltage on the coil.

With the air cores the results were highly satisfactory because distortions were absent. There were no hysteresis or eddy current losses to vitiate the readings and the permeability remained constant. It was only after a great deal of experimental work had been carried on, however, that the best type of core and winding was arrived at. Further investigation on the subject of reluctance of solenoids with a view toward developing mathematical relations should, I believe, be carried out with air cores which can be bent into a torus. The design of such cores will depend to a large extent on whether the investigation is to be made with a ballistic galvanometer, or by the use of a static voltmeter with the solenoid excited by alternating current. The latter is, I believe, by far the better method since it eliminates a large number of variable factors, simplifies calculations, and once adjusted and calibrated, it gives consistent and accurate readings. There are certain precautions to be taken, however, in using a static voltmeter which are necessary if consistent results are to be obtained. The instrument must be made free from static disturbances by grounding the case and one terminal as shown on plate I. The leads to be instrument should consist of small wire, #30 or smaller, to prevent vibration, and they should be twisted to avoid inductive disturbances. A high resistance, (one megohm) should be inserted in serieswwith the instrument when used on alternating current to prevent the occurrence of resonance. The insertion of this resistance does not affect materially the deflection, but the instrument should be calibrated with the resistance in the circuit. The use of a mounting similar to that on plate "I" will reduce vibrations to a minimum. The use of an air drying agent in the case was found unnecessary. The calibration of the instrument was checked at intervals with the curve on plate II and was found to check with great accuracy. The calibration was accomplished by connecting a low voltage A.C. voltmeter in parallel with static voltmeter and source of voltage. The readings of the static voltmeter were taken with the A.C. meter continually in the circuit. One of the troubles experienced in using the static voltmeter was a varying zero reading. It was found advisable to check the zero reading frequently.

To carry out the experiment of bending an air core solenoid, using a static voltmeter reading up to seven volts, the solenoid should be carefully designed. In the first place, the diameter must be great enough so that a compact secondary coil is possible. A diameter of five centimeters is about right. In order that the ampere turns per centimeter of length shall be as great as possible, thereby decreasing the secondary turns necessary, a ribbon copper wire should be used, wound on edge. The use of a ribbon wire will greatly facilitate the bending operation and will prevent the turns from crowding over one another at the inner side of the torus. The wire can be wound in a lathe on a wodden form turned to size.

For a five centimeter core, the length should be about twenty-five centimeters or $\frac{L}{D}$ should be about five. This will give a variation of about 4.5 percent in the voltage reading from the straight to the closed loop position. After the coil is wound it should be slipped off the form and the outside turns on each end should be tied by a cord thru the center of the solenoid, so that all the turns are in contact. The solenoid may now be bent to form a closed loop with an internal circumference equal to the length of the winding in the straight position. The cord will prevent the length from changing. The secondary coil should be wound on a narrow fibre spool which just slips over the solenoid.

The wire used in the primary coil should carry about ten amperes continuously. A ribbon, .1" x .015" will answer nicely. Such a wire will allow twenty-five turns per centimeter. With ten amperes, H equals:

$$.4 \times \pi \times 25 \times 10 = 314.2 \text{ gauss.}$$

Using a five centimeter core :

$$\phi \text{ equals } H \times A \text{ equals } \frac{314.2 \times \pi \times 25}{4} \text{ equals}$$

6170 maxwells.

Since the flux wave is sinusoidal,

ϕ_M: equals 6170 x 1.413 equals 8720 maxwells.

$E_{R.M.S.}$ equals $4.44 \times f \times N \times \phi_M \times 10$

N equals $E \times 10^8 \div 4.44 \times f \times \phi_M = 7 \times 10^8 \div 4.44 \times 60 \times$ 8720 equals 301.5 turns.

For full scale deflection on the static voltmeter, a twenty-five centimeter solenoid with the above winding on the primary would require about three hundred turns of, say, #36 wire on the secondary. Such a solenoid, will, I believe, give excellent results if used with the static voltmeter mentioned above. With a length of twenty-five centimeters, the voltage will vary about 4.5 percent and if the length is cut to fifteen centimeters, a 9 percent variation will result. With this length, the core can easily be bent to form a torus, and since eddy currents and hysteresis will be eliminated, highly satisfactory results should be obtained.

This investigation of iron and air-core solenoids proves, I believe, that the use of alternating current in conjunction with a static-voltmeter is an excellent method of securing data whereby definite conclusions can be arrived at. Magnetic investigations have hitherto been caried out almost entirely using ballistic galvanometers, and although this method has its advantages, it is my firm belief that where great accuracy is desired the static voltmeter method is superior. The fact that such an instrument consumes practically no current, and thereby prevents distortions and gives a true indication of the voltage induced in an exploring coil, gives it a tremendous advantage over other methods. Two months of experimentation with a billistic galvanometer proved that it was entirely unfitted for bending tests using iron cores. When a long iron solenoid was excited with direct current, it was found that on opening the circuit fully ninety percent of the flux failed to collapse. The slightest disturbance, however, caused the flux to break down in varying amounts. It was necessary, therefore to remove all residul flux after every reading, a procedure which took considerable time. With alternating current no such precautions were necessary since the flux was forced regularly thru a cycle of values.

In short then, the investigation showed that the external reluctance of long solenoids is practically zero because of the large area of the return path; that in air core solenoids, $\frac{L}{D}$ should be at least nine if the error in using the relation: $H = .4 \times \pi \times N \times I$, is to be less than one percent; that errors in spacing and diameter of turns should be carefully guarded against; that resistance in the circuit distorts the flux wave of iron core solenoids, causing it to be flat-topped for high densities and peaked for low densities; that by the use of simple formulae for the magnetic circuit only average values can be accurately determined when there is iron in the circuit; that the permeability of iron varies between wide limits when subjected to tensile and compressive stresses; that eddy currents in laminated cores cause irregularities in the flux distribution unless good insulation between laminations is provided; that the flux in long iron core solenoids fails to collapse when the circuit is opened; but breaks down when the core is disturbed; that the reluctance of long solenoids with iron cores increases when the core is bent and regains its former value when a closed loop is formed with perfect metallic contact; that the alternating current, in conjunction with a static voltmeter, is a better method for the investigation of the magnetic fields of iron core solenoids than the ballistic galvanometer method, if precautions are taken to avoid static and inductive disturbances. Further, the investigation showed the numerous difficulties which present themselves in problems of research, and that approximate results are readily obtained; but when accuracies of one-tenth percent or less are attempted, the difficulties encountered are so great that extreme care must be exercised, and a full and complete knowledge of the factors and variations which tend to vitiate the readings must be had, if consistent and reliable results are to be obtained.

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Zeleny and Erickson

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Shepardson.

"THE THEORY OF ELECTRICITY" -----Livens.

"SOLENOIDS"------------------Underhill.

"TESTING OF ELECTRO-MAGNETIC MACHINERY"

VOL.I and II. Swenson and Frankenfield.

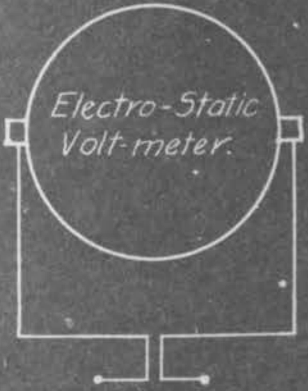


Fig. 1.

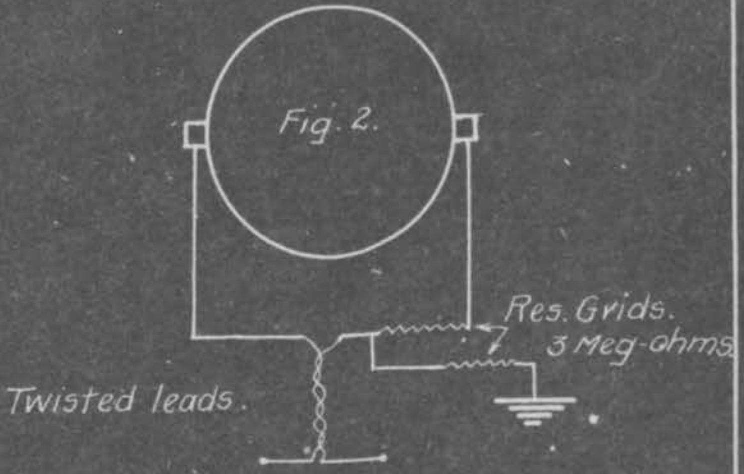


Fig. 2.

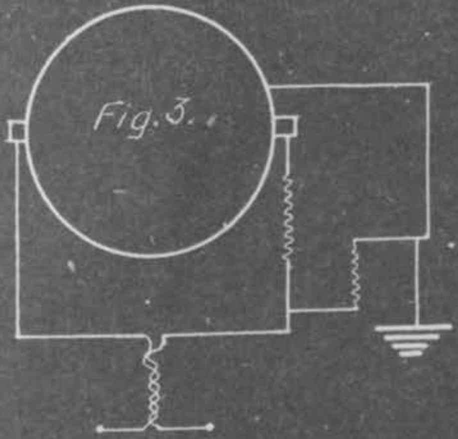


Fig. 3.

Fig 4. Method of mounting E.S.V. to eliminate vibration.

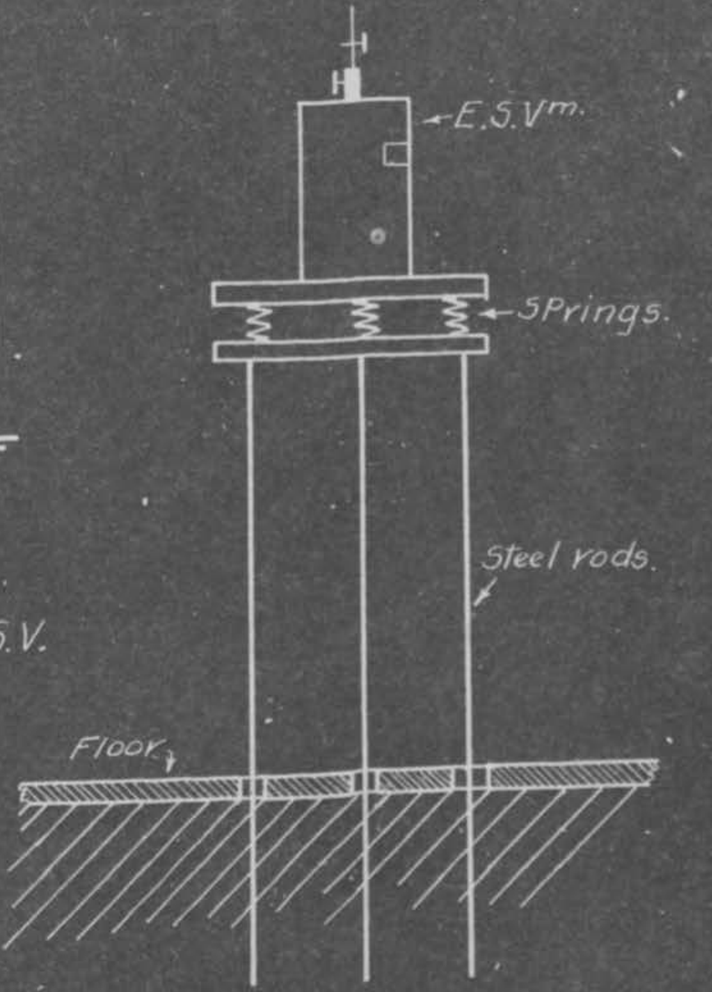
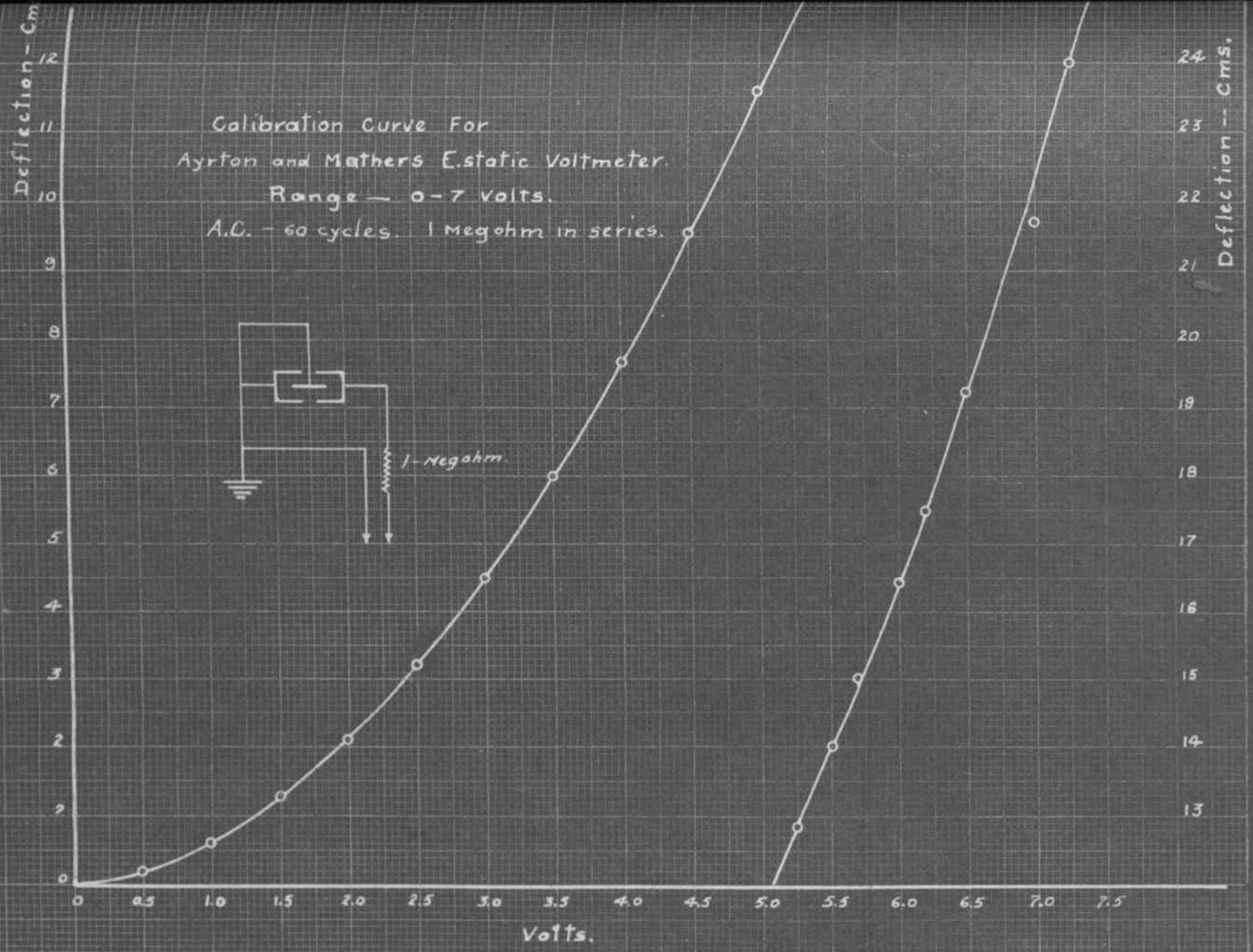


Fig. 4.

