THE UNIVERSITY OF MINNESOTA

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Report

of

Committee on Thesis

The undersigned, acting as a Committee of the Graduate School, have read the accompanying thesis submitted by Samuel Arnold Berg for the degree of Electrical Engineer They approve it as a thesis meeting the requirements of the Graduate School of the University of Minnesota, and recommend that it be accepted in partial fulfillment of the requirements for the degree of Electrical Engineer.

Geo. D. Shepardson
Chairman

F. W. Springer

John T. Tate

Date ____________________

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The Influence of the Variation of Air Path on the Reluctance of Iron and Air Core Solenoids.

A Thesis Submitted to the
Faculty of the Graduate School of the
University of Minnesota

by

Samuel Arnold Berg

assisted by

Hugo William Wahlquist

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C-O-N-T-E-N-T-S

PREFACE.

Since the electrical industry has always and will always depend almost wholly upon electromagnetic phenomena, it seemed desirable to find out something more specific and quantitative concerning the laws of the magnetic field. Solenoids have been the center of interest of many investigators but always in the more practical than in the theoretical sense. With a view to expanding the knowledge regarding the magnetic circuit, we have carried on experiments with the hope that we might reduce to mathematical laws some of the phenomena of the magnetic field, or at least to discover some characteristic behaviors or reactions of that field.

After some months of investigation, it was determined to select from the enormously large field suggested by our subject, the specific problem of the investigation of the fields of solenoids, giving particular heed to the variation of reluctance and leakage of the magnetic path, which might be varied by bending the solenoid into various positions. The air path of the magnetic flux may be made to vary by bending the core into arcs with varying radii. That there is a definite relation between the length of air gap or the radius of the bent core and the variation of reluctance of the resulting magnetic circuits is a thing to be ascertained by experiment and by reducing these relations to positive mathematical expressions.

There is, of course, the fundamental and basic equation from which all mathematical expressions in the realm of magnetism must spring, namely,

$$\text{Flux} = 1.257 \ N \ I \mu A \div L$$

which is the equation for a simple magnetic circuit. A ring, uniformly wound, obeys the laws expressed in the above equation. When we leave ideal cases, however, and consider magnetic circuits consisting of a combination of iron and of long distances of air, then simple mathematics fail, simple variations fail, and variables of staggering number enter into the problem. Small air gaps in the magnetic circuit have been treated with satisfaction by Ewing and others, but the effect of long air paths on the magnetic circuit involves conditions that render the problem rather difficult to attack mathematically. The mathematics involved is by far the most complex in electrical engineering, because of the many variables as well as the variations among the variables. C. H. Livens, of the University of Sheffield, in his masterly volume, "The Theory of Electricity", says, after 706 pages of the most intricate maze of mathematical discussion of the theory of electricity, "Although the present exposition is essentially a mathematical one, much of the purely analytical mathematics usually associated with the subject has been omitted."

On this particular subject, namely, the variation of reluctance and changes in leakage with varying positions of the bent core, nothing has been said by Ewing, Underhill, and others who seem to have been particularly interested in the investigations of magnetic nature. The specific problem in hand seems to have been outside of the scope of their work, since they were unable to surmount the difficulties of experimental nature. Because the apparatus at our disposal was peculiarly adapted to the task in hand, we were enabled to obtain definite data on this subject.

PREPARATION.

The investigation of a magnetic field lead us finally to examine the variation of reluctance of the air path and leakage of the magnetic field of cores that were excited by alternating current and were bent into positions determined by various definite radii of circles. The pursuit of the subject of magnetism and the examination of what others had done in that field lead us naturally into the work done by that master, Ewing. His scholarly book, "Magnetic Properties of Iron and Other Metals", influenced us at first to emulate his method of magnetic investigation by means of the ballistic galvanometer. This lead us into a maze of work consisting of eliminating vibrations, calibrating the instrument every day, determining constants, and other details of similar and tedious nature. We found that this method was particularly useless for our work, because of the fact that every time the core was bent into a new position, the permeability changed and was hopelessly inconstant. The fact has been pointed out by Ewing that iron changes its permeability very considerably when it is subjected to shocks or taps. Because of the very nature of our problem, we were compelled to lift the coil, move it to a different position, and take a reading. The result of disturbing the core and forcing it into a new position was to change the permeability and magnetic conditions so that no uniform observations could be obtained.

Another method which we quickly discarded was the magnetometric. This method was unsuited to investigating closed rings and was, moreover, subject to the same limitations as was the ballistic method. We also abandoned the potentiometer method of measuring the alternating voltages of minute amounts because of troubles and inaccuracies due to phase lag and wave distortion. After a bitter month of puttering with the galvonometer, which from the very outset was entirely inadequate and unsuited to our needs, we were encouraged to carry out the investigation by the fact that there was in the possession of the Electrical Engineering Department an electrostatic voltmeter capable of measuring as low as one volt and as high as seven volts of alternating current. This was the very thing that made the pursuit of our problem possible.

The Ayrton and Mathers electrostatic voltmeter, it was decided, was to be used in our work to record voltages in the secondary of an exploring coil that enclosed the primary windings of the solenoids. The connections used are shown on Plate VI. It was planned to send an alternating current of constant value under 110 volt pressure through the winding of the solenoid. A secondary coil of a sufficient number of turns to get a good reading on the electrostatic voltmeter would be connected in series with the meter. Any change in reluctance or any leakage would then be registered by the voltmeter which, of course, changed in proportion to the change in flux or effective lines of force.

It was our next task to reduce this extremely sensitive instrument to definiteness of action and reliability of indication. Some difficulties were encountered which at first seemed to promise nothing but tribulations. There was, for example, great trouble experienced due to the mechanical vibrations of the instrument. The proximity of the Corliss engine in the powerhouse caused a considerable trembling of the instrument. To remedy this we mounted the instrument on iron stakes which were driven directly into the ground. On these stakes we mounted the instrument as shown on Plate I, figure IV. No further troubles due to mechanical vibrations was experienced.

We found, in the second place, that on connecting the instrument to a coil which was in turn connected inductively to a source of alternating current, that the moving element would be strongly affected by the movement of a person walking across the room at some distance from it. The dragging of a machine across the floor directly above would cause the moving element to disappear for some thirty seconds until the accumulated charge could escape. The motion of the shaper upstairs and the running of the Fort Wayne alternator in the laboratory also rendered it inaccurate. The instrument, depending as it does, on the electrostatic difference of potential between the two vanes or surfaces, is easily and strongly influenced by the slightest electromagnetic disturbance. To eliminate the disturbances we proceded in a definite manner. First, we twisted the leads which were made from #30 d.c.c. copper wire. Static and electromagnetic disturbances were still in evidence, though less marked. It was found that when taking voltage readings, there was a certain point where the capacity and inductance of the circuit were in the exact proportion to produce resonance. At this point, the moving element would, of course, move violently off scale. After some study and consultation, it was found that the insertion of a resistance into the circuit would obviate the resonance difficulty. A mathematical solution showed that this should be a resistance of about a megohm and one half. A grid was made from India ink and paper which when inserted into the circuit eliminated the source of annoyance.

Accumulated charges still vitiated our readings. It was quickly found that if a free passage was afforded over which charges would leak off, the instrument was unaffected except by unusually large disturbances such as the sudden starting of a nearby induction motor or by the tremendous field set up by the wireless transmission set. As shown on Plate I, we also grounded the case. By means of the connection shown in Figure III on Plate I, we had no difficulty from disturbances excepting that of the wireless station. It was by the latter that we were most constantly annoyed, because the station was frequently operated. During laboratory hours, it might be stated, there was a destructive variation of voltage that made investigation impossible. However, the instrument was made to behave beautifully during certain periods, and when conditions were favorable, it rendered dependable and faithful service. Calibration curves were made from time to time to check the constancy of the meter. It remained surprisingly constant, following exactly the curve plotted on Plate II.

PROCEDURE.

The first move after good working conditions had been obtained with the instrument, was to make a long core, which will be referred to as core #I, which could be considered to be of infinite length as far as end demagnetizing effects were concerned. A wire was annealed electrically and was cut into 200 cm. lengths and gathered into a diameter of three tenths of an inch or .762 cm. This was wound with a layer of flat copper wire .015" X .098" making a total of 810 turns. Coils of 300 turns of #28 and one of 765 turns of #36 wire were wound so that they would fit snugly over the primary core. The drawings are shown on Plate III.

After the solenoid had been connected as shown on Plate VI, a magnetization curve was obtained which enabled us to determine just where the permeability suffered the greatest change. Plate IV, data table II. Having obtained the point at which the iron underwent the greatest change of permeability, a piece of apparatus was laid out on which the cores could be bent with ease and precision. On a standard platform were laid out curves of radii varying from infinity to 9 cm. The curves were lined with screws or pegs which held the core in any desired position. This bending plate is shown clearly on Plate V. On this plate was placed the core in the straight position i.e. where the radius equals infinity. At each radius readings were taken of amperes, which were kept constant, and of the electrostatic voltmeter, as shown on Plate VI. The deflections of the voltmeter varied as the square of the voltage as may be seen from the calibration curve.

At this point some almost insurmountable difficulties were encountered. At the very first, readings were obtained which gave a very suspicious indication that the reluctance of the air path was less at 400 cm. than at the position where the radius was infinite. Moreover, the readings varied in a truly amazing fashion as may be seen from glancing at Plate VII, data Table III. On investigating this trouble, it was discovered that the twist or distortion of the core changed the permeability to such a tremendous extent that the readings were absolutely vitiated. To demonstrate the effect of twist, the core was kept in the same straight line and twisted in the clockwise direction, noting, as this was done, the voltmeter reading. It will be observed from scanning the data in Table III, how striking was the change, when it is seen that the induced volts dropped from 6.85 to less that 5.5, when it should have remained constant. Ewing points out and substantiates by experiment that the distortions and changes in permeability due to twist are considerable. The changes that were observed were staggering. Another source of annoyance was eddy currents. Their presence was heralded by means of the uneven flux distribution along the core when a leakage test was made. Curve on Plate VII. The readings were erratic and demonstrated that this type of core would not do for the work. Subsequent work with this core showed that either the investigation must be carried on with an air core that would be free from these magnetic evils, or that a core must be built in such a manner that no effects due to twist or eddy currents would occur.

To eliminate eddy currents and twist distortion from the cores was the next problem. With this end in view some electric transformer sheet steel was obtained and cut into pieces .5" X 29.5". Eleven of these strips were used for the core of new solenoid. Each strip was separated by an oiled linen cloth. When the solenoid #2 (descriptions of which will be found on Plate III) was assembled and wound, it was found that each lamination was insulated perfectly for 110 volts and that bending could be done smoothly and without much friction. A magnetizing curve was made and shown on Plate X, data Table VI. That eddy currents were missing will be seen from the smoothness of the leakage curve on Plate VIII, curve #1, data Table IV.

Now it was possible to get readings of leakage and of changes of voltage with different bent positions of the core. The core was first placed in a straight position with radius equal to infinity, and readings were taken placing the exploring coil at various positions along the core. These readings gave the flux densities as well as the relative voltage changes due to variations in reluctance. The curve on Plate VIII, curves #2, and #3, data Table IV, shows the variation of flux and voltage as the core was bent from a straight position to the positions of 30 and 12 cm. and to such a position that the air gap was .25". This flux distribution was also examined with a higher density of excitation. The curves are shown on Plate IX, data Table V.

Still another core, known as #3, and shown on Plate III, was made up of a single lamination of the sheet steel. This core, which was far from being free of trouble due to twist and bending, was somewhat of an improvement in that respect and was free to a large extent from eddy currents. Readings were taken of this core with the exploring coil in the middle of radius, of curvature, of amperes, and of electrostatic voltmeter deflections. The curve on Plate XI, data Table VII, shows the variation of voltage and flux and reluctance with the variation of radius. This core gave no distortions in leakage and seemed to be singularly free of eddy current losses. The curves show quite distinctly that the reluctance increases quite sharply as the core assumes a circular position. The reluctance increases up to a radius of 17.5 cm. at which point it drops off sharply again. That it does not assume the same reluctance as it had at a radius of infinity is due partially to the fact that there is always an air gap no matter how hard the ends are pressed together and partly to the fact that the compression and tension of the twist affects the permeability destructively.

To calculate the flux density and the reluctance from readings of voltage, the following formulas and processes were followed:

(1) $E = 4.44 \; f \; N \; \phi_m 10^{-8}$

f = frequency of alternation

N = number of effective turns

E = effective volts

then $\phi_m = \dfrac{E \times 10^{8}}{4.44 \times f} \times N$

also

(2) $\text{Reluctance} = \dfrac{\text{M. M. F.}}{\phi_m}$

$= \underline{1.257 \times N \times I} \div \phi_m$

In all calculations of reluctance, the M.M.F. per cm. of length, of course, was kept constant. In calculating the total reluctance it was kept in mind that the total reluctance consisted of the reluctance of the external circuit as well as of the reluctance of the internal or core reluctance.

Thus

$$\phi_m = \frac{1.257\ N\ I}{R_e + R_i} = \frac{M.M.F.}{\Sigma R}$$

where

R_i = internal reluctance

R_e = external reluctance

To check the conclusions arrived at with the iron cores, the air core was made as shown in Figure 4, Plate III. This core would obviously be entirely free from eddy currents, hysteresis losses, and bending distortions. Some difficulty was experienced in getting an exploring coil of sufficient number of turns to give an appreciable reading on the electrostatic voltmeter. This was finally solved by placing around the primary a coil of 7270 turns obtained from the secondary of a high tension spark coil.

A flux distribution curve was made from the air core as shown on Plate XII, data Table VIII. The distribution, as will be seen, is even and without variations, increasing rapidly from a density of 40 gauss at the ends to a maximum of 90 gauss in the center.

Starting with this core 115 cm. in length, it was desired to examine the variation of reluctance with the variation of solenoid length. To do so, the winding and core was cut off to the desired length, and then, keeping the secondary coil in the center, readings were taken of the amperes, the deflection of the voltmeter, and of the length of the solenoid. From these data calculations could be made of reluctance, using the formulas given above. Plate XIII, data Table IX, shows the variation of reluctance and flux density with a constant magnetomotive force per unit length. As will be seen from the curves, the flux density and the reluctance remain constant up to 30 cm. when the reluctance increases sharply and the flux density drops off.

On Plate XIV, the total reluctance and reluctance per cm. are shown. The total reluctance, as will be shown, is a constantly increasing value, increasing as the length of the core increases except at the vicinity of the origin where it has a bend upward. This upward bend is due to the fact that the total reluctance consists of two reluctances, the internal and the external, and the internal being constant is having added to it the constantly increasing external reluctance. The external reluctance increases because of the fact that the area of the path is growing less and less, and since the reluctance varies directly as the length and as the square of the radius of the area, although the length is growing less, the area is growing less still faster, causing an increase in reluctance.

On Plate XV, it will be noticed that the total reluctance is plotted together with a tangent drawn tangent to the total reluctance curve at such a point that the external reluctance can be neglected. At such a point the reluctance can be considered as due entirely to the internal path in the core, since the area of the external path is infinite.

There still remained the examination of the variation of the reluctance with the variation of the radius of the air core. After several experiments, it was found, as will be corroborated by the data Table XIV, and Plate XVI, that for each solenoid length up to 20 cm. the reluctance remained constant throughout all radial positions. Since the length which could be readily bent into a circle was limited, shorter solenoids than 20 cm. were not investigated. As is shown on Plate XVI, each solenoid retained its constant reluctance through all radial changes.

In order to investigate the exact variation of the/shorter air core length, especially those of lsee than ten diameters, a solenoid was built consisting of 89 turns of #16 d.c.c. wire on a core 8.68 cm. in diameter. The winding was marked off into 1 cm. lengths in order to facilitate special investigations. An exploring coil was made up of 500 turns of #36 wire and designed so that it would slip snugly over the primary. The primary leads were brought through the same end of the winding. The whole core was shellaced and rendered solid and substantial in its finish. By making this core of large diameter and short length, it was possible to obtain a low ratio of L over D, namely, a maximum of 1.895.

An investigation of the flux distribution was made with the exploring coil of five hundred twins, using 12 amperes as the exciting current through the primary. This excitation gave the maximum throw on the electrostatic voltmeter, serving, thus, to increase the accuracy of readings. Table XI, Plate XVIII, shows the flux distribution over the entire length of the core under investigation. It will be seen that the flux increase is uniform and smooth.

Since only a radius of infinity was possible with the rigid construction, it was convenient to examine the solenoid cut into various lengths. A clip was fashioned from brass, which clamped the solenoid at intervals of one centimeter where bare spots were scraped through the insulation of the wires. The core length was then varied by slipping the contact at various points along the core. At each point, readings were taken of the deflection of the voltmeter, making sure that the current was constant. Knowing the voltages, since they could be taken from the calibration curve on Plate II, it was possible to calculate the flux and the reluctance per cm. of the various magnetic paths. Thus, from formula (1), p.13, namely:

$$\varphi_M = \frac{E \times 10^{+8}}{4.44 \times f} \times N,$$

the fluxes at various points were calculated. Knowing the flux it was then simply a matter of solving the equation, (2) p. 13 for reluctance per cm. of winding. The total reluctance was, then, the value of the reluctance per cm. multiplied by the length of winding. Table XII, and Plate XIX, show these relations clearly. On Plate XIX, it will be noticed, there is a curve for the reluctance of the air core. This curve was arrived at by making use of the formula, reluctance equals $L \div A$, which in the case in hand was $16 \div 1 \times 3.1414 \times (4.30)^2 = 2.76$ oersteds at 16 cm. Now, when the total reluctance of the magnetic circuit was plotted, it embraced values of reluctance that involved both the external and the internal reluctance, as has been previously pointed out. It is apparent, then, that the difference between the ordinates of the two curves would represent the value of the external reluctance. This value, expressed in per cent of the total, is what is represented for various lengths on Plate XX. It will be noted that the external reluctance may be considered negligible in cores of slightly over 8.5 diameters in length.

