THE UNIVERSITY OF MINNESOTA

GRADUATE SCHOOL

Report

of

Committee on Examination

This is to certify that we the undersigned, as a committee of the Graduate School, have given James Francis Mackell final oral examination for the degree of Master of Arts. We recommend that the degree of Master of Arts be conferred upon the candidate.

Minneapolis, Minnesota

May 28, 1921

W. F. G. Swann
Chairman

A. L. Underhill

Anthony Zeleny

Henry A. Erikson

W. T. Ryan

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THE UNIVERSITY OF MINNESOTA

GRADUATE SCHOOL

Report

of

Committee on Thesis

The undersigned, acting as a Committee of the Graduate School, have read the accompanying thesis submitted by James Francis Mackell for the degree of Master of Arts. They approve it as a thesis meeting the requirements of the Graduate School of the University of Minnesota, and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Arts.

W. F. G. Swann
Chairman

W. T. Ryan

A. L. Underhill

May 28, 19~~18~~ 21

INFLUENCE OF THE EARTH'S POTENTIAL GRADIENT UPON THE MEAN IONIC DENSITY OF THE ATMOSPHERE AS MEASURED BY THE EBERT ION COUNTER.

A Thesis

Submitted to the Graduate Faculty

of the

University of Minnesota

by

James F. Mackell

In partial fulfillemt of the requirements

for the

degree of

Master of Arts.

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When a conducting body is placed above the earth's surface and connected to earth by a conductor, an induced charge will appear upon the conducting body. This charge will, under ordinary conditions be negative owing to the positive sign of the atmospheric electrical field. The magnitude of the charge depends of course upon the magnitude of the field in question and upon the geometry of the conducting body.

Historical Introduction.

One of the most common methods of determining the ionic density of the atmosphere is by the use of the so-called Ebert Ion Counter, designed by Professor Ebert some twenty years ago. The general method employed is as follows: Air is drawn into a cylindrical condenser, the central member of which is charged to a potential of sufficient magnitude to enable it to attract to itself all the ions of opposite sign which enter the tube, the outer system being earthed. The charge upon the central system then falls off and the rate of discharge may be noted by the effect produced upon a charged electroscope or electrometer system. In this way the ionic density for the positive and negative ions may be determined if the rate of air flow and the capacity of the apparatus are known.

It is of course obvious that with the usual sign of the earth's potential gradient, a negative charge will appear at the top of the ion counter where the air is drawn into the tube. This may be expected to have an effect upon the

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number of ions entering the tube in unit time. Professor W.F.G. Swann[1] has shown from mathematical theory that the effect is to reduce the number of negative ions entering the instrument, while in the case of the positive ions, just as many ions are caught by the charge on the top of the instrument as corresponds to the excess of positive ions which are drawn to the system. He shows that if ΔN be the error in the number of ions which enter per second, and N the number which would have entered per second in the absence of the charge Q on the inside of the top of the apparatus, then

$$\frac{\Delta N}{N} = \frac{4\pi Q v}{\text{Flow of air per second}}$$

where v is the specific velocity of the ions.

In the same paper Professor Swann describes a series of four experiments in which he made a direct measurement of Q for known values of the potential gradient. His calculations show that in the case of the instrument used, the error should be of the order of 26% for a positive potential gradient of 70 volts per meter, and he naturally concludes that an error of this size might play an important role in accounting for the difference usually found between the ionic densities of the positive and negative ions. It is also apparent that inasmuch as this effect might be expected to become greater with increase of the potential gradient, the error in the case of high atmospheric potentials would become enormous. It is obviously of importance that a matter of this kind affecting measurements made by

1) Terrestial Magnetism and Atmospheric Electricity, Vol. 19, p.205, 1914.

observatories for periods extending over many years and costing thousands of dollars should receive the most searching inquiry.

The ordinary Ebert ion counter at present on the market is provided with a conical umbrella-like attachment,(the cap), which is fixed to the upper end. The original purpose of this cap was to keep insects and other foreign matter out of the apparatus. It however plays an important part in influencing the conditions as regards the induced charge on the open end of the apparatus,---not, it is true, as drastic a role as might at first sight appear; for, although it shields the open end from the potential gradient, it itself takes the place of this open end and acquires a charge which has an influence on the measurements. Swann's theory applies to an opening of any shape even to one constituted by the mouth of the cap,provided that the appropriate experimentally measured value of @ be established. Since in his experiment the purpose was merely to illustrate the theory by obtaining the order of magnitude of the correction, and since moreover, most of the work up to that time had been done with instruments without a cap, he for simplicity used an instrument of this type.

In 1916 E.H.Nichols[2)] published the results of some investigations carried out at Kew Observatory in order to test the conclusions reached by Swann. He analyzed the data at Kew taken over a period of several years, setting the measured density values for the negative ions at low potentials against the

2) Terr. Mag. and Atmos. Elec. Vol.21,p.87,1916.

corresponding densities for high potentials. He also carried out a set of experiments in which an attempt was made to eliminate the inductive effect by giving the outer system a positive charge after insulating it. Considerable effort was also made by Nichols to investigate the lines of equipotential in the neighborhood of the apparatus and this is discussed at some length in his paper. A summary of Nichols' conclusions is as follows:

1. An examination of the Kew data in the manner stated above does not show that an appreciable inductive effect is apparent.

2. Experiments made with two Ebert electrometers to find the effect on the recorded negative charge of partly neutralizing the induced charge by means of a battery showed no effect to substantiate Swann's theory.

3. Other experiments investigating the equipotential lines near the instruments, the wind velocity and the stream lines of the air motion in the vicinity of the opening, failed to support Swann's theory in the case where the cap was used, whereas, in the case where the cap was removed, the results indicated that the theory was sound, so that the theory seems to apply only to this case.

In Swann's[3)] reply to this criticism as regards the cap, he points out that his theory is applicable to any form of opening, even to one in which the cap takes part in determining the form of the opening, but that the experimentally measured

3) Terr. Mag. and Atmos. Elec. Vol.21, p.99, 1916.

value of Q for the opening must be determined in order that his formula may be applied, and it is not legitimate to draw quantitative conclusions as to what the theory has to say as regards an instrument with a cap from data obtained by using an instrument without one.