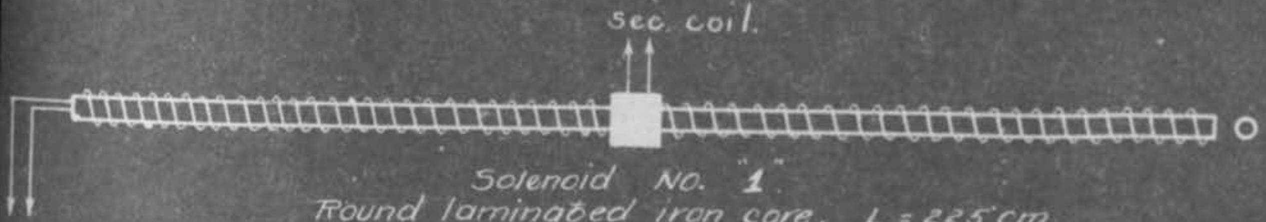
PLATE I.

PLATE II.

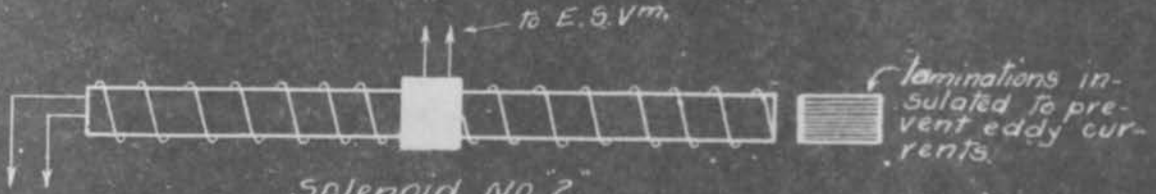


Calibration Curve For
Ayrton and Mathers E.static Voltmeter.
Range — 0-7 volts.
A.C. - 60 cycles. 1 Megohm in series.

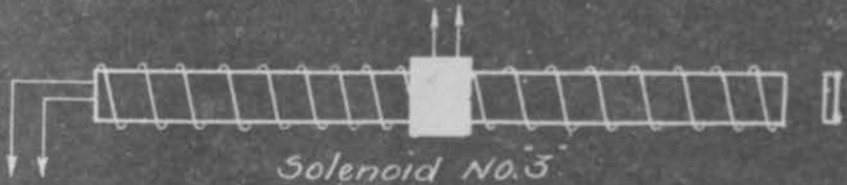
CORE and SOLENOID DATA.



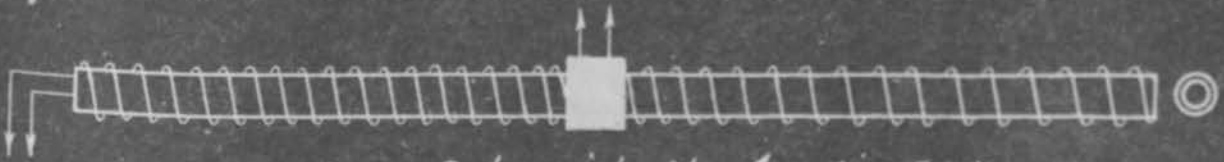
Solenoid NO. "1"
Round laminated iron core. L = 225 cm.
area of core → .1755 Sq. cm.
Primary turns → 810. Sec. turns → 300.



Solenoid NO. "2."
Rectangular core: 11 laminations
$\frac{1}{2}$" x .02" x 29.5" each. L = 29.5" = 75 cm.
area of core → .71 sq. cm.
Primary turns → 251. Sec. Turns → 580.



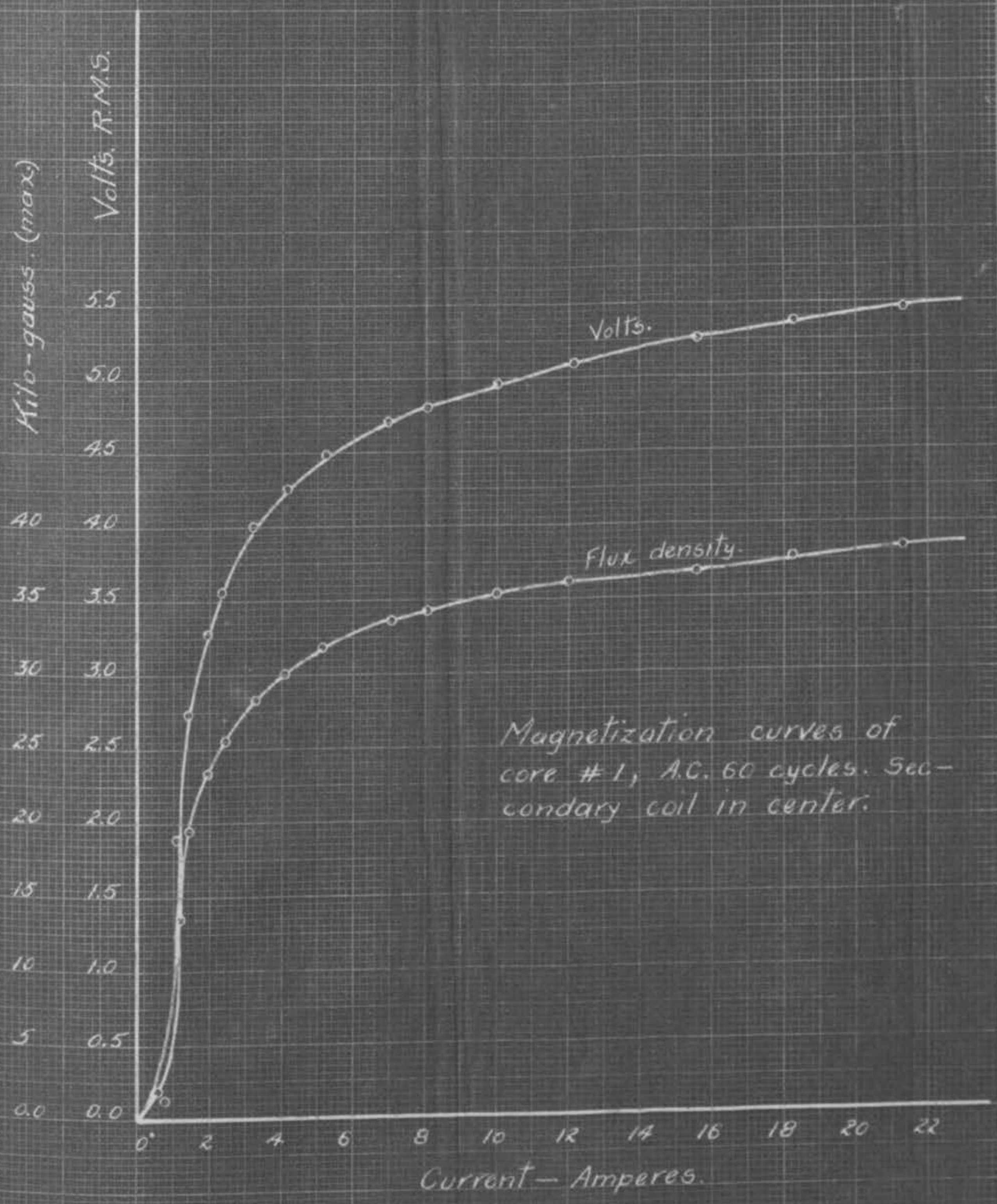
Solenoid NO. "3"
Single strip core. L = 75 cm.
Area of core → .0645 sq. cm.
Primary turns → 250. Sec. T. = 2550.



Solenoid No. 4. Air core.
length → 115 cm. Diam. core = 2.22
Primary turns → 409. Sec. T. = 7,270 #36.

All the above cores are wound with strap copper: .
Turns per inch = 9 = 3.54 Per. cm. .097" X .015" = 1855 C.M.

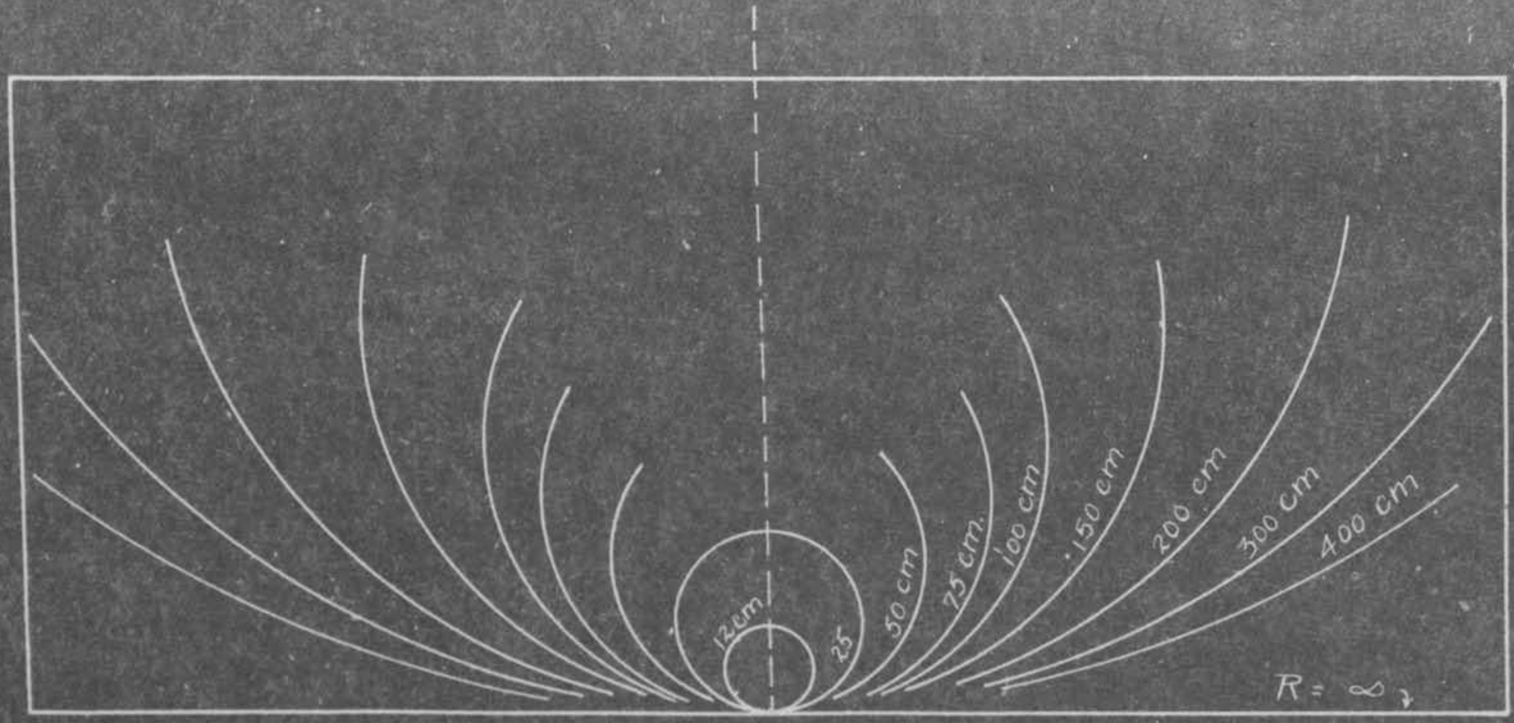
PLATE III.



Magnetization curves of core #1, A.C. 60 cycles. Secondary coil in center.