On the following Plate XXI, will be found curves of reluctance per cm. length of winding, total flux, and per cent toroidal flux. The data appears in Table XIII. The flux and reluctance were calculated as before. Here, as on Plate XIV, the reluctance per cm. increases swiftly for short lengths, with a consequent drop of flux to zero. The per cent toroidal curve was obtained by determining the flux that would have existed had the core been toroidal in shape, in other words it is the flux that would exist were the magnetic path to have the reluctance of the internal path alone.

Attention was next given to the further investigation of the iron core solenoid #2 (see Plate III). Having obtained the flux distribution and magnetization curves, it was of interest to examine the variation of flux and reluctance with the variation in air path length or radius of bend with using different lengths of core. Placing a secondary coil of one hundred turns over the primary, and using 8.5 amperes for exciting current, data was obtained as shown in Table XIV. The electrostatic voltmeter was connected as before, with the precaution of examining the zero point after each reading. The flux and reluctances were calculated in the usual manner.

Plate XXII, shows clearly the changes in reluctance that are undergone as the different lengths of cores are bent in radial curves from an infinite radius to the closed position. It will be noticed that the curves for the cores for lengths of 70, 50, 40, and 30 cm. suffer a hump as the position approaches the curved position. This brings out clearly the fact that the reluctance exists largely in, and increases greatly with, the existence of the short air gap. It is also apparent that the reluctance at all closed positions, no matter what the length of core, drops to the same low value that existed when the core was long and of infinite radius. The values of Table XIV, were used on the Plate.

Plate XXIII, shows the variation of flux under the same conditions of the preceding Plate. Here, it is shown that the flux drops off appreciably and abruptly when the core is bent into such a position that a short air gap exists. The values of closed core fluxes, in all cases, tend to approach the original value existing with a long core of infinite radius.

The reluctance per cm. of winding was calculated for various lengths of core, as shown on Plate XXIV. The Plate shows, too, the variation of total reluctance and the flux. The short lengths of core suffered the great increases in reluctance per cm. It is of interest to note that the reluctance per cm. (.004) remains in the 70 cm. core, an amount that is considerably less than the reluctance (.18) of an air core of the same length.

CONCLUSIONS.

The most practical experience gained from the investigation was the knowledge acquired concerning the nature of the core that must be used in the work. It was demonstrated quite strikingly that the long bare wire core was utterly useless for the work. The twist and strain due to the bending of the core caused violent changes in permeability that rendered the results absolutely worthless. The process of perfecting the core was a slow one. Each core had to be assembled carefully, the primary coil had to be carefully and accurately and closely wound, and the secondary coil had to be built compactly and of just the sufficient number of turns to give the greatest effect within the range of the voltmeter. Several cores were built only to be thrown into the discard after disappointing experiments had proved their worthlessness. With the completion of the insulated, oiled laminated core and the air core, the experiments could be carried out with a great degree of satisfaction.

The adaptation of the electrostatic voltmeter capable of low readings was also the result of careful investigation and thought. The elimination of mechanical vibration and of electrostatic disturbances rendered it usable. The fact that the potential of the secondary coil could be measured without in the least affecting or distorting the field was one that facilitated the investigation beyond all mention. Ewing's method of ballistic and magnetometric measurements fall into the discard as far as this particular work is concerned. The alternating field is a novel means of excitation as it is used in the investigation of the magnetic problem. To employ formulas for flux and reluctance had been restricted largely to direct current excitation, yet in this case alternating current source furnished not only accurate data for magnetic circuit computations, but also a rapid means of arriving at results.

The progress in core preparation was one of importance. The difference between the first core, #1, and core #2 was highly in evidence, making the difference between failure and success in the investigation. In the investigation of core #2, the laminated, oil insulated separation was the solution to the problem of eliminating eddy current losses. When flux densities were plotted of this core, it was found, as will be seen by referring to Plate VIII, and Plate IX, that as the rebending approached the closed loop state, the resultant curves approached the same straight line condition that a tightly welded loop would have. This is as it should be. Since a perfectly closed magnetic ring would show a straight line, it was interesting to note how the leakage curves that were derived in this case approached the closed circuit position. The reason that the values in Tables IV and V, of the data do not assume the original value may be ascribed to the fact that there always existed an air gap in the closed circuit loop. If the core were welded, without producing eddy current losses, we would expect to observe the fact that the flux distribution would be uniform.

In the examination of core #3, the curves displayed the fact that the reluctance increased perceptibly as the core was bent from infinity radius to the closed position. It will be noted, however, that the reluctance dropped off sharply after the 17 cm. radius point was reached. We find, then, that the reluctance of the circuit increases as the core is bent from the straight position. It will be noted, moreover, that the reluctance increases until the ends are pretty well in contact, at which point it begins to drop off. Compression and tension in the core, however, undoubtedly lowered the permeability, so that this might have been responsible for the fact that the flux did not increase to its true value of its unstressed condition.

From the smoothness and regularity of the air core flux distribution curves on Plate III, the air core under investigation promised to be free from the magnetic evils that attended the iron cores. The variation of the reluctance with the variation of core length is shown clearly on Plate XIII. It is apparent that the reluctance is constant up to a core length of 30 cm. which is a length of 15 diameters. That this is so is due to the fact that with cores of long lengths practically the entire reluctance is within the core itself, since the path of the external reluctance has an infinite area. However, this important fact is demonstrated by this investigation that as the curve for the total reluctance approaches the origin, the curve deviates from a straight line and becomes bent. Now, the bend is due to the added increase of the external reluctance whose path has been narrowed and whose resistance has been thereby increased.

This fact is strikingly brought out on Plate XV, A line, representing the reluctance of the core alone, was arrived at in this manner. At a point far out on the total reluctance curve a line was drawn through the origin and tangent to the curve at the above mentioned curve. Because of the fact that way out there where the ratio of the diameter to the length was small, the total reluctance could be considered to exist in the core itself. Now, any deviation or increase from this line would be due entirely to external reluctance. It will be noted that external reluctance does not begin to affect the total until the core becomes shorter than 15 diameters in length. It will also be noted that on Plate XV, the external reluctance may be easily measured by the simple expedient of noting the difference between the ordinates of the total reluctance curve and the reluctance of the core alone. Now a glance at Plate XVII, will show exactly what per cent of the total reluctance of the circuit the external reluctance actually forms for any length of core. The lengths can be easily changed to length of core in terms of diameters by a simple sliderule process, thus getting the actual per cent of the total reluctance in the external circuit or path for any known ratio of diameter to length.

From Plate XVI, it will be noted that the reluctance of the air core circuit underwent no change as it was varied from a position of infinite radius to the closed position. This bears out the fact previously brought out, that the external reluctance was an inappreciable part of the total reluctance for diameters above 15. Difficulties in bending cores of shorter lengths prevented following up and verifying the indication that shorter cores would undoubtedly show a change in reluctance with varying radii, since they fall within the vicinity where the external reluctance is an appreciable and the governing part of the total reluctance. The curve on Plate XVII, shows how the external reluctance increases with shorter solenoids.

In order to examine more closely the reluctance changes in shorter cores that are less than 15 diameters in length, a core was built according to the specifications before mentioned. Because the maximum relation of length to diameter was not over 1.895, it was possible to examine the reluctance change in the vicinity of the origin. The total reluctance of the magnetic circuit was arrived at by the method outlined in the Procedure. Total reluctance, as will be seen from the curves on Plate XIX, was plotted to a large scale. The curve showing the reluctance of the air core itself, was derived from the formula,

$$\text{reluctance} = L \div \mu A.$$

For a length of 16, then, a value of 2.76 was obtained. A line drawn from the origin to this point would represent the reluctance due to the air core itself. Since the total reluctance embraced the external as well as the internal reluctance, it was a matter of merely getting the difference between the ordinates of the two curves to arrive at the external reluctance of the circuit. This external reluctance was plotted as a per cent of the total reluctance on the next Plate, XX. Thus, since the abscissas are the ratios of L to D, we can tell at a glance just what per cent of the total reluctance of any solenoid exists in the external circuit. Once the external quantity is arrived at, it is an easy matter to determine how much flux can be considered to exist in a toroid or closed magnetic circuit where only the reluctance due to the internal path controls the value of the flux.

The curves on Plate XXII, demonstrate an interesting characteristic of the behavior of the reluctance of a magnetic circuit, consisting of iron and air. It will be noticed that the curves for iron cores of 70, 50, 35, and 30 cm. are fairly constant up to a point where a small air gap exists. At this position there exists a hump. This hump shows clearly that the reluctance increases with suddenness as the lines of force are compelled to crowd through the constricted area of a small air gap. Another fact is demonstrated by these curves. As the cores of long dimensions are bent gradually into the circular position and are finally closed, the reluctance drops to the value it had in the straight position where the radius was infinite. Thus, it is demonstrated that the reluctance of the air path in the straight core of long dimension is zero, since it is equal to the value of the reluctance when the core is in the closed position. Moreover, it is obvious now that the external reluctance is zero when a core of long dimension is in the straight position. This is so because in this position it has the same reluctance as when it is in the closed position where, obviously, the reluctance is wholly internal.

Plate XXIV, carries out and shows strikingly that the foregoing statements are based on fact. Here will be seen that the flux in the magnetic circuit will regain, in the closed position, its original value that a long core enjoys in the straight condition. Of course, the union must be made so that there will be no air gap. The curves show distinctly that there was a small air gap. This accounts for the fact that the curves stop at points short of their original value. These curves also demonstrate the fact that, as the core becomes shorter and shorter, the air forms so large a part of the total reluctance due to the shortness of path, that any change in position, except that due to actual contact of ends, causes no appreciable change in the reluctance. At contact, of course, the reluctance of all cores, no matter what their length, falls to the low value of a long core in the straight position.

In short, then, the investigation demonstrated that particular cores and particular instruments were adapted to this kind of investigation; that the reluctance of iron cores increases up to a certain point as it is bent into various positions, and this point is extremely close to the closed position, and that after passing this point, the reluctance falls off to a value determined by the internal reluctance alone; that the reluctance of air cores remains constant no matter what their position, providing that the core is greater than fifteen diameters in length; that after the core has become smaller than 15 diameters, the reluctance increases quite rapidly; that the total reluctance at these core lengths is comprised largely of external reluctance; and that by means of curves such as those on Plates XVII and XX, the percentage of the total reluctance which is external reluctance can be arrived at quantitatively.

Suggestions for further investigations.

There is endless room for following up further significant relations between the various factors that enter into the changes of the magnetic fields of solenoids,especially the variation of internal reluctance with bending.

An air core of less than ten diameters in length might be made which could be easily bent up into circles, thus enabling one to examine more closely the behavior of the reluctance of short air cores undergoing various changes of air path. In case of iron cores, more attetion could be given to actual flux values rather than to relative changes.

The problem offers a splendid chance for the application of mathematics to these foregoing relations which appear to vary in a definite manner.

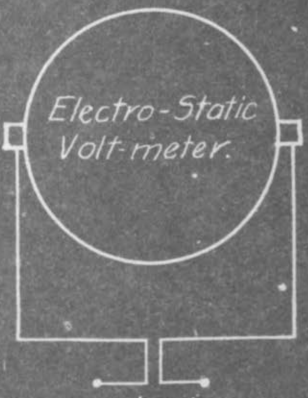


Fig. 1.

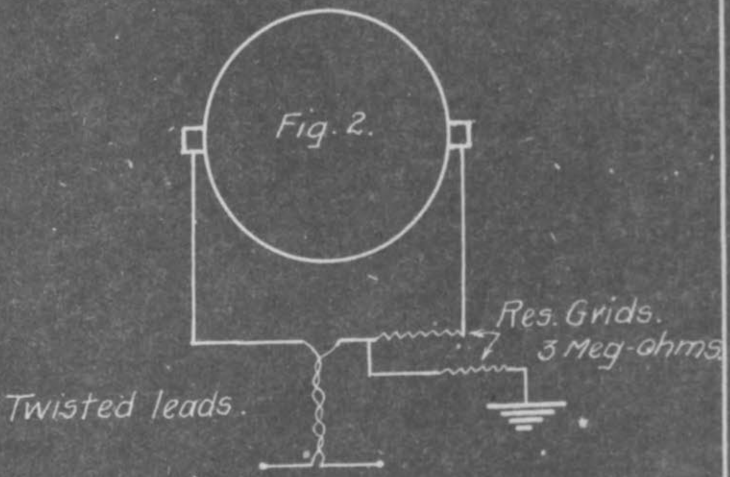


Fig. 2.

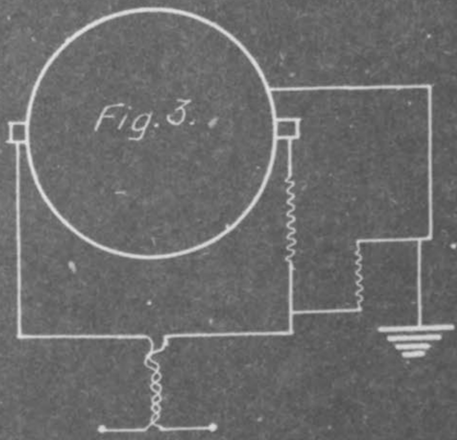


Fig. 3.

g4. Method of mounting E.S.V.
to eliminate vibration.

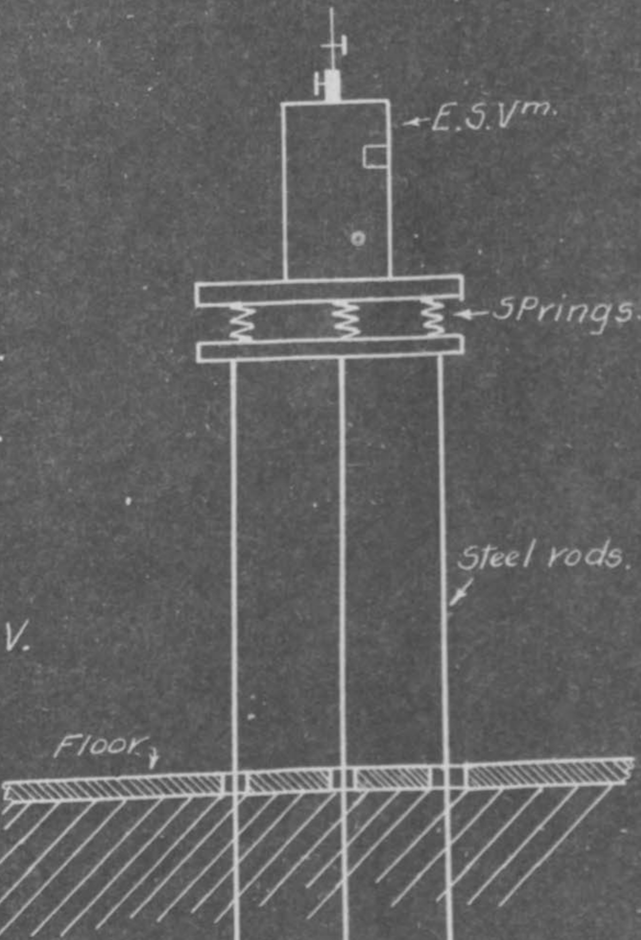
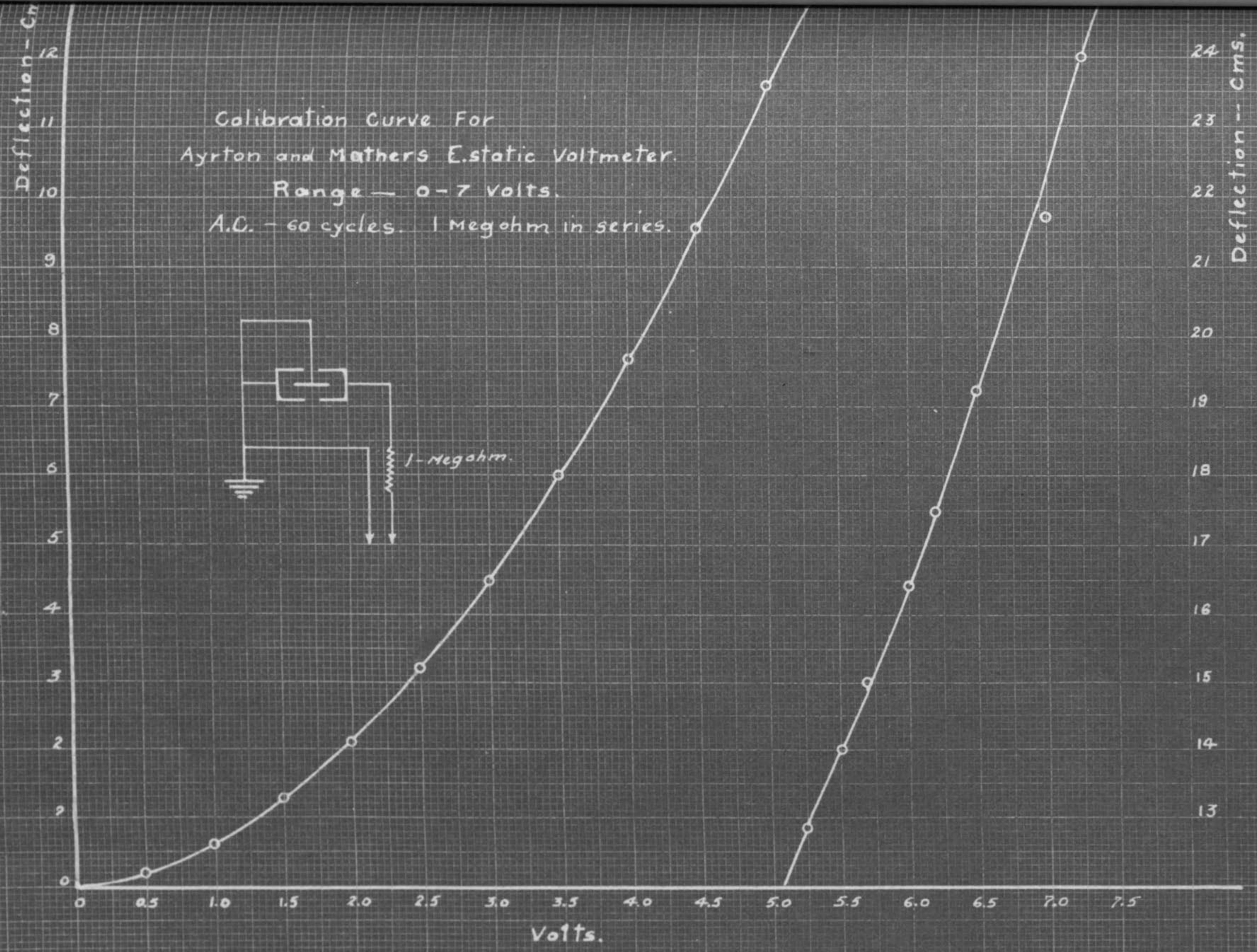


Fig. 4.