Although Swann's experiments upon an instrument without a cap do not claim to apply to an instrument with one, it appears that some of Nichols' data intended as evidence that the error is small in the case of the instrument with a cap actually support the reverse conclusion when interpreted as it appears they should be. Thus as regards Nichols' experiments concerned with the determination of the equipotential surfaces in the neighborhood of the cap, and from which it was argued that the field at the open end would not be sufficient to prevent the entrance of ions which are carried by air with an appreciable velocity, it appears as Swann has pointed out that the point involved is not as to whether an ion can or can not enter under the opposing influence of the field, but whether the rate at which the ions enter is appreciably reduced by the field. Taking Nichols' own data in this matter, Swann argues that these when properly interpreted, lead to the conclusion that, even with the cap, the error due to the potential gradient may be of an order of magnitude fully as great as he had obtained with an instrument without a cap.

In view of the above considerations and of the very large amount of work that has been done, and is being done, with instruments of this type, it seemed altogether fitting that a direct set of measurements be made with a capped instrument to consider the point in question. With this idea in view, the present work was commenced in the spring of 1920 at the suggestion of, and under the direction of Professor W.F.G. Swann, at the University of Minnesota.

While measurements were being made and while data organization was in progress, a communication was received from Dr.Harold Norinder of the University of Upsula,Upsula Sweden, to the effect that a similar piece of work was being carried on at that place, and the data produced indicated a substantiation of Swann's theory. While this paper was being prepared and after the material submitted herewith had been collected and organized, Dr.Norinder's paper appeared in final form. Thus a valuable means is offered the author for comparative purposes.

Dr. Norinder[4] carried out his experiments in two distinct parts,one set in the laboratory in which he produced an artificial potential gradient between parallel plates,and another set in the open air. In both instances he found a decided effect in the case of the negative ions but no appreciable effect in the case of the positive ions. His values for the apparent error range from four per cent to fifty-eight per cent but,while his results show a tendency of the error to follow

4) Arkiv for Mathematik,Astronomi,and Fysik. Bd.15, No.2.

the potential gradient, the evidence does not appear strong enough to be conclusive in this respect. However, it appears from his paper that there is no doubt that an appreciable effect is present in the case of the negative ions, while in the case of the positive ions no error is apparent.

Experimental Procedure.

Two ion counters of the Ebert type were made and care was taken to have them as nearly exact duplicates as possible. The general shape and proportions of these instruments are shown by figure 1. The dimensions were approximately the same as those of the Ebert type commonly sold by instrument makers. Chamber E was a metal cubical box with an edge of 10 centimeters. The outer cylinder,B, was 50 centimeters long and 3 centimeters in diameter, while the inner cylinder,C, was 57 centimeters long and 0.5 centimeters in diameter. The inner cylinder,C, was set into hard rubber insulation,F, and surrounded by a guard ring,G,set in hard rubber,which in turn was bound by a brass ring soldered to the lower part of the chamber,E.

The two instruments were mounted about two meters apart on a board about five meters long with cross arms two meters long so as to enable a hemispherical cage to be placed over either instrument.(Figure 2.) The cage was two meters in diamater and was made of wire netting with a mesh of about two centimeters. As it was found inconvenient to mount the instruments over the ground, and as it obviously makes no difference whether the ion counter cylinders occupy vertical or horizontal positions, so long as their lengths are parallel to the field, the board was mounted vertically in the open air at a distance of one meter from the East wall of the Physics Building and about midway between the ground and the roof. The instruments were thus in a vertical position with respect to the wall of the building, about eight meters from the ground and the same distance from the roof, (Figure 2 and photograph). Exhaust pipes leading to the lower part, E, of either instrument led to a suction fan inside the building, and the velocity of the air at the opening of the capped ends was observed at intervals by means of an annemometer. This was found to be fairly constant and of the order of two meters per second. The wire cage was supported by a truss from the roof and could be placed over either instrument at will by a system of pulleys. This cage was earthed by a wire leading to a water pipe as were the outer parts of the ion counters. The inner systems were insulated and protected by guard rings, which were kept at the same potentials as the inner systems themselves.

Conducting wires shielded in metal pipes, (Figure 2), from outside effects led to an electrometer on the inside. The electrometer and keys were of course shielded in a metal case kept at zero potential. The needle of the electrometer was kept at a potential of $\pm$125 volts, while the case of the electrometer was maintained at a potential of $\pm$ 250 volts. One pair of quadrants was connected to one inner system, and the other pair to the other inner system, (Figure 2). By means of the key, K_1, the quadrants were connected to the case and were charged to a potential of $\pm$250 volts. In carrying out an experiment, the cage being absent from both instruments, K_1 was opened thus separating the quadrants from the case and leaving a potential of $\pm$ 250 volts upon the inner systems and the quadrants. K_2 was then opened, separating the quadrants. The fan being in operation, an approximate balance was established by arranging the caps so that the respective air flows into the instruments were such as to result in a very small rate of deflection of the electrometer needle. This balance was made perfect to within three or four per cent of the effect of the potential gradient which was under investigation,and was therefore within the range of probable experimental error.

The cage was placed over first one of the instruments and then the other. In the case of the negative ions,the application of the cage resulted in a considerable rate of deflection, ΔR, of such a sign as to indicate an increase in the rate at which ions were being received by the instrument. In the case of the positive ions, no appreciable change was produced by the cage. Readings were always taken over a fixed range of the scale, and the whole insulated system was brought to a potential of $\pm$250 volts after each observation so as to prevent it from ever departing far from its initial value.

For the purpose of expressing the error in the measured ionic content, it was necessary to perform experiments in which a measurement was made of R, the total rate of alteration of potential of one instrument when the other was earthed. This was done with first one, and then the other earthed,both being unshielded in each case. It was further necessary to determine reduction factors to reduce the rates of deflection, ΔR and R to comparable units since it would not be quite correct to assume that the capacity of one of the electrometer systems was the same when the other was insulated and when it was earthed.

Since in the main experiments, the procedure was adopted of measuring the time for a spot of light to move over a fixed range of the scale, which was the same for all experiments, all that was necessary was to determine for each case the quantities of electricity corresponding to this deflection. In order to do this an experiment was performed as follows: The needle of the electrometer system was kept at a potential of 125 volts and the electrometer case at zero potential. A calibrated Gerdien condenser was then connected to one of the quadrants as shown in figure 3.