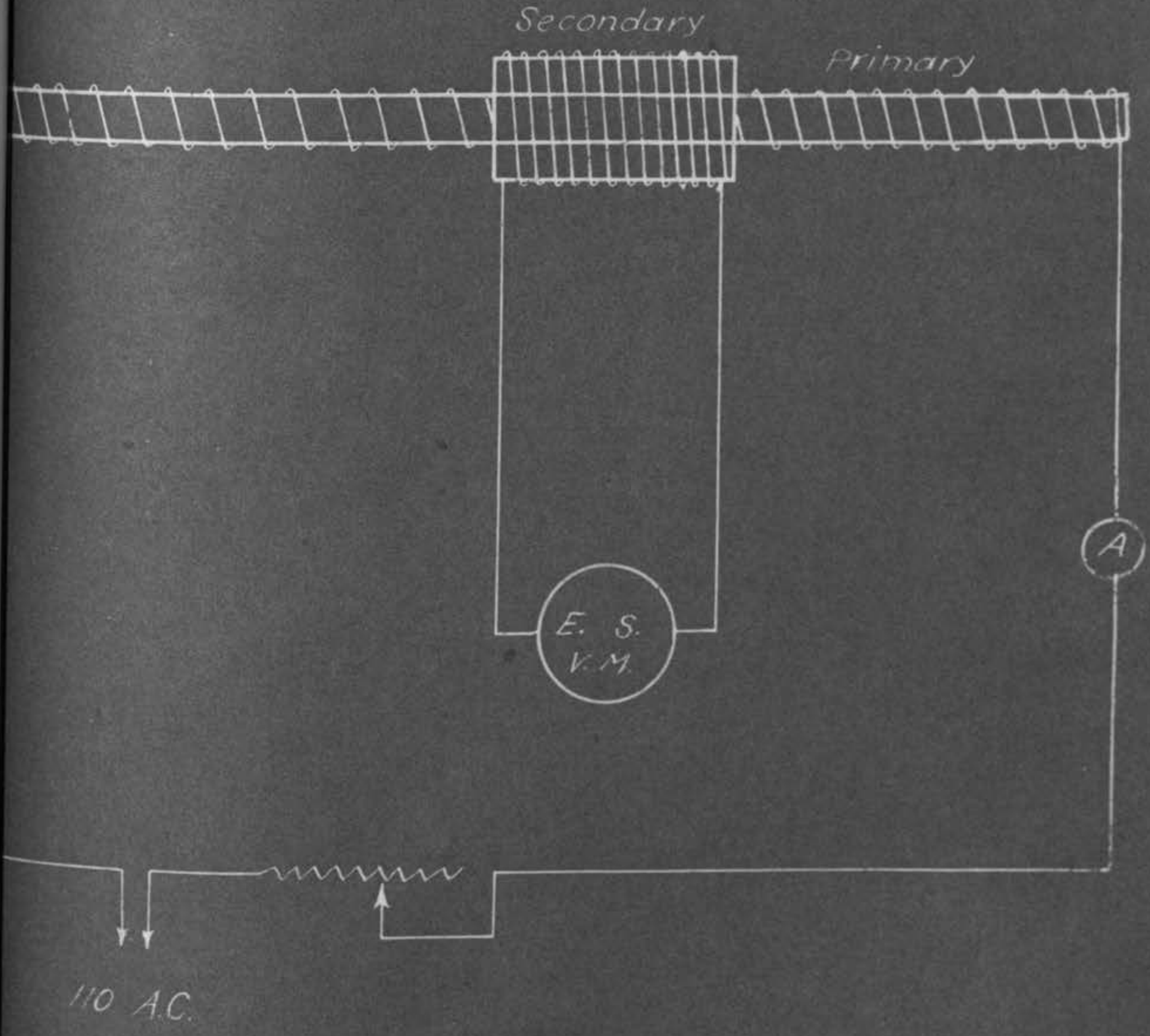
PLATE IV.

PLATE V.



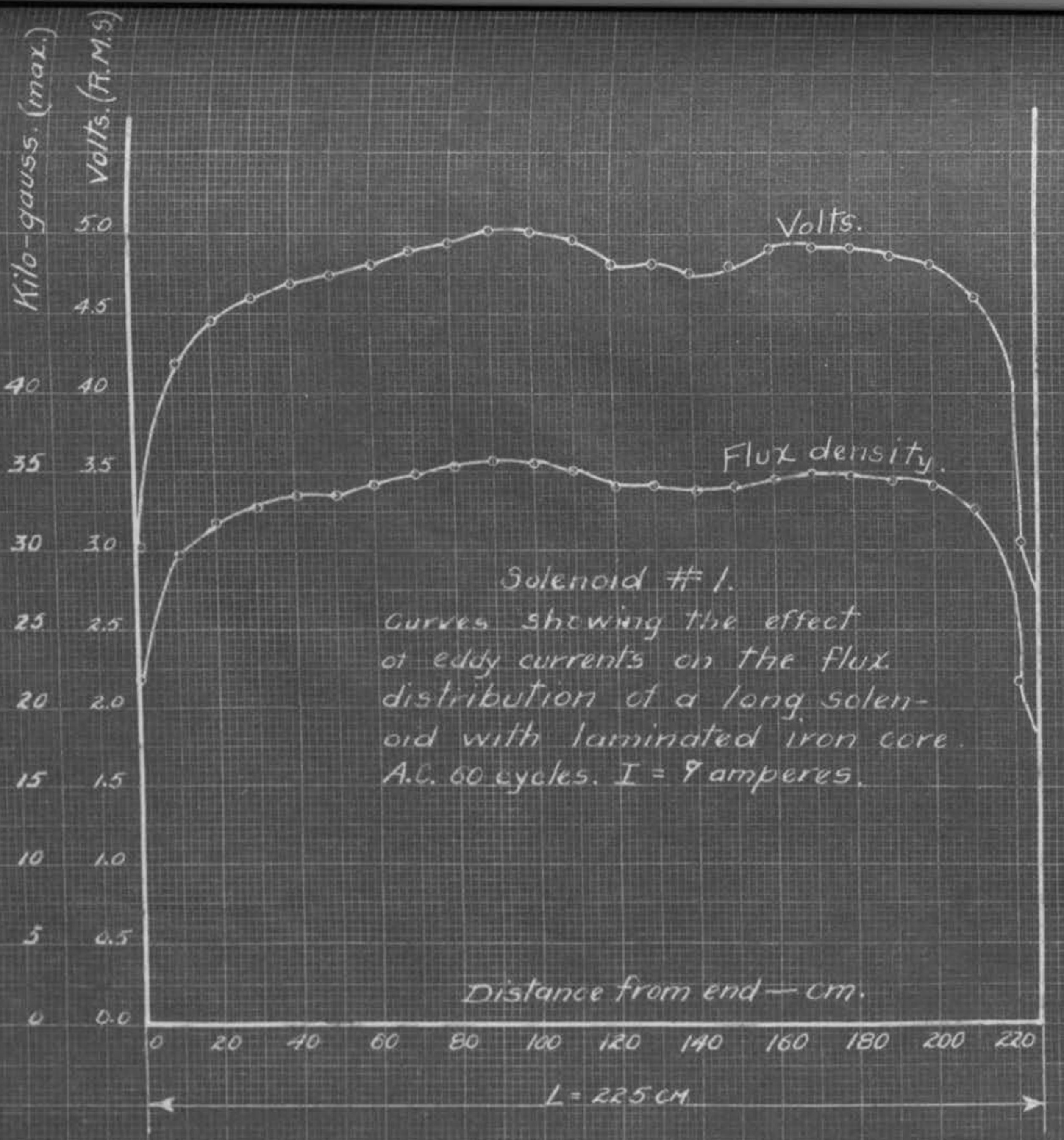
Bending Plate.

Used to vary the radius of curvature
of solenoids.



Connections of core
and voltmeter.

PLATE VI



Solenoid #1.
Curves showing the effect of eddy currents on the flux distribution of a long solenoid with laminated iron core. A.C. 60 cycles. I = 9 amperes.

PLATE VII.

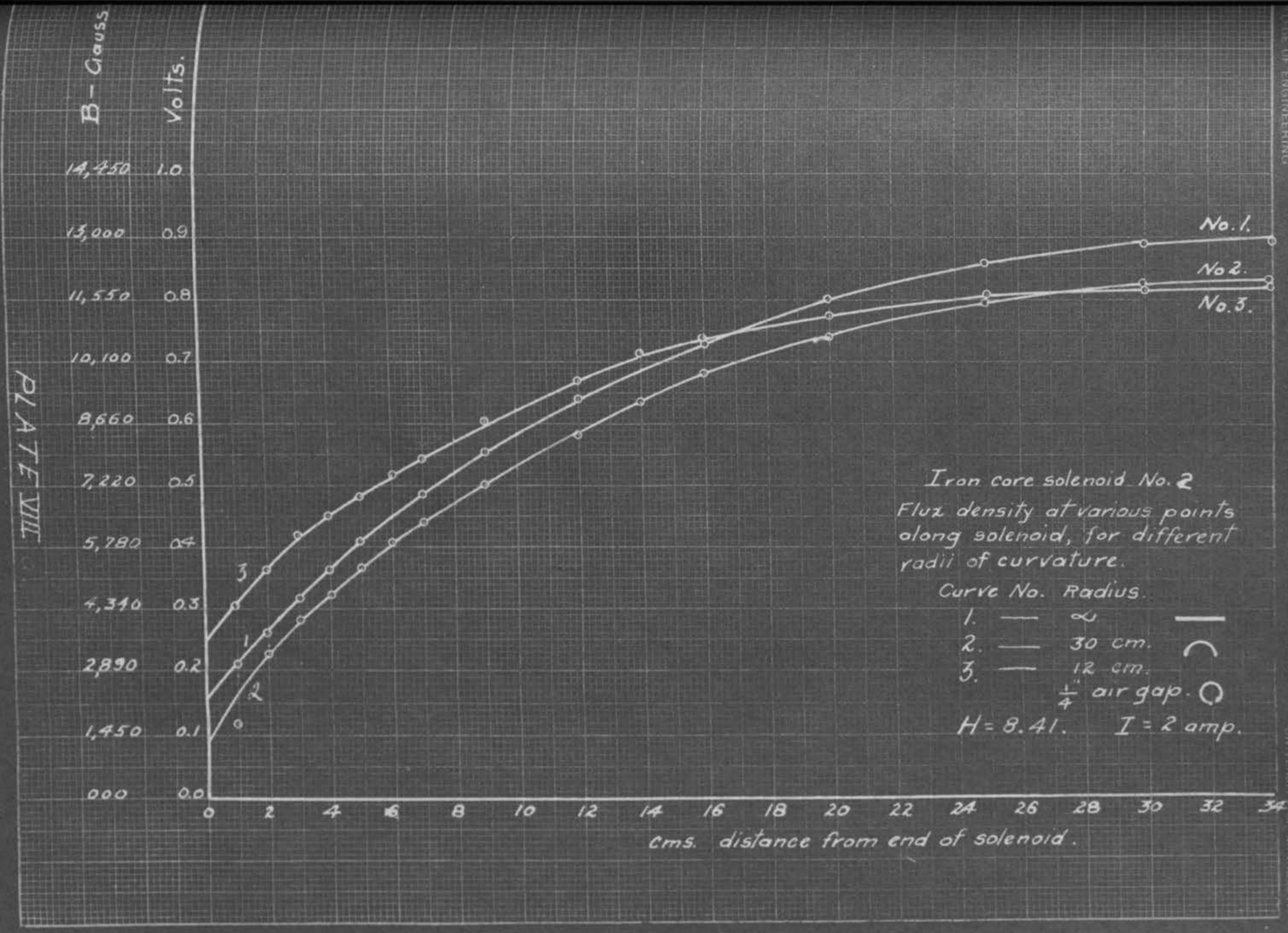
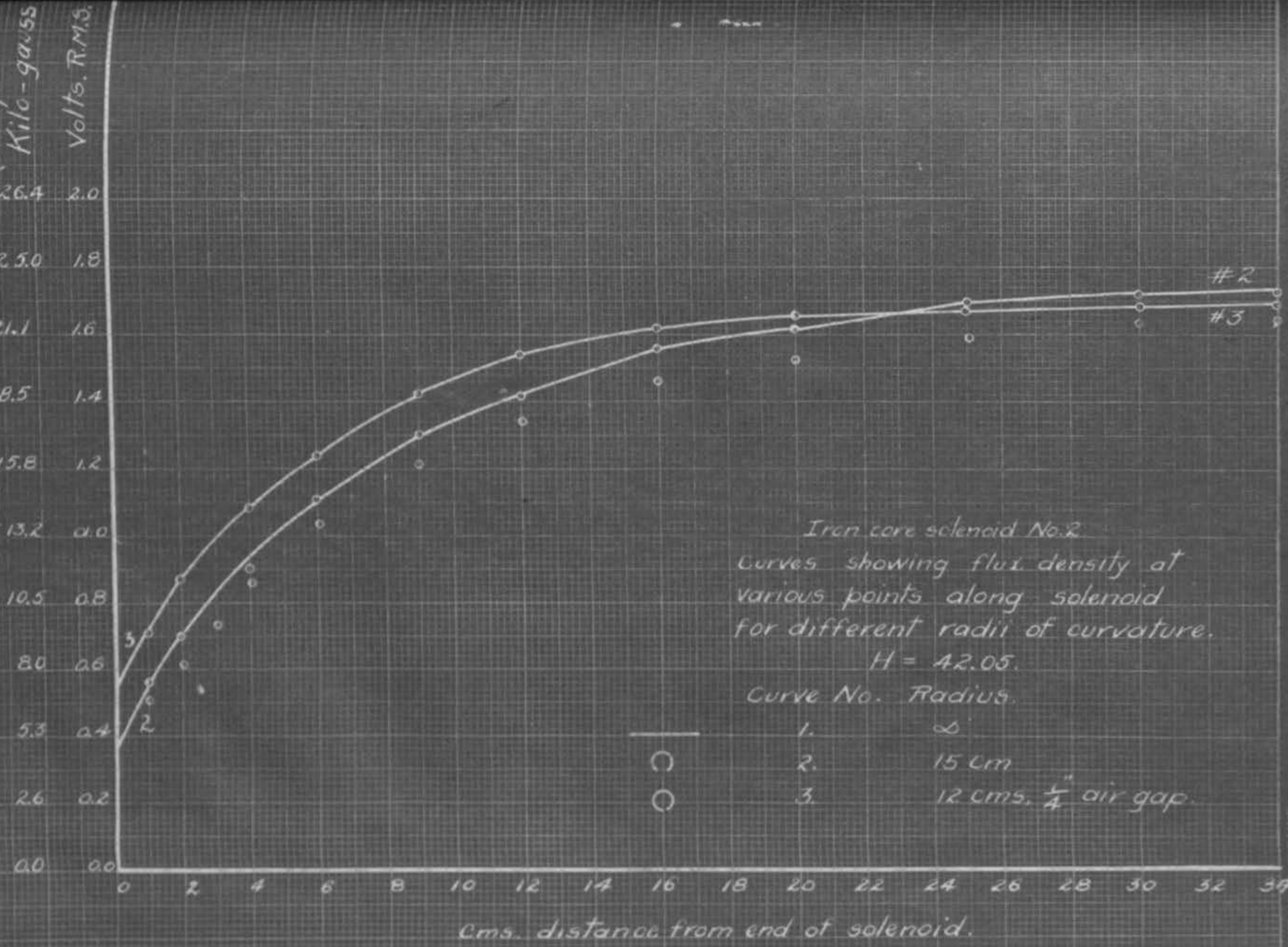