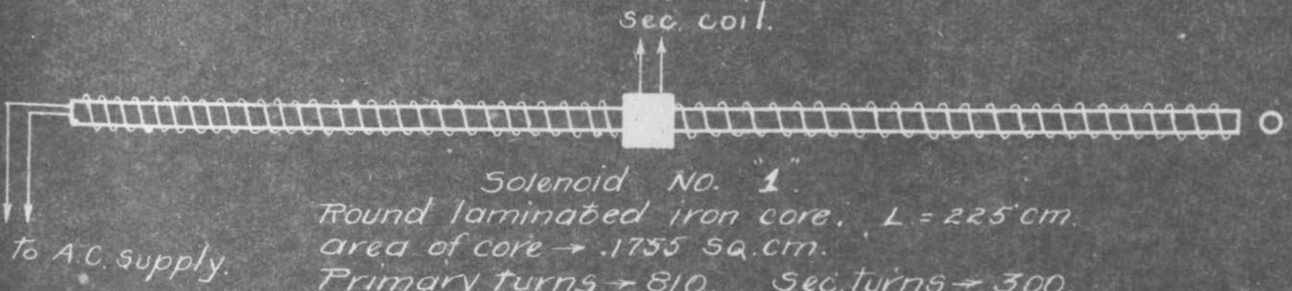
PLATE I.



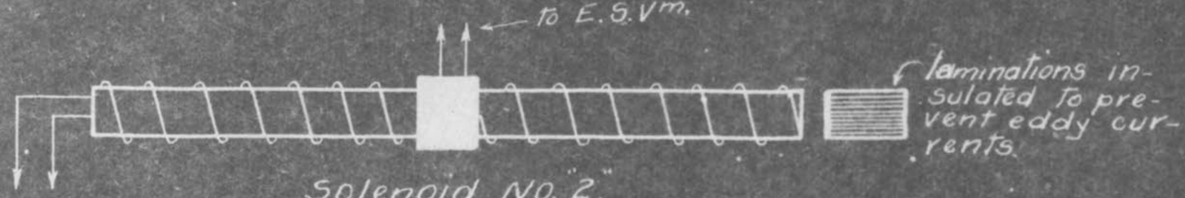
Calibration Curve For
Ayrton and Mathers E.static Voltmeter.
Range — 0-7 volts.
A.C. – 60 cycles. 1 Megohm in series.

PLATE II.

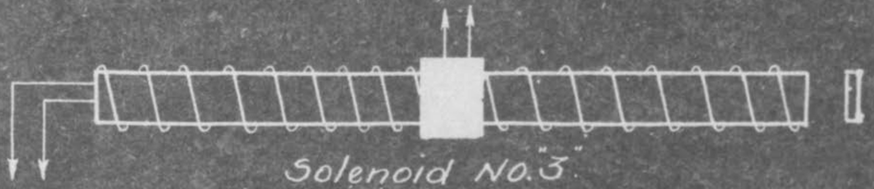
CORE and SOLENOID DATA.



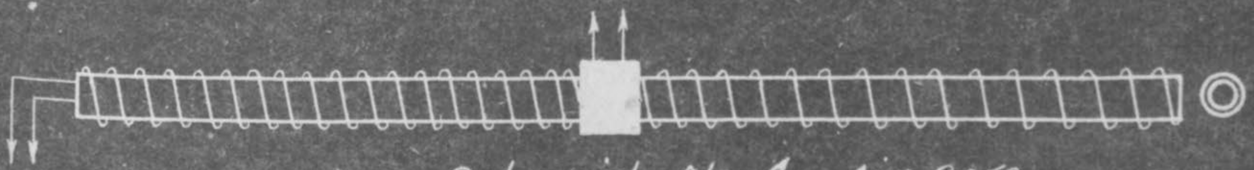
Solenoid No. "1".
Round laminated iron core. L = 225 cm.
area of core → .1755 sq.cm.
Primary turns → 810. Sec. turns → 300.



Solenoid No. 2.
Rectangular core: 11 laminations
$\frac{1}{2}$" x .02" x 29.5" each. L = 29.5" = 75 cm.
area of core → .71 sq.cm.
Primary turns → 251. Sec. turns → 580.



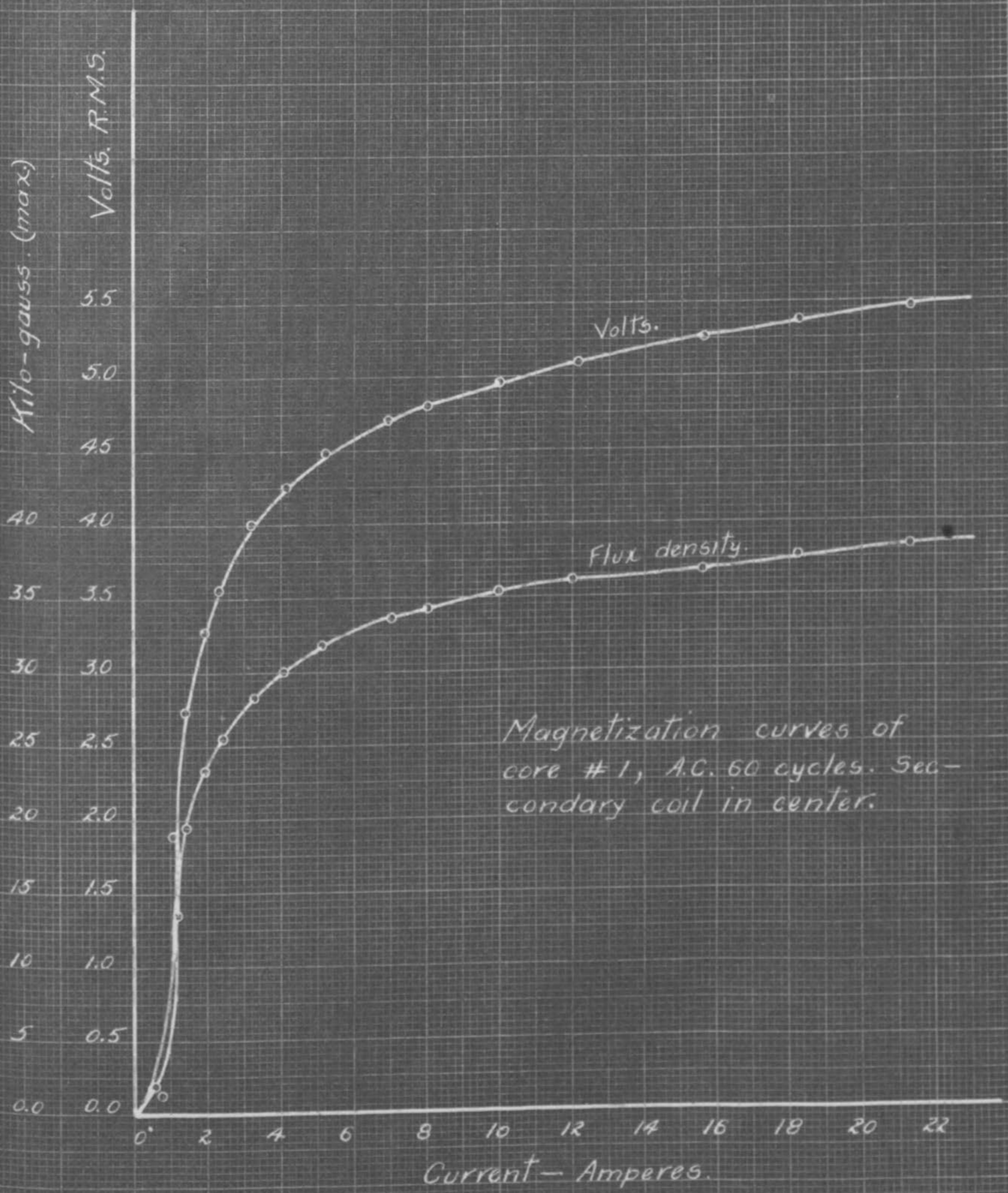
Solenoid No. "3".
Single strip core. L = 75 cm.
Area of core → .0645 sq.cm.
Primary turns → 250. Sec. T. = 2550.



Solenoid No. 4. Air core.
length → 115 cm. Diam. core = 2.22
Primary turns → 409. Sec. T. = 7,270 #36.

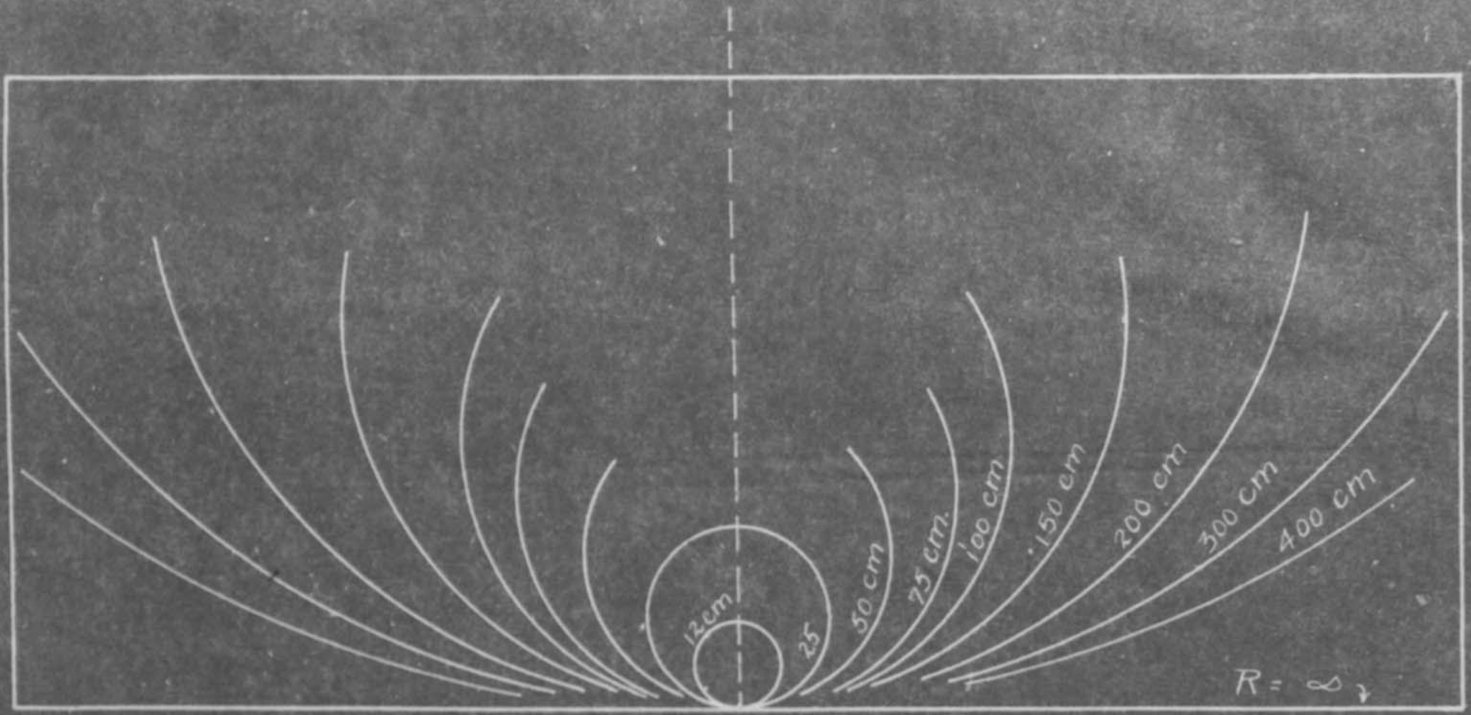
All the above cores are wound with strap copper: .
Turns per inch = 9 = 3.54 per.cm. .097" x .015 = 1855 C.M.

PLATE III.



Magnetization curves of core # 1, A.C. 60 cycles. Secondary coil in center.

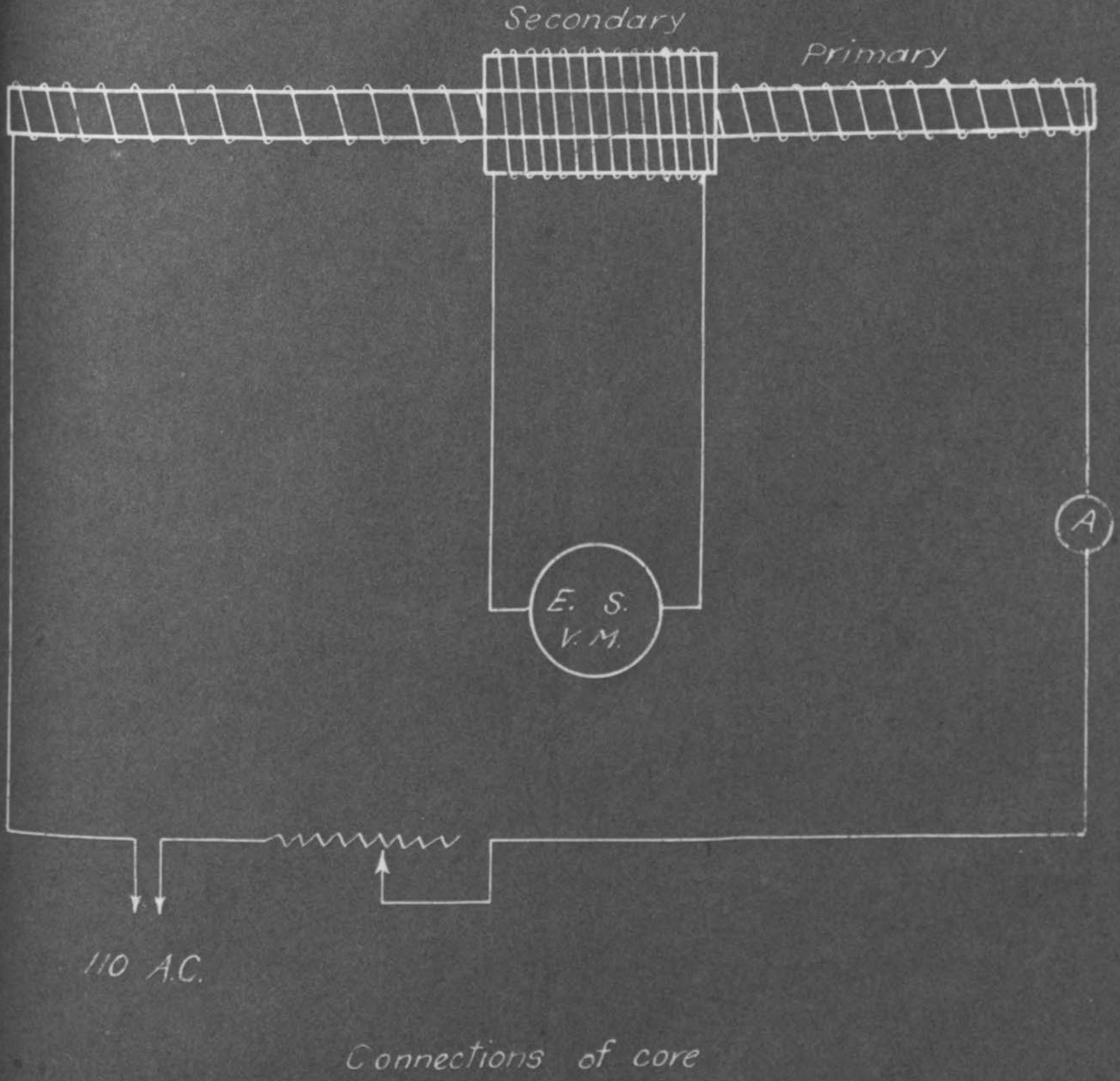
PLATE IV.