Suppose it was desired to determine the quantities corresponding to the cases where both electrometer quadrants were insulated; The procedure was to disconnect the quadrants and vary the potential applied to the condenser until the standard deflection, 20 centimeters was obtained. Then if C represents the capacity of the condenser, V the potential applied to the condenser, and v the rise of potential of the electrometer, the quantity, q_1, of electricity corresponding to the rise, v, is given by $C (V - v)$. Repetition of the experiment with the condenser connected first to one quadrant system and then to the other, the quadrant not connected to the condenser being in each case earthed, gave the values, q_2 and q_3 , corresponding to these cases in the main experiments. The values obtained in this way were as follows:

Both systems insulated,-------------- $q_1 = 0.0400$

A insulated, B earthed,-------------- $q_2 = 0.0411$

B insulated, A earthed,-------------- $q_3 = 0.0417$

The expressions for the percentage error in the measured ionic content resulting from the potential gradient are given by

$$\frac{q_1 / t_1}{q_2 / t_2} \times 100 \text{ , and } \frac{q_1 / t_1}{q_3 / t_3} \times 100;$$

for A and B respectively; t_1, t_2, and t_3 being the respective times for the spot of light to move over the fixed range of the scale. The error in the above case is estimated on the uncorrected values of the quantities. If calculated from the corrected quantity values, we have for the correct percentage error,

$$\left(\frac{\Delta N}{N}\right)_A = 1 / \left(1 + \frac{q_2 \; t_1}{q_1 \; t_2}\right) \text{ and } \left(\frac{\Delta N}{N}\right)_B = 1 / \left(1 + \frac{q_3 \; t_1}{q_1 \; t_3}\right) .$$

which are the values recorded in the tables.

The error introduced by the cage owing to the induced charge which will obviously appear upon it has been investigated by Professor Swann[5] and has been shown to be comparatively of small order of magnitude, amounting to about four per cent for a wind velocity of one meter per second. It may be observed moreover that if the error were at all appreciable, it would only serve to make the results shown by the present experiments

5) Terr.Mag. and Atmos. Elec. ,Vol.19,No.4,pp. 210-212.

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too small rather than too large in percentage of error due to inductive action.

The effective value of the potential gradient influencing the apparatus was measured at the time of each observation by means of a pollonium collector fastened at the center of a wire supported vertically with respect to the ground, one and one-half meters from the wall of the building and about five meters to the side of the apparatus. The wire led to a calibrated electroscope which was read at intervals during each observation, and a mean value was taken in each case. Since the electroscope was not sensitive to values less than 30 volts per meter, no observations were obtained for gradient values less than this.

Discussion of Results Obtained.

The results obtained may be readily seen by a study of tables I, II, III, and IV, and the accompanying curves, 1, 2 and 3. The potential gradient values given in tables I and II for the negative ions are grouped by tens, i.e.,30 to 39, 40 to 49, etc.volts per meter. It may also be pointed out in this connection that the values given for the measured potential gradient are the means of at least two readings taken during each of the observations. In some of the cases of course the gradient varied phenomenally during a reading, but in the most drastic cases of this sort the readings were discarded because of the large probable error in the mean. The values recorded here are therefore those in which the fluctuations were small.

The days picked for taking observations were in the main fine clear days on which the humidity conditions were nearly perfect for such work. It was found that in mid-summer the high relative humidity and other objectionable features introduced errors so great that no reliability could be pleced upon the data. For each of the ranges of ten volts per meter of the measured potential gradient enough readings were secured to obtain fairly accurate means for all the days. The readings were taken in the main between the hours of 9:30 A.M. and 6 P.M. of each day and in most cases during the greater part of this interval as it was desired to make as great a use as possible of the good days.

For the sake of comparison with results obtained for the ionic content of the air per unit volume by investigators in the past, the values of N_+ and N_- obtained with the respective instruments shielded are included in the tables. In calculating these, the velocity as obtained by an annemometer was used. This was measured only once for each set of experiments as it was experimentally verified that this value did not vary more than ten per cent during all the observations. The mean value was found to be 2.0 meters per second.

A glance at the tables will show that the values for $\Delta N_- / N_-$ have a decided tendency to follow the potential gradient, mean values ranging from 16.9 per cent to 26.5 per cent between the measured values of 30 volts per meter and 120 volts per meter of potential gradient. The values for the lower instrument are slightly higher than the corresponding values for the upper system, but in general they follow the same course. The fact that the two instruments were not located at the same spot and that the wall of the building can not be considered absolutely plane may help to explain this small discrepancy. It must also be admitted that this small difference is none too great to be attributed to experimental error, but the fact that it is always in the same direction precludes this explanation as likely.

With regard to the objection offered by Nichols, (1), cited on page 4, this was based upon the fact that the mean values for the positive and negative ionic content did not show any marked variation with the potential gradient, such as might be expected if the gradient affected N_- and not N_+ , it will be observed that the ionic content examined by Nichols was the ratio $(N - \Delta N)_+ / (N - \Delta N)_-$. A glance at the table of mean values, (Table IV), shows that this quantity is practically independent of the gradient as Nichols found, but, that this does not carry with it the conclusion that the ΔN_-'s are negligible is borne out by the same observation; the solution of the apparent paradox being contained in the fact, which becomes here experimentally demonstrated, that N_+ / N_- appears itself to be a function of the potential gradient.

The fact that the curves in all cases show a very decided rise from the origin to the first observed gradient point, while from that point on they seem to follow a comparatively straight

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line may need a word of explanation. This may due to the manner in which the values for the potential gradient were obtained. The observed values for the potential gradient are of course the differences of potential between that at the pollonium collector one and one-half meters from the wall of the building and that of the case of the electroscope which was earthed to a gas pipe. These values were assumed to be the same as those at the capped ends of the ion counters, which amounts to the same thing as assuming that the wall of the building was at the same potential as the gas pipe. One might assume from the nature of the data obtained that the wall of the building had a potential greater than zero, and, working on this assumption, the slope of the curves should be used for determining the correction values insofar as they relate to the theory outlined above, although the whole error is of course that indicated by the experiment.