PLATE VIII

Iron core solenoid No. 2

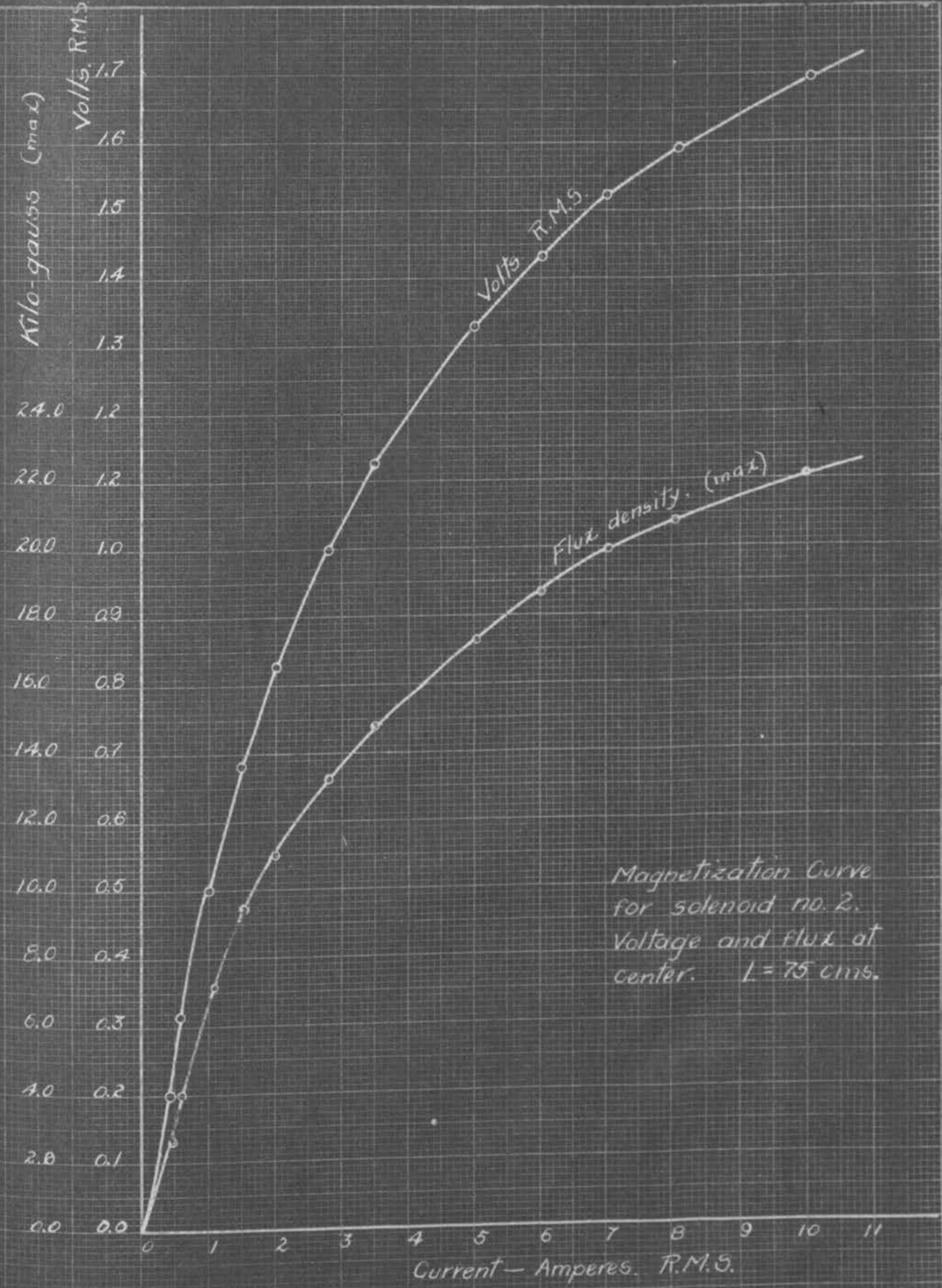
Flux density at various points along solenoid, for different radii of curvature.

Plate. IX



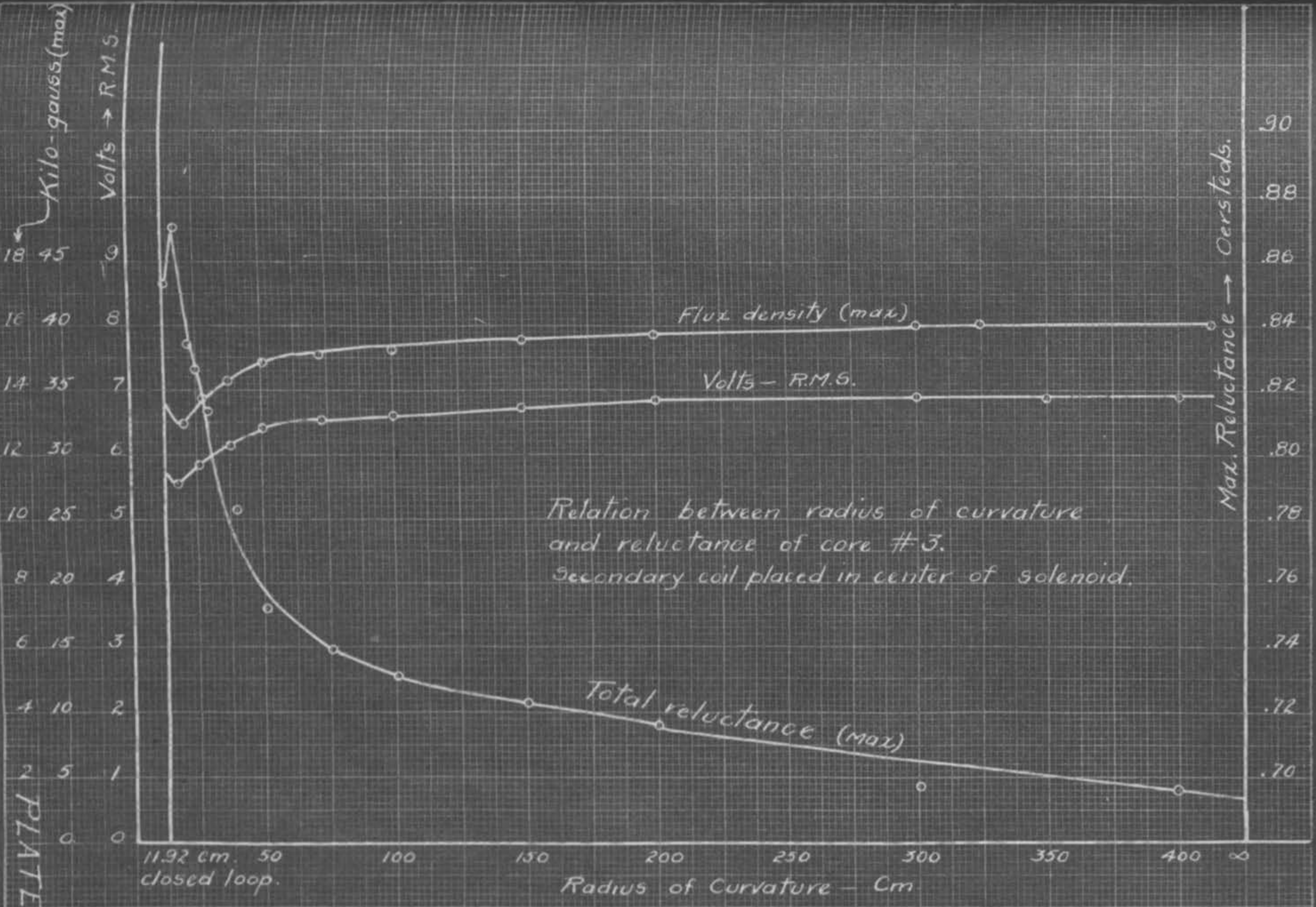
Iron core solenoid No.2
Curves showing flux density at various points along solenoid for different radii of curvature.
H = 42.05.

Curve No.	Radius.
1.	∞
2.	15 Cm
3.	12 cms. $\frac{1}{4}''$ air gap.



Magnetization Curve
for solenoid no. 2.
Voltage and flux at
center. L = 75 cms.

PLATE X



Relation between radius of curvature and reluctance of core #3.
Secondary coil placed in center of solenoid.

PLATE XI.

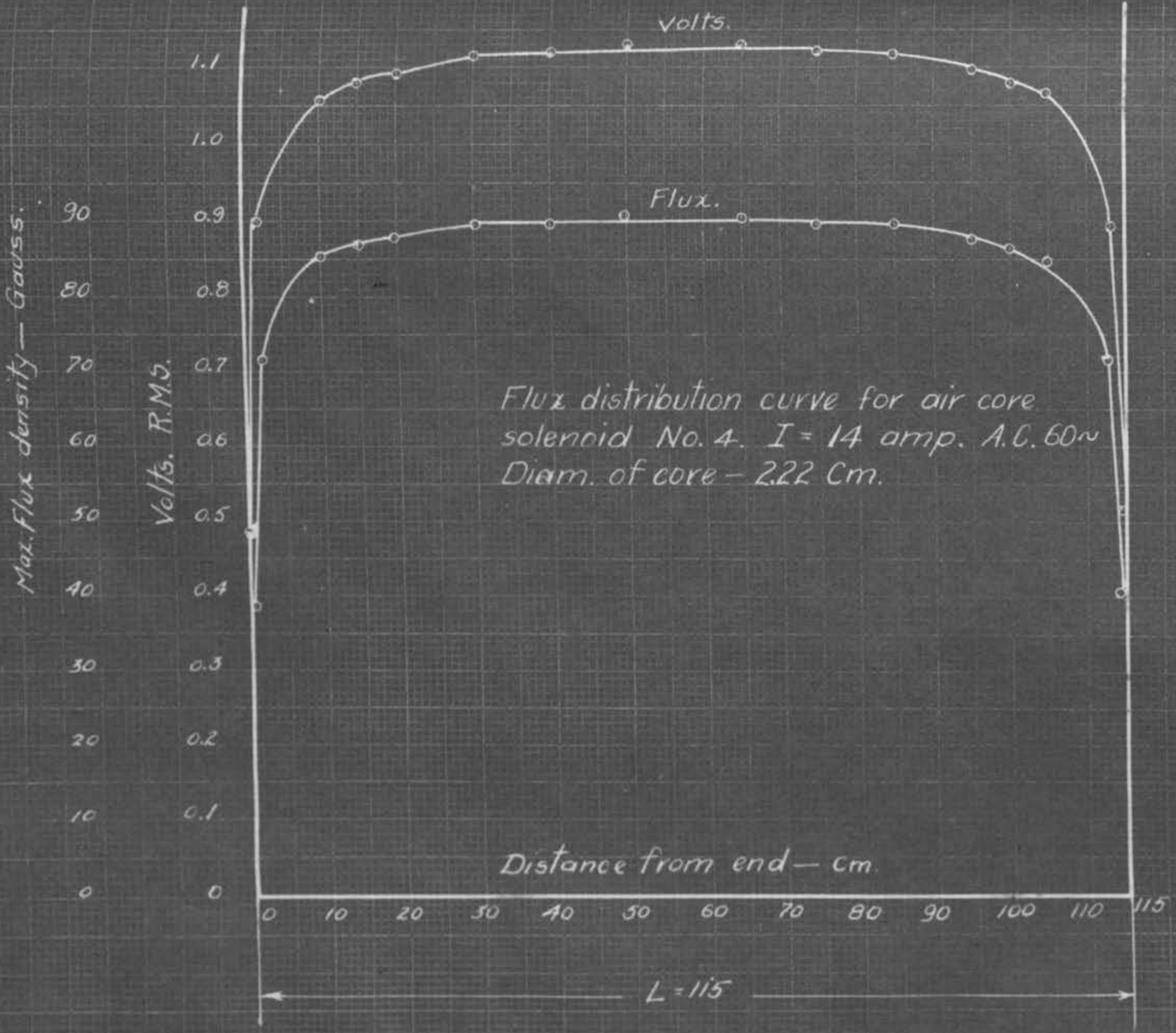
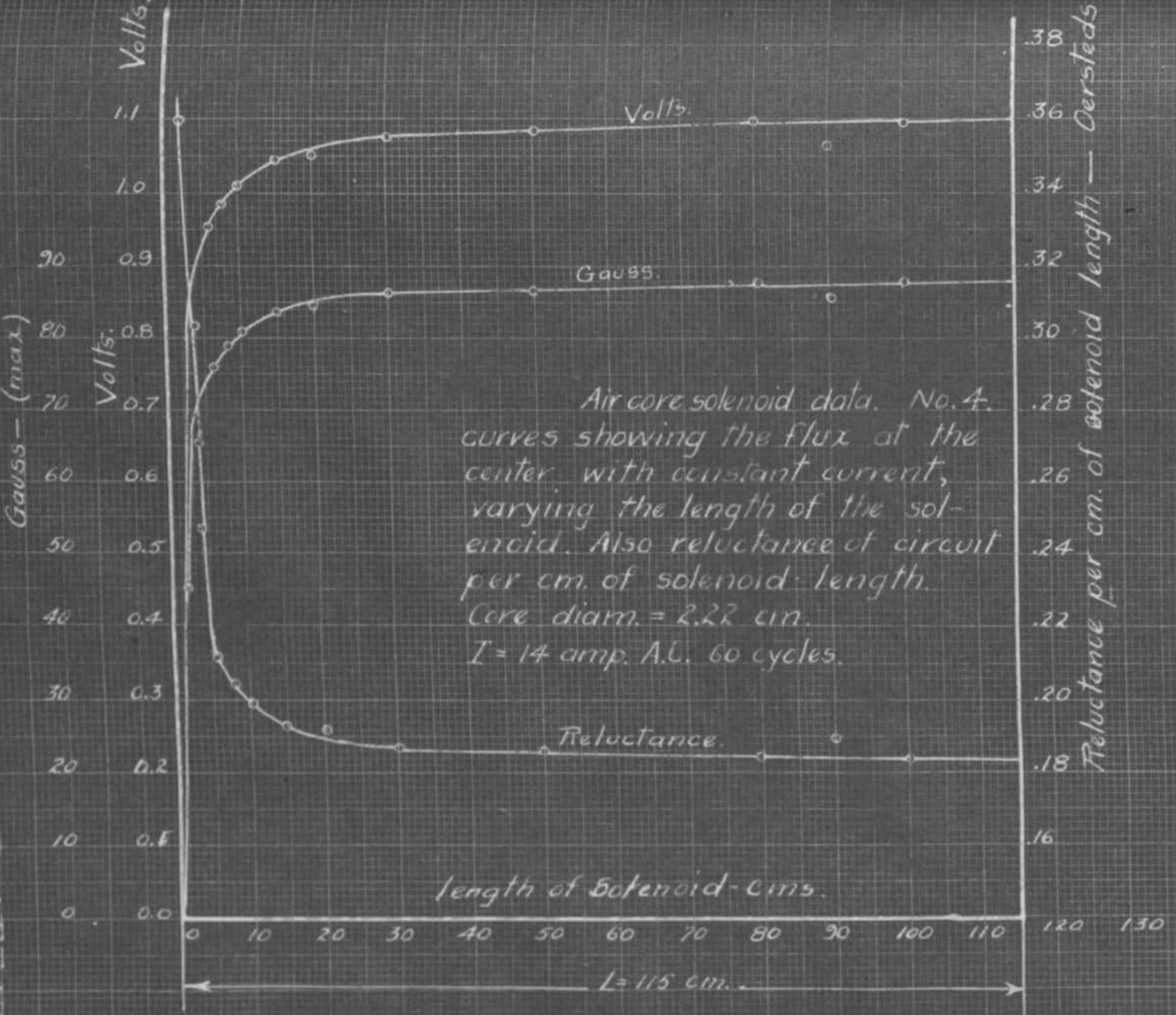