Bending Plate.

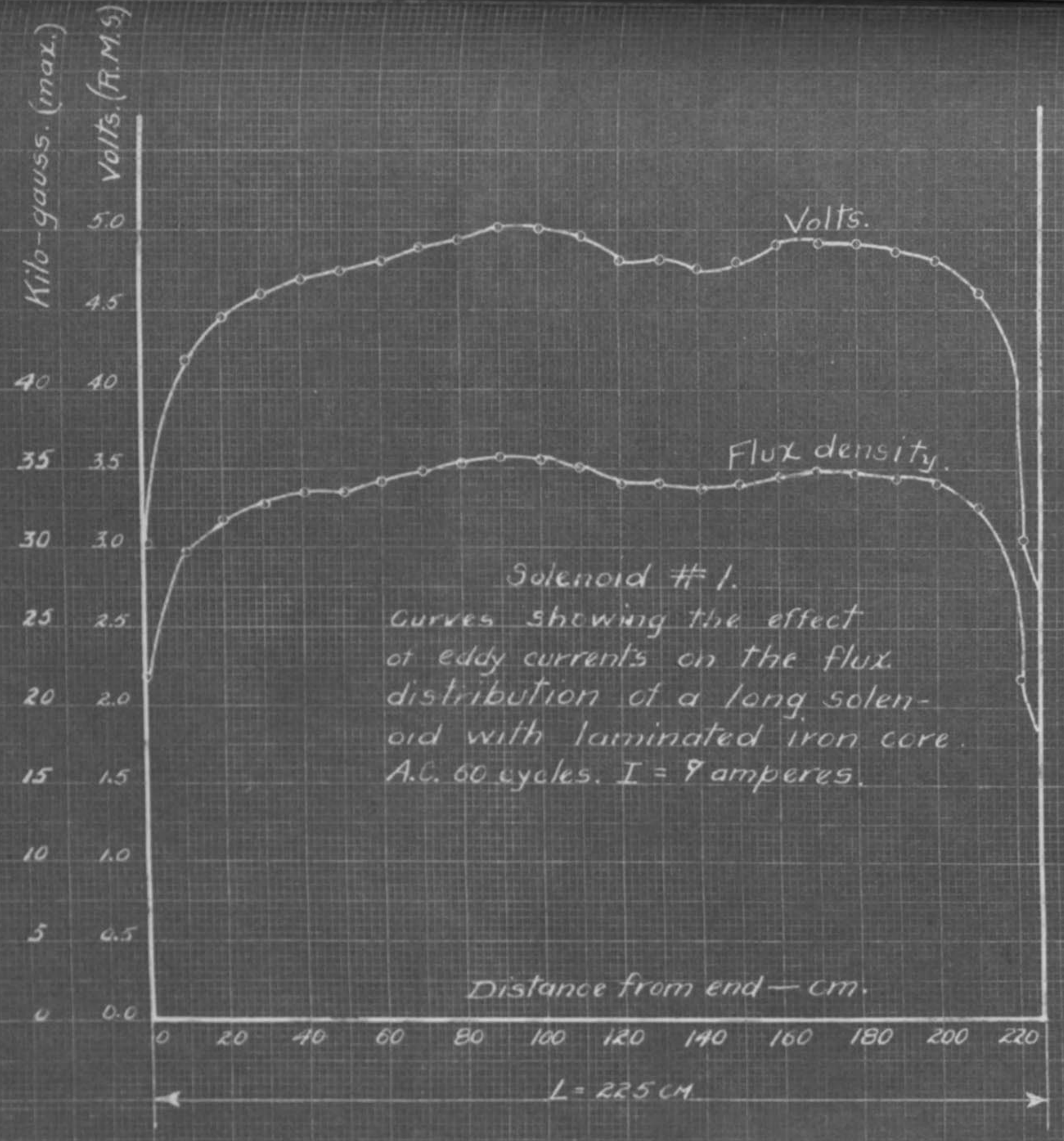
Used to vary the radius of curvature
of solenoids.

PLATE V.



Connections of core
and voltmeter.

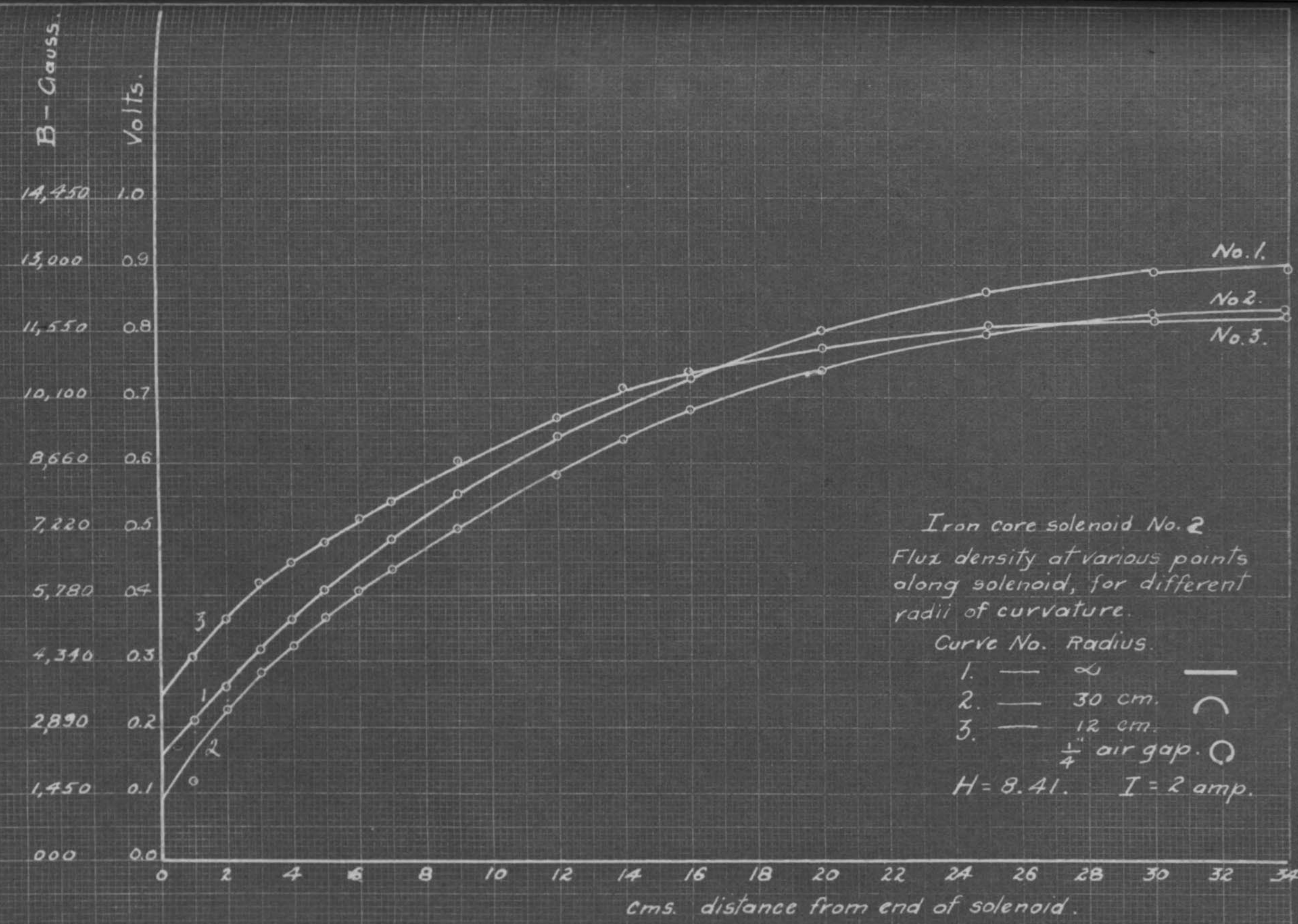
PLATE VI



Solenoid #1.
Curves showing the effect of eddy currents on the flux distribution of a long solenoid with laminated iron core. A.C. 60 cycles. I = 9 amperes.

PLATE VII.

PLATE VIII



Iron core solenoid No. 2

Flux density at various points along solenoid, for different radii of curvature.

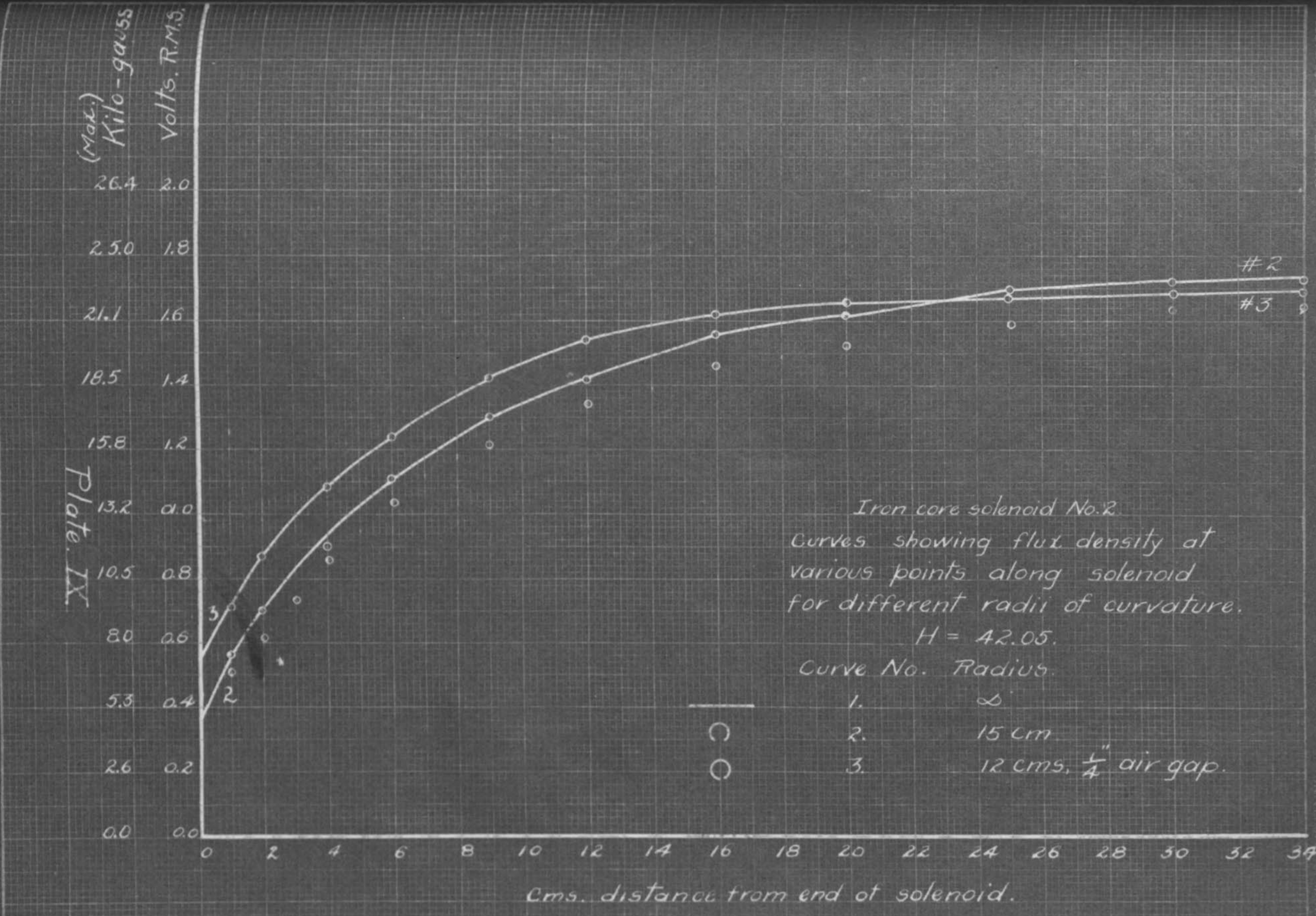
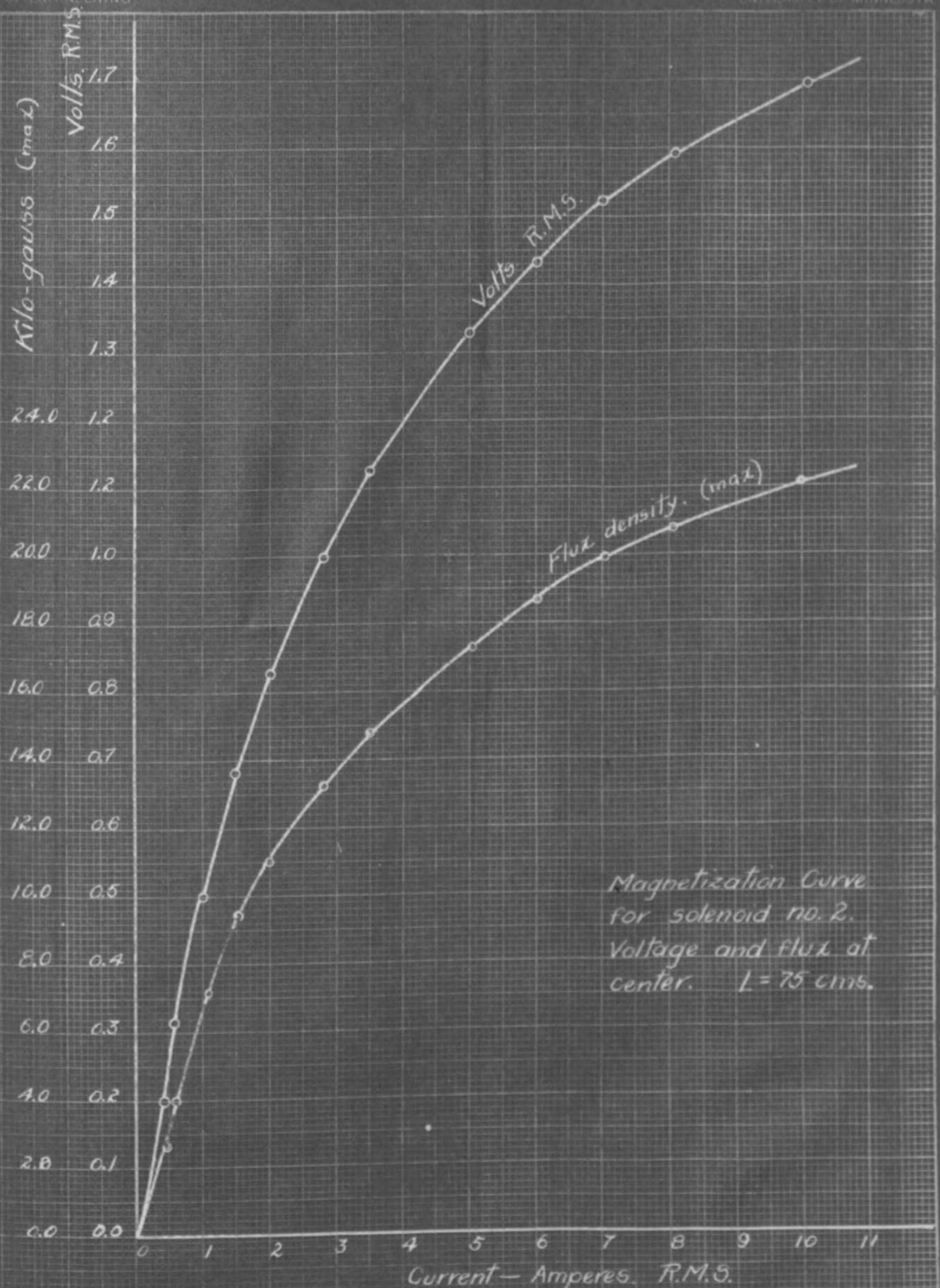
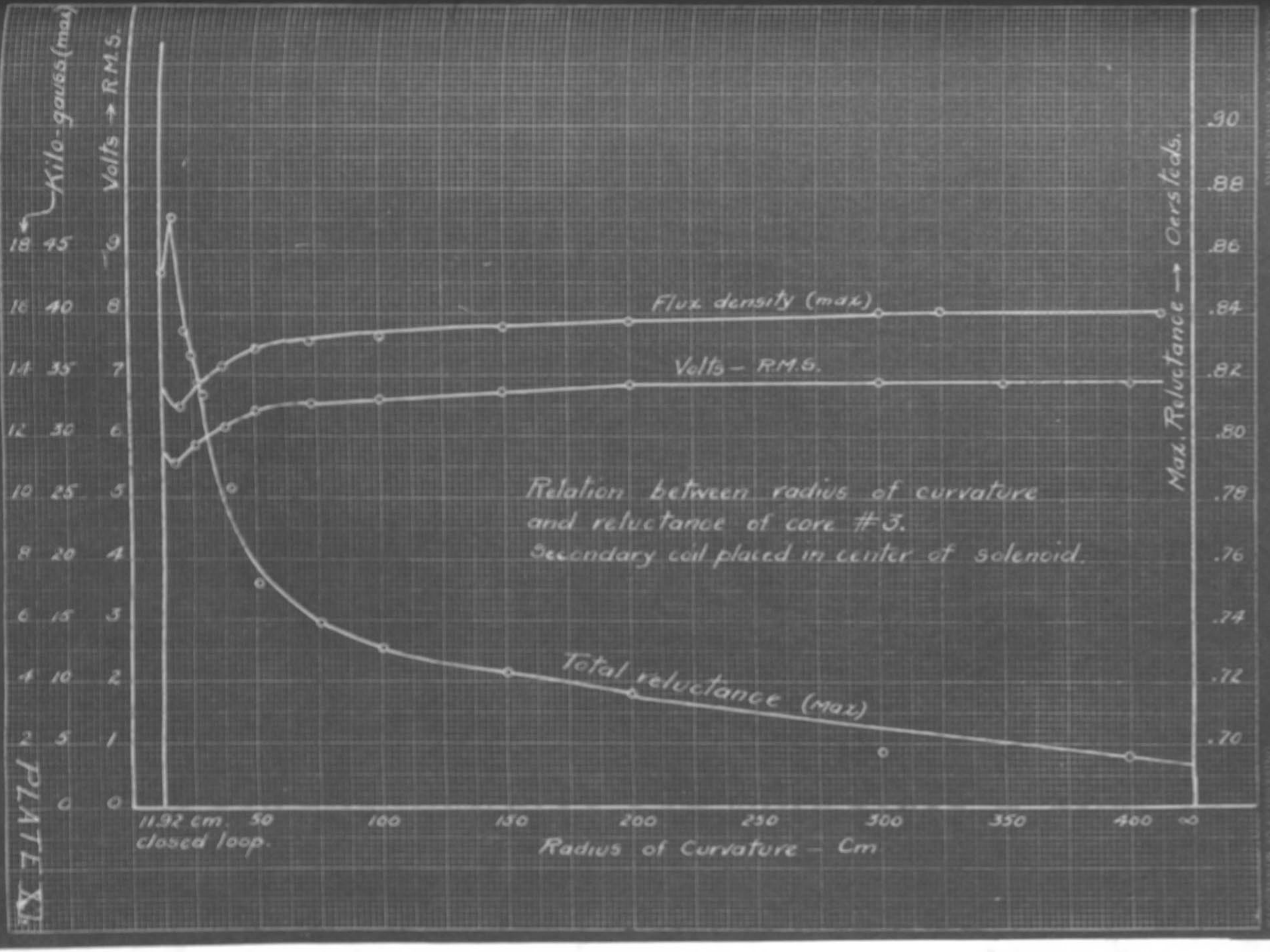