Another explanation has been suggested by Professor Swann along the following lines: It is obvious that, owing to the charge on the open end of the ion counter, there will be a certain region around the wall into which no ions penetrate. This "critical region" as it may be called will be bounded by a surface across which the normal component of the air-velocity is just equal to the product of the ionic mobility and field strength resolved perpendicular to the surface. The critical region differs from the remainder of the space around the open end in that the ionic density in it is zero, whereas in the remainder of the space it is constant and equal to the value in regions far removed from the ion counter. For very high fields, the region in question will extend completely over the top of the ion counter so that no ions will be able to penetrate. The existence of this critical region complicates the simple theory to some extent. The general effect is to increase still further the loss of ions accounted for on the simple theory, the additional loss being of a type which increases less rapidly with the field that would correspond to a linear law. It is probable that the curves 1 , 2, and 3 fail to pass through the origin because of a combination of this loss with that predicted by the simple theory.

Summary of Conclusions.

1. The data obtained show that a shield, such as the cage used in the present experiment, over the ion counter does affect the results.

2. The effect for the negative ions is in the right direction and of great enough order of magnitude to substantiate Swann's theory. Moreover, it appears that the cap on the instrument does not necessarily eradicate the error.

3. The values of $\Delta N_+ / N_+$ are practically zero, which is in accordance with the theory.

4. The mean values for $(N - \Delta N)_+ / (N - \Delta N)_-$ are substantially the same for all values of the potential gradient, which is in agreement with the data submitted by Nichols, and mentioned before in this thesis, but the values for N_+ / N_- vary, which leads one to infer that this latter value is itself a function of the potential gradient.

In closing, the author takes pleasure in acknowledging the assistance of Prof.W.F.G.Swann, who suggested the problem and who at all times was ready to help with valuable suggestions for overcoming the many difficulties encountered. In designing the apparatus, the assistance of Mr.C.H.Dane,mechanician, is greatly appreciated.

Department of Physics,
University of Minnesota,
December, 1920.

(TABLE 2.)

Data Concerning Upper System, "A", for Negative Ions.

Gradient, V/Meter.	Date of Obs. 1920	t_2 A alone, unshielded. seconds.	t_1 Both acting, A shielded. seconds.	q_2/t_2 (E.S.U./sec.) $\times 10^{-6}$	q_1/t_1 (E.S.U./sec.) $\times 10^{-6}$	$(N_-)_A$ per cc.	$(\Delta N_-)_A$ per cc.	$\frac{(\Delta N_-)}{(N_-)_A} \times 100$.
	10-23	80	412					
30	10-25	70	383					
to	10-30	75	370					
39	11-1	78	375					
	Mean	76	385	538	104	950	154	16.2
	10-23	93	377					
40	10-25	82	378					
to	10-30	70	360					
49	11-1	86	368					
	Mean.	83	373	493	107	889	159	17.9
	10-23	88	378					
50	10-25	82	313					
to	10-30	77	353					
59	11-1	88	341					
	Mean.	84	343	489	114	888	164	18.6
	10-23	83	333					
60	10-25	80	311					
to	10-30	70	350					
69	11-1	84	321					
	Mean.	81	328	505	120	921	173	18.8
	10-23	90	304					
70	10-25	82	289					
to	10-30	82	305					
79	11-1	87	300					
	Mean.	85	298	484	132	908	191	21.0
	10-23	88	291					
80	10-25	76	263					
to	10-30	75	247					
89	11-1	84	270					
	Mean.	81	270	505	148	967	219	22.6
	10-23	87	264					
90	10-25	80	251					
to	10-30	70	249					
99	11-1	79	264					
	Mean.	80	254	513	157	992	232	23.4
	10-23	82	259					
100	10-25	80	233					
to	10-30	72	270					
109	11-1	85	224					
	Mean.	78	248	526	161	1016	238	23.4
	10-23	97	248					
110	10-25	76	225					
to	10-30	74	246					
120	11-1	92	215					
	Mean.	85	234	484	169	967	250	25.9

(TABLE II.)

Data Concerning Lower System, "B", for Negative Ions.)

Gradient, V/M.	Date of Obs. (1920)	t_3 B alone, unshielded. (seconds)	t_1 Both acting, A shielded. (seconds.)	q_3 / t_3 (E.S.U. / sec. $\times 10^{-6}$)	q_1 / t_1 (E.S.U./ sec $\times 10^{-6}$)	$(N_-)_B$ (per cc.)	$(\Delta N_-)_B$ (per cc.)	$\frac{(\Delta N_-)}{(N_-)_B}$ $\times 100$.
	10-20	125	429					
30	10-30	90	395					
to	11-5	88	380					
39	11-6	78	401					
	Mean	94	401	442	100	802	148	17.7
	10-20	112	423					
40	10-30	88	383					
to	11-5	85	365					
49	11-6	83	365					
	Mean	92	384	454	104	826	154	18.6
	10-20	132	365					
50	10-30	90	383					
to	11-5	88	355					
59	11-6	102	368					
	Mean.	105	368	396	110	749	163	21.8
	10-20	120	360					
60	10-30	76	345					
to	11-5	86	275					
69	11-6	76	368					
	Mean.	92	337	454	118	847	175	20.7
	10-20	122	295					
70	10-30	88	305					
to	11-5	92	320					
79	11-6	83	302					
	Mean.	96	305	434	131	836	194	23.2
	10-20	125	290					
80	10-30	80	300					
to	11-5	88	300					
89	11-6	71	271					
	Mean.	91	289	459	138	883	204	23.1
	10-20	125	282					
90	10-30	85	256					
to	11-5	93	270					
99	11-6	79	263					
	Mean.	95	267	438	152	873	225	25.8
	10-20	115	235					
100	10-30	78	249					
to	11-5	82	200					
109	11-6	82	215					
	Mean.	92	226	454	177	934	262	28.1
	10-20	125	205					
110	10-30	80	240					
to	11-5	85	235					
120	11-6	75	220					
	Mean.	87	225	479	178	973	264	27.1

(TABLE III.)

Data Concerning Both Systems, "A" and "B" for Positive Ions.
(Values indicate means of two or more observations.)
(Blanks indicate no appreciable rate of deflection.)