Volts.
Flux.
Flux distribution curve for air core
solenoid No. 4. I = 14 amp. A.C. 60~
Diam. of core - 2.22 Cm.
Max. Flux density — Gauss.
90
80
70
60
50
40
30
20
10
0
Volts. R.M.S.
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Distance from end — cm.
0
10
20
30
40
50
60
70
80
90
100
110
115
L = 115

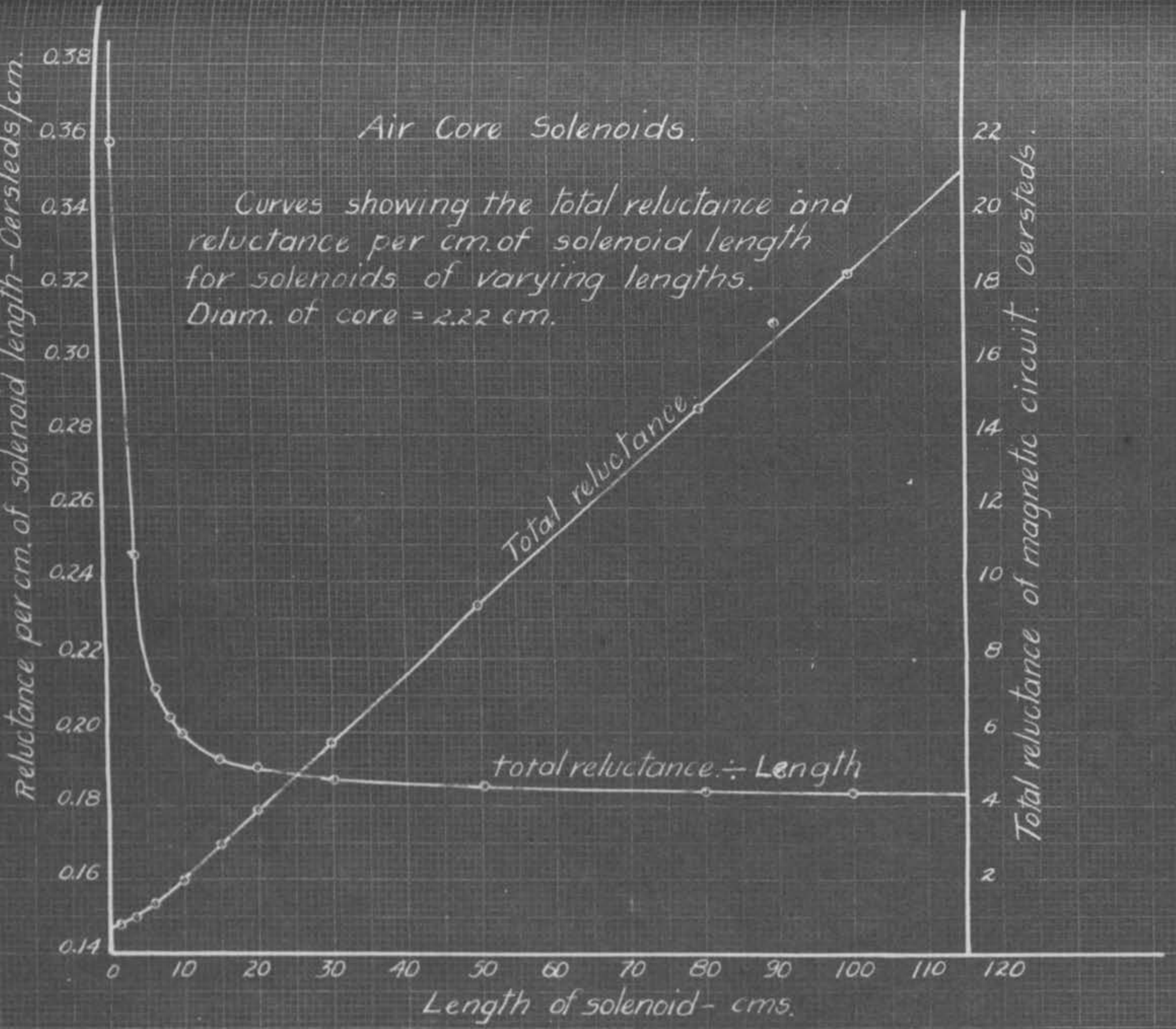
PLATE XII



Air core solenoid data. No. 4. curves showing the flux at the center with constant current, varying the length of the solenoid. Also reluctance of circuit per cm. of solenoid length.
Core diam. = 2.22 cm.
I = 14 amp. A.C. 60 cycles.

PLATE XIII

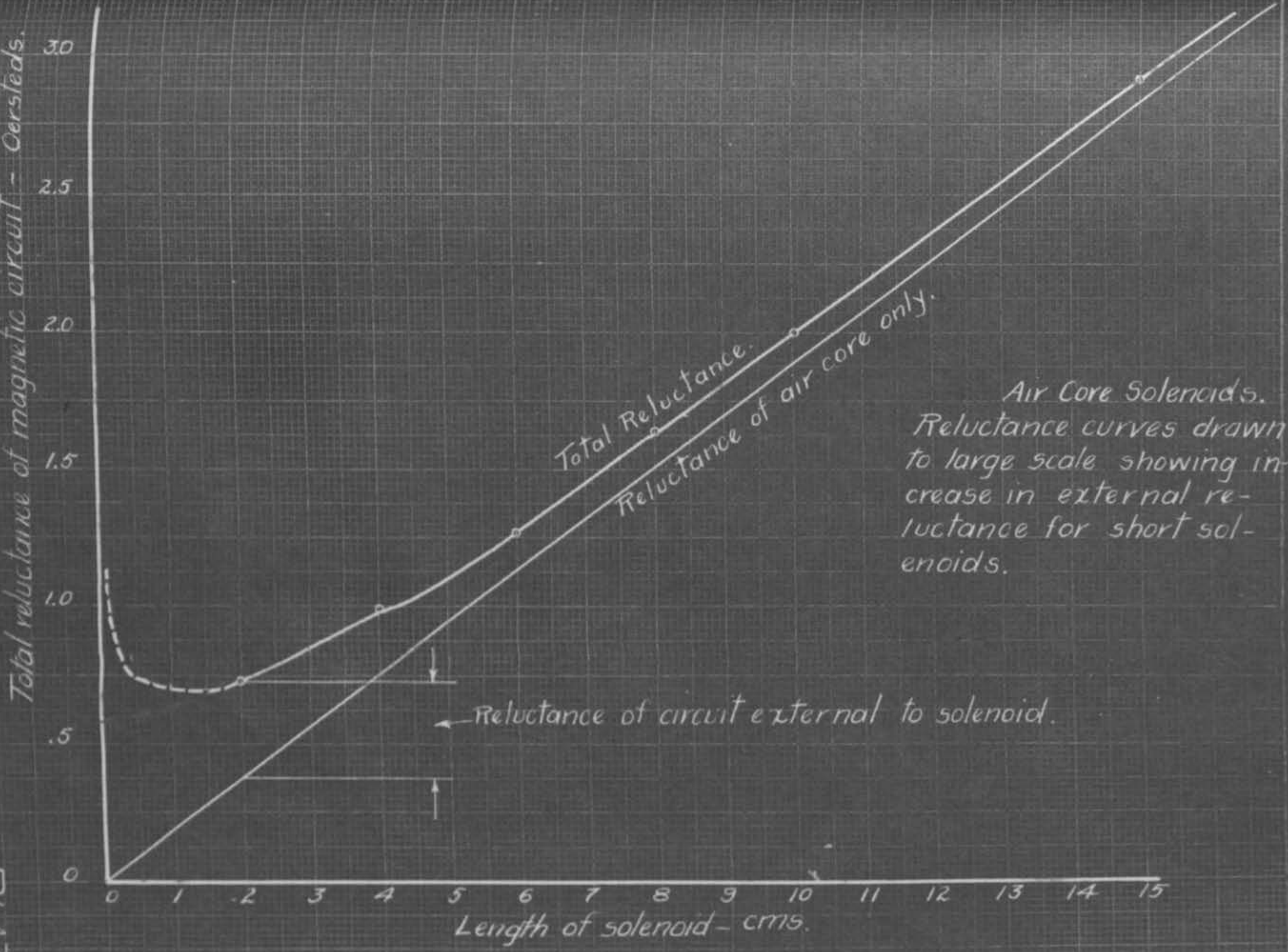
PLATE XIV.



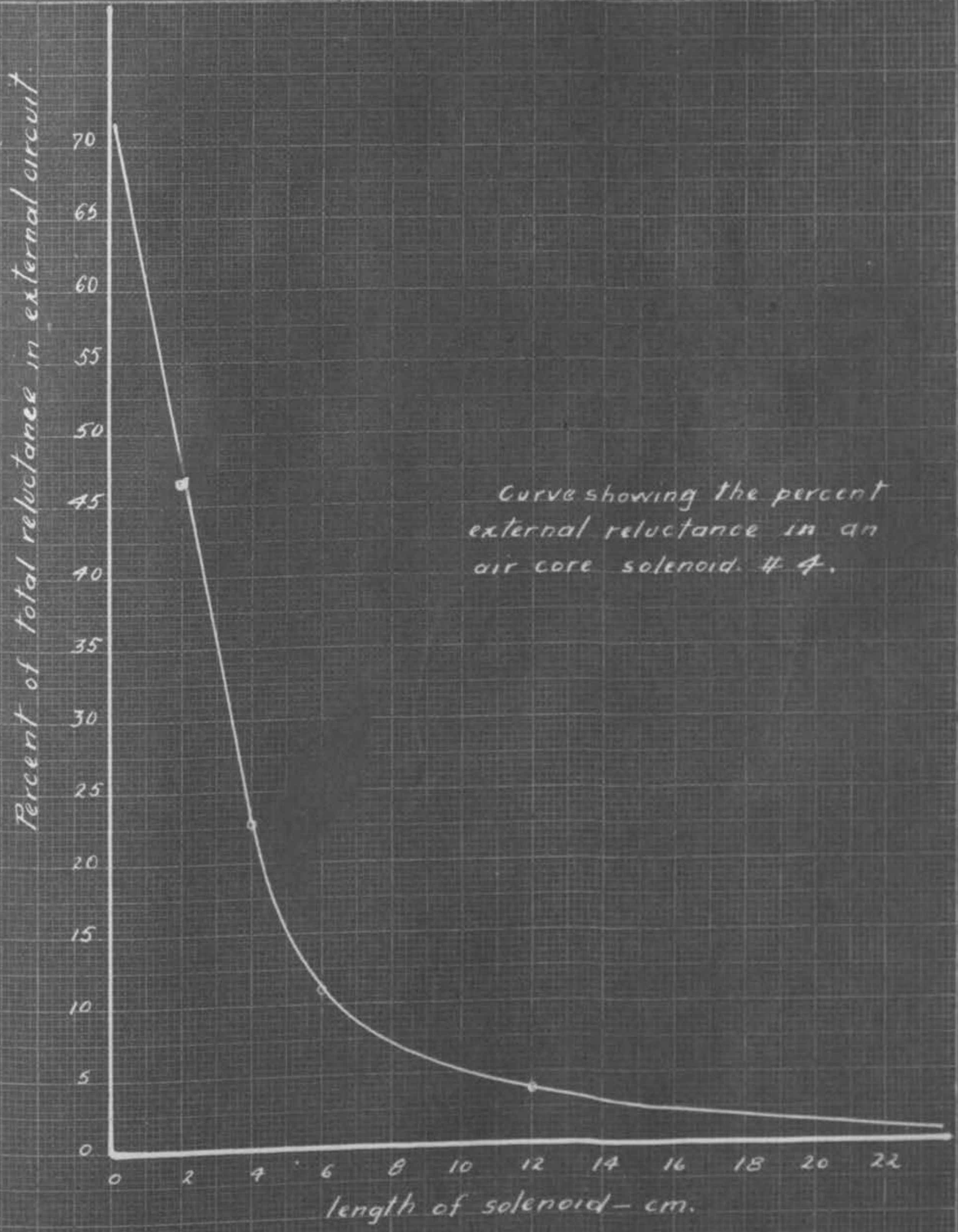
Air Core Solenoids.

Curves showing the total reluctance and reluctance per cm. of solenoid length for solenoids of varying lengths.
Diam. of core = 2.22 cm.