(Max.) Kilo-gauss
Volts. R.M.S.
26.4 2.0
25.0 1.8
21.1 1.6
18.5 1.4
15.8 1.2
13.2 1.0
10.5 0.8
8.0 0.6
5.3 0.4
2.6 0.2
0.0 0.0
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34
Cms. distance from end of solenoid.
#2
#3
3
2
Iron core solenoid No. 2.
Curves showing flux density at various points along solenoid for different radii of curvature.
H = 42.05.
Curve No. Radius.
1. ∞
2. 15 cm
3. 12 cms. 1/4" air gap.

Plate. IX.



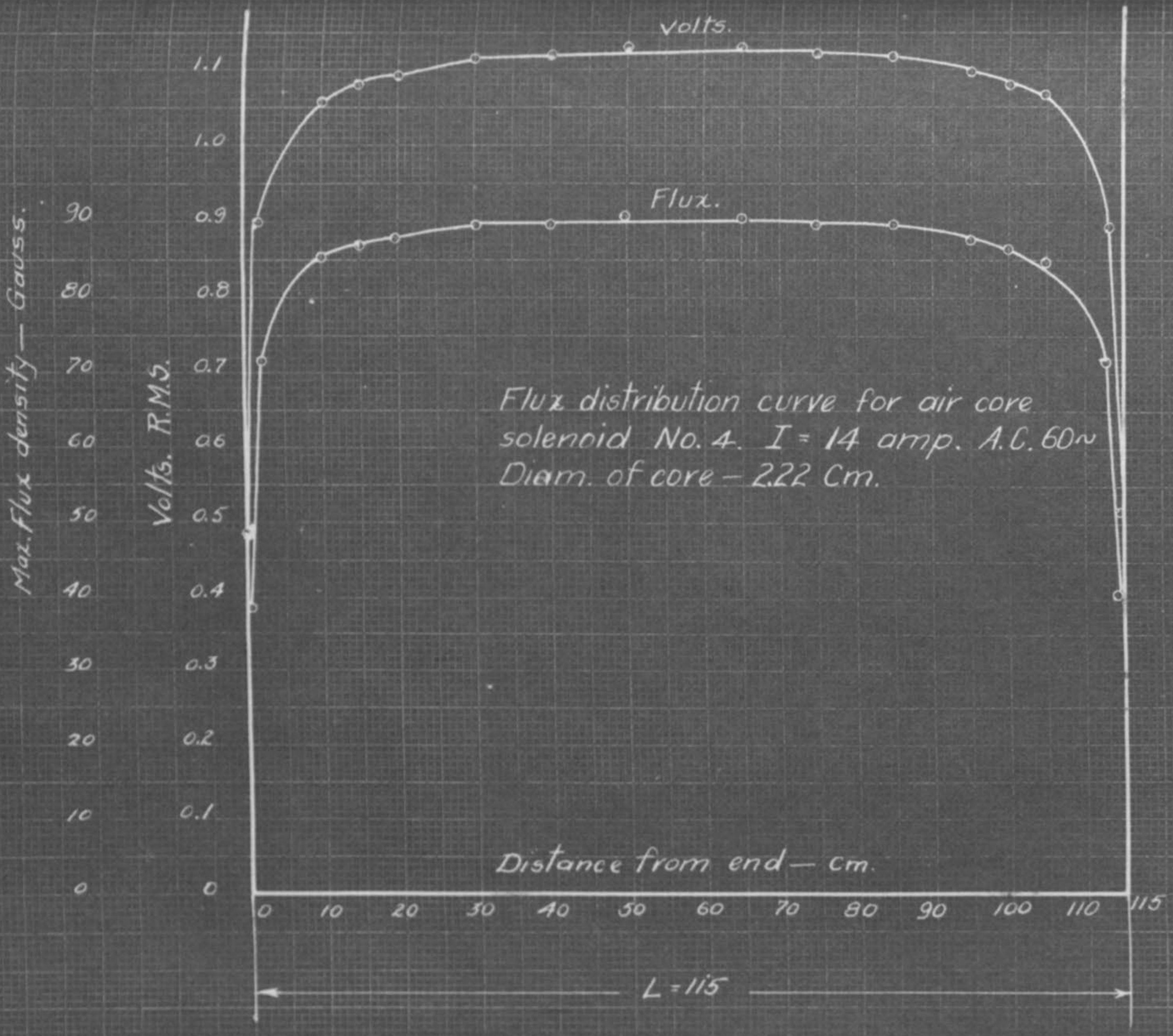
Magnetization Curve
for solenoid no. 2.
Voltage and flux at
center. L = 75 cms.

PLATE X



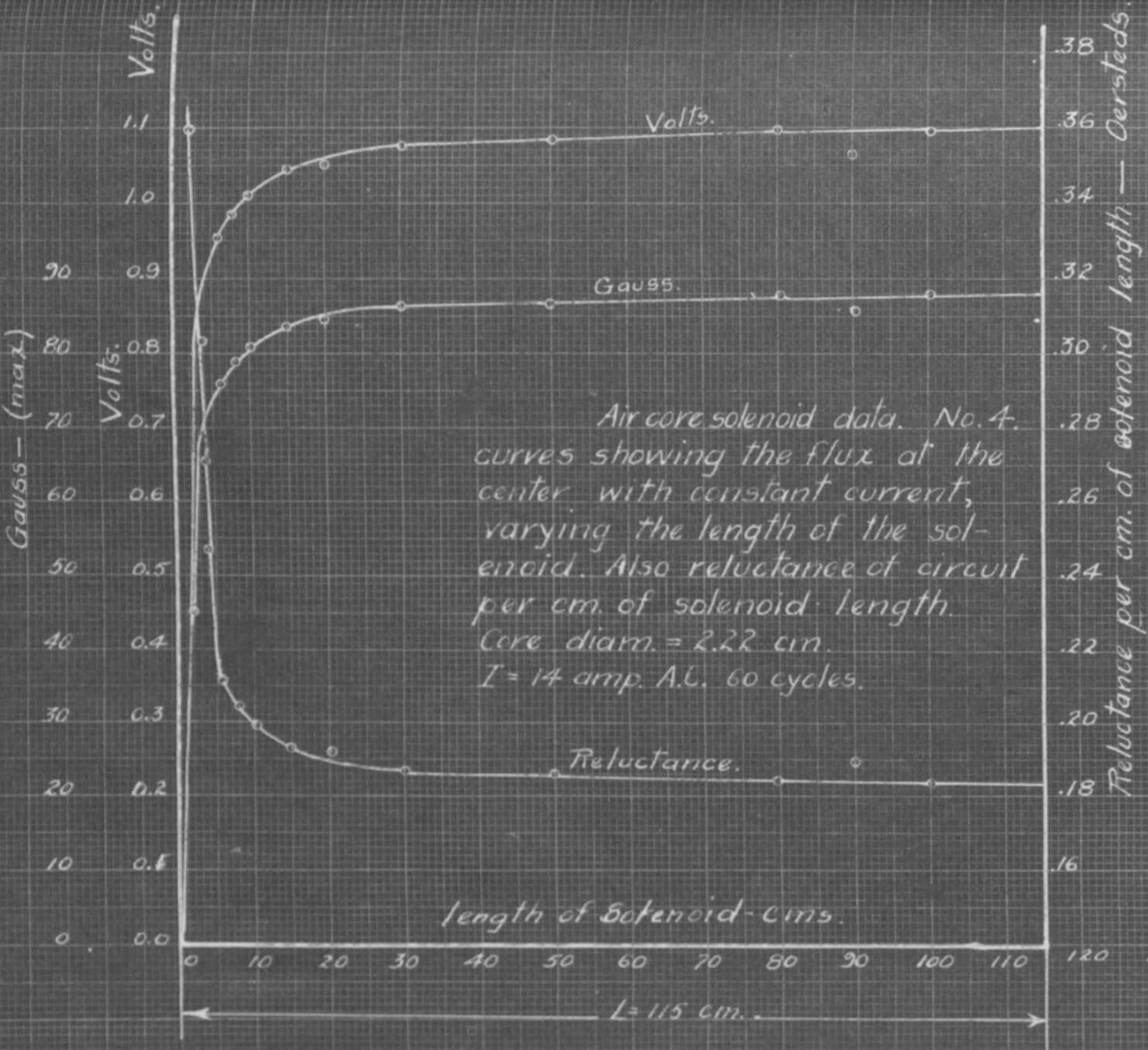
Relation between radius of curvature
and reluctance of core #3.
Secondary coil placed in center of solenoid.

PLATE XI.



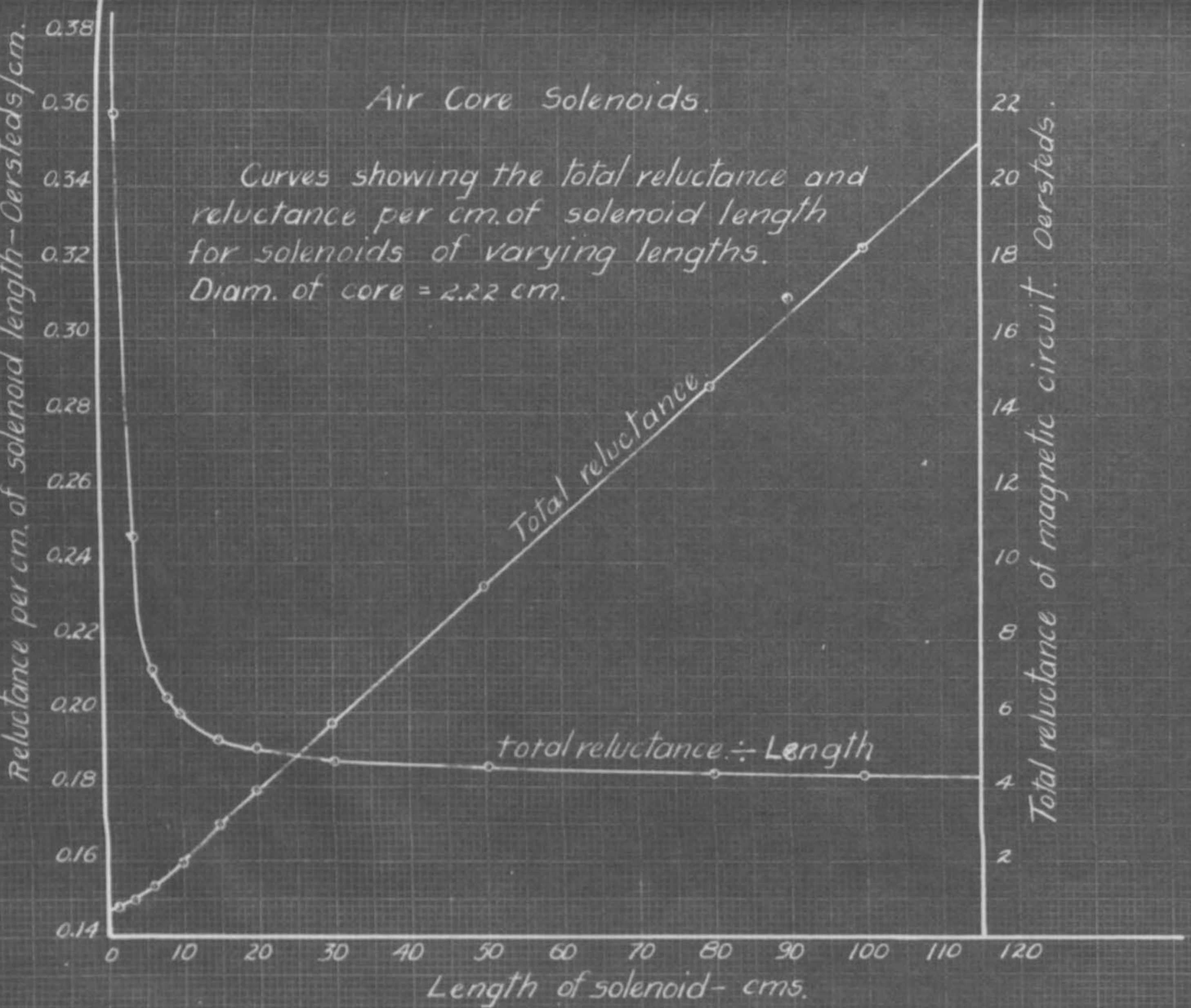
Flux distribution curve for air core solenoid No. 4. I = 14 amp. A.C. 60~
Diam. of core – 2.22 Cm.

PLATE XII



Air core solenoid data. No. 4.
curves showing the flux at the center with constant current, varying the length of the solenoid. Also reluctance of circuit per cm. of solenoid length.
Core diam. = 2.22 cm.
I = 14 amp. A.C. 60 cycles.

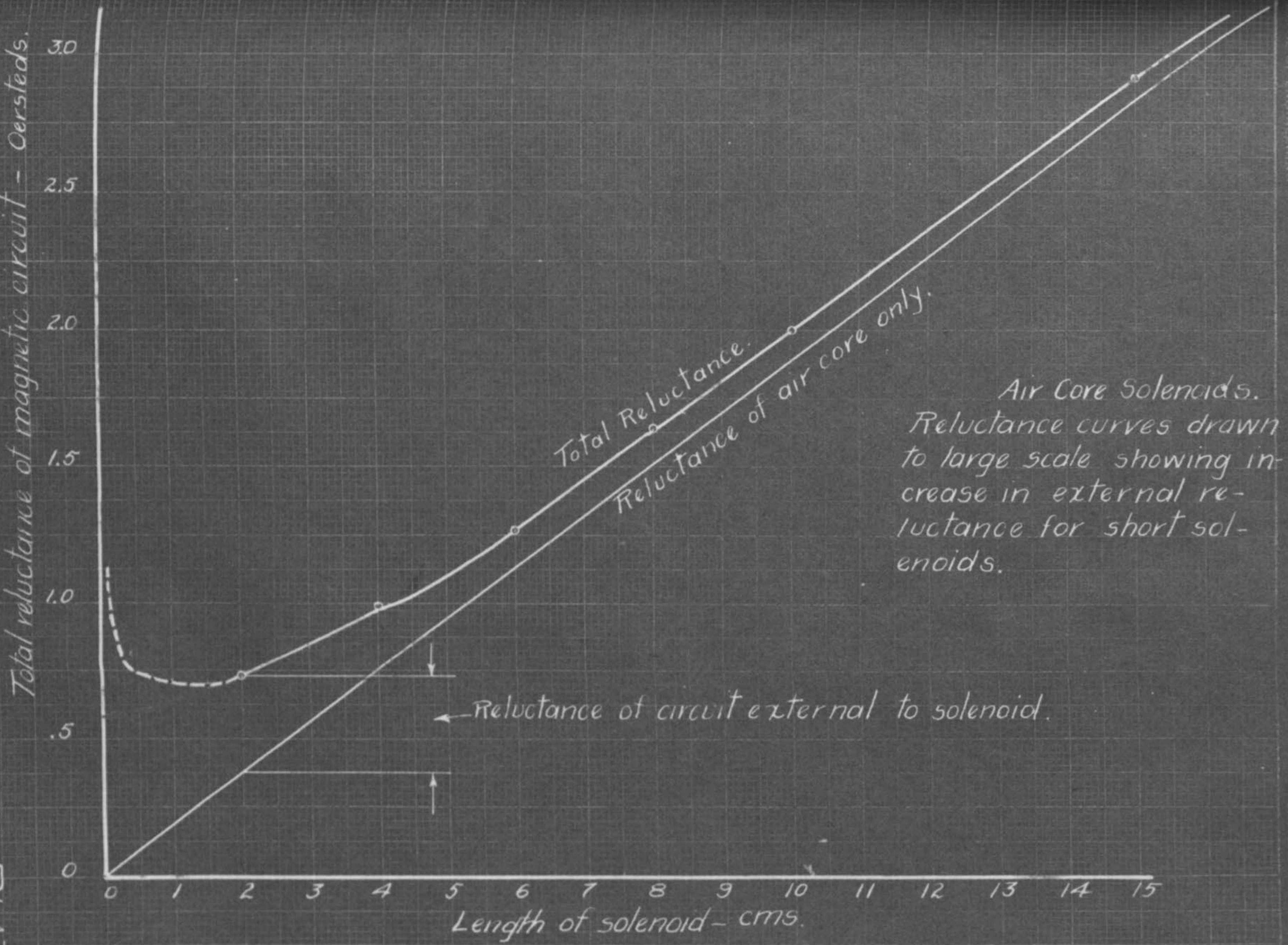
PLATE XIII.