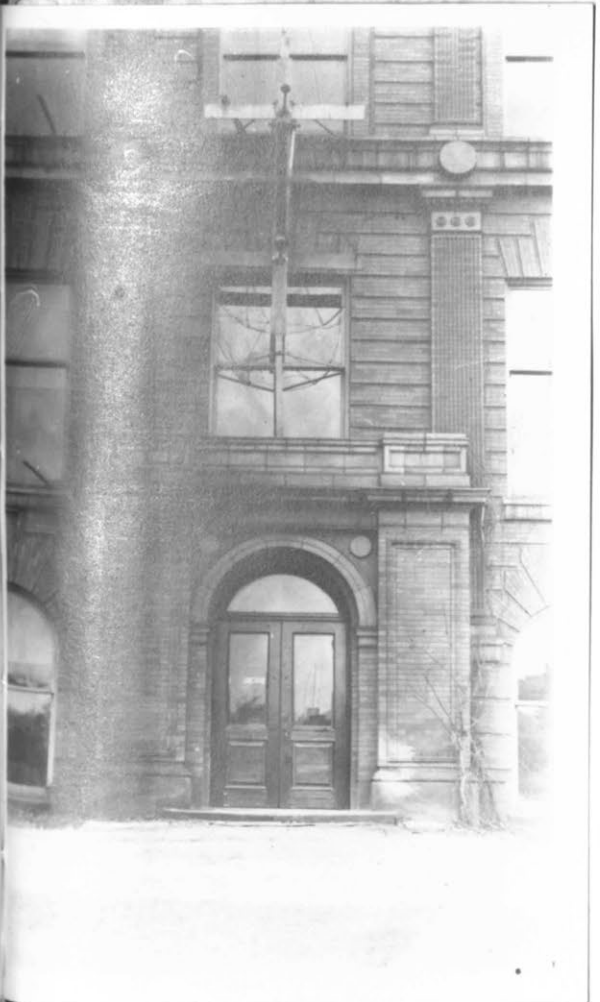
Gradient, V /M.	Date of Obs.	t_2 A alone, unshielded. (seconds)	t_1 Both acting, "A" sh'd. (seconds)	q_2 / t_2 E.S.U./ sec. $\times 10^{-6}$	q_1 / t_1 E.S.U./ sec. $\times 10^{-6}$	$(N_+)_A$ per cc.	$(\Delta N_+)_A$ per cc.	$\frac{(\Delta N_+)}{(N_+)}_A \times 100.$
30 to 50	11-8	52	----					
	11-15	56	----					
	Mean	54		-760	0.0.	1125	0.0	0.0
51 to 70	11-8	54	1320					
	11-15	50	985					
	Mean	52	1152	798	0.24	1168	5.0	0.4
71 to 90	11-8	53	-1475					
	11-15	53	----					
	Mean	53	-1475	-777	-8.00	1150	2.0	0.2
91 to 110	11-8	48	1442					
	11-15	50	1580					
	Mean	49	1511	830	16.0	1229	2.0	0.2
111 to 130	11-8	54	----					
	11-15	50	-864					
	Mean	52	-864	781	-24.0	1156	9.0	0.8

		t_3 A alone, unsh'd. (seconds)	t_1 Both act'g A sh'd. (seconds)	q_3 / t_3 E.S.U./ sec. $\times 10^{-6}$	q_1 / t_1 E.S.U./ sec. $\times 10^{-6}$	$(N_+)_B$ per cc.	$(\Delta N_+)_B$ per cc.	$\frac{(\Delta N_+)}{(N_+)}_B \times 100.$
30 to 50	11-8	51	-1125					
	11-15	50	-----					
	Mean	50.5	-1125	817	-8.0	1199	2.0	0.2
51 to 70	11-8	55	-----					
	11-15	55	------					
	Mean	55	--------	756	0.0	1118	3.0	0.3
71 to 90	11-8	52	1325					
	11-15	56	-----					
	Mean	54	1325	771	8.0	1141	2.0	0.2
91 to 110	11-8	50	978					
	11-15	60	1050					
	Mean	55	1014	756	28.0	1118	5.0	0.4
111 to 130	11-8	54						
	11-15	58						
	Mean	56	----	--------	-----	----	---	-----

(TABLE IV.)

Table of Mean Values for all Observations for Both Systems.

Gradient, volts/ meter.	N_- per cc.	ΔN_- per cc.	$\frac{\Delta N_-}{N_-} \times 100$	N_+ per cc.	ΔN_+ per cc.	$\frac{\Delta N_+}{N_+} \times 100$	$\frac{(N - \Delta N)_+}{(N - \Delta N)_-}$	$\frac{N_+}{N_-}$
30 to 39	876	151	16.9	1162	-4.0	0.4	1.63	1.34
40 to 49	857	156	18.2					
50 to 59	818	164	20.2	1143	4.0	0.4	1.67	1.34
60 to 69	884	174	19.7					
70 to 79	872	193	22.1	1145	0.0	0.0	1.63	1.28
80 to 89	925	212	22.9					
90 to 99	933	228	24.6	1173	22.0	2.0	1.64	1.25
100 to 109	975	250	25.7					
110 to 120	970	257	26.5	1156	0.0	0.0	1.61	1.20