PLATE XV

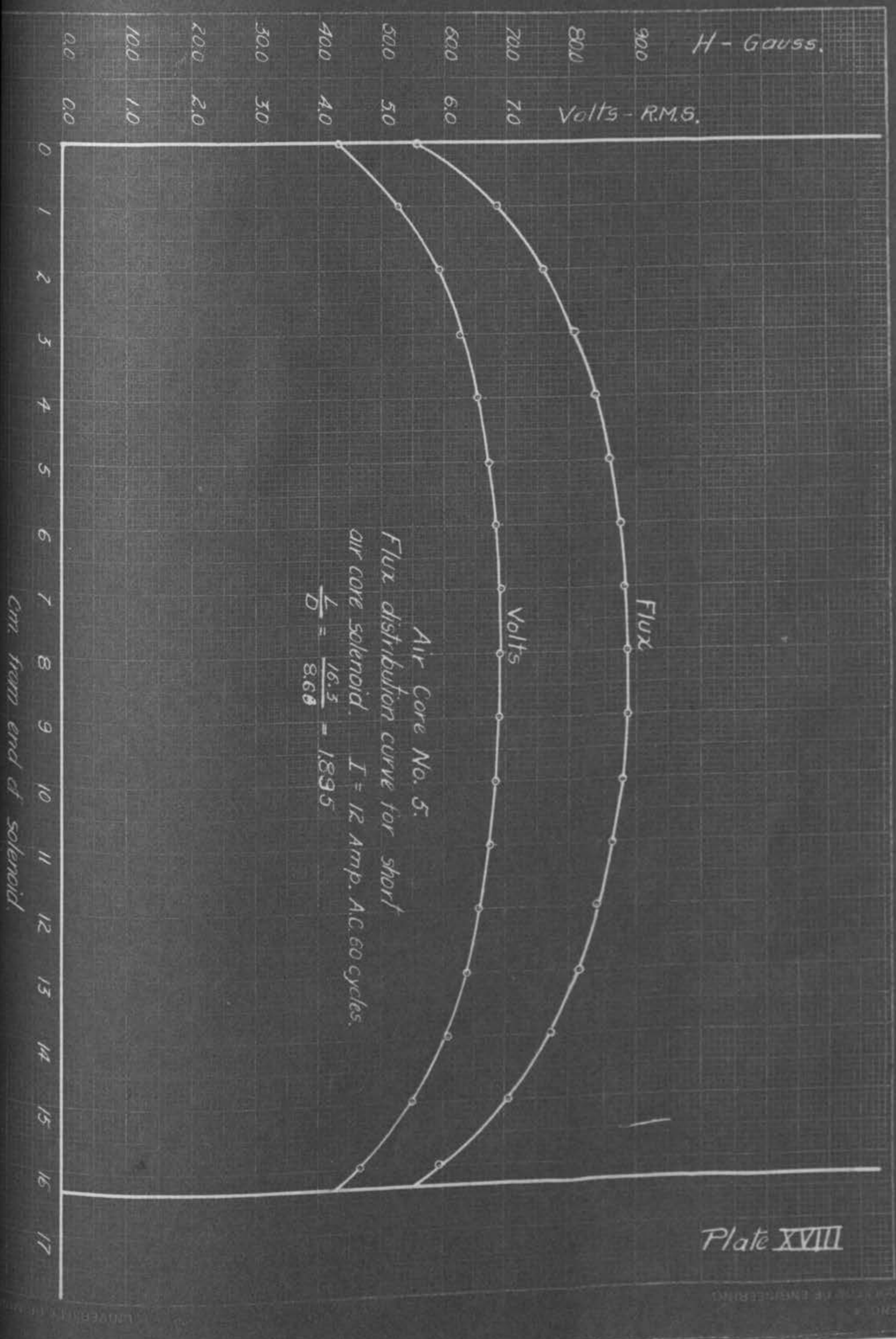


Air Core Solenoids.
Reluctance curves drawn to large scale showing increase in external reluctance for short solenoids.



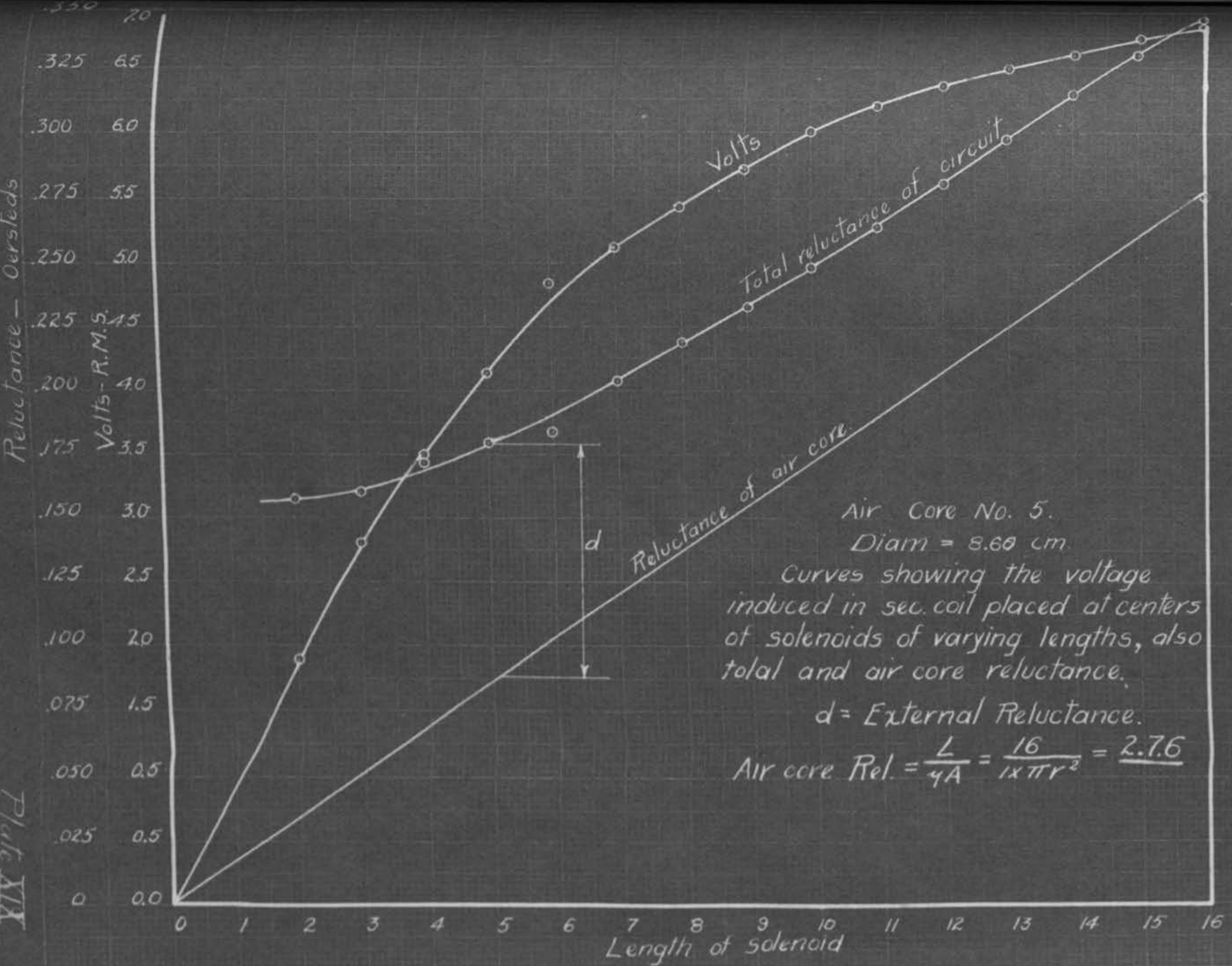
Curve showing the percent external reluctance in an air core solenoid #4.

Plate XVIII



Air Core No. 5.
Flux distribution curve for short
air core solenoid. $I = 12$ Amp. A.C. 60 cycles.
$\frac{L}{D} = \frac{16.3}{8.68} = 1.895$

Plate XVIII



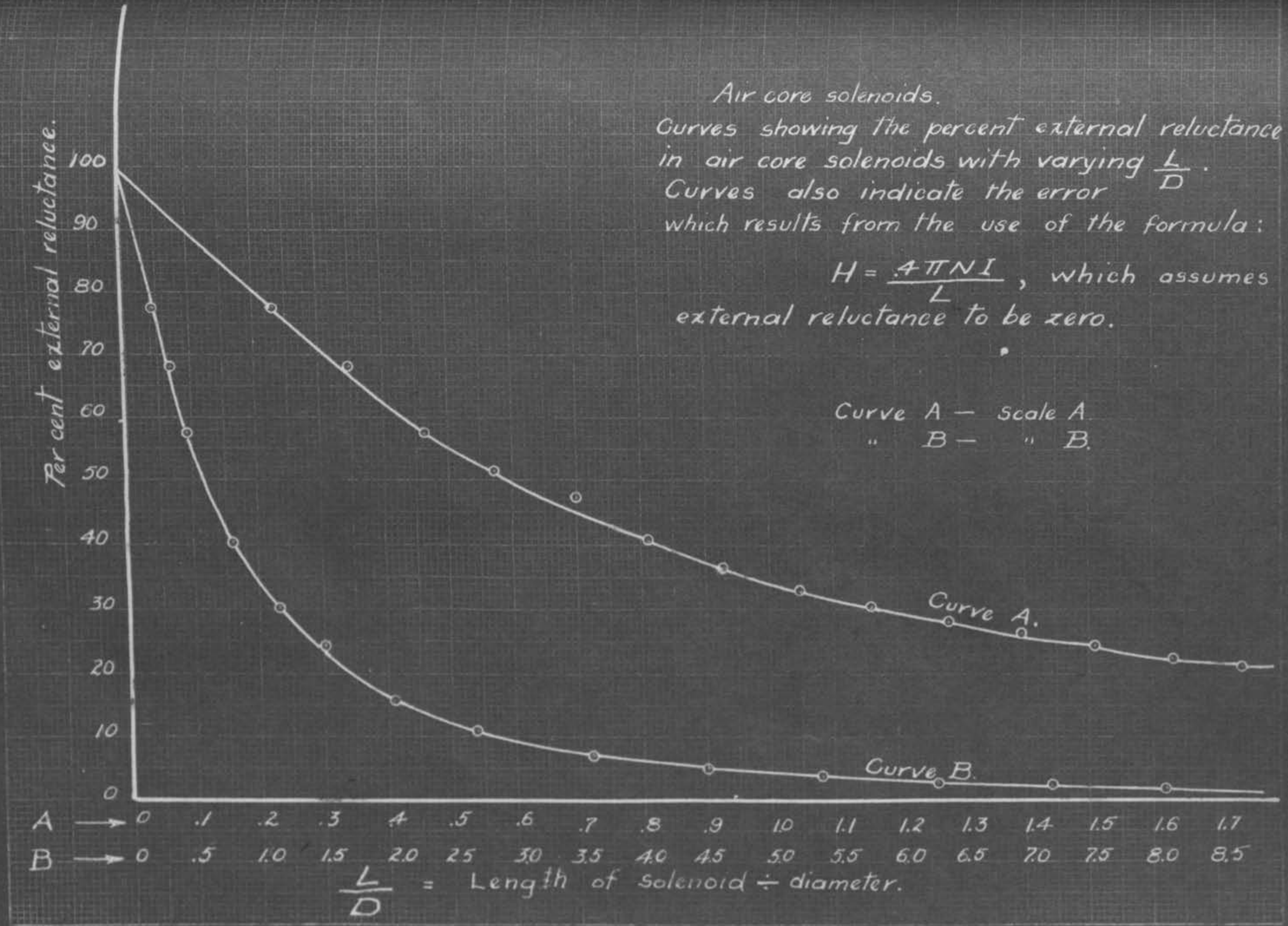
Air Core No. 5.
Diam = 8.60 cm.

Curves showing the voltage induced in sec. coil placed at centers of solenoids of varying lengths, also total and air core reluctance.

d = External Reluctance.

$$\text{Air core Rel.} = \frac{L}{4A} = \frac{16}{1 \times \pi r^2} = \underline{2.7.6}$$

Plate XIX



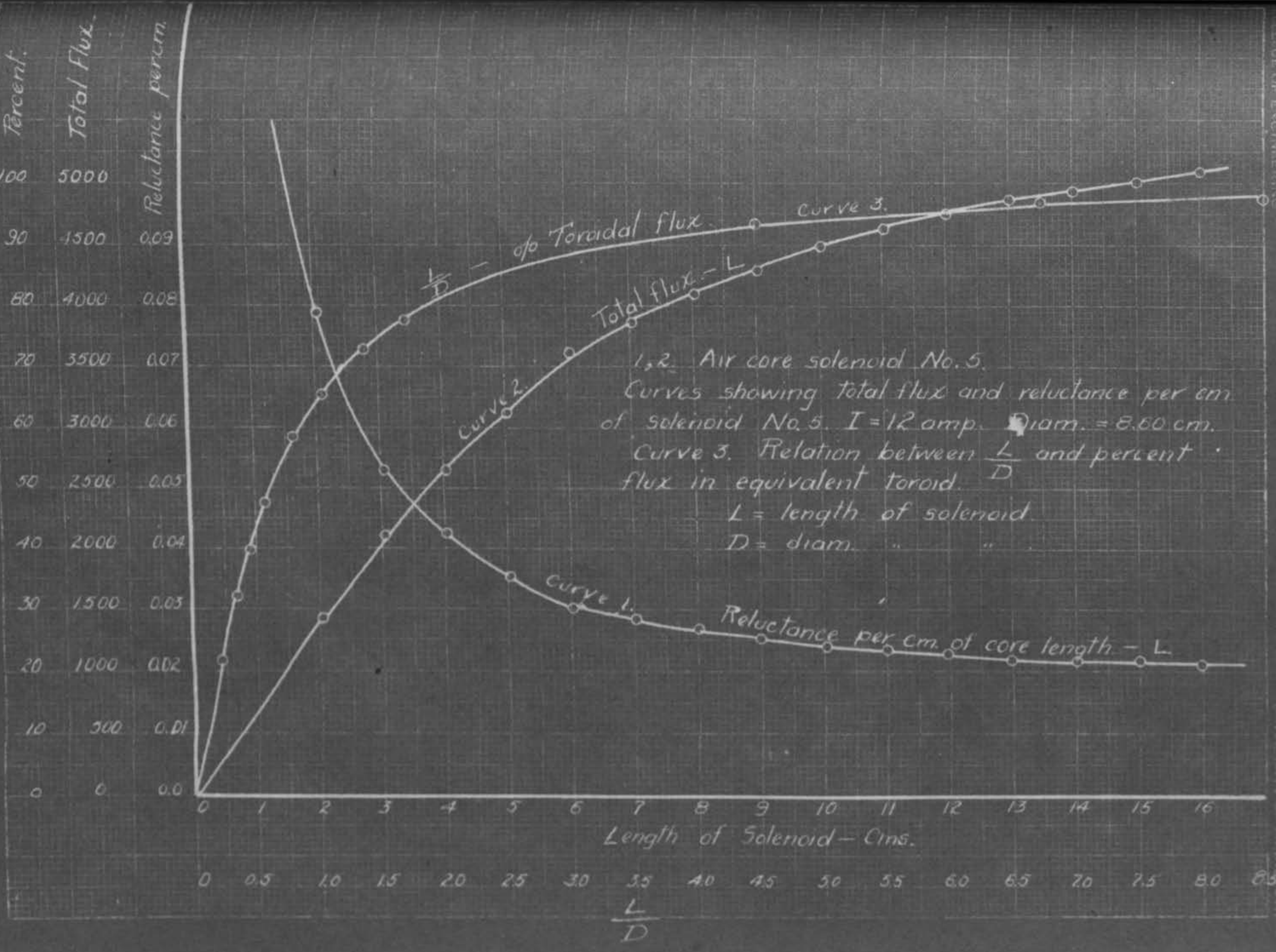
Air core solenoids.