Air Core Solenoids.

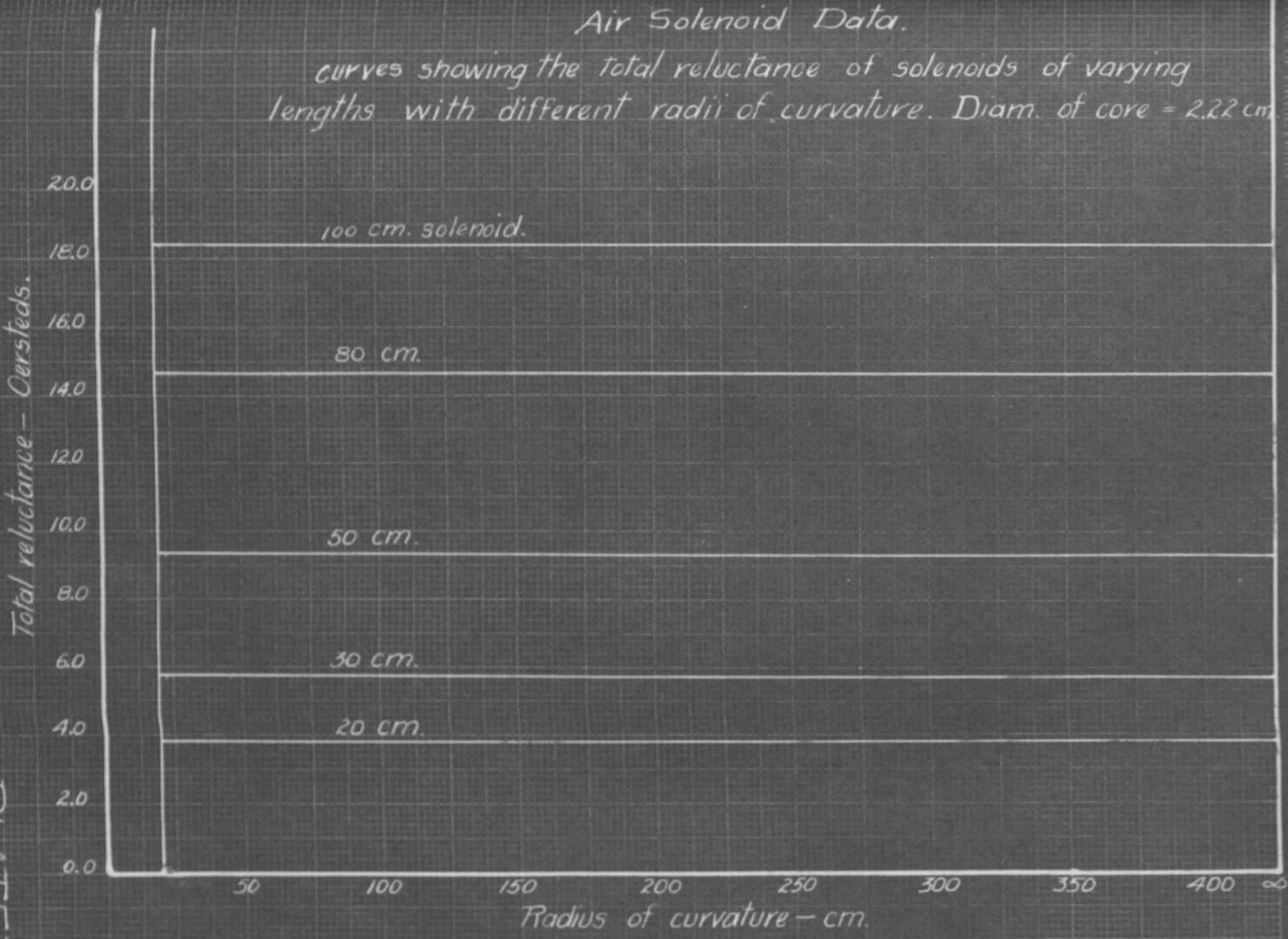
Curves showing the total reluctance and reluctance per cm. of solenoid length for solenoids of varying lengths.
Diam. of core = 2.22 cm.

PLATE XIV.



Air Core Solenoids.
Reluctance curves drawn to large scale showing increase in external reluctance for short solenoids.

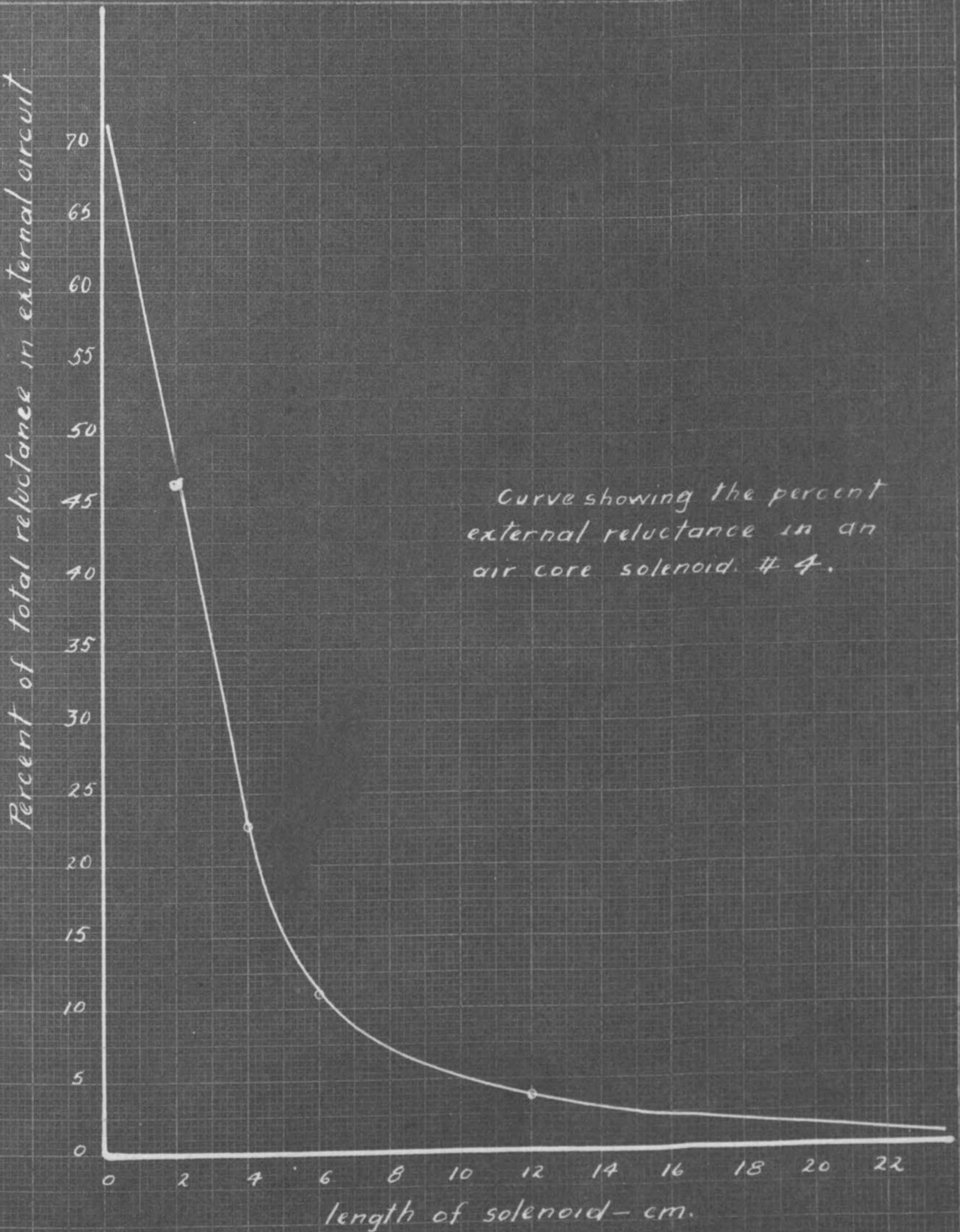
PLATE XV



Air Solenoid Data.

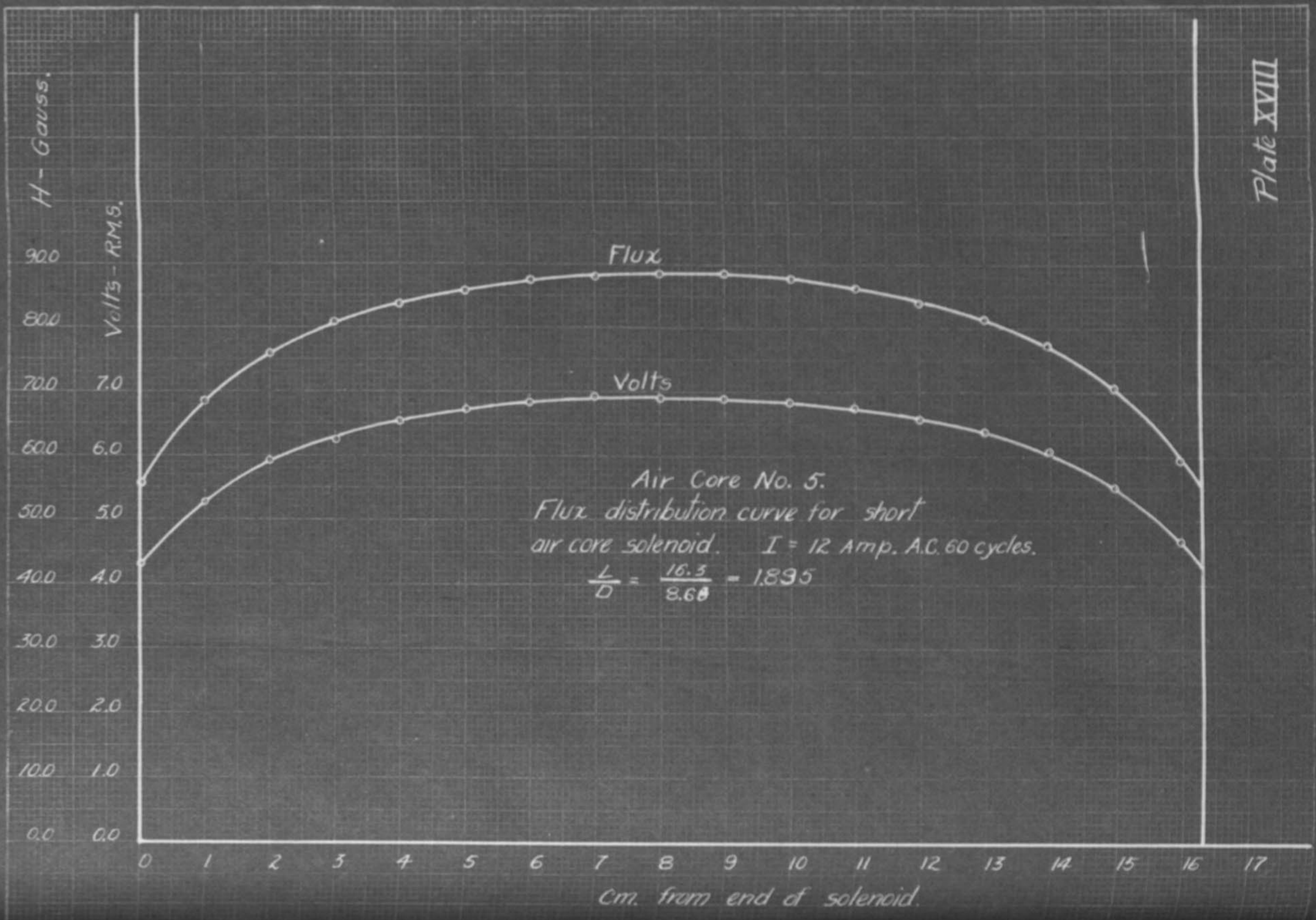
curves showing the total reluctance of solenoids of varying lengths with different radii of curvature. Diam. of core = 2.22 cm.

PLATE XVI.



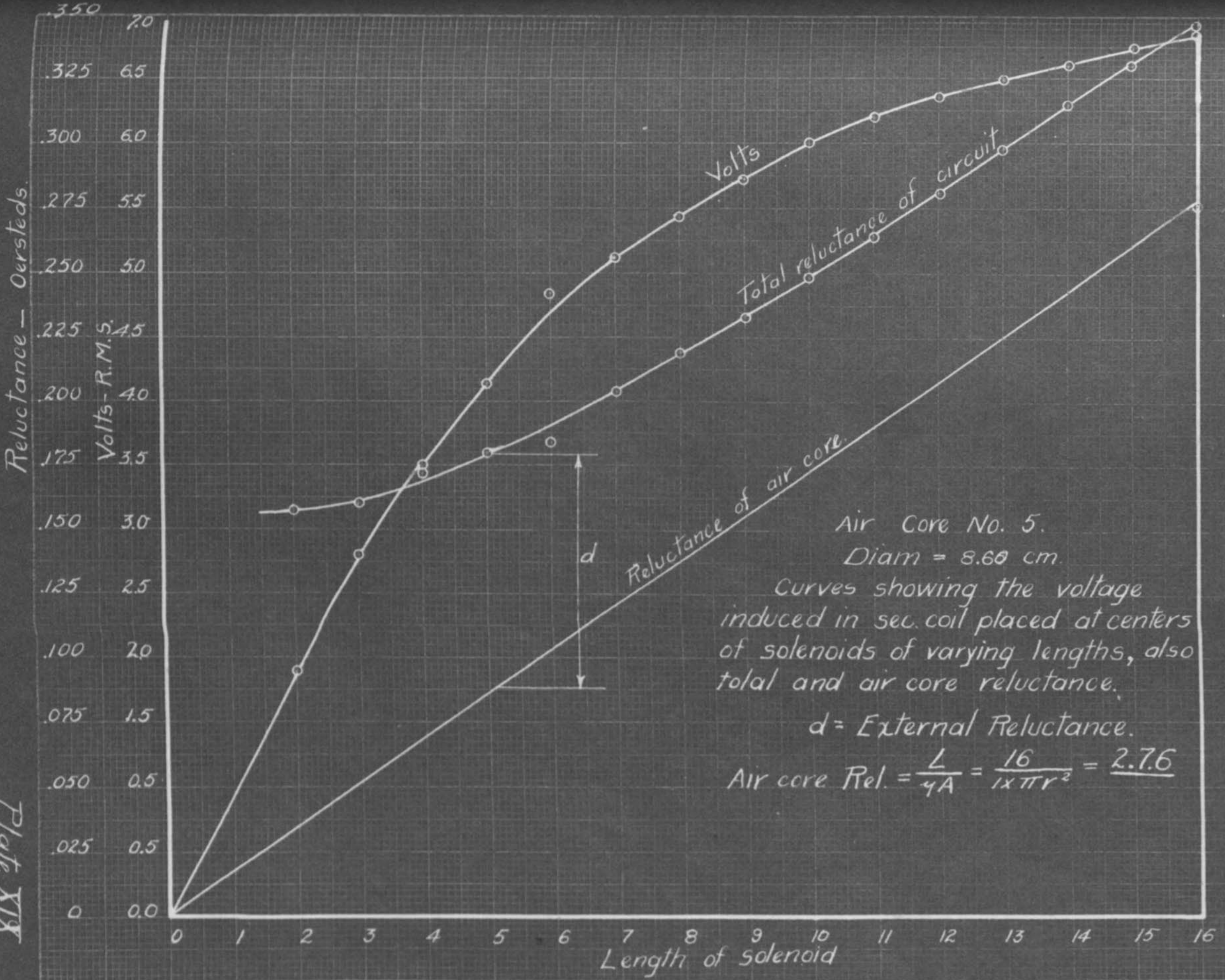
Curve showing the percent external reluctance in an air core solenoid. # 4.