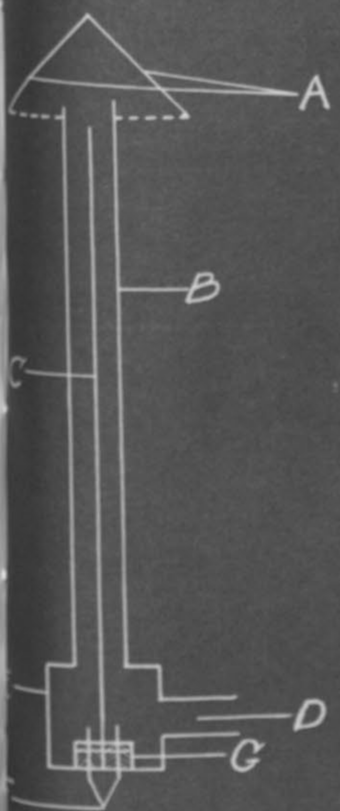


Figure 1

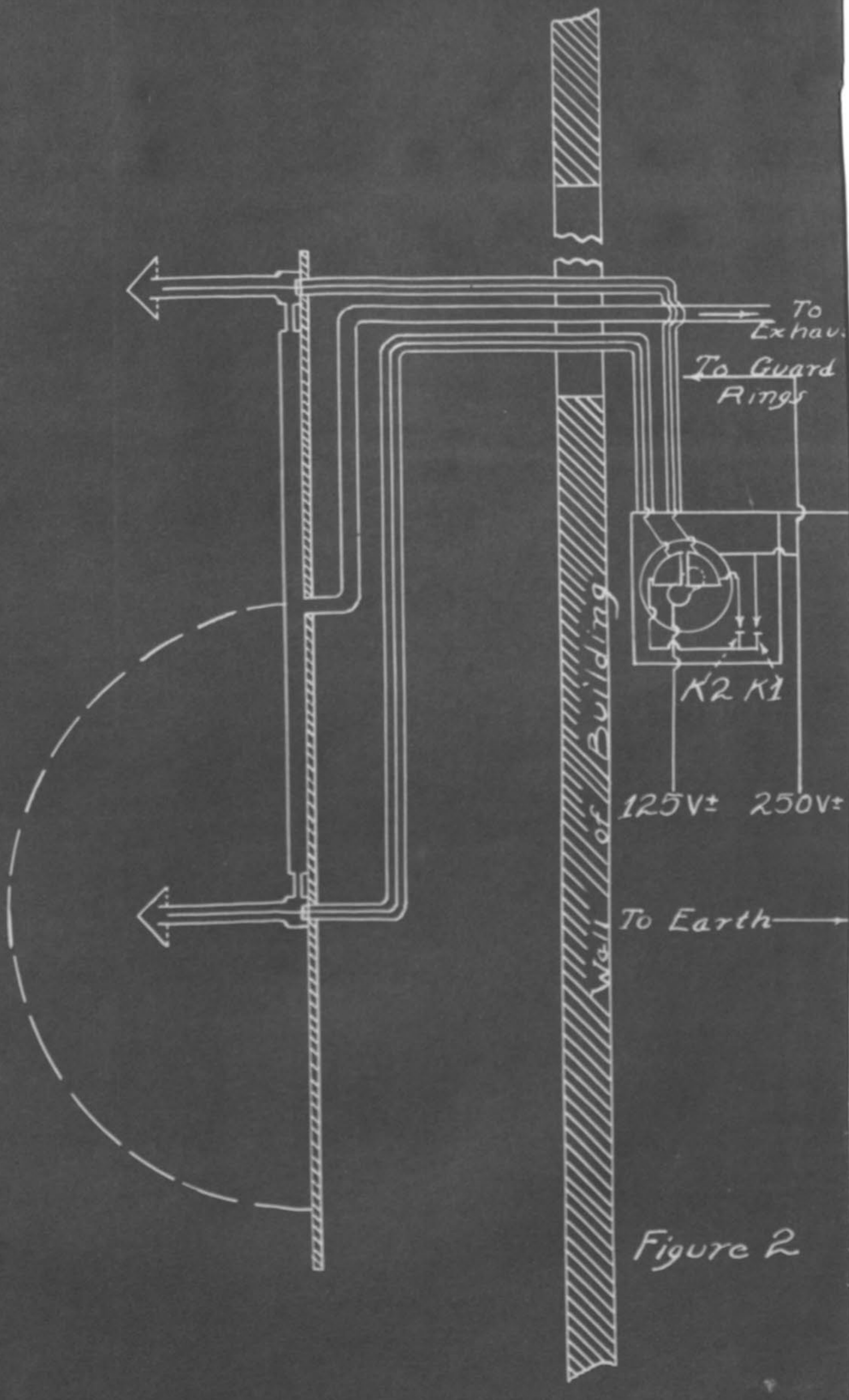


Figure 2

Table V
Mean Values of q

Circuits A+B	C	V	v	C(V-v)	q
Both systems at same potential and insulated	E.S.U. 193.31	Volts 0.187	Volts 0.125	E.S.U. 0.0400	q_1
A insulated B earthed	193.31	0.189	0.125	0.0411	q_2
B insulated A earthed	193.31	0.190	0.125	0.0417	q_3