Curves showing the percent external reluctance in air core solenoids with varying $\frac{L}{D}$. Curves also indicate the error which results from the use of the formula:

$$H = \frac{.4\pi NI}{L}$$, which assumes external reluctance to be zero.

Curve A — Scale A.
" B — " B.

Plate XX



1, 2. Air core solenoid No. 5.
Curves showing total flux and reluctance per cm of solenoid No. 5. $I = 12$ amp. Diam. = 8.60 cm.
Curve 3. Relation between $\frac{L}{D}$ and percent flux in equivalent toroid.
L = length of solenoid.
D = diam. " "

Plate XVI

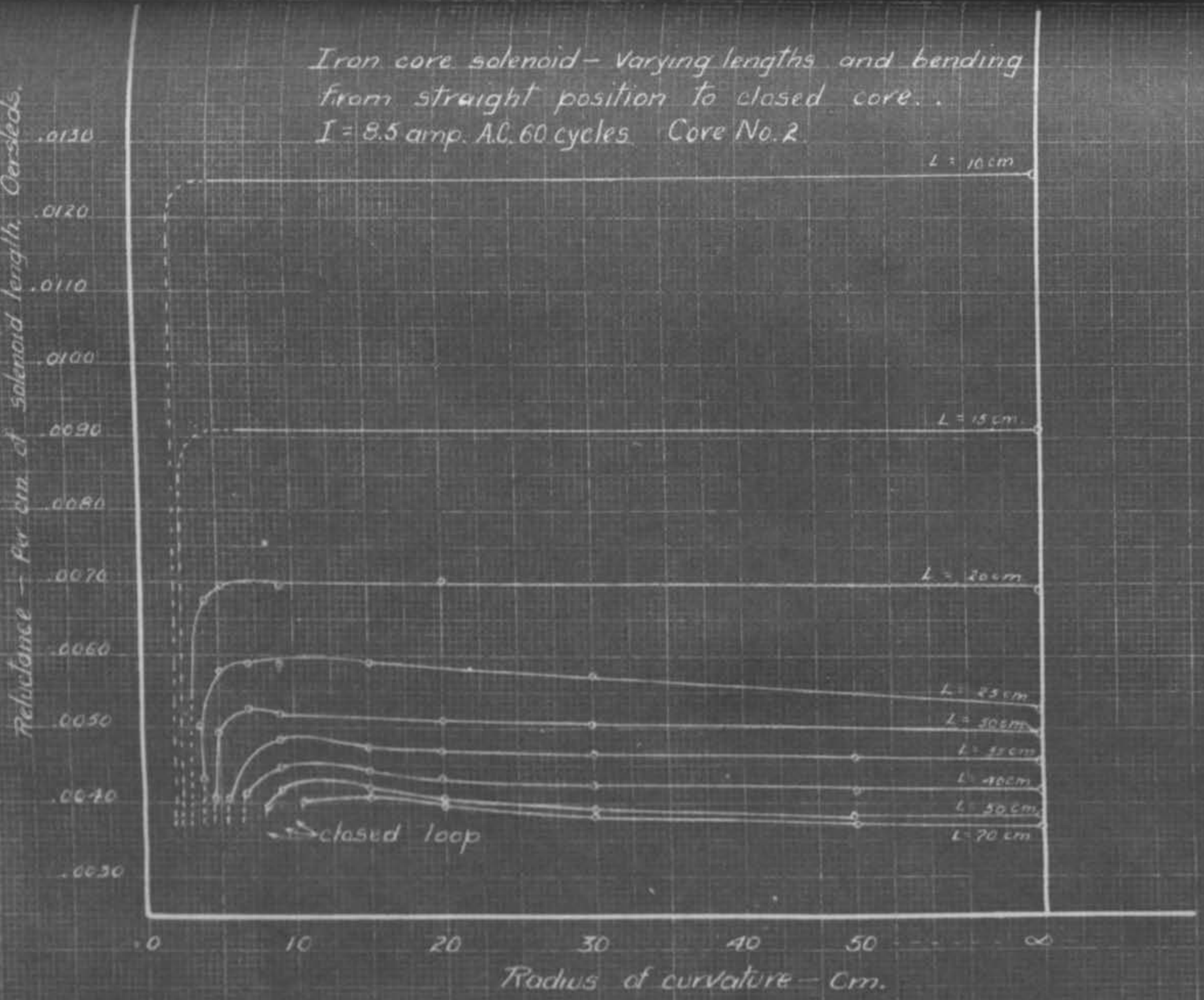

Iron core solenoid - Varying lengths and bending
from straight position to closed core.
I = 8.5 amp. A.C. 60 cycles Core No. 2
L = 10 cm
L = 15 cm
L = 20 cm
L = 25 cm
L = 30 cm
L = 35 cm
L = 40 cm
L = 50 cm
L = 70 cm
closed loop
Reluctance - Per cm. of solenoid length. Oersteds.
.0130
.0120
.0110
.0100
.0090
.0080
.0070
.0060
.0050
.0040
.0030
0
10
20
30
40
50
∞
Radius of curvature - Cm.

Plate XXII

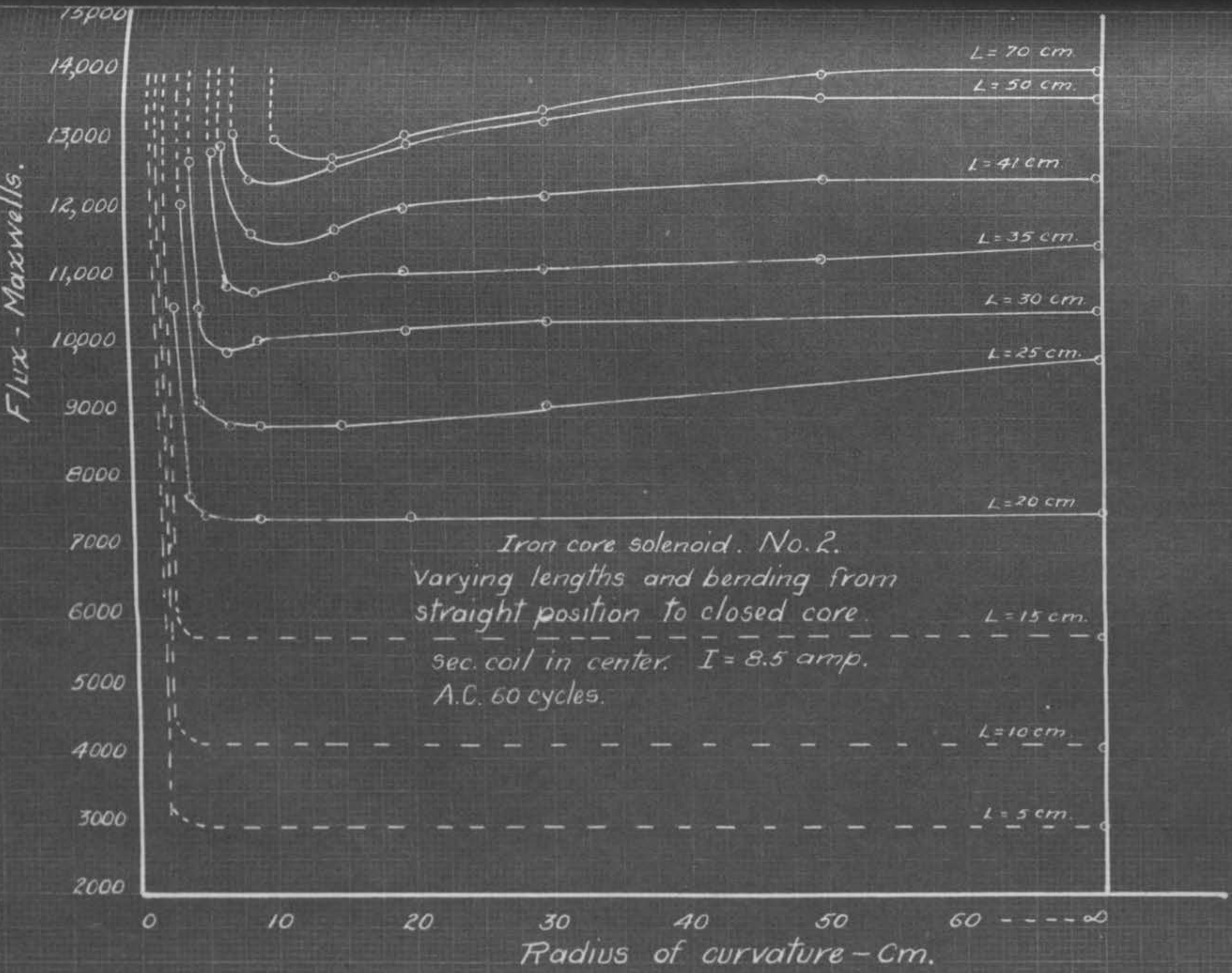
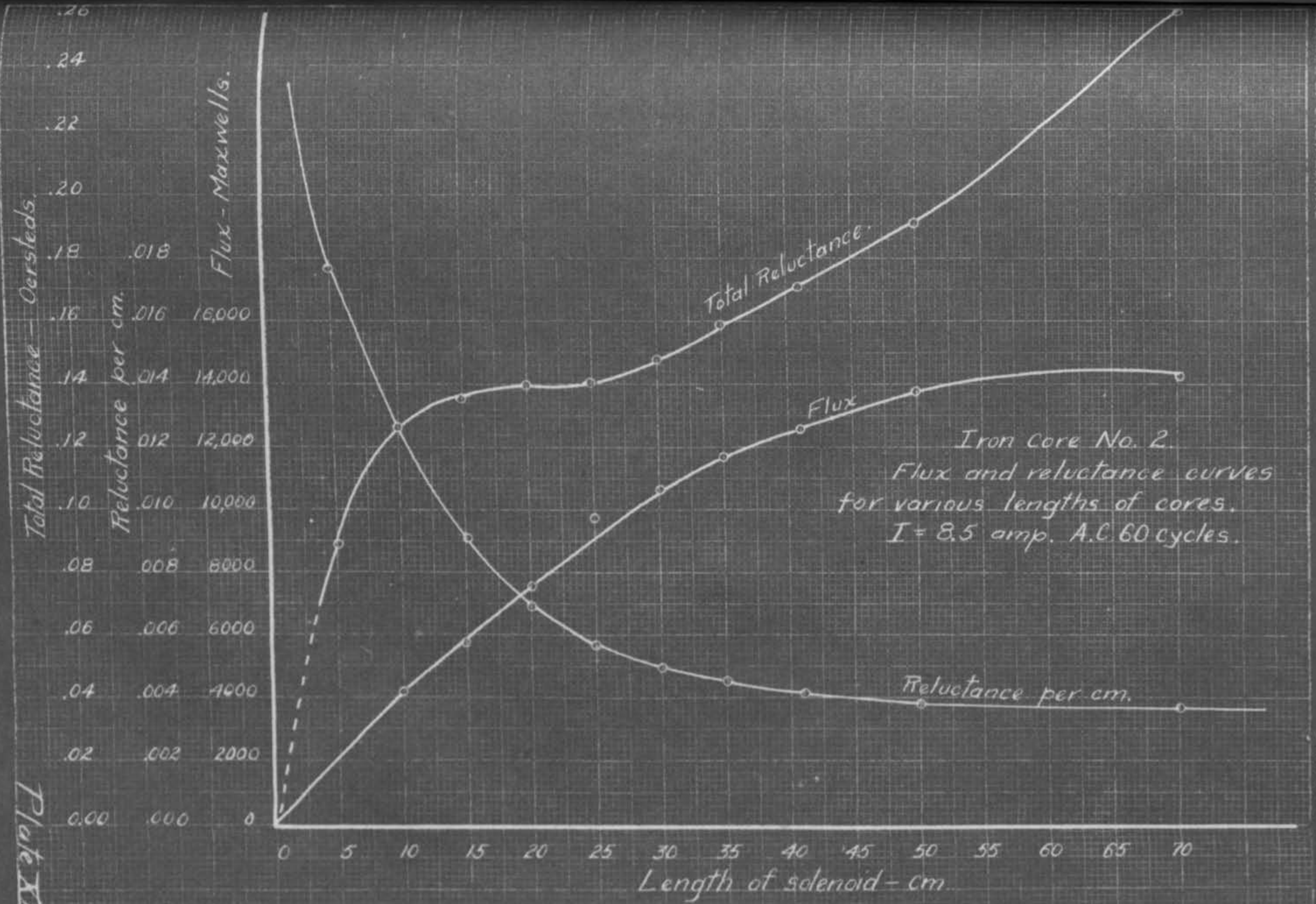

Iron core solenoid. No. 2.
Varying lengths and bending from straight position to closed core.
Sec. coil in center. I = 8.5 amp.
A.C. 60 cycles.