Plate XVII



Air Core No. 5.
Flux distribution curve for short
air core solenoid. $I = 12$ Amp. A.C. 60 cycles.
$\frac{L}{D} = \frac{16.3}{8.60} = 1.895$

Plate XVIII

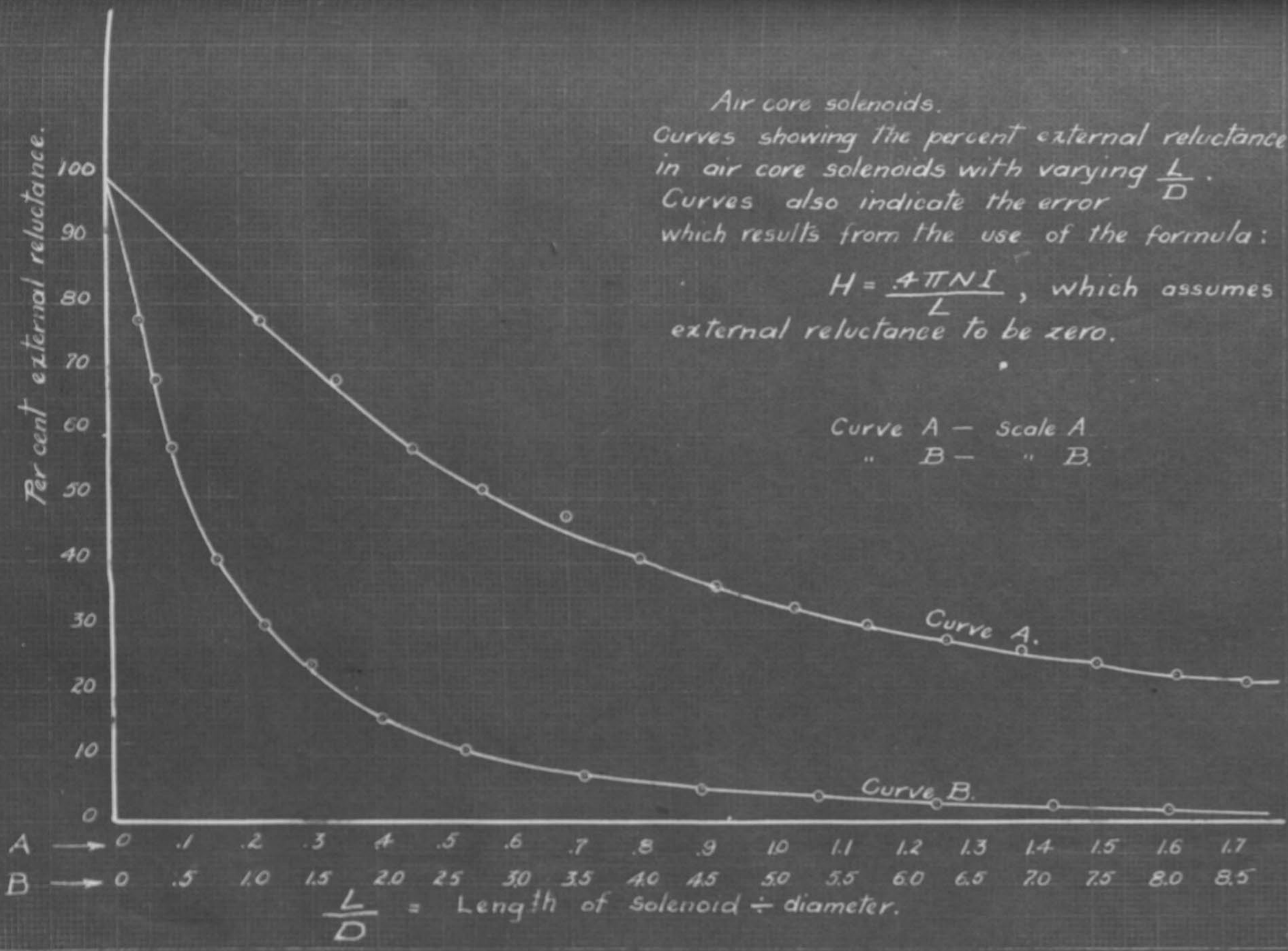


Air Core No. 5.
Diam = 8.60 cm.
Curves showing the voltage induced in sec. coil placed at centers of solenoids of varying lengths, also tolal and air core reluctance.

d = External Reluctance.

Air core $Rel. = \frac{L}{\mu A} = \frac{16}{1 \times \pi r^2} = \underline{2.76}$

Plate XIX



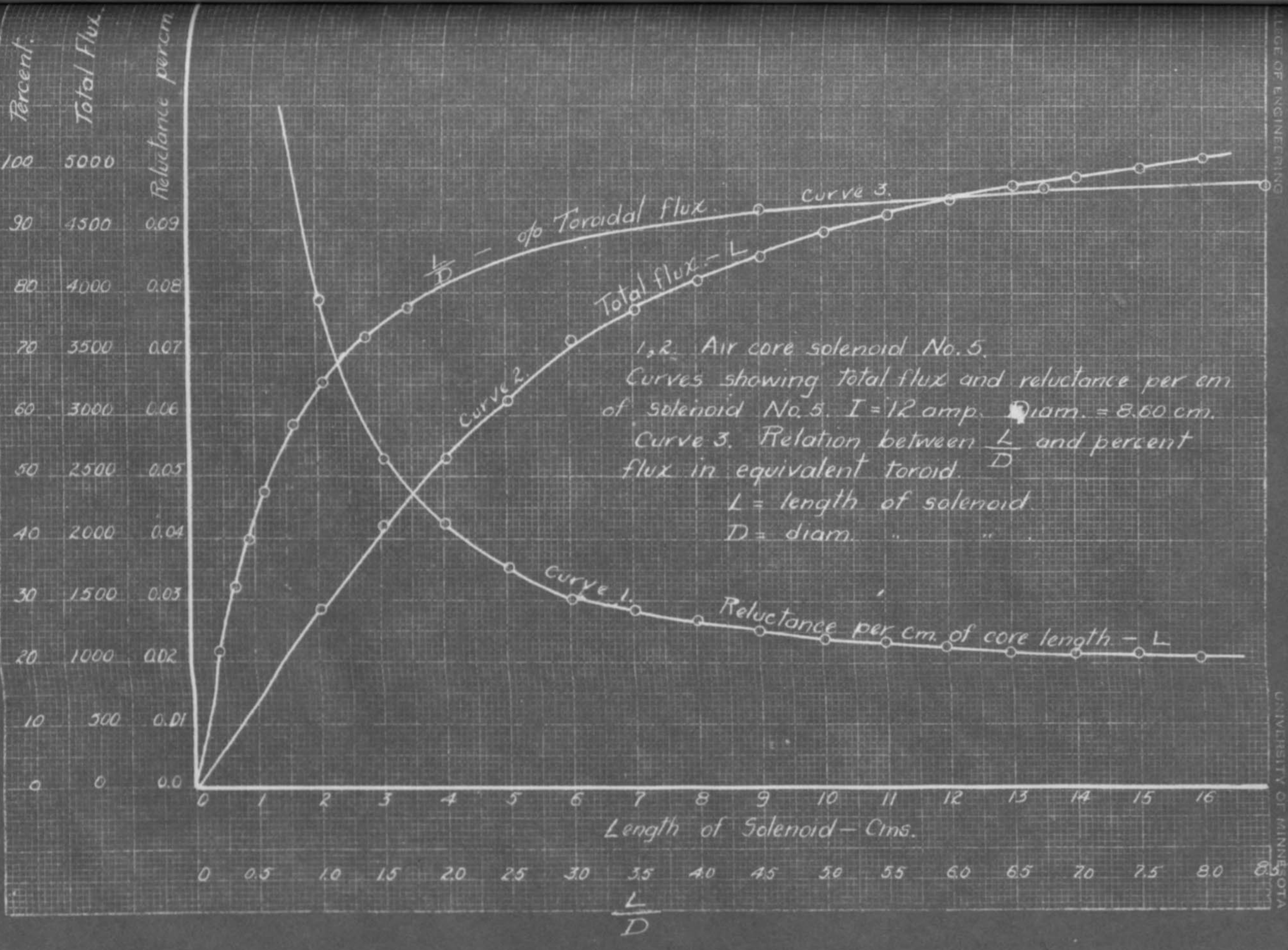
Air core solenoids.

Curves showing the percent external reluctance in air core solenoids with varying $\frac{L}{D}$. Curves also indicate the error which results from the use of the formula:

$$H = \frac{.4\pi N I}{L}$$, which assumes external reluctance to be zero.

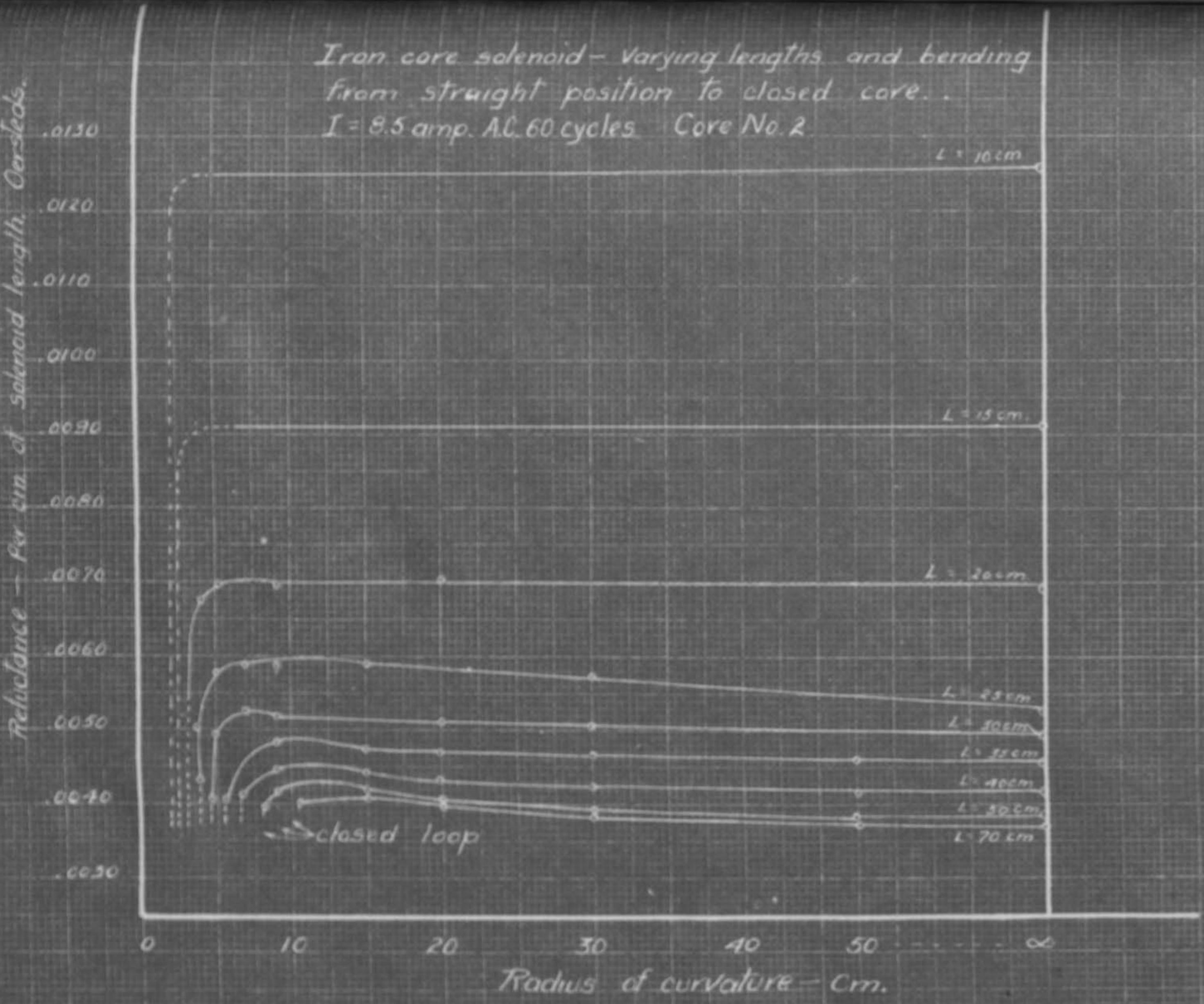
Curve A — Scale A
" B — " B.

Plate XX



1, 2. Air core solenoid No. 5.
Curves showing total flux and reluctance per cm of solenoid No. 5. $I = 12$ amp. Diam. = 8.60 cm.
Curve 3. Relation between $\frac{L}{D}$ and percent flux in equivalent toroid.
L = length of solenoid
D = diam. " "

Plate XXI



Iron core solenoid – Varying lengths and bending from straight position to closed core. I = 8.5 amp. A.C. 60 cycles Core No. 2

Plate XXII

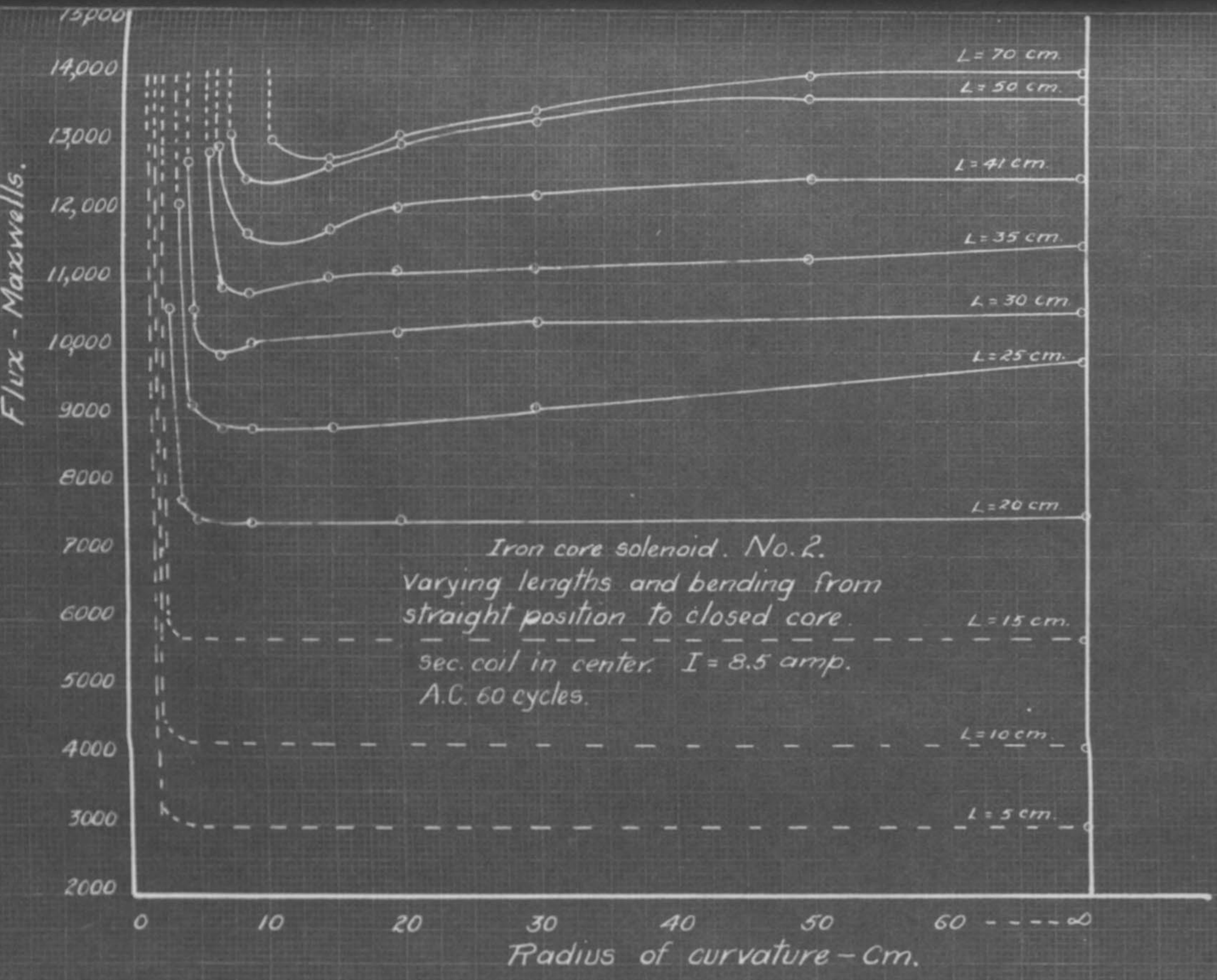
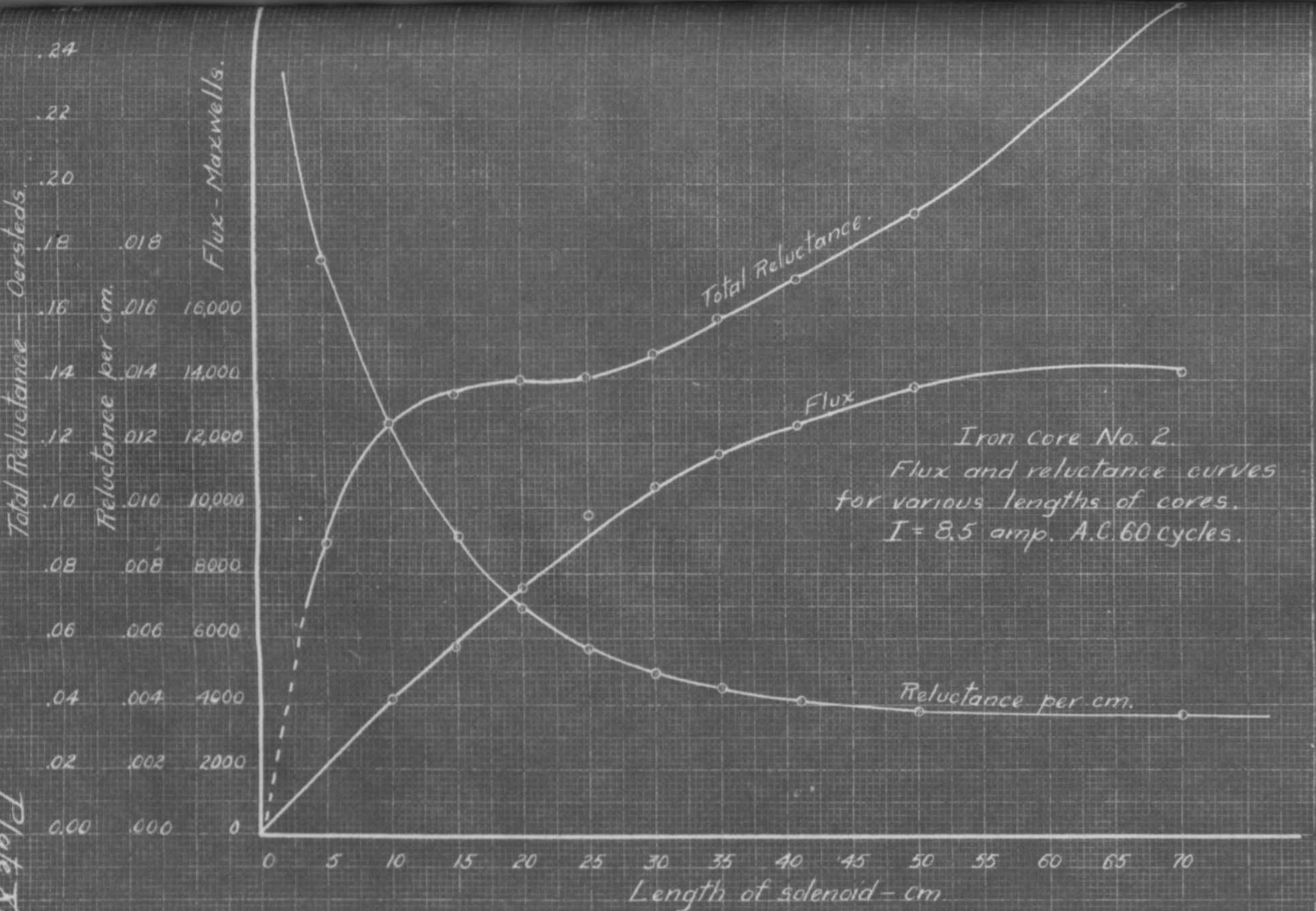