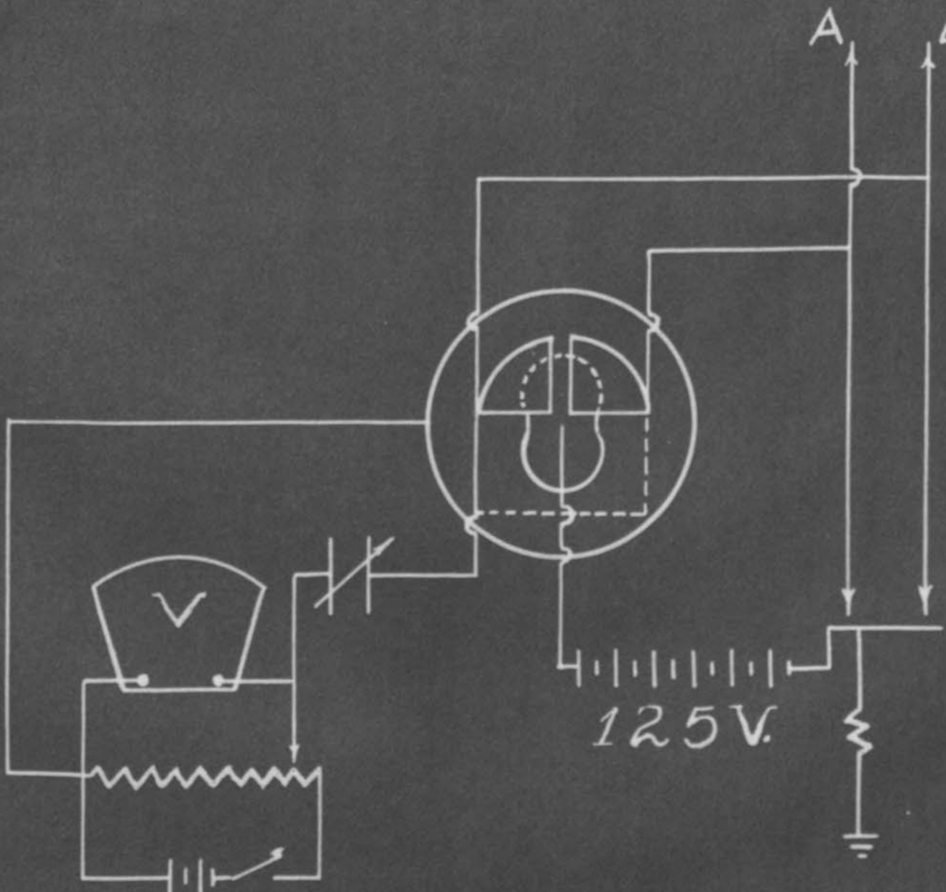
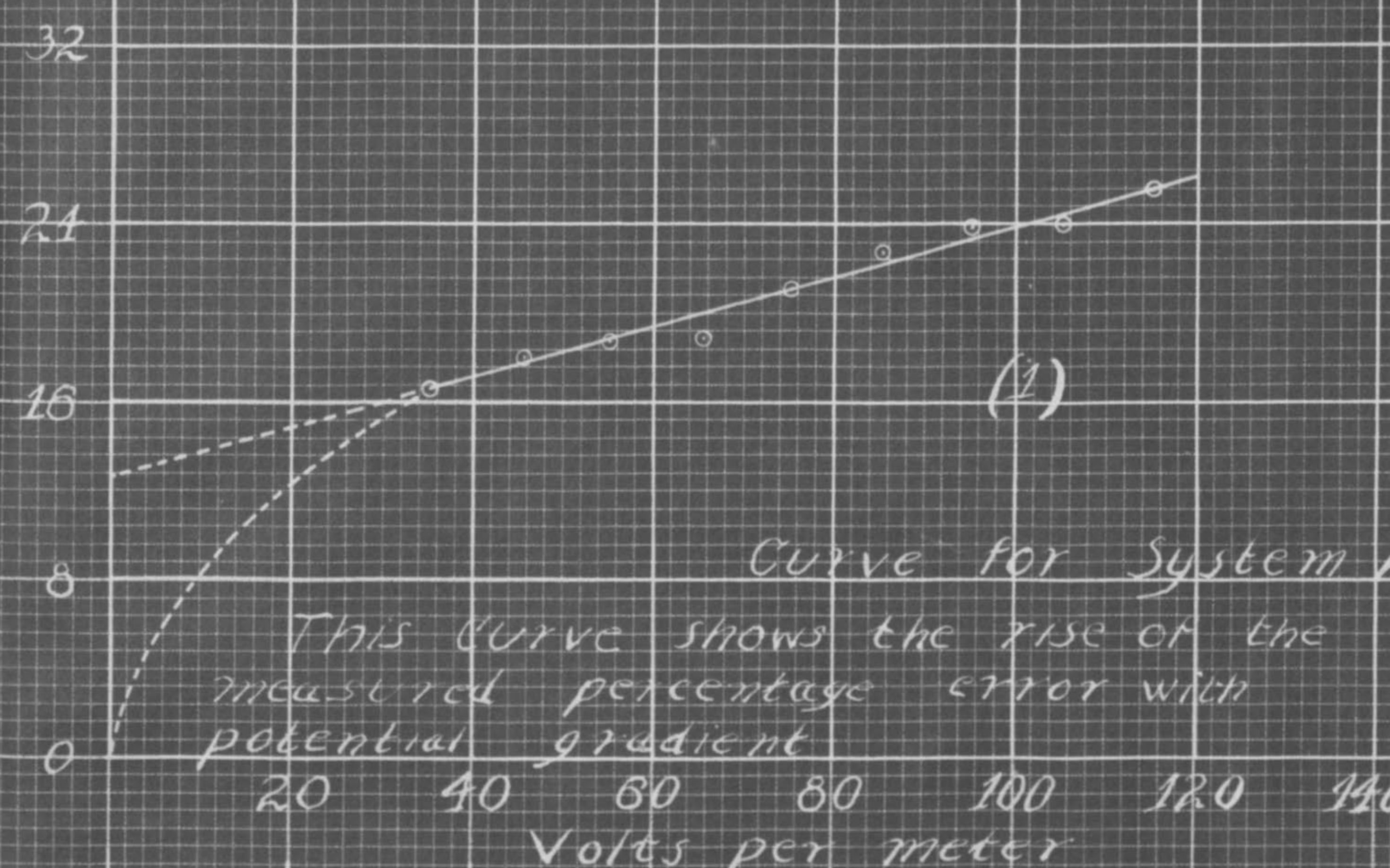
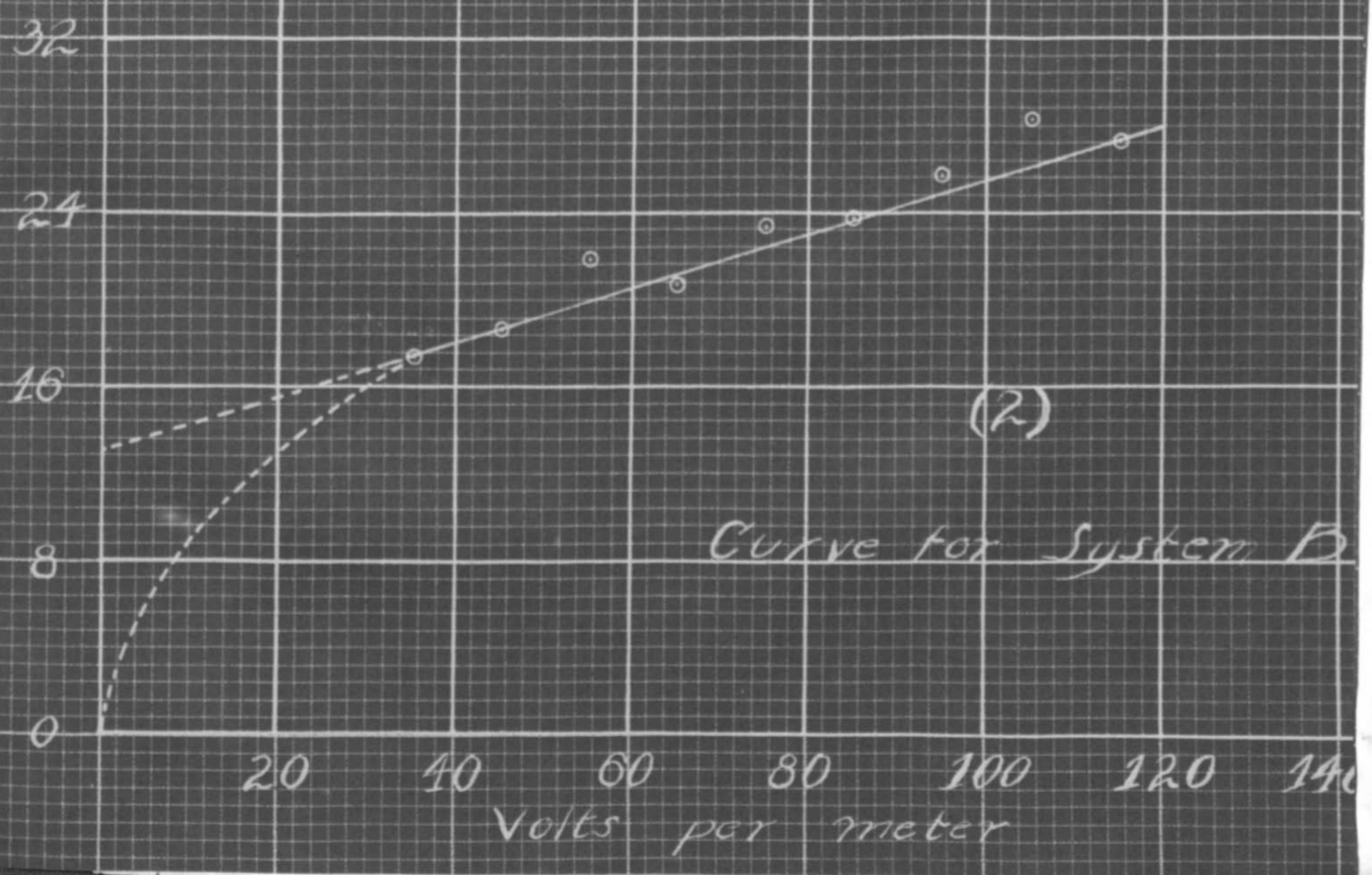
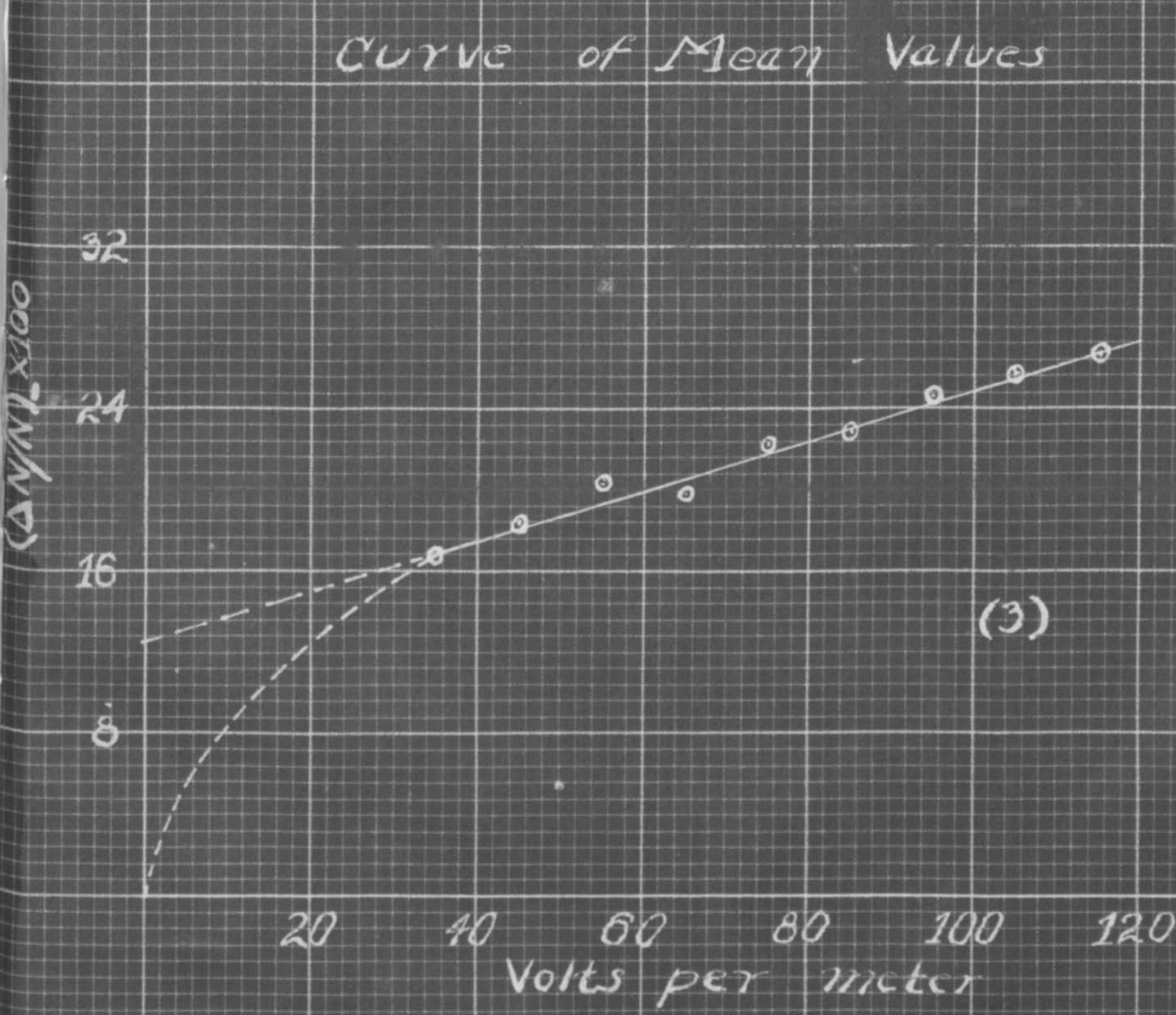


Figure 3

Showing arrangement of apparatus for determining q_1, q_2, and q_3


32
24
16
8
0
(1)
Curve for System A
This Curve shows the rise of the measured percentage error with potential gradient
20
40
60
80
100
120
140
Volts per meter


32
24
16
8
0
(2)
Curve for System B
20
40
60
80
100
120
140
Volts per meter



Curve of Mean Values
32
24
16
8
$(\Delta N/N) \times 100$
(3)
20
40
60
80
100
120
Volts per meter