Plate XXIII



Iron Core No. 2.
Flux and reluctance curves
for various lengths of cores.
I = 8.5 amp. A.C. 60 cycles.

Plate XXIV

Table I. Plate II.

Calibration of Elec. Static voltmeter.

Volts. R.M.S.	Deflection.
0.00	0.00 CM.
0.25	0.09 "
0.50	0.22 "
0.75	0.38 "
1.00	0.62 "
1.25	0.92 "
1.50	1.28 "
1.75	1.58. "
2.00	2.12 "
2.25	2.65 "
2.50	3.22 "
2.75	3.85 "
3.00	4.49 "
3.25	5.22 "
3.50	6.00 "
3.75	6.87 "
4.00	7.72 "
4.25	8.62 "
4.50	9.55 "
4.75	10.62 "
5.80	11.62 "
5.25	12.88 "
5.50	14.05 "
5.75	15.30 "
6.00	16.48 "
6.25	17.90 "
6.50	19.22 "
6.75	20.50 "

Table IIIa.

Effect of torsion on flux distribution.

Twist.	cm. def.	volts.
0°	21.32	6.85
90°	19.10	6.45
180°	18.30	6.35
270°	17.60	6.20
360°	16.00	5.90
540°	14.90	5.70
720°	14.00	5.50

Table II. Plate IV.

Magnetization curve of core No. 1. AC. 60 cycles. Sec. coil in center.

Amp.	cm. d.	Volts	Φ_M Max.	B_M Gauss
0.8	0.10	0.10	125	710
1.15	1.96	1.90	2380	13500
1.40	3.85	2.72	3400	19300
1.90	5.45	3.28	4100	23200
2.33	6.36	3.55	4440	25200
2.78	7.11	3.78	4720	26800
3.19	7.78	4.00	5000	28400
4.14	8.80	4.23	5280	30000
5.10	9.66	4.47	5600	31800
5.96	10.25	4.60	5750	32700
6.85	10.73	4.70	5880	33400
7.99	11.23	4.80	6000	34100
9.98	11.90	4.93	6170	35100
12.10	12.47	5.08	6350	36100
13.80	12.87	5.13	6420	36500
15.60	13.38	5.25	6570	37400
18.20	13.75	5.35	6680	38000
21.35	14.30	5.45	6820	38800

Table III Plate VIII.

Leakage data on core No. 1. $I = 9$ amp. A.C. 60 cycles.

Cm. from end	cm. defl.	Volts	B_m Gauss.
00	04.50	3.00	21800
10	8.30	4.20	29800
20	9.45	4.47	31800
30	9.90	4.60	32600
40	10.20	4.68	33200
50	10.37	4.72	33500
60	10.70	4.80	34100
70	11.05	4.88	34600
80	11.35	4.95	35300
90	11.61	5.02	35600
100	11.55	5.00	35500
110	11.30	4.95	35200
120	10.70	4.80	34100
130	10.35	4.82	34200
140	10.42	4.75	33800
150	10.72	4.80	34100
160	11.05	4.90	34800
170	11.13	4.90	34800
180	11.06	4.90	34800
190	10.82	4.85	34400
220	4.70	3.05	21900

Table IV Plate VIII

Core No. 2. bending data. Voltage of sec. coil. with ; varying radius of curve.

Position of sec. coil.	∞ radius Volts	30 cm. rad Volts	15 cm. rad Volts	$\frac{1}{4}$" air g. Volts
0 cm.	0.220	.090	0.10	0.250
1 "	0.260	0.118	0.19	0.308
2 "	0.318	0.225	0.24	0.362
3 "	0.362	0.280	0.28	0.419
4 "	0.410	0.320	0.33	0.450
5 "	0.450	0.365	0.38	0.481
6 "	0.482	0.406	0.42	0.516
7 "	0.519	0.440	0.45	0.541
8 "	0.552	0.470	0.48	0.575
9 "	0.582	0.5015	0.52	0.601
10 "	0.640	0.527	0.54	0.622
12 "	0.690	0.582	0.59	0.670
14 "	0.728	0.635	0.64	0.715
16 "	0.860	0.679	0.68	0.738
25 "	0.860	0.797	0.78	0.809
30 "	0.888	0.822	0.80	0.812
35 "	0.888	0.832	0.82	0.838

Table V Plate IX

Core No. 2. High saturation. I=10

Position, Sec. coil.	Volts 15 cm. r.
0.00 cm	0.36
1 "	0.551
2 "	0.691
3 "	0.811
4 "	0.912
5 "	1.08
6 "	1.12
7 "	1.18
8 "	1.22
9 "	1.295
10 "	1.32
12 "	1.42
14 "	1.51
16 "	1.55
25 "	1.70
30 "	1.72
35 "	1.73

Table VI Plate X

Magnetization Curve data for core No. 2.

amperes	Volts	B_m Gauss	O_m Max.
0.47	0.20	2650	1880
0.655	0.315	4160	2950
0.663	0.330	4350	3090
1.095	0.540	7120	5060
1.059	0.520	6860	4870
1.160	0.565	7470	5300
1.200	0.581	7680	5450
1.280	0.615	8120	5760
1.460	0.680	8960	6370
1.600	0.720	9500	6740
1.760	0.762	10050	7140
1.945	0.810	10700	7580
2.000	0.832	11 000	7800
2.400	0.917	12100	8600
2.850	1.000	13200	9380
3.500	1.121	14800	10500
5.000	1.32	17300	12381
6.000	1.42	18700	13300
7.000	1.52	20,000	14200
10.000	1.680	20,800	15,700

Table V Plate IX

Core No 2. High saturation.

Position of Sec.	$\frac{1}{4}$" air g. Volts
0	0.56
1	0.72
2	0.87
3	0.99
4	1.10
5	1.20
6	1.23
7	1.31
8	1.39
9	1.42
10	1.48
12	1.53
14	1.60
16	1.61
25	1.68
30	1.68
35	1.68

Table IX. Plates XIII - XIV - XV.

Air core solenoid data. Varying lengths. $I = 14$ amperes - D.C. Solenoid No. 4.					Reluctance, Oersteds.		
length-cm.	Throw.cm.	Volts	ϕ, maxwells	B, Gauss	Per cm.	Total	
2	3.16	.562	174	45.0	.359	.718	
4	4.60	.817	253	65.4	.247	.990	
6	5.35	.953	295	76.2	.212	1.265	
8	5.56	.988	306	79.1	.204	1.630	
10	5.69	1.010	313	80.9	.200	1.995	
15	5.89	1.045	324	83.8	.193	2.900	
20	5.91	1.050	325	84.0	.192	3.84	
30	6.06	1.075	333	86.1	.187	5.63	
50	6.08	1.082	335	86.5	.186	9.32	
80	6.17	1.097	340	87.9	.184	14.70	
90	6.00	1.065	330	85.3	.189	17.00	
100	6.16	1.093	339	87.6	.184	18.4	

Table VIII. Plate XII

Flux distribution of solenoid No. 4. $I = 14$ amp. D.C. Ballistic galvanometer.				
Pos. Sec. c.	Deflection	Volts	ϕ_M Maxwell	B Gauss.
0	2.72	.481	149	38.5
2	5.02	.895	277	71.6
5	6.05	1.073	333	86.1
10	6.00	1.064	330	85.3
15	6.06	1.08	334	86.4
20	6.16	1.093	339	87.6
30	6.30	1.120	347	89.7
40	6.28	1.120	346	89.5
50	6.38	1.130	351	90.7
60	6.12	1.100	341	88.2
75	6.28	1.120	346	89.5
85	6.30	1.120	347	89.7
95	6.16	1.093	339	87.6
105	6.00	1.064	330	85.3
113	5.02	.895	277	71.6
115	2.72	.481	149	38.5

Table X

Plate XIV

Reluctance of Air core No. 4.			
core	R_e	R_t	% R_e
cms.	.000	0	0
0	All	0	100
2	.355	.75	46.6
4	.225	1.00	22.5
6	.150	1.35	10.4
8	.125	1.615	7.75
12	.100	2.35	4.25
15	.075	2.90	2.60
20.	.060	4.00	1.50

TABLE XI PLATE XVIII

Flux distribution of Core #5 $I = 12$ A.C. 60~

cm. from end	cm. def.	Volts	ϕ_M
16	10.25	4.68	3510
15	14.05	5.50	4130
14	16.70	6.08	4565
13	18.55	6.36	4780
12	19.70	6.58	4940
11	20.70	6.73	5070
10	21.40	6.82	4960
9	21.85	6.90	5200
8	21.85	6.90	5200
7	21.85	6.90	5180
6	21.31	6.81	5130
5	20.55	6.70	5050
4	19.50	6.52	4920
3	18.07	6.25	4730
2	16.18	5.91	4470
1	13.15	5.29	4020
0	8.79	4.31	3250

TABLE XII PLATE XIX

$I = 12$ A.C. 60~

Variation of reluctance with length of core

Length-cm	cm. def.	Volts	ϕ_M	Rel/cm.	Rel.-total
16	20.7	6.85	5145	.0217	.347
15	20.7	6.74	5060	.0220	.330
14	19.9	6.61	4960	.0225	.315
13	19.2	6.48	4870	.0229	.298
12	18.7	6.35	4770	.0234	.281
11	17.3	6.20	4650	.0240	.264
10	16.5	6.00	4500	.0248	.248
9	15.4	5.72	4300	.0259	.233
8	12.6	5.43	4080	.0273	.219
7	12.6	5.12	3840	.0290	.203
6	11.1	4.85	3640	.0306	.183
5	8.5	4.14	3110	.0359	.179
4	6.2	3.50	2600	.0429	.172
3	4.1	2.80	2105	.0530	.159

TABLE XIII PLATE XX

Variation of percent external reluctance with L÷D in air cores

Length	L÷D	Rel-total	Rel-external	%
16	1.86	.347	.0700	20.2
15	1.74	.330	.0700	21.2
14	1.63	.315	.0717	22.7
13	1.51	.298	.0725	24.3
12	1.40	.281	.0733	26.3
11	1.28	.264	.0742	28.1
10	1.16	.248	.0750	30.2
9	1.04	.233	.0772	33.1
8	.93	.219	.0805	36.7
7	.82	.203	.0825	40.6
6	.70	.183	.0870	47.4
5	.57	.179	.0925	51.5
4	.46	172	.099	57.5
3	.35	.159	.108	68.0
2	.23	158	.1225	77.5

TABLE XIV   PLATES XXII + XXIII

Variation of flux + reluctance with variation of rad. of curvature
$I = 8.5$

Radius	cm. def.	Volts	Φ_M	Rel./cm.	Rel.-total
		L =	70 cm.	.00369	
∞	6.85	3.77	14,170	.00372	.258
50	6.70	3.73	14,010	.00385	.260
30	6.30	3.60	13520	.00397	.269
20	6.00	3.50	13150	.00406	.278
15	5.75	3.42	12840	.00400	.284
11.12 closed	5.95	3.47	13040		.280
		L =	50 cm.		
∞	5.50	3.34	12,560	.00416	.170
50	5.48	3.34	12560	.00416	.170
30	5.25	3.27	12290	.00424	.174
20	5.10	3.22	12100	.00431	.176
15	4.90	3.14	11800	.00442	.181
9.2	4.80	3.12	11720	.00444	.182
7.9 closed	5.90	3.46	13000	.00491	.164
		L =	41 cm.		
∞	6.40	3.64	13690	.00381	.1905
50	6.40	3.64	13690	.00381	.1905
30	6.15	3.56	13380	.00390	.1950
20	5.90	3.46	13000	.00401	.2000
15	5.70	3.41	12800	.00407	.2040
9.2	5.40	3.32	12490	.00418	.2090
6.53 closed	6.00	3.50	13150	.00397	.1985
		L =	35 cm.		
∞	4.70	3.07	11520	.00452	.158
50	4.55	3.02	11350	.00459	.160
30	4.40	2.97	11160	.00468	.164
20	4.40	2.97	11160	.00468	.164
15	4.35	2.95	11090	.00471	.164
9.2	4.20	2.89	10860	.00481	.168
7.0	4.20	2.89	10860	.00481	.168
5.58 closed	5.80	3.44	12910	.00404	.142
		L =	30 cm.		
∞	4.00	2.82	10,600	.00492	.148
30.00	3.82	2.75	10320	.00505	.152
20.00	3.80	2.73	10250	.00510	.153
9.20	3.70	2.69	10100	.00517	.155
7.00	3.67	2.67	9970	.00523	.157
5.00	4.02	2.82	10600	.00492	.148
4.78 closed	5.70	3.41	12800	.00408	.122

TABLE XIV cont.

Radius	cm. def.	Volts	Φ_M	Rel./cm.	Rel-total
		L=	25 cm.		
∞	3.2	2.49	9860	.00529	.132
30.00	3.0	2.43	9130	.00572	.143
15.00	2.9	2.36	8880	.00588	.147
9.20	2.9	2.36	8880	.00588	.147
7.00	2.9	2.36	8880	.00588	.147
5.00	3.0	2.40	9030	.00578	.145
3.99 closed	5.2	3.24	12170	.00430	.108
		L=	20 cm.		
∞	2.17	2.00	7525	.00694	.139
20	2.12	1.98	7440	.00702	.142
9.2	2.15	1.99	7480	.00698	.140
5.0	2.15	1.99	7480	.00698	.140
4.0	2.25	2.06	7740	.00675	.135
3.19 closed	4.00	2.82	10600	.00493	.099
		L=	15 cm.		
∞	1.3	1.53	5750	.00908	.135
		L=	10 cm.		
∞	.72	1.1	4130	.0126	.126
		L=	5 cm.		
∞	.5	.79	2960	.0176	.088

TABLE XV PLATE XXIV

Variation of reluctance with core length

I= 8.5 Rad = ∞

Length	Volts	Φ_M	Rel/cm.	Rel-total
70	3.77	14170	.00369	.258
50	3.64	13690	.00381	.190
41	3.34	12560	.00416	.170
35	3.07	11520	.00452	.158
30	2.82	10600	.00492	.147
25	2.49	9350	.00560	.140
20	2.00	7525	.00694	.139
15	1.53	5750	.00908	.135
10	1.10	4130	.01261	.126
5	.79	2960	.01765	.088