Iron core solenoid. No. 2.
Varying lengths and bending from straight position to closed core.
Sec. coil in center. I = 8.5 amp.
A.C. 60 cycles.
Flux - Maxwells.
15,000
14,000
13,000
12,000
11,000
10,000
9000
8000
7000
6000
5000
4000
3000
2000
Radius of curvature - Cm.
0
10
20
30
40
50
60 - - - - ∞
L = 70 cm.
L = 50 cm.
L = 41 cm.
L = 35 cm.
L = 30 cm.
L = 25 cm.
L = 20 cm.
L = 15 cm.
L = 10 cm.
L = 5 cm.

Plate XXIII



Iron Core No. 2.
Flux and reluctance curves
for various lengths of cores.
I = 8.5 amp. A.C. 60 cycles.

Plate XXIV

Table I. Plate II.

Calibration of Elec. static voltmeter.

Volts. R.M.S.	Deflection.
0.00	0.00 CM.
0.25	0.09 "
0.50	0.22 "
0.75	0.38 "
1.00	0.62 "
1.25	0.92 "
1.50	1.28 "
1.75	1.58. "
2.00	2.12 "
2.25	2.65 "
2.50	3.22 "
2.75	3.85 "
3.00	4.49 "
3.25	5.22 "
3.50	6.00 "
3.75	6.87 "
4.00	7.72 "
4.25	8.62 "
4.50	9.55 "
4.75	10.62 "
5.00	11.62 "
5.25	12.88 "
5.50	14.05 "
5.75	15.30 "
6.00	16.48 "
6.25	17.90 "
6.50	19.22 "
6.75	20.50 "

Table IIIa.

Effect of torsion on flux distribution.

Twist.	cm. def.	volts.
0°	21.32	6.85
90°	19.10	6.45
180°	18.30	6.35
270°	17.60	6.20
360°	16.00	5.90
540°	14.90	5.70
720°	14.00	5.50

Table II. Plate IV.

Magnetization curve of core No. 1. AC. 60 cycles. sec. coil in center.

Amp.	cm. d.	Volts.	ϕ_M Max.	B_M Gauss
0.8	0.10	0.10	125	710
1.15	1.96	1.90	2380	13500
1.40	3.85	2.72	3400	19300
1.90	5.45	3.28	4100	23200
2.33	6.36	3.55	4440	25200
2.78	7.11	3.78	4720	26800
3.19	7.78	4.00	5000	28400
4.14	8.80	4.23	5280	30000
5.10	9.66	4.47	5600	31800
5.96	10.25	4.60	5750	32700
6.85	10.73	4.70	5880	33400
7.99	11.23	4.80	6000	34100
9.98	11.90	4.93	6170	35100
12.10	12.47	5.08	6350	36100
13.80	12.87	5.13	6420	36500
15.60	13.38	5.25	6570	37400
18.20	13.75	5.35	6680	38000
21.35	14.30	5.45	6820	38800

Table III Plate VII.

Leakage data on core No. 1.
I = 9 amp. A.C. 60 cycles.

Cm. from end	cm. defl.	Volts	B_M Gauss.
00	04.50	3.00	21800
10	8.30	4.20	29800
20	9.45	4.47	31800
30	9.90	4.60	32600
40	10.20	4.68	33200
50	10.37	4.72	33500
60	10.70	4.80	34100
70	11.05	4.88	34600
80	11.35	4.95	35300
90	11.61	5.02	35600
100	11.55	5.00	35500
110	11.30	4.95	35200
120	10.70	4.80	34100
130	10.35	4.82	34200
140	10.42	4.75	33800
150	10.72	4.80	34100
160	11.05	4.90	34800
170	11.13	4.90	34800
180	11.06	4.90	34800
190	10.82	4.85	34400
220	4.70	3.05	21900

Table IV Plate VIII

Core No.2. bending data. Voltage of sec. coil. with varying radius of curve.

Position of sec. coil.	∞ radius Volts	30 cm. rad Volts	15 cm. rad. Volts	$\frac{1}{4}$" air g. Volts
0 cm.	0.220	.090	0.10	0.250
1 "	0.260	0.118	0.19	0.308
2 "	0.318	0.225	0.24	0.362
3 "	0.362	0.280	0.28	0.419
4 "	0.410	0.320	0.33	0.450
5 "	0.450	0.365	0.38	0.481
6 "	0.482	0.406	0.42	0.516
7 "	0.519	0.440	0.45	0.541
8 "	0.552	0.470	0.48	0.575
9 "	0.582	0.5015	0.52	0.601
10 "	0.640	0.527	0.54	0.622
12 "	0.690	0.582	0.59	0.670
14 "	0.728	0.635	0.64	0.715
16 "	0.860	0.679	0.68	0.738
25 "	0.860	0.797	0.78	0.809
30 "	0.888	0.822	0.80	0.812
35 "	0.888	0.832	0.82	0.838

Table V Plate IX

Core No. 2. High saturation. I=10

Position, Sec. coil.	Volts 15 cm. r.
0.00 cm	0.36
1 "	0.551
2 "	0.691
3 "	0.811
4 "	0.912
5 "	1.08
6 "	1.12
7 "	1.18
8 "	1.22
9 "	1.295
10 "	1.32
12 "	1.42
14 "	1.51
16 "	1.55
25 "	1.70
30 "	1.72
35 "	1.73

Table VI Plate X.

Magnetization Curve data for core No. 2.

amperes	Volts	B_m Gauss	O_m Max.
0.47	0.20	2650	1880
0.655	0.315	4160	2950
0.663	0.330	4350	3090
1.095	0.540	7120	5060
1.059	0.520	6860	4870
1.160	0.565	7470	5300
1.200	0.581	7680	5450
1.280	0.615	8120	5760
1.460	0.680	8960	6370
1.600	0.720	9500	6740
1.760	0.762	10050	7140
1.945	0.810	10700	7580
2.000	0.832	11000	7800
2.400	0.917	12100	8600
2.850	1.000	13200	9380
3.500	1.121	14800	10500
5.000	1.32	17300	12381
6.000	1.42	18700	13300
7.000	1.52	20,000	14200
10.000	1.680	20,800	15,700

Table V Plate IX

Core No 2. High saturation.

Position of Sec.	$\frac{1}{4}$" air g. Volts
0	0.56
1	0.72
2	0.87
3	0.99
4	1.10
5	1.20
6	1.23
7	1.31
8	1.39
9	1.42
10	1.48
12	1.53
14	1.60
16	1.61
25	1.68
30	1.68
35	1.68

Table IX. Plates XIII - XIV - XV.

Air core solenoid data. Varying lengths. I = 14 amperes - D.C. Solenoid No. 4.					Reluctance, Oersteds.		
length-cm.	Throw.cm	Volts	ϕ, maxwells	B, Gauss	Per cm.	Total	
2	3.16	.562	174	45.0	.359	.718	
4	4.60	.817	253	65.4	.247	.990	
6	5.35	.953	295	76.2	.212	1.265	
8	5.56	.988	306	79.1	.204	1.630	
10	5.69	1.010	313	80.9	.200	1.995	
15	5.89	1.045	324	83.8	.193	2.900	
20	5.91	1.050	325	84.0	.192	3.84	
30	6.06	1.075	333	86.1	.187	5.63	
50	6.08	1.082	335	86.5	.186	9.32	
80	6.17	1.097	340	87.9	.184	14.70	
90	6.00	1.065	330	85.3	.189	17.00	
100	6.16	1.093	339	87.6	.184	18.4	

Table VIII. Plate XII

Flux distribution of solenoid No. 4. I = 14 amp. D.C. Ballistic galvanometer.				
Pos. sec. c.	Deflection	Volts	ϕ_H Maxwell	B Gauss.
0	2.72	.481	149	38.5
2	5.02	.895	277	71.6
5	6.05	1.073	333	86.1
10	6.00	1.064	330	85.3
15	6.06	1.08	334	86.4
20	6.16	1.093	339	87.6
30	6.30	1.120	347	89.7
40	6.28	1.120	346	89.5
50	6.38	1.130	351	90.7
60	6.12	1.100	341	88.2
75	6.28	1.120	346	89.5
85	6.30	1.120	347	89.7
95	6.16	1.093	339	87.6
105	6.00	1.064	330	85.3
113	5.02	.895	277	71.6
115	2.72	.481	149	38.5

Table X

Plate XIV

Reluctance of Air core No. 4.			
core	R_e	R_t	% R_e
cms.	.000	0	0
0	All	0	100
2	.355	.75	46.6
4	.225	1.00	22.5
6	.130	1.25	10.4
8	.125	1.615	7.75
12	.100	2.35	4.25
15	.075	2.90	2.60
20	.060	4.00	1.50

TABLE XI PLATE XVIII

Flux distribution of Core #5 $I = 12$ A.C. 60~			
cm. from end	cm. def.	Volts	Φ_M
16	10.25	4.68	3510
15	14.05	5.50	4130
14	16.70	6.08	4565
13	18.55	6.36	4780
12	19.70	6.58	4940
11	20.70	6.73	5070
10	21.40	6.82	4960
9	21.85	6.90	5200
8	21.85	6.90	5200
7	21.85	6.90	5180
6	21.31	6.81	5130
5	20.55	6.70	5050
4	19.50	6.52	4920
3	18.07	6.25	4730
2	16.18	5.91	4470
1	13.15	5.29	4020
0	8.79	4.31	3250

TABLE XII PLATE XIX

$I = 12$ A.C. 60~ Variation of reluctance with length of core					
Length-cm	cm. def.	Volts	Φ_M	Rel/cm.	Rel.-total
16	20.7	6.85	5145	.0217	.347
15	20.7	6.74	5060	.0220	.330
14	19.9	6.61	4960	.0225	.315
13	19.2	6.48	4870	.0229	.298
12	18.7	6.35	4770	.0234	.281
11	17.3	6.20	4650	.0240	.264
10	16.5	6.00	4500	.0248	.248
9	15.4	5.72	4300	.0259	.233
8	12.6	5.43	4080	.0273	.219
7	12.6	5.12	3840	.0290	.203
6	11.1	4.85	3640	.0306	.183
5	8.5	4.14	3110	.0359	.179
4	6.2	3.50	2600	.0429	.172
3	4.1	2.80	2105	.0530	.159

TABLE XIII PLATE XX

Variation of percent external reluctance with L ÷ D in air cores.

Length	L ÷ D	Rel-total	Rel-external	%
16	1.86	.347	.0700	20.2
15	1.74	.330	.0700	21.2
14	1.63	.315	.0717	22.7
13	1.51	.298	.0725	24.3
12	1.40	.281	.0733	26.3
11	1.28	264	.0742	28.1
10	1.16	.248	.0750	30.2
9	1.04	.233	.0772	33.1
8	.93	.219	.0805	36.7
7	.82	.203	.0825	40.6
6	.70	.183	.0870	47.4
5	.57	.179	.0925	51.5
4	.46	172	.099	57.5
3	.35	.159	.108	68.0
2	.23	158	.1225	77.5

TABLE XIV PLATES XXII & XXIII

Variation of flux & reluctance with variation of rad. of curvature
$I = 8.5$

Radius	cm. def.	Volts	Φ_m	Rel./cm.	Rel.-total
		L =	70 cm.	.00369	
∞	6.85	3.77	14,170	.00372	.258
50	6.70	3.73	14,010	.00385	.260
30	6.30	3.60	13520	.00397	.269
20	6.00	3.50	13150	.00406	.278
15	5.75	3.42	12840	.00400	.284
11.12 closed	5.95	3.47	13040		.280
		L =	50 cm.		
∞	5.50	3.34	12,560	.00416	.170
50	5.48	3.34	12560	.00416	.170
30	5.25	3.27	12290	.00424	.174
20	5.10	3.22	12100	.00431	.176
15	4.90	3.14	11800	.00442	.181
9.2	4.80	3.12	11720	.00444	.182
7.9 closed	5.90	3.46	13000	.00491	.164
		L =	41 cm.		
∞	6.40	3.64	13690	.00381	.1905
50	6.40	3.64	13690	.00381	.1905
30	6.15	3.56	13380	.00390	.1950
20	5.90	3.46	13000	.00401	.2000
15	5.70	3.41	12800	.00407	.2040
9.2	5.40	3.32	12490	.00418	.2090
6.53 closed	6.00	3.50	13150	.00397	.1985
		L =	35 cm.		
∞	4.70	3.07	11520	.00452	.158
50	4.55	3.02	11350	.00459	.160
30	4.40	2.97	11160	.00468	.164
20	4.40	2.97	11160	.00468	.164
15	4.35	2.95	11090	.00471	.164
9.2	4.20	2.89	10860	.00481	.168
7.0	4.20	2.89	10860	.00481	.168
5.58 closed	5.80	3.44	12910	.00404	.142
		L =	30 cm.		
∞	4.00	2.82	10,600	.00492	.148
30.00	3.82	2.75	10320	.00505	.152
20.00	3.80	2.73	10250	.00510	.153
9.20	3.70	2.69	10100	.00517	.155
7.00	3.67	2.67	9970	.00523	.157
5.00	4.02	2.82	10600	.00492	.148
4.78 closed	5.70	3.41	12800	.00408	.122

TABLE XIV cont.

Radius	cm. def.	Volts	Φ_M	Rel./cm.	Rel.-total
		L=	25 cm.		
∞	3.2	2.49	9860	.00529	.132
30.00	3.0	2.43	9130	.00572	.143
15.00	2.9	2.36	8880	.00588	.147
9.20	2.9	2.36	8880	.00588	.147
7.00	2.9	2.36	8880	.00588	.147
5.00	3.0	2.40	9030	.00578	.145
3.99 closed	5.2	3.24	12170	.00430	.108
		L=	20 cm.		
∞	2.17	2.00	7525	.00694	.139
20	2.12	1.98	7440	.00702	.142
9.2	2.15	1.99	7480	.00698	.140
5.0	2.15	1.99	7480	.00698	.140
4.0	2.25	2.06	7740	.00675	.135
3.19 closed	4.00	2.82	10600	.00493	.099
		L=	15 cm.		
∞	1.3	1.53	5750	.00908	.135
		L=	10 cm.		
∞	.72	1.1	4130	.0126	.126
		L=	5 cm.		
∞	.5	.79	2960	.0176	.088

TABLE XV PLATE XXIV

Variation of reluctance with core length

$I = 8.5$ Rad. = ∞

Length	Volts	Φ_M	Rel./cm.	Rel-total
70	3.77	14170	.00369	.258
50	3.64	13690	.00381	.190
41	3.34	12560	.00416	.170
35	3.07	11520	.00452	.158
30	2.82	10600	.00492	.147
25	2.49	9350	.00560	.140
20	2.00	7525	.00694	.139
15	1.53	5750	.00908	.135
10	1.10	4130	.01261	.126
5	.79	2960	.01765	